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INFORMATION ACQUISITION IN DELIBERATIVE DEMOCRACIES

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JEL Codes: D72, D82, D83.

Keywords: Elections, Information Acquisition, Deliberation.

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Information Acquisition in Deliberative Democracies

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We examine the impact of deliberation on political learning and election outcomes. A rational, common-valued electorate votes under majority rule, after potentially acquiring costly private information and sharing it freely through public deliberation. Our findings suggest that deliberation can lead to free-riding on information gathering, but also encourage the emergence of informed political experts. Overall, deliberation may legitimize purely electoral outcomes and yield more accurate decisions. However, deliberation may also reduce electoral accuracy. We provide conditions for these results and contribute to the understanding of the strengths and limitations of deliberative democracies.

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1 Introduction

Democracies entail procedures for making decisions collectively, involving the participation of either all individuals impacted by the decision or their chosen representatives. One such procedure is voting, of course; but it may not be the only one. Deliberation, or "decision-making by means of arguments offered by and to participants who are committed to the values of rationality and impartiality" is another procedure, often combined with voting (Elster, 1998, p.5). Proponents of a deliberative democracy emphasize the creation of spaces where citizens can freely debate, apart from traditional aspects of a democracy such as voting under majority rule.[1] The core idea is that, through reasoned deliberation, participants can reach more legitimate and informed decisions.

[1] Ackerman and Fishkin (2008) propose the creation of a "Deliberation Day" in the United States, a national holiday before national elections in which citizens would meet and discuss the political agenda. Participation would be voluntary and participants would receive some compensation for their civic efforts.

From a normative viewpoint, deliberation as a democratic procedure seems uncontroversial. However, some defenses of deliberative democracies fail to analyze deliberation instrumentally, i.e., less as an intrinsically desirable feature, and more as a means to produce the desirable result. In response, scholars have examined the effects of deliberation in democracies from a game-theoretic perspective, accounting for the incentives of citizens to share truthfully their private information and to vote. Most of the literature challenges the normative support of deliberative democracies through the assumption that citizens are *not* impartial: they have private interests apart from caring about the common good. Although a valid point, we believe that such relaxation may distort the idea of a deliberative democracy as a normative benchmark in political philosophy and blur a logical assessment on whether deliberation can produce more legitimate or informed decisions.

In this paper, we model a democratic society of rational and common-valued citizens who can share information in public at no cost prior to voting, along the lines of the "ideal public sphere" (Habermas, 1991). Crucially, however, we account for the fact that acquiring information prior to sharing is costly. How does *ex-post* deliberation then influence the incentives to acquire costly information *ex ante*? How does this, in turn, affect election outcomes? Specifically, when information acquisition is costly, can deliberative democracies induce more accurate decision-making? And, if *ex-ante* deliberation is judged socially as a way of improving the legitimacy of electoral decisions, can deliberation justify the use of the majority rule even if deliberation is not possible, in the sense that the same outcomes are generated with and without deliberation?

For our endeavor, we start from Martinelli (2006) and consider a society made up of a finite (and for simplicity, odd) number of *ex-ante* equal citizens who have to choose via voting with the majority rule one of two alternatives. All citizens agree that one of the two alternatives is best for one of the two binary states of the world, so our setup is of common value. However, citizens may disagree about the likelihood they attach to each state, since they receive different private, conditionally independent signals about the true state. The accuracy of any such state-conditional signal depends on the effort an individual incurs to become informed. Information acquisition costs are modeled via an increasing, convex function of accuracy. To include the possibility of deliberation, we then depart from Martinelli (2006) by assuming that prior to voting, citizens can (publicly) communicate at no cost to any other citizen whatever information they acquired first-hand and however they acquired such information.

The idea that electoral democracies might discourage information acquisition and lead to uninformed decision-making is not new. As conjectured already by Downs (1957), citizens may not find it worthwhile to acquire costly information about the consequences of a policy they are voting for. The reason for this is that citizens do not expect the information they acquire to be consequential for the election outcome with a high probability, as they deem unlikely the events in which their vote will be pivotal.

Martinelli (2006) nonetheless proves that although costly information acquisition discourages private learning, majority voting can still induce information aggregation. Intuitively, in a world of common-valued citizens, information is a public good. If marginal costs of being informed are low, acquiring information is a cheap contribution to the common good. By contrast, the marginal benefit of doing so is significant, because the probability of being pivotal is never zero. In fact, depending on the technology citizens use to acquire information, as well as on the size of the electorate, majority voting can lead to perfectly accurate decision-making, even if voters are individually poorly informed.

The above insights rest on a crucial assumption: citizens' privately acquired information can only be expressed through voting. With information sharing (i.e., with deliberation), two effects could arise. First, at the information acquisition and deliberation stages, some citizens may choose *not* to acquire costly information themselves and instead rely on the information of others. Deliberation could then increase the incentives for citizens to free-ride on the information acquired by someone else.

Second, at the voting stage, the citizens who *did* acquire information may be pivotal *de facto* with a very large probability. The reason for this is that free-riding citizens will replicate the vote of the civic-minded citizens based on the latter's acquired information, provided civic-minded citizens are trusted by the free-riding citizens. Deliberation then could increase the incentives for civic-minded citizens to become even more informed at the information acquisition stage in anticipation. In other words, better information would buy more voting power. It is not straightforward to identify which effect will dominate, making it difficult to predict how deliberation will affect electoral outcomes.

The timeline of the game we analyze is the following: In Stage 0, nature draws the state of the world. In Stage 1 (*information acquisition stage*), each citizen chooses the accuracy of the information about the state they want to acquire, at a cost that increases with accuracy. Then citizens privately observe their signals of the chosen accuracy. In Stage 2 (*deliberation stage*), citizens send public messages to each other, evaluate the truthfulness of such messages, and update their beliefs about the state of the world. In Stage 3 (*voting stage*), each citizen casts a vote, the alternative with most votes is implemented, and payoffs are realized.

The above dynamic game is difficult to analyze if we consider complex communication protocols. Focusing on public communication, which seems a natural consideration in deliberative democracies, the bulk of our game is tantamount to a static game, called *(voting) game with deliberation*. In it, all citizens simultaneously choose their accuracy of information and receive their private signals. Then, they *(i)* transmit their first-hand information truthfully to everybody else; and *(ii)* vote sincerely, i.e., they vote for the alternative that is most likely given all the information at their disposal. This simplification allows us to compare results of the game with deliberation to the game without deliberation established in Martinelli (2006), which serves as benchmark.

To justify our simplification, we prove the following auxiliary results for the dynamic game. First, sincere voting in Stage 3 weakly dominates all other voting strategies if communication is public and voters believe that the information sent to them is truthful (Lemma 1). Second, truthful communication in Stage 2 is weakly dominant, provided that messages are commonly believed (Lemma 2). These two lemmas imply that we do not need to explicitly model Stage 2 or 3, and we can therefore focus on Stage 1.[2]

The above lemmas rely on the (behavioral) assumption that citizens will believe that any message sent to them is truthful; we say that messages are *commonly believed*. Although disputed in models where citizens have private interests, we see this assumption as less debatable in our model of pure common value. Proceeding with such an assumption enables the above discussed simplification and is in keeping with the normative description of a deliberative democracy. Using these two lemmas, we can extend any Nash equilibrium of the voting game with deliberation (which we call an *equilibrium*) to a perfect Bayesian equilibrium of the whole dynamic game in which citizens send truthful messages, believe all messages, and vote sincerely (Corollary 1).[3] This means that one can see the assumption that messages are commonly believed as a refinement of the set of all perfect Bayesian equilibria of the dynamic game.

Our main results are the following. First, the only equilibrium with symmetric accuracy in the game without deliberation is *not* an equilibrium of the game with deliberation if acquiring information is cheap enough (Theorem 1). In this case, free-riding in deliberative democracies may kill collective cost sharing. By contrast, when not only information is costly, but the marginal cost to acquire information increases with information accuracy, then the symmetric equilibrium without deliberation is also an equilibrium with deliberation (Theorem 2). This equilibrium is typically non-fully revealing, i.e., all citizens have the same imperfect information accuracy prior to voting (yet they might receive different signals). Moreover, if there exists a symmetric equilibrium of the game with deliberation at all, then the information accuracy chosen by citizens must be the same accuracy chosen in the symmetric equilibrium without deliberation (Proposition 1).

Without deliberation, voters vote according to their private information, so there is no consensus. With deliberation, although the same information accuracies may be individually acquired, all citizens have a more accurate posterior about the state of the world since information is pooled together. The fact that all citizens end up with the same information when deliberation precedes voting means that all citizens vote alike. The finding that consensus is achieved through deliberation aligns with the foundational principles of deliberative democratic theory (Habermas, 1985).

Greater confidence that the adopted policy is the right one can have intrinsic social

[2]Lemma 2 implies that if citizens acquire first-hand information, we can assume that they will simply share it with all other citizens. Lemma 1 implies that we can abstract from other voting strategies.

[3]We discuss relaxations of the commonly believed assumption in Section 4.

value, especially when the consequences of a policy take a long time to materialize. Yet, under majority rule the outcome will be the same with and without deliberation for any realization of signals, provided we focus on the symmetric equilibrium and such equilibrium exists in the latter case. If one considers electoral outcomes without deliberation to be illegitimate, this latter result then suggest that a deliberative process can help to legitimize these outcomes through a counterfactual with deliberation.

Second, there *always* exists a class of equilibria in the game with deliberation (Theorem 3) that *never* exists in the game without deliberation (Proposition 2). In this class, labeled political specialization, one citizen, labeled political expert, contributes to the common good by acquiring information first-hand so that all other citizens use this acquired information to vote alike. This suggests that unanimous decision-making in deliberative democracies may arise not only from deliberation *per se* but also from incentives to monopolize information. Although in our main setup we focus equilibria with one political expert, in Section 4 we show that our results extend to equilibria with multiple experts.

Third, depending on the information costs and on the electorate size, the probability of implementing the right alternative in the political specialization equilibrium will be smaller or greater than in the symmetric equilibrium (with or without deliberation). Simply put, we cannot unambiguously say that deliberative democracies lead to more (Proposition 3) or less (Proposition 4) accurate decision-making collectively, especially for large electorates, even in a common-value setup like ours.

We contribute to the discussion on the strengths and limitations of deliberative democratic theory by highlighting that *(i)* even if deliberation is carried over freely by rational and impartial citizens, it may not follow logically that collective-decision making is improved, or *(ii)* a lack of deliberation does not necessarily mean that voting outcomes are illegitimate. These results might have practical implications insofar as there has been a deliberative wave across OECD countries which gained momentum especially in the first decade of the twenty-first century (OECD, 2020). In fact, our findings identify an adverse effect of promoting deliberative processes without facilitating information acquisition to the participants.[4]

Our insights may also have implications beyond the impact of deliberative processes in elections. In today's information society, it is costly to obtain accurate, first-hand

[4]OECD (2020) identified almost 300 representative deliberative processes in OECD countries, from 1980 to 2020, with most starting around 2010. Recent examples in Spain include the Citizen's Climate Assembly of Catalonia (2024), the Citizen's Jury of Besaya (2021) and the Citizen's Convention on Mental Health in the Valencian Community (2021). Specific methodologies vary, but in general, citizens, randomly selected through a civic lottery, engage in learning and deliberation before voting on proposals for public administration. Although some guidance is provided in most cases, citizens are encouraged to conduct their own research, gathering additional information to enrich the discussions. While recommendations are not always implemented, many proposals lead to tangible policy changes. Lack of accessible information can thus create knowledge disparities and impact policy effectiveness.

information about public policies or candidates, especially with the popularization of paywalls from online news outlets.[5] Meanwhile, sharing information on social networks has virtually no cost. Whether social media impact elections negatively or positively is unclear, especially with false information, or polarization. Our results abstract away from fake news or polarized voters, focusing instead on the balance between the decreasing cost of communication and the increasing cost of information acquisition.

Related literature

This is not the first paper to analyze the impacts of deliberation on voting outcomes through a game-theoretical perspective. We elaborate on the references for this topic in the following paragraphs. Our main innovation and contribution to this literature is to identify a key determinant of information aggregation in deliberative democracies: The fact that information is costly to acquire.

Austen-Smith (1990), Austen-Smith and Feddersen (2002, 2005) and Doraszelski, Gerardi, and Squintani (2003) study the impact of communication in voting outcomes within committees. A common finding is that, provided that citizens' preferences for the voted alternatives are not too different, deliberation can never reduce the probability of the majority decision selecting the right alternative. This is because preference heterogeneity reduces the trustworthiness of the information shared collectively. We show instead that, with costly information acquisition, deliberation may induce additional errors in collective decision-making, even when the citizens' preferences are aligned. This is because costly information acquisition may produce political experts, i.e., citizens who are responsible for learning about the alternatives and who might end up acquiring too little information. Since political experts may end up being pivotal, they may induce less informed decision-making.[6]

Another common finding in the cited papers is that decisions backed by the majority rule without deliberation are not necessarily illegitimate, in the sense that information commonly revealed after citizens vote would also be revealed if citizens could deliberate before casting their votes. We partially confirm this finding: the equilibrium in the majority decision without deliberation that aggregates information could also exist when citizens deliberate, but deliberation introduces stronger conditions on the cost function for such equivalence.

We draw upon insights from the literature on costly information acquisition in elec-

[5] At the early stages of the digital transformation of society, news outlets relied more ad revenue from website visitors than on paid content. However, search engines and news aggregators diverted advertising away from these outlets, leading to a shift in business models. In response, many outlets adopted paywalls to generate revenue and limit aggregators from reposting original content. For instance, in commenting a study from Høst (2016) that documents the number of Norwegian news outlets with online presence that implemented paywalls from 2011 to 2015, Skjeret, Steen, and Wyndham (2019) notes that while less than one percent of Norwegian news outlets had paywalls in 2011, nearly two-thirds had implemented them by 2015.

[6] Deliberation can strictly improve the probability of majority decision selecting the right alternative in the cited papers. Our paper also finds this result, although in a significantly different setting.

tions without communication. Martinelli (2006) is the closest paper to our work. In both models, voters incur a cost to increase the precision of their private information about an alternative to be voted upon later, under majority rule. In our setup, however, voters can send messages to each other. We prove that deliberation introduces equilibria that do not exist without deliberation.[7]

Gerardi and Yariv (2007, 2008) explore committee models with communication, the latter including costly information acquisition, but assume binary information levels.[8] Our model allows citizens to choose from a continuum of signals' precisions, enabling novel comparative statics regarding the marginal impact of information costs on the number of citizens acquiring information and the overall information level. It also produces a counterpart result: While the cited papers show that communication renders the voting rule irrelevant, we show that using majority rule can make communication irrelevant for outcomes (but not always).

Another related paper is Chan, Lizzeri, Suen, and Yariv (2018). In this paper, members of a committee choose when to cast votes on alternatives whose payoffs depend on an unknown state of the world. While waiting to decide, members observe realizations of public information about this state. Their main setup models the cost of information gathering as the cost of delaying decisions (with members discounting time), yet it also allows for an analogous interpretation in terms of explicit information acquisition costs.[9] However, this interpretation does not account for the endogenous choice of information quality among committee members, as our model does. On the one hand, we confirm their finding that deliberation can lead to less accurate decisions under majority rule. On the other hand, we identify novel equilibria in which a single citizen holding all deliberation power can induce more accurate collective decision-making. This is possible because, in our setting, one citizen can acquire perfectly revealing information, contrasting with the exogenous information accuracy in Chan et al. (2018).

Li (2001) considers a model of costly individual information gathering and public sharing in committees. Like our paper, committee members with common-valued preferences choose from a continuum of signals' precisions. However, his setup assumes sincere voting and truthful reporting. We prove instead that voting sincerely and reporting information truthfully are properties of equilibria in our setting. Additionally, he focuses on symmetric equilibrium, while we examine both symmetric and asymmetric equilibria, and identify conditions for which the latter produces more accurate social decisions than the former.

Our paper also enriches the understanding of how elections aggregate information and

[7]Gersbach, Mamageishvili, and Tejada (2022, 2024) study the role of costly information acquisition in committees, but do not consider the possibility of communication between members.

[8]Goeree and Yariv (2011) conduct an experiment to test the implications of the cited papers.

[9]See their Footnote 6.

impact democratic performance.[10] There have been exciting contributions to this literature recently. For example, Ekmekci and Lauermann (2022) study the performance of large elections under population uncertainty. They identify conditions on the distribution of the electorate size for which information does not fully aggregate asymptotically. We also identify conditions for which equilibria without full information aggregation exist, but our conditions relate to the information acquisition cost function. In another example, Barelli, Bhattacharya, and Siga (2022) study aggregation failures under a very general set of common-valued utilities and private information structures, although, unlike us, information is exogenously given.

Bouton, Llorente-Saguer, Macé, and Xefteris (2024) considers the possibility of privately (and exogenously) informed voters choosing from rich (for instance, continuous) ballot spaces. When voters are able to better reveal their private information through their votes, they can align better their influence on the electoral outcome with their private knowledge. Our result on the existence of equilibria with a political expert bears a similar logic to their continuous voting model, since buying more accurate information also buys more possibilities for a voter to be pivotal.

Apart from the analysis of information aggregation in elections, our paper contributes broadly to the literature of public goods provision. Indeed, information serves as a public good in our model, since private signals and their inference are made available to all voters in equilibrium. Moreover, our assumption on costly acquisition deepens the incentives for free-riding on the provision of information by other players. Bramoullé and Kranton (2007) study the provision of public goods in networks, although in a complete information environment. They find the existence of equilibria with specialization, i.e., equilibria in which only some individuals contribute and others free ride. Our results of existence of equilibria with a political expert is an analogue to their findings. Like us, Bramoullé and Kranton (2007) also find that specialization can be good or bad for society.

In our model, messages can be sent for free, so lying is costless. Yet, because there is no conflict of interest, we find that an equilibrium exists with full information disclosure among citizens (a separating equilibrium in cheap talk games, as commonly coined since Crawford and Sobel, 1982). Our assumption about messages being commonly believed resembles the assumption of naivety from receivers in Chen (2011).

Finally, Besley (2023) examines deliberation as an instrument to strengthen citizens' compliance with a social policy. Deliberation is treated as an exogenous process that affects citizens' beliefs about a right alternative to later be implement by a government. We thus provide a microfoundation for his assumption and confirm his finding that de-

[10]Condorcet (1785) conjectured that majority outcomes may be more reliable when more citizens exert their voting rights; in fact, the wisdom of majority may be infallible in arbitrarily large elections. The first formal explorations of Condorcet's thoughts (e.g. see Black, 1958; Grofman and Feld, 1988; Young, 1988; Ladha, 1992) generally considered settings in which information is spread exogenously among voters.

liberation strengthens compliance, since deliberation leads to all citizens having the same posterior belief prior to voting in our setting.

2 Setup

In this section, we describe our basic model and derive some preliminary results.

2.1 Model

An electorate of $2n+1$ citizens must choose one of two alternatives, A and B via voting with the majority rule. We let $N := \{1, ..., 2n+1\}$ denote the set of all citizens. All citizens agree that each of the alternatives is more suitable for one of two states of the world. Such state cannot be directly observed. *Ex ante*, citizens attach equal probabilities to any of the two possible states and this is common knowledge.

A citizen's utility $U(d, z)$ depends on the alternative chosen, $d \in \{A, B\}$, and on the realized state, $z \in \{z^A, z^B\}$. We normalize utilities so that $U(A, z^A) = U(B, z^B) = 1$ and $U(A, z^B) = U(B, z^A) = 0$ for all citizens. This means that citizens derive the same utility from implementing alternative A in state z^A and alternative B in state z^B. Citizens also derive the same utility level, yet a lower one, from implementing alternative A in state z^B or alternative B in state z^A.

Prior to voting, citizens must decide on the accuracy of information about the state they want to acquire. The accuracy is some value $x \in [0, 1/2]$ affecting the probability distribution over a binary signal set $\{s^A, s^B\}$ such that $\mathbb{P}[s^A|z^A, x] = \mathbb{P}[s^B|z^B, x] = \frac{1}{2} + x$. Thus, the higher the choice of x, the higher the informativeness of any observed signal. However, acquiring information is costly: choosing x reduces the citizen's utility by some amount $C(x)$. We follow Martinelli (2006) and impose standard regularity conditions.

Assumption 1 (Regular cost function)**.** $C : [0, 1/2] \to \mathbb{R}_+ \cup \{+\infty\}$ *is strictly increasing, strictly convex, twice continuously differentiable in* $(0, 1/2)$*, with* $C(0) = C'(0) = 0$.[11]

Once citizens have acquired *first-hand* information about the state of the world through their signals, they can send a message to other citizens through some exogenously given communication protocol. In these messages, citizens can specify both *(a)* how much information about the state they have acquired and *(b)* what signal they have received. In our baseline model, we assume that every citizen sends public messages to all other citizens. Formally, citizen i sends message $m_i^j = (x_i^j, s_i^j) \in [0, \frac{1}{2}] \times \{s^A, s^B\}$ to all other

[11]Strict convexity rules out the possibility of multiple symmetric equilibrium in Martinelli (2006) as well as here. The condition $C(0) = C'(0) = 0$ ensures that there exists an equilibrium with positive information acquisition.

citizens $j \neq i$. This is a justifiable assumption from the normative perspective of deliberation in the "ideal public sphere" (Habermas, 1991). In Section 4, we elaborate on the role of public communication on our results and discuss other communication protocols.

Assumption 2 (Deliberation)**.** *Every message is observed by all citizens.*

After observing the messages sent to them as well as to the other citizens, citizens update their beliefs about the node of the game that has been reached. In our setup, updating about the state of world alone is not sufficient. Citizens also update their beliefs about how much information other citizens might have, given the received messages. We momentarily bypass the discussion of whether citizens can trust each other's messages by assuming the following:[12]

Assumption 3 (Commonly believed messages)**.** *It is common knowledge that citizens believe that the information sent to them is truthfully reported by other citizens.*

Assumption 3 reflects a behavioral tenet that aligns with our common value setup. When messages are commonly believed, citizens are allowed to lie to others (on or off equilibrium path), yet this will not be learned by the citizens receiving such messages. Then the updating process regarding which game nodes have been reached is straightforward: each citizen will allocate probability one to the node matching both her first-hand information (which is privately acquired) and her second-hand information (which she has received from other citizens).

At the final stage, citizens cast their votes (no abstention occurs).[13] The alternative that receives more votes is implemented, and payoffs are realized. The following summarizes the timeline of our game, which consists of three main stages:

0. Nature draws the state of the world $z \in \{z^A, z^B\}$.

1. ***Information acquisition stage:*** each citizen $i \in N$ chooses accuracy of information $x_i \in [0, 1/2]$ and observes signal $s_i \in \{s^A, s^B\}$ with precision $\frac{1}{2} + x_i$.

2. ***Deliberation stage:*** citizens send messages to each other, observe messages sent to them and update their beliefs about which game node has been reached.

3. ***Voting stage:***

    (a) Each citizen casts one vote, and the alternative with more votes is implemented.

[12]We discuss relaxations of such assumption in Section 4.

[13]Allowing for abstention is not consequential for our results, because we focus on scenarios where, as part of equilibrium play, all citizens hold the same information. Therefore, information acquisition levels in equilibrium do not depend on whether we rule out abstention.

(b) Citizen i obtains payoff $U(d,z) - C(x_i)$ under state z if $d \in \{A, B\}$ is implemented and she chose x_i in Stage 1.

In the above dynamic game, a strategy for citizen i consists of the following elements: *(i)* an information accuracy $x_i \in [0, \frac{1}{2}]$; *(ii)* a public message $m_i(x_i, s_i)$ for any chosen $x_i \in [0, \frac{1}{2}]$ and any signal $s_i \in \{s^A, s^B\}$ received; *(iii)* a mapping α^i from her information set to a probability distribution over the set of alternatives. Since citizens update their beliefs about the node of the game that has been reached, the appropriate solution concept is Perfect Bayesian Equilibrium (PBE). We focus on PBE with information transmission at the deliberation stage (conditional on information acquisition at the information acquisition stage), so we rule out babbling at the deliberation stage.[14]

2.2 Preliminary analysis

In the following, we prove two lemmas that will enable us to simplify the game introduced above. Before doing so, some definitions will be useful. First, we adapt the definition of sincere voting of Austen-Smith and Banks (1996) to our setup.

Definition 1. *Citizen i's mapping α_i is sincere if it always selects an alternative which maximizes her expected utility given her information.*

A sincere voting rule is uniquely defined for any given information unless both alternatives yield the same expected utility given such information.

The next lemma proves that, under a public communication protocol (Assumption 2) and commonly believed messages (Assumption 3), voting sincerely is a dominant strategy.[15]

Lemma 1 (Sincere voting)**.** *Voting sincerely is weakly dominant for all citizens in the voting stage.*

Next, we show that, under Assumptions 2 and 3, if all citizens vote sincerely in the voting stage, it must also be that in the deliberation stage all citizens report truthfully both the signals they received and the information accuracies they acquired in the information acquisition stage.

Lemma 2 (Truthful reporting)**.** *Following the information acquisition stage, at the deliberation stage no citizen has strict incentives to send a message different from her chosen accuracy and signal realization.*

It remains to check whether, given truthful reporting and sincere voting, there exist equilibrium accuracy levels in the information acquisition stage. Henceforth, we assume

[14]With babbling, our setup with deliberation produces the results of a setup without deliberation.
[15]Unless said otherwise, all proofs for the stated results are in Appendix A.

that voters send truthful messages in the deliberation stage and vote sincerely in the voting stage no matter the history of the game. This enables us to focus on a reduced version of the dynamic game in which each citizen can perfectly observe the signals and the information levels of the other citizens and voting is sincere. The timeline of this static game, called **(voting) game with deliberation**, is:

0. Nature draws the state of the world $z \in \{z^A, z^B\}$.

1. Acquisition, deliberation, and voting stage:

   (a) Each citizen $i \in N$ chooses the accuracy of information $x_i \in \left[0, \frac{1}{2}\right]$.

   (b) Each citizen observes signal $s_i \in \{s^A, s^B\}$ with precision $\frac{1}{2} + x_i$.

   (c) Each citizen observes the signals and information accuracies of other citizens.

   (d) Each citizen votes sincerely, given $(x_i, s_i)_{i \in N}$.

   (e) Citizen i obtains payoff $U(d, z) - C(x_i)$ under state z, if $d \in \{A, B\}$ is implemented and x_i was chosen by such a citizen.

In the game with deliberation, a strategy for citizen i is merely a choice of information accuracy x_i, with $x_i \in \left[0, \frac{1}{2}\right]$. Being a static game, it is natural to use Nash equilibrium as the equilibrium concept. Lemmas 1 and 2 enable us to obtain the following result:

Corollary 1. *Let $(x_i^*)_{i \in N}$ be a Nash equilibrium of the game with deliberation. Then, a PBE of the dynamic game exists in which citizens acquire information levels $(x_i^*)_{i \in N}$, send truthful messages, vote sincerely, and messages are commonly believed.*

To simplify the exposition and focus on the information acquisition part of an equilibrium, we select a tie-breaking rule for the case when the citizens' posterior is the same for both states.[16] To introduce our tie-breaking rule, some further notation comes in handy. Let $\boldsymbol{x} := (x_i)_{i \in N}$ represent a vector of information qualities and $\boldsymbol{s} := (s_i)_{i \in N}$ represent a vector of signals, one for each citizen. Let also $s^A(\boldsymbol{s})$ and $s^B(\boldsymbol{s})$ represent the set of components of $\boldsymbol{s}$ that are equal to s^A and s^B, respectively. The state-conditioned probabilities of $\boldsymbol{s}$ under $\boldsymbol{x}$ are as follows:[17]

$$\mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^A] = \prod_{j \in s^A(\boldsymbol{s})} \left(\frac{1}{2} + x_j\right) \prod_{k \in s^B(\boldsymbol{s})} \left(\frac{1}{2} - x_k\right) \tag{1}$$

$$\mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^B] = \prod_{j \in s^A(\boldsymbol{s})} \left(\frac{1}{2} - x_j\right) \prod_{k \in s^B(\boldsymbol{s})} \left(\frac{1}{2} + x_k\right). \tag{2}$$

[16] Different tie-breaking rules such as randomizing between states or choosing arbitrarily one of the states when a posterior tie occurs would not lead to different results.

[17] In Equations (1) and (2) we adopt the convention that the empty product equals one.

Assumption 4 (Tie-breaking rule). *For any citizen $i \in N$, if her posterior is completely uninformative, i.e., if $\mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^A] = \mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^B]$, then such a citizen votes for the alternative that is most suitable for the state that matches the signal she received.*

Assumption 4 implies that in the game with deliberation, it is common knowledge among all citizens that any citizen i will vote for alternative A if $\mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^A] > \mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^B]$, will vote for alternative B if $\mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^B] > \mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^A]$, and will vote for alternative s_i if $\mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^A] = \mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^B]$. Since we are considering a public communication protocol, i.e., a extreme form of deliberation, all citizens will vote for the same alternative unless $\mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^A] = \mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^B]$.

Our next task is to explore the set of Nash equilibria of the game with deliberation. In any Nash equilibria of this (static) game, which we denote typically as $(x_i^*)_{i\in N}$, each citizen i chooses $x_i^* \in \left[0, \frac{1}{2}\right]$ to maximize her *ex-ante* payoff, given that other citizens choose $\boldsymbol{x}_{-i}^*$. This payoff can be written as a function of $(x_i)_{i\in N}$ as

$$G(x_i, \boldsymbol{x}_{-i}) := \frac{1}{2}\left[P_\alpha(A|x_i, \boldsymbol{x}_{-i}, z^A) + P_\alpha(B|x_i, \boldsymbol{x}_{-i}, z^B)\right] - C(x_i), \tag{3}$$

where $P_\alpha(A|\boldsymbol{x}, z^A)$ and $P_\alpha(B|\boldsymbol{x}, z^B)$ represent the probabilities of alternative A and B winning the election under state z^A and z^B respectively, given $\boldsymbol{x}$ and the common sincere voting rule α.

2.3 Benchmark without deliberation

In the **(voting) game without deliberation**, Nature draws the state of the world, then citizens choose the accuracy of the information they acquire and observe the realization of their own signal, but the chosen accuracy and the realized signal are kept private. Thus, it is an analogue of our dynamic game without a deliberation stage.[18] Under sincere voting, the analysis of the game without deliberation reduces to the analysis to the static game we described in Section 2.2, but without the possibility for citizens to send public messages to other citizens.[19]

The game without deliberation has been studied by Martinelli (2006) and Gersbach et al. (2024). We identify three key findings from these papers. The first is the existence of a symmetric equilibrium with positive information acquisition.

Remark 1 (Martinelli, 2006; Theorem 1). *The game without deliberation has a unique symmetric equilibrium, denoted by $(x_i^*)_{i\in N}$ with $x_i^* = x^*$ for every $i \in N$, where $x^* \in (0, 1/2)$ is the solution to the following equation*

$$\binom{2n}{n}\left(\frac{1}{4} - (x^*)^2\right)^n = C'(x^*). \tag{4}$$

[18] Whether messages are commonly believed is irrelevant with no deliberation.

[19] Martinelli (2006) shows that voting sincerely is dominant in the game without deliberation.

A rough intuition for this finding is as follows. Suppose that every citizen except i is acquiring the same positive accuracy $x^* > 0$. Then she affects the probability of the right alternative winning the election only in cases where she is pivotal, i.e., in histories at which n other citizens have observed signal s^A and the remaining n other citizens have observed signal s^B. Thus, $P_\alpha(A|x_i, x^*, z^A)$ and $P_\alpha(B|x_i, x^*, z^B)$, the probabilities of alternative A and B winning the election under state z^A and z^B respectively, given her choice x_i and other citizens' choices x^*, are linear in the probabilities of reaching a history in which she is pivotal and obtaining signal s^A and s^B respectively. As such, citizen i's payoff (Equation (3)) is proportional to

$$\left(\frac{1}{2}+x_i\right)\left[\frac{1}{2}\binom{2n}{n}\left(\frac{1}{2}+x^*\right)^n\left(\frac{1}{2}-x^*\right)^n+\frac{1}{2}\binom{2n}{n}\left(\frac{1}{2}+x^*\right)^n\left(\frac{1}{2}-x^*\right)^n\right]-C(x_i). \tag{5}$$

The regularity assumptions on the cost function ensure that there exists a unique $x_i \in (0, 1/2)$ that maximizes the above equation, and that the maximum is obtained through the first-order condition. As the choice of citizen i was arbitrary, it must be the case that $x_i = x^*$ for every $i \in N$. Simple rearrangements lead to Equation (4).

The second finding is that the equilibrium accuracy in the symmetric equilibrium of the game without deliberation decreases strictly with the electorate size. This is because the probability of a tie goes down as population rises.

Remark 2 (Gersbach et al., 2024; Proposition 1)**.** *The symmetric equilibrium accuracy $x^* = x^*(n)$ decreases strictly in n.*

The third finding is that, although the accuracy of information x^* chosen in the symmetric equilibrium decreases with the size of the electorate, the probability that majority rule yields the right alternative need not approach zero as n grows large. In fact, it can approach one for particular information acquisition technologies, so successful aggregation of information is possible.

Remark 3 (Martinelli, 2006; Theorem 2)**.** *Along the sequence of symmetric equilibria $(x^*(n))_{n\in\mathbb{N}}$, the probability of choosing the right alternative converges to $\Phi(2\sqrt{2k})$, where $\Phi(\cdot)$ denotes the standard normal distribution function and k is a solution to $\sqrt{2}\phi(2\sqrt{2k}) = kC''(0)$, with $k = 0$ if $C''(0) = \infty$ and $k = \infty$ if $C''(0) = 0$ ($\phi(\cdot)$ denotes the standard normal density). In particular,*

$$\lim_{n\to\infty} P_\alpha(d|x^*(n), z^d) = 1 \ \ \forall\, d \in \{A, B\}, \quad \textit{if } C''(0) = 0. \tag{6}$$

3 Equilibria

In this section, we derive sufficient conditions on the information cost function for both the non-existence and existence of the symmetric equilibrium that exists without deliberation in the game with deliberation (Theorems 1 and 2, respectively). It is not obvious whether, and if so under what circumstances, this symmetric equilibrium will survive under deliberation, given that we rule out babbling at the deliberation stage.

We also show that there is at most one symmetric equilibrium in the game with deliberation (Proposition 1). After that, we identify a class of asymmetric equilibria with information acquisition that *always* exists in the game with deliberation (Theorem 3) and *never* exists in the game without deliberation (Proposition 2).

Finally, we show that whether in our voting environment this class of asymmetric equilibria exhibits a higher probability of selecting the right alternative hinges on specific properties on the cost function and the size of the electorate (Propositions 3 and 4).

3.1 Example

Before moving on to the main results, it is instructive to take a closer look at an example of our model where the electorate consists of three citizens ($n = 1$) and the information acquisition cost function is quadratic, i.e., $C(x) = ax^2$.

Example 1. *Consider* $n = 1$ *and* $C(x) = ax^2$. *For this specific example, we can solve Equation* (4) *explicitly to obtain*

$$x^* = \frac{-a + \sqrt{a^2+1}}{2}. \tag{7}$$

Without loss of generality, we focus on citizen 1*'s best response to citizens* 2 *and* 3 *acquiring information level* x^*. *There are two situations in which citizen* 1*'s accuracy can affect the electoral outcome. First, when her signal is not informative enough to offset two identical signals from citizens* 2 *and* 3, *but she becomes pivotal when citizens* 2 *and* 3 *get opposing signal realizations. Second, when citizen* 1*'s signal is informative enough to compensate two identical signals from citizens* 2 *and* 3.[20]

The first situation occurs if $(1/2 - x_1)(1/2 + x^*)^2 \geq (1/2 + x_1)(1/2 - x^*)^2$ *or, equivalently, if* $x_1 \leq \Delta^2_{x^*}$, *where*

$$\Delta^2_{x^*} \equiv \frac{\left(\frac{1}{2} + x^*\right)^2}{\left(\frac{1}{2} + x^*\right)^2 + \left(\frac{1}{2} - x^*\right)^2} - \frac{1}{2}. \tag{8}$$

Note that $0 < \Delta^2_{x^*} < 1/2$, *since* $x^* \in (0, 1/2)$. *In this situation, citizen 1's payoff*

[20] Recall that any voter will vote for alternative A if $\mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^A] > \mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^B]$, will vote for alternative B if $\mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^B] > \mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^A]$, and will vote for alternative s_1 if $\mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^A] = \mathbb{P}[\boldsymbol{s}|\boldsymbol{x}, z^B]$.

(Equation (3)) from choosing $x_1 \in [0, \Delta^2_{x^*}]$ while others choose x^* is

$$G^0(x_1, x^*, x^*) = \mathbb{P}[s_2 = s_3 = s^\omega] + \mathbb{P}[s_2 \neq s_3]\left(\frac{1}{2} + x_1\right) - C(x_1),$$
$$= \left(\frac{1}{2} + x^*\right)^2 + 2\left(\frac{1}{2} + x^*\right)\left(\frac{1}{2} - x^*\right)\left(\frac{1}{2} + x_1\right) - C(x_1), \qquad (9)$$

where $\mathbb{P}[s_2 = s_3 = s^\omega]$ represents the probability of citizens 2 and 3 obtaining the signal s^ω that matches the true state $\omega \in \{A, B\}$. The first-order condition of the above problem leads to the same first-order condition of the game without deliberation (Equation (4)) with $n = 1$. Since there is a unique point that satisfies such condition, it must be the case that $x_1 = x^*$. This value is attainable since $x^* < \Delta^2_{x^*}$.

The second situation occurs if $x_1 \in (\Delta^2_{x^*}, 1/2]$. In this situation, citizen 1's payoff is

$$G^1(x_1, x^*, x^*) = \left(\frac{1}{2} + x_1\right) - C(x_1). \qquad (10)$$

Since $C'(0) = 0$, we obtain $[G^1]'(0, x^*, x^*) = 1 > 0$, so $x_1 = 0$ cannot be a best reply to x^*. Thus, citizen 1's best response to x^* is either $x_1 \in (0, 1/2)$ if $C'(1/2) > 1$ or $x_1 = 1/2$ if $C'(1/2) \leq 1$. For our specific example, $x_1 = 1/(2a)$ if $a > 1$ and $x_1 = 1/2$ if $a \leq 1$. It is straightforward to compute $\Delta^2_{x^*}$ and verify that $1/(2a) > \Delta^2_{x^*}$. Thus, the optimal choice of x_1 in $(\Delta^2_{x^*}, 1/2]$ is attained at $x_1 = 1/(2a)$ if $a > 1$ and at $x_1 = 1/2$ if $a \leq 1$.

Whether $x_1 = x^*$, $x_1 = 1/(2a)$ (for $a > 1$), or $x_1 = 1/2$ (for $a \leq 1$) will depend on the respective payoffs $G^0(x^*, x^*, x^*)$, $G^1(1/(2a), x^*, x^*)$ for $a > 1$ and $G^1(1/2, x^*, x^*)$ for $a \leq 1$. In Figures 1.A and 1.B, we plot $G^0(x, x^*, x^*)$ as in Equation (9) (blue line) and $G^1(x, x^*, x^*)$ as in Equation (10) (red line), for the two possible configurations of a.[21] Visual inspection identifies that

(i) For $a > 1$, choosing $x_1 = x^*$ is a better response for citizen 1 than $x_1 = 1/(2a)$. Hence, the symmetric equilibrium of the game without deliberation exists in the game with deliberation. In fact, this is the unique symmetric equilibrium of the latter game.

(ii) For $a \leq 1$, choosing $x_1 = 1/2$ is a better response for citizen 1 than $x_1 = x^*$. In this case, the symmetric equilibrium in the game without deliberation is not an equilibrium of the game with deliberation.

Regarding item (ii), it remains to find an equilibrium for the game with deliberation. A candidate is $x_1 = 1/2$ and $x_2 = x_3 = 0$. We claim this is an equilibrium indeed. First, suppose that citizens 2 and 3 do not want to increase their accuracy of information beyond their prior. If citizen 1 chooses any $x_1 > 0$, then the signal this latter citizen receives

[21] We prove the following observations for this specific example in Appendix B.

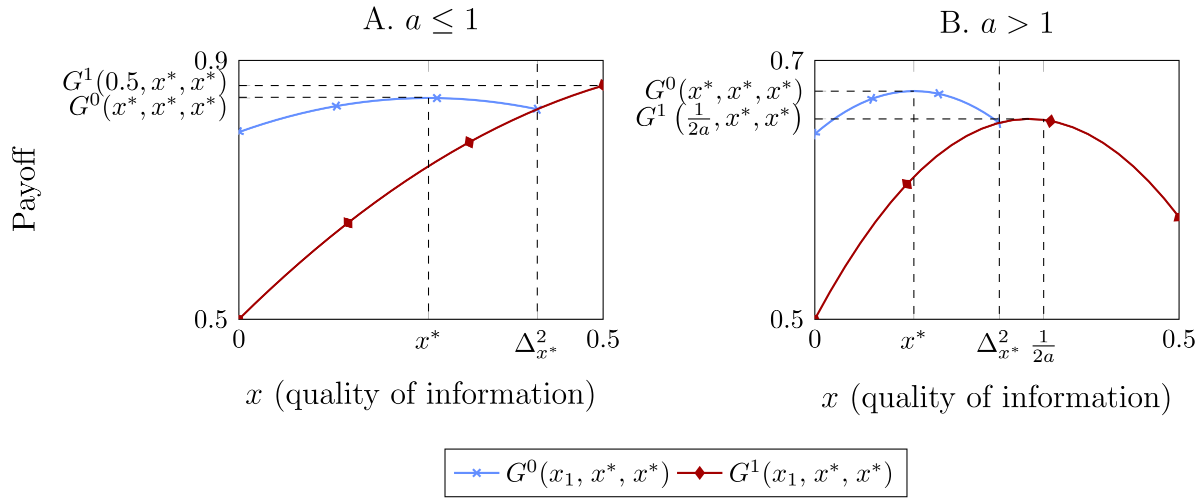

Figure 1: Comparison of payoffs from Example 1

determines the alternative chosen. For $a \leq 1$, $x_1 = 1/2$ *is a best reply. Second, suppose that* $x_1 = 1/2$*. Then citizen* $i \in \{2, 3\}$ *does not wish to deviate. Choosing* $x_i \in (0, 1/2]$ *is not optimal since choosing* $x_i = 0$ *would be a profitable deviation. Choosing the former only increases the learning costs with no informational benefit.*

Regarding item (i), we argue that $x_1 = 1/(2a)$ *and* $x_2 = x_3 = 0$ *is also an equilibrium. Clearly, it is optimal for citizen* 1 *to acquire* $1/(2a)$ *when all other citizens are not investing in their information accuracy. For her part, citizen* $i \in \{2, 3\}$ *does not wish to deviate either. Choosing* $x_i \in (0, 1/(2a))$ *is not optimal, since choosing* $x_i = 0$ *would be a profitable deviation, for the same reason as above. Choosing* $x_i \in (1/(2a), 1/2]$ *is not optimal either since choosing* $x_i = 1/(2a)$ *would be a profitable deviation. In the latter case citizen* i *would become the political expert, and we already know that the optimal level of information acquisition conditional on being the sole political expert is* $1/(2a)$*. It therefore remains to see if citizen* i *would like to deviate to* $x_i = 1/(2a)$*.*

If $s_1 = s_i = s$*, then all citizens vote according to the common signal* s*. Thus, the alternative corresponding to such signals is chosen with probability one. If signals* $s_1 \neq s_i$ *differ, all citizens vote according to their own signal (Assumption 4). This implies that citizen* 1*'s and citizen* i*'s signals cancel each other out and the alternative chosen depends on citizen* $j \notin \{1, i\}$*. Hence, the probability of alternative* A *(alternative* B*) winning under state* z^A *(state* z^B*) coincides with the probability of citizen* j *obtaining signal* s^A *(signal* s^B*). But this latter probability is* $1/2$*, since* $x_j = 0$ *and citizen* j*'s signal is uninformative. Overall, citizen* i*'s payoff from switching to* $x_1 = 1/(2a)$ *is*

$$G\left(x_1 = \frac{1}{2a}, x_i = \frac{1}{2a}, x_j = 0\right) = \left(\frac{1}{2} + \frac{1}{2a}\right) - C\left(\frac{1}{2a}\right).$$

Sticking to $x_i = 0$ instead would lead to the same probability of the right alternative being chosen, but at zero cost to citizen i. Thus, switching to $1/(2a)$ is not profitable for this citizen.

Since there can be multiple equilibria when $a > 1$, we compare them in terms of the probability of choosing the right alternative (electoral accuracy). Straightforward computations show that such a probability is greater in the asymmetric equilibrium than in the symmetric equilibrium, although the difference decreases with parameter a (see Figure 2).

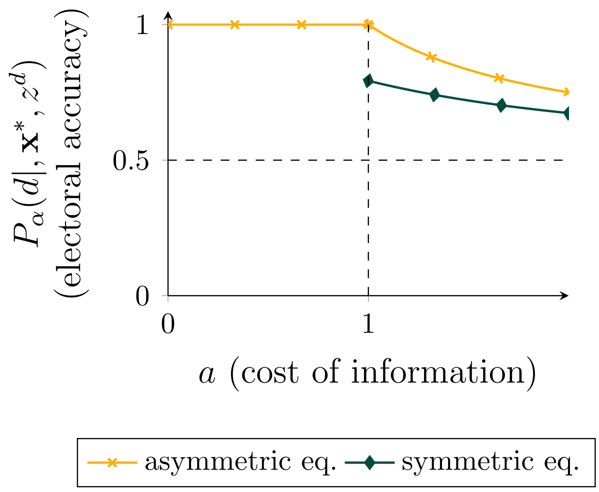

Figure 2: Comparison of electoral accuracy between equilibria from Example 1

In this example, not only can we solve the equations of interest explicitly, but we also evidence that (i) the unique symmetric equilibrium in the game without deliberation exists in the game with deliberation, albeit only under some conditions on parameter a (it has to be large); (ii) there always exist equilibria in which a single citizen acquires information first-hand, while such equilibria never exist in the game without deliberation; and (iii) this equilibria with political specialization may lead to more accurate decision-making through elections. In later sections, we will generalize (i) and (ii) and prove that (iii) might not always hold, i.e., that equilibria with specialization might lead to lower electoral accuracy.

3.2 Symmetric equilibria

We start the analysis of symmetric equilibria by proving that the game with deliberation has at most one symmetric equilibrium and that, in this equilibrium, the chosen accuracy must be the same accuracy chosen in the game without deliberation.

Proposition 1. *In the game with deliberation, if $(x_i = x)_{i \in N}$ is an equilibrium, then $x_i = x^*$, where x^* is the unique solution to Equation* (4).

Here is the intuition for this result. Suppose all other citizens except, say, citizen 1, acquire some information accuracy $x > 0$. We show later that $x = 0$ can never be an equilibrium. We can also rule out $x = 1/2$, since in such case citizen 1 would prefer to acquire zero information and free ride on the (perfect) information acquired by the others. Thus, we can assume that $x \in (0, 1/2)$.

If citizen 1 is to acquire accuracy x_1, such a choice will be beneficial to her only when she is pivotal. This will occur when exactly n citizens observe signal s^A and n other citizens observe signal s^B. This means that citizen 1's payoff given $(x_1, (x_j = x)_{j \neq 1})$ is the same as her payoff absent deliberation (Equation (5)). The solution to citizen 1's problem is therefore pinned down by the same equation that pins down the unique symmetric equilibrium of the voting game without deliberation, i.e., Equation (4).

Our first main result extends the observations made at Example 1 in the following way. Like in the example, we derive conditions on a level of information costs below which the symmetric equilibrium of the game without deliberation does not exist in the game with deliberation. This level is pinned down by multiplying some parameter $a > 0$ times any cost function that satisfies Assumption 1.

Theorem 1 (Non-existence of symmetric equilibria). *Consider some information acquisition cost function $\widetilde{C}(x)$ satisfying Assumption 1 and $\widetilde{C}'(1/2) < +\infty$. Let $C(x) \equiv a\widetilde{C}(x)$ be the information acquisition cost function, where $a > 0$. For every $n \in \mathbb{N}$, there exists*

$$\underline{a}(n) \in \left(0, \frac{1}{\widetilde{C}'(1/2)}\right)$$

such that if $a \leq \underline{a}(n)$, then the symmetric equilibrium of the game without deliberation is not an equilibrium of the game with deliberation.

To prove the above result, we first show that, as information costs reduce across-the-board through a reduction in a, the individual accuracy in the symmetric equilibrium converges to its highest value (that is, 1/2). At the same time, we also show that the probability of the majority voting for the right alternative remains bounded above by a value lower than one. A deviation for one citizen from x^* to full information acquisition leads to an electoral accuracy of one. Convexity of $\widetilde{C}$ and boundedness of $\widetilde{C}'(1/2)$ then ensure that such a deviation is profitable for small values of a.[22]

To derive the second main result of this subsection, we impose an additional assumption on the information acquisition cost function. Assumption 5 below requires that marginal learning costs are convex, i.e., they increase at an increasing rate as information accuracy increases. An interpretation of Assumption 5 is that information regarding the state of the world is complex, so not only is the marginal cost of information an in-

[22]Convexity and boundedness of the derivative ensure that the cost function does not explode at 1/2. Equation (4) allows us to rewrite the upper bound on the electoral accuracy under x^* as a function of the marginal cost $\widetilde{C}'$. Thus, we can express an upper bound on the payoff from a citizen sticking to same accuracy of other citizens as a relation between the differences in marginal costs at x^* and 1/2.

creasing function of its accuracy, but the rate of change also increases for more accurate information.

Assumption 5 (Convex marginal cost)**.** *C is regular (satisfies Assumption 1) and C'' is continuously differentiable in $(0, 1/2)$, with $C'''(x) \geq 0$ for all $x \in [0, 1/2]$.*

Our second main result extends the observations made at Example 1 in the following way. Like in the example, we derive conditions on a level of information costs above which the symmetric equilibrium of the game without deliberation exists in the game with deliberation. As before, this level is pinned down by multiplying some parameter $a > 0$ times any cost function that satisfies Assumption 5.

Theorem 2 (Existence of symmetric equilibrium)**.** *Consider some information acquisition cost function $\tilde{C}(x)$ satisfying Assumption 5. Let $C(x) \equiv a\tilde{C}(x)$ be the information acquisition cost function of the game, where $a > 0$. For every $n \in \mathbb{N}$, there exists*

$$\bar{a}(n) \in \left[\frac{1}{\tilde{C}'(1/2)}, +\infty\right)$$

such that if $a > \bar{a}(n)$, then the symmetric equilibrium of the game without deliberation is an equilibrium of the game with deliberation.

The proof of Theorem 2 is involved and relies on a number of technical lemmas, stated formally in Appendix A. Here we convey the intuition for some of the steps of the proof, which mirror the footsteps of Example 1.

To this end, consider that $2n$ citizens choose to acquire accuracy x^*, the one acquired in the equilibrium of the game without deliberation. We shall examine the best response of the remaining citizen, say, citizen 1, given that all others citizens choose x^*. As in Example 1, there can be situations in which citizen 1's signal is not informative enough to compensate a certain number of identical signals from other citizens. To account for this possibility, consider $\boldsymbol{s}_{-1}^m$ to be a sequence of $m \in \{2, 4, ..., 2n\}$ equal signals $s \in \{s^A, s^B\}$ of accuracy x. Then define Δ_x^m to be the accuracy of one signal $s' \neq s$ guaranteeing that the posterior belief after observing $(\boldsymbol{s}_{-1}^m, s')$ is equal to the prior. Formally,

$$\Delta_x^m \equiv \frac{\left(\frac{1}{2} + x\right)^m}{\left(\frac{1}{2} + x\right)^m + \left(\frac{1}{2} - x\right)^m} - \frac{1}{2}. \tag{11}$$

For completeness, we also define $\Delta_x^0 := 0$ and $\Delta_x^{2n+2} := 1/2$.

For $k \in \{1, ..., n\}$, it is clear that Δ_x^{2k} is increasing in both k and x. Intuitively, one needs signal s' to be more accurate to compensate either more signals of the same accuracy or the same number of signals of greater accuracy. Moreover, a signal of accuracy $x_1 = x^*$ can never compensate two opposite signals of the same accuracy. Therefore, $x^* \in [0, \Delta_{x^*}^2]$.

We can partition set $(0, 1/2]$ in intervals $\{(\Delta_{x^*}^{2k}, \Delta_{x^*}^{2k+2}]\}_{k=0}^{n}$ and examine what would be the optimal accuracy x_1 acquired by citizen 1 in each interval.[23] For $(0, \Delta_{x^*}^{2}]$, the first-order condition of the payoff maximization problem leads to same first-order condition of the game without deliberation, which yields x^*. For $k \in \{1, ..., n\}$, $x_1 \in (\Delta_{x^*}^{2k}, \Delta_{x^*}^{2k+2}]$ means that x_1 is accurate enough so that citizen 1's signal $s \in \{s^A, s^B\}$ offsets $2k$ opposite signals $s' \neq s$, but not $2k+2$ of them.

Under Assumption 5, we show that *(i)* for every $k \in \{1, ..., n\}$, there exists some $a_1(n, k) > 0$ sufficiently large so that for $a > a_1^*(n, k)$, the expected payoff of citizen 1 over $(\Delta_{x^*}^{2k}, \Delta_{x^*}^{2k+2}]$ is decreasing in x_1 when x_1 approaches the upper bound $\Delta_{x^*}^{2k+2}$. In Example 1, for the case $a > 1$, function $G^1(x_1, \boldsymbol{x}_{-1}^*)$ is decreasing in x_1 when x_1 approaches $1/2$ from the left. We also show that *(ii)* for $a > a_2^*(n, k)$, function $G^{k-1}(x_1, \boldsymbol{x}_{-1}^*)$ decreases faster near the upper bound $\Delta_{x^*}^{2k}$ than the rate of increase of function $G^k(x_1, \boldsymbol{x}_{-1}^*)$ near the lower bound, and to the right of $\Delta_{x^*}^{2k}$.

In our setup, these two observations imply that if parameter a is above a certain threshold that depends on n and on function $\widetilde{C}(x)$, then for each $k' \in \{0, \ldots, n-1\}$ the optimal payoff in $\cup_{k=0}^{k'}(\Delta_{x^*}^{2k}, \Delta_{x^*}^{2k+2}]$ is at least as large as the optimal payoff if we allow x to be chosen from $\cup_{k=0}^{k'+1}(\Delta_{x^*}^{2k}, \Delta_{x^*}^{2k+2}]$. In particular, the payoff from $x_1 = x^*$ over interval $(0, \Delta_{x^*}^{2}]$, which corresponds to the payoff from the symmetric equilibrium, is greater than any other payoff from the optimal choice of x_1 over other intervals $(\Delta_{x^*}^{2k}, \Delta_{x^*}^{2k+2}]$, $k \neq 0$.

Theorem 2 shows that the game with deliberation has one symmetric equilibrium if the cost of acquiring information is sufficiently high and the marginal cost is convex. It also shows that in such an equilibrium all citizens acquire the information accuracy they would acquire in the game without deliberation. However, under the symmetric equilibrium of Theorem 2, the posterior belief that each citizen has about the state of the world is greater than in the game without deliberation. Although each individual acquires private signals of same accuracy, information becomes public. One interpretation, following Besley (2023), is that this benefits a social planner seeking to strengthen compliance with the chosen policy after its deliberation and implementation, since with deliberation citizens would have greater confidence that the government implemented the right policy.

Yet, under majority rule, the electoral accuracy is the same as the one in the game without deliberation when focusing on the symmetric equilibrium. That is, the right alternative is implemented as often with deliberation as without deliberation. One interpretation is that electoral outcomes without deliberation are not necessarily illegitimate; the same electoral decisions can be generated if citizens could deliberate prior to casting their votes, provided that acquiring information is not too cheap.

[23] There is always an optimal choice, since we prove citizen 1's payoff is continuous in each interval.

3.3 Asymmetric equilibria

In this section we identify a class of asymmetric equilibria that always exists in the game with deliberation, and never in the game without deliberation. Since in any equilibrium from this class only one citizen (the political expert) acquires information to improve her prior, these equilibria are called *equilibria with a political expert.* In Section 4, we show how our rsults extend to account for multiple experts.

Definition 2 (Equilibrium with a political expert)**.** *An equilibrium with a political expert is an equilibrium of the game (with or without deliberation) in which exactly one citizen i acquires positive accuracy $x_i > 0$ and all other citizens $j \neq i$ choose $x_j = 0$.*

Like in Example 1, whether such equilibria lead to a fully informed political expert depends on the cost function, more specifically, on $C'(1/2)$.

Theorem 3 (Existence of equilibria with a political expert)**.** *In the game with deliberation, there is always an equilibrium with a political expert. Any such equilibria leads to fully revealing information if and only $C'(1/2) \leq 1$.*

Following the arguments from Example 1, it is clear that it is a best response for some citizen i to invest in accurate information when everybody else acquires zero information. Whether such a citizen (the political expert) will become fully informed about the state of the world will depend on whether $C'(1/2) \leq 1$. Also from Example 1, we know that if some citizen i chooses $x_i > 0$, then any citizen $j \neq i$ finds it optimal not to choose $x_j \in (0, x_i) \cup (x_i, 1/2]$. The rest of the proof of Theorem 3 consists in showing that it is never a best response for j to choose $x_j = x_i$ either.

An equilibrium involving political experts is a distinctive aspect of deliberation, as proved in Proposition 2 below. Provided learning is costly, there can be no political experts without deliberation.

Proposition 2. *There is no equilibrium with a political expert in the game without deliberation.*

We point out that equilibria with a political expert do not simply arise from the fact that, in contrast to Martinelli (2006), we look for asymmetric equilibria. Such equilibria of the game with deliberation exist because *(i)* public communication allows citizens to rely on the information acquired by the expert at no cost, and *(ii)* political experts monopolize the electoral decision, i.e., they are pivotal with probability one since all other citizens vote along the political expert.[24]

[24]For Proposition 2 to hold, there can be no abstentions. With the possibility of abstentions, asymmetric equilibria (and, in particular, equilibria with a political expert) would also exist without deliberation, yet for different reasons. Moreover, achieving such equilibria would require coordination during the voting stage, which could be challenging without deliberation. With deliberation, by contrast, one does not need to worry about communicating turnout intentions, since asymmetric equilibria nevertheless exist in our common-value setup, rendering abstentions not critical for our results.

3.4 Deliberation and electoral accuracy

A consequence of Theorem 3 is that the accuracy of information a political expert acquires in equilibrium is independent of the size of the electorate. Such accuracy merely depends on the behavior of $C'(x)$ near $x = 1/2$. If $C'(1/2) \leq 1$, in particular, then political experts acquire (and share) fully revealing information. In this case, electoral accuracy is maximum.

On the other hand, we know from Remark 3 that for a large electorate and if $C''(0) > 0$, the electoral accuracy of the symmetric equilibrium (if it exists) is bounded away from one. If, moreover, $C'(1/2) \leq 1$, it then follows that the electoral accuracy of the equilibria with a political expert is strictly higher than the electoral accuracy of the symmetric equilibria, no matter the size of the electorate.

Proposition 3. *Suppose $C'(1/2) \leq 1$. Then the electoral accuracy of the equilibria with a political expert of the game with deliberation is weakly higher than the electoral accuracy of any other equilibria. Moreover, electoral accuracy in an equilibrium with a political expert is strictly higher than the electoral accuracy of the symmetric equilibrium (when it exists) for any $n \in \mathbb{N}$ if $C''(0) > 0$.*

Proposition 3 identifies a setup in which deliberation may lead to a more informed electoral decision. This occurs since, without deliberation and under $C''(0) > 0$, the symmetric equilibrium does not lead to a fully accurate decision.

By contrast, if $C'(1/2) > 1$ an equilibrium with a political expert features an electoral accuracy below one. If, in addition, $C''(0) = 0$, then also from Remark 3, we know that large electorates would reach arbitrarily accurate decisions without deliberation. With deliberation, either the symmetric equilibrium does not exist, or if it does, it does not improve upon the case without deliberation. In the former case, deliberation erupts one equilibrium in which information aggregates perfectly as the electorate size increases. In the latter case, deliberation introduces equilibria with limited information gathering, no matter n. Therefore, deliberation may decrease electoral accuracy.

Proposition 4. *Consider some information acquisition cost function $\widetilde{C}$ satisfying Assumption 5, with $\widetilde{C}'(1/2) > 1$ and $\widetilde{C}''(0) = 0$. Let $C(x) \equiv a\widetilde{C}(x)$ be the information acquisition cost function of the game, where $a > 0$. There exists $n^* \in \mathbb{N}$ and $\bar{a} : \mathbb{N} \to [1, +\infty)$ such that, if $n \geq n^*$ and $a > \bar{a}(n)$,*

- *(i) both the symmetric equilibrium and the equilibria with a political expert coexist in the game with deliberation;*
- *(ii) the electoral accuracy of the equilibria with a political expert is strictly lower than the electoral accuracy of the symmetric equilibrium.*

In light of Propositions 3 and 4 we can say that the effect of deliberation on electoral accuracy is ambiguous, especially for large electorates.[25]

4 Discussion

In this section we discuss the validity of our insights when we either allow some features of our baseline model to vary or we extend our previous analyses.

4.1 Non-public communication

We carried over our analysis in the preceding sections under the double assumption that there is a public communication protocol (an extreme version of public deliberation) and that messages are commonly believed. On the one hand, we have shown that the latter assumption is not restrictive if we impose the former. The reason for this is that (Nash) equilibria of the game with deliberation are also a PBE of the dynamic game in which citizens truthfully share their private information (Corollary 1). This means that commonly believed messages are consistent with the strategic behavior of the citizens under a public communication protocol.

However, if we consider less universal communication protocols, the assumption of commonly believed messages may be less reasonable, and Lemmas 1 and 2 might not carry over. To see why, consider the following example:

Example 2. *There are three citizens ($n = 1$). Citizen 1 can communicate with citizens 2 and 3, but the latter two citizens cannot communicate with each other. Suppose that citizen 1 chooses accuracy $x_1 = 0.3$ and receives signal $s_1 = s^A$, while citizens 2 and 3 choose accuracies $x_2 = x_3 = 0.2$ and receive signals $s_2 = s_3 = s^B$. This is summarized in Figure 3.*

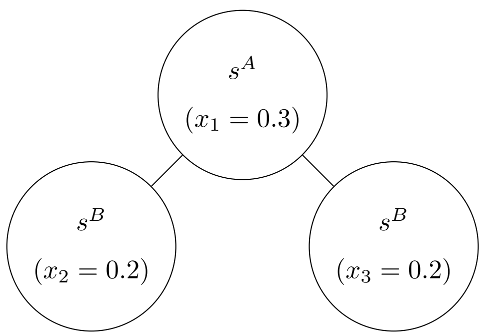

Figure 3: Non-public communication of Example 2 with the accuracies and signals.

[25] We focused on electoral accuracy and abstracted from a welfare analysis that considers also information acquisition costs. Still, the average cost of information in an equilibrium with a political expert goes to zero with a large electorate, so the insights from Proposition 3 remain from a welfare perspective. Using Remark 3, so do the insights from Proposition 4.

Suppose now that the three citizens submit truthful messages and that messages are commonly believed, i.e., Assumption 3 holds. With the private information they possess, citizens 2 *and* 3 *think that* $z = z^A$ *is the most likely state of the world. However, citizen* 1*, who has better private information, thinks that* $z = z^B$ *is actually more likely. Then, if citizen* 3 *votes sincerely, i.e., if she votes for* A*, it is strictly better for citizen* 2 *to vote for* B *than to vote for* A*, since by doing so she makes citizen* 1 *pivotal. In other words, in this example, voting sincerely might not be dominant for all citizens. Therefore, an equivalent to Lemma 1 cannot be proved for general communication protocols in the same way as it was proved for a public communication protocol (Assumption 2).*

Furthermore, if we merely assume that all citizens vote sincerely and that Assumption 3 holds, then citizen 1 *strictly prefers to submit a truthful message to one of the citizens and the message* (x_2, s^B) *to the other one, instead of submitting a truthful message to both citizens. Similarly as before, this ensures that citizen* 1 *is pivotal, which is better for everyone since she has more information. Hence, the proof of Lemma 2 cannot be extended to non-public communication protocols either.*

An interesting avenue for further research would be to examine other communication protocols implementing a less extreme form of deliberation.[26] Yet, there are specific cases that can be studied already borrowing from the proof techniques developed in this paper. For example, suppose that a majority of citizens has access to a public communication protocol exclusive to such a majority, with the remaining citizens using other, less connected communication protocols. Then, *(i)* an equilibrium with a political expert from the majority exists; and *(ii)* an equilibrium in which all citizens of the majority acquire the same level information (and all members excluded from the majority acquire zero information and vote their signals) exists if information is costly enough and does not exist if information is cheap.[27]

4.2 Equilibria with multiple experts

The analysis of our baseline setup focused on two polar classes of equilibria: one class in which all citizens acquire the same non-zero information accuracy, and one class in which only one citizen—the political expert—acquires a positive information accuracy. However, it is in principle also possible that there exist equilibria in which only a strict

[26]Note the analogy of the communication protocol with a network structure. A public communication protocol resembles a full network, while a no-deliberation setting resembles an empty network. Thus, our paper deals with extreme network settings. It would be interesting to study deliberation under costly information acquisition in other networks, such as star-like constellations or clusters of citizens in the form of bubbles. This is left for further research.

[27]Item *(ii)* is an immediate consequence of Theorem 2 and Theorem 1, respectively. To see why, it suffices to consider the majority as a society itself and then note that majority members account for more than half of the votes and that deliberation guarantees that they will all vote alike. This renders the vote of the citizens outside the majority immaterial for outcomes, forcing them to acquire zero information.

subset of citizens with more than one member obtains accurate information. In particular, we can extend our previous analysis of the game with deliberation to examine *equilibria with homogeneous experts*, i.e., equilibria in which an arbitrary number of citizens acquire the same non-zero accuracy and the remaining citizens do not acquire any information by themselves.

Assume that there exists an equilibrium of the above kind for an electorate of size $2n+1$ in which only k citizens, with $1 < k < 2n+1$, acquire the same accuracy, denoted $x^*(k;n) > 0$. On the one hand, if $x^*(k;n)$ defines an equilibrium for $2n+1$ citizens, then it must define a symmetric equilibrium for an electorate of size k, since the equilibrium conditions for an electorate of size k are a strict subset of the equilibrium conditions for an electorate of size $2n+1$ where $2n+1-k$ citizens acquire zero information.[28] The same logic implies that $x^*(k;n)$ must define an equilibrium with homogeneous experts in any society of at least k citizens. Thus, if such an equilibrium exists with deliberation, the accuracies and the electoral outcome are determined by the expert group size, not by the electorate size. This allows us to write $x^*(k)$ instead of $x^*(k;n)$. From Remark 2 it follows that $x^*(k)$ decreases in k. Hence, the greater the number of experts, the lower the individual accuracy the experts acquire about the state of the world.

On the other hand, k can only be an odd number. The reason stems from the following statistical fact: "given k' signals of any accuracy, with k' odd, adding an extra signal does not enhance the probability of choosing right". Indeed, an extra signal in a given odd subset of the electorate with the same information accuracy can only affect the collective decision if it leads to a tie among all signals, in which case the individual posteriors are uninformative and both alternatives are equally likely to be correct. But then all citizens choosing some positive information accuracy would be better off by deviating to acquire zero information.[29]

From the above discussion, as well as from the results in the previous sections, we can therefore conclude that equilibria with $1 < k < 2n+1$ homogeneous experts might also exist, provided k is odd and political information is complex enough to acquire (i.e., if the cost function satisfies Assumption 5). Indeed, the proof strategy for proving the existence of such equilibria with multiple experts boils down, in a first part, to deriving conditions for which $x^*(k)$ (the accuracy that defines the symmetric equilibrium of the game without deliberation for an electorate of size k) defines a symmetric equilibrium of the game with deliberation for an electorate of the same size. This has been done in Theorem 2 already.

It therefore remains to show, in the second part of the proof, that a citizen who is not an expert, and thus who is acquiring zero information, say some citizen j, is content

[28]The converse is not be true. An information accuracy defining a symmetric equilibrium might not define an equilibrium with homogeneous experts of a larger electorate.

[29]This argument also proves that, with deliberation, there is no symmetric equilibrium if electorate size is even.

with such a choice. In the case $k = 1$, this has been done in the proof of Theorem 3. For $k > 1$, we need to extend such proof and distinguish several cases, depending on how many signals of accuracy $x^*(k)$ can signal x_j, with $x_j \in [0, 1/2]$, compensate. To do so, we can partition set $[0, 1/2]$ in intervals $[0, \Delta^1_{x^*(k)}] \cup \{(\Delta^{2m+1}_{x^*(k)}, \Delta^{2m+3}_{x^*(k)}]\}_{m=0}^{n-1}$ and examine what would be the optimal accuracy x_j acquired by citizen j in each interval, where

$$\Delta^{2m+1}_x \equiv \frac{\left(\frac{1}{2}+x\right)^{2m+1}}{\left(\frac{1}{2}+x\right)^{2m+1}+\left(\frac{1}{2}-x\right)^{2m+1}} - \frac{1}{2}.$$

Note that $\Delta^1_{x^*(k)} = x^*(k)$. Take $x_j \in [0, \Delta^1_{x^*(k)})$. Clearly, choosing x_j in this interval cannot be optimal since choosing zero instead would be a profitable deviation. This follows from the fact that $\widetilde{C}(x)$ is increasing and that if citizen j acquires an accuracy lower than $x^*(k)$ then the alternative chosen by all citizens (including citizen j) will continue to be the same as the one implemented if citizen j chooses zero accuracy. Similarly to the proof of Theorem 3, one can check that $x_j = x^*(k)$ is not a best response either since deviating to zero would again be profitable. Hence, the optimal choice of x_j in $[0, \Delta^1_{x^*(k)}]$ is $x_j = 0$. Then, following the logic of the proof of Theorem 2, we can show that if parameter a is above a certain threshold that depends on k, the optimal choice of x_j within $[0, 1/2]$ must lie in $[0, \Delta^1_{x^*(k)}]$. All of the above yields the following result:[30]

Proposition 5. *Consider some information acquisition cost function $\widetilde{C}(x)$ satisfying Assumption 5. Let $C(x) \equiv a\widetilde{C}(x)$ be the information acquisition cost function of the game, where $a > 0$. For every odd $k \in \mathbb{N}, 1 < k < 2n + 1$, there exists $\bar{a}(k) < +\infty$ such that if $a > \bar{a}(k)$, then the symmetric equilibrium of the game without deliberation in an electorate of size k defines an equilibrium of the game with deliberation with k homogeneous experts.*

That is, if acquiring information is challenging and all citizens can publicly deliberate, access to first-hand information prior to deliberation can vary widely: it may range from a symmetric distribution to a highly unequal one where only a single individual obtains first-hand information, along with all intermediate scenarios.

By contrast, a result that follows immediately from the observations made above as well as from Theorem 1, and thus requires no proof, is the following:

Corollary 2. *Consider some information acquisition cost function $\widetilde{C}(x)$ satisfying Assumption 1 and $\widetilde{C}'(1/2) < +\infty$. Let $C(x) \equiv a\widetilde{C}(x)$ be the information acquisition cost function of the game, where $a > 0$. Then there exists $\underline{a}(n) > 0$ such that if $a < \underline{a}(n)$, then the game with deliberation has no equilibrium with $k > 1$ homogeneous experts.*

The above result implies that the insight derived in the previous sections that deliberation can kill the emergence of symmetric equilibria can be extended to account for

[30] The specific details of the proof are available upon request.

equilibria where multiple citizens become (equal) experts. That is, if it is very cheap to acquire information, all citizens but one will have an incentive to free ride on information acquisition and a single political expert will emerge, who will be the sole responsible for political learning. Following the logic behind Proposition 3, this reinforces the validity of the insight that deliberation can decrease electoral accuracy if $C'(1/2) < 1$ as was shown in Section 3.4.

Finally, the fact that, with deliberation, equilibria with a different number of political experts can exist raises the question which expert group size is optimal for electoral accuracy (or welfare). Our analysis above implies that answering this question is tantamount to answering the question which electoral size is optimal without deliberation.[31] According to Martinelli (2006), there is not a unique answer to this latter question, and either large values of group expert size would be optimal (if $C''(0) = 0$) or one expert could be optimal (if $C''(0) > 0$).[32]

4.3 Deliberation with asymmetric preferences

Our baseline setup considers common and symmetric preferences, which suffices to generate new insights about the role of deliberation in democracies where political learning is costly, even as a normative benchmark. But we can apply our techniques to examine the impact of deliberation with asymmetric preferences as well.

Without deliberation, Martinelli (2006; Theorem 4) identifies conditions for which the symmetric equilibrium ceases to exist when citizens' preferences are biased towards one of the alternatives. One particular condition is the electorate size being large. With deliberation, we can show that there exist equilibria in which one citizen becomes expert, even under Martinelli's nonexistence conditions.[33] Since the expert's accuracy does not depend on the electorate size, this result reaffirms our finding that deliberation can under some circumstances increase electoral accuracy by creating incentives for political experts to arise.

5 Concluding Remarks

We introduced and examined a model that enables investigating the effect of deliberation on information acquisition in elections. Our setup is non-ideological, in the sense that all citizens agree on which alternative is right for each of the two possible states of the world. Citizens can purchase costly information to obtain a signal about the state, and then can transmit all this information to the other citizens for free.

[31] The latter question is analyzed using our setup (without deliberation) by Gersbach et al. (2022) and Gersbach et al. (2024), but these papers allow for monetary transfers.

[32] See Section 5.2 in Martinelli (2006).

[33] The proof is available upon request.

If citizens believe the messages they receive, then neither have they incentives to send false messages nor to vote different from sincerely. This observation allowed us to focus on information acquisition and side-pass the dynamic aspects of the problem at hand, and thus study the game called (voting) game with deliberation against the benchmark of another static game called the (voting) game without deliberation (Martinelli, 2006).

The resulting game with deliberation is amenable to be analyzed analytically, yet doing so is not straightforward, which makes our contribution noteworthy from a technical perspective. However, a better understanding is needed regarding how alternative credibility assumptions might shape equilibrium behavior. For example, if citizens do not trust messages that state high accuracy, the symmetric equilibrium might exist more often than in our model. Questions of this sort might be interesting to study further.

Focusing on the game with deliberation, our analysis highlighted two types of equilibria: *(i) symmetric equilibra* and *(ii) equilibria with political expert(s).* Considering these two classes of equilibria has sufficed to show that if information acquisition is costly, deliberation is not unambiguously good for electoral outcomes. It has also allowed us to derive other novel insights about the role of deliberation in elections and to improve knowledge when the absence of deliberation is most critical for the quality of democracy, all of which are the core of our substantive contribution.

Nevertheless, we do not offer a full characterization of the set of equilibria of the game with deliberation, and thus neither for the full dynamic game. A richer set of equilibria might enable the derivation of other insights about deliberation in the context of democratic elections.

Finally, it would interesting to examine the impact of deliberation when citizens have heterogeneous priors. In most deliberative processes, citizens' invitations to participate happen randomly through a civic lottery. Although the symmetric prior is a reasonable assumption from a normative point, understanding which of our results generalize to the heterogeneous priors case would generate more robust policy implications.

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Appendix A

Proof of Lemma 1. Henceforth we let α^i_ω denote the probability of citizen i voting for $d = \omega$, given the realization of signal s^ω and some history following the other stages.

Consider first an environment in which all citizens observe their chosen accuracies and realized signals. Let us rewrite the probability of the majority choosing the right alternative in a given equilibrium in a way that evidences the impact of citizen i's voting strategy. To do so, it is instructive to first define, from citizen i's perspective, the probability of having exactly n other citizens voting for A and n citizens voters voting for B at some public history. This is given by

$$\mathrm{piv}_\alpha(\boldsymbol{s},\boldsymbol{x}) \equiv \sum_{\substack{M\subseteq N\setminus\{i\},\\ |M|=n}} \left(\prod_{\substack{j\in M,\\ s_j=s^A}} \alpha^j_A(s_{-j},\boldsymbol{x}) \prod_{\substack{j\in M,\\ s_j=s^B}} [1-\alpha^j_B(s_{-j},\boldsymbol{x})] \right) \left(\prod_{\substack{j\notin M\cup\{i\},\\ s_j=s^A}} [1-\alpha^j_A(s_{-j},\boldsymbol{x})] \prod_{\substack{j\notin M\cup\{i\},\\ s_j=s^B}} \alpha^j_B(s_{-j},\boldsymbol{x}) \right).$$

Now rewrite the probability of alternative A being elected in state s^A as

$$\begin{aligned}\mathbb{P}_\alpha(A|\boldsymbol{x},z^A) &= \sum_{\boldsymbol{s}} \mathbb{P}[\boldsymbol{s}|\boldsymbol{x},z^A] P_\alpha(A|\boldsymbol{s},\boldsymbol{x})\\ &= \left(\frac{1}{2}+x_i\right)\sum_{s_{-i}} \mathbb{P}[s_{-i}|x_{-i},z^A] P_\alpha(A|s^A,s_{-i},\boldsymbol{x}) + \left(\frac{1}{2}-x_i\right)\sum_{s_{-i}} \mathbb{P}[s_{-i}|x_{-i},z^A] P_\alpha(A|s^B,s_{-i},\boldsymbol{x}),\end{aligned}$$

where $P_\alpha(A|s_i,s_{-i},\boldsymbol{x})$ is the probability of alternative A winning the election given $(\boldsymbol{s},\boldsymbol{x})$. For $s_i = s^A$, this is

$$P_\alpha(A|s^A,s_{-i},\boldsymbol{x}) = \sum_{\substack{M\subseteq N,\\ |M|\geq n+1}} = \left(\prod_{\substack{j\in M,\\ s_j=s^A}} \alpha^j_A(s_{-j},\boldsymbol{x}) \prod_{\substack{j\in M,\\ s_j=s^B}} [1-\alpha^j_B(s_{-j},\boldsymbol{x})] \right) \left(\prod_{\substack{j\notin M,\\ s_j=s^A}} [1-\alpha^j_A(s_{-j},\boldsymbol{x})] \prod_{\substack{j\notin M,\\ s_j=s^B}} \alpha^j_B(s_{-j},\boldsymbol{x}) \right).$$

Rearranging the right-hand side of the above equation, we obtain

$$\begin{aligned}&\alpha^i_A(s_{-i},\boldsymbol{x}) \sum_{\substack{M\subseteq N\setminus\{i\},\\ |M|\geq n}} \left(\prod_{\substack{j\in M,\\ s_j=s^A}} \alpha^j_A(s_{-j},\boldsymbol{x}) \prod_{\substack{j\in M,\\ s_j=s^B}} [1-\alpha^j_B(s_{-j},\boldsymbol{x})] \right) \left(\prod_{\substack{j\notin M,\\ s_j=s^A}} [1-\alpha^j_A(s_{-j},\boldsymbol{x})] \prod_{\substack{j\notin M,\\ s_j=s^B}} \alpha^j_B(s_{-j},\boldsymbol{x}) \right)\\ &+ [1-\alpha^i_A(s_{-i},\boldsymbol{x})] \sum_{\substack{M\subseteq N\setminus\{i\},\\ |M|\geq n+1}} \left(\prod_{\substack{j\in M,\\ s_j=s^A}} \alpha^j_A(s_{-j},\boldsymbol{x}) \prod_{\substack{j\in M,\\ s_j=s^B}} [1-\alpha^j_B(s_{-j},\boldsymbol{x})] \right) \left(\prod_{\substack{j\notin M,\\ s_j=s^A}} [1-\alpha^j_A(s_{-j},\boldsymbol{x})] \prod_{\substack{j\notin M,\\ s_j=s^B}} \alpha^j_B(s_{-j},\boldsymbol{x}) \right),\\ &= \alpha^i_A(s_{-i},\boldsymbol{x})\mathrm{piv}_i(s^A,s_{-i},\boldsymbol{x}) + P_\alpha(v_A \geq n+1|s^A,s_{-i},\boldsymbol{x}),\end{aligned}$$

where $P_\alpha(v_A \geq n+1|s^A, s_{-i}, \boldsymbol{x})$ is the probability of alternative A getting at least $n+1$ votes from other citizens, that is,

$$\sum_{\substack{M \subseteq N \setminus \{i\}, \\ |M| \geq n+1}} \left(\prod_{\substack{j \in M, \\ s_j = s^A}} \alpha_A^j(s_{-j}, \boldsymbol{x}) \prod_{\substack{j \in M, \\ s_j = s^B}} [1 - \alpha_B^j(s_{-j}, \boldsymbol{x})] \right) \left(\prod_{\substack{j \notin M, \\ s_j = s^A}} [1 - \alpha_A^j(s_{-j}, \boldsymbol{x})] \prod_{\substack{j \notin M, \\ s_j = s^B}} \alpha_B^j(s_{-j}, \boldsymbol{x}) \right),$$

where v_A represents the number of votes for A. Note that $P_\alpha(v_A \geq n+1|s^A, s_{-i}, \boldsymbol{x})$ does not depend on i's voting strategy. For $s_i = s^B$ same logic leads to

$$P_\alpha(A|s^B, s_{-i}, \boldsymbol{x}) = [1 - \alpha_B^i(s_{-i}, \boldsymbol{x})]\text{piv}_i(s^B, s_{-i}, \boldsymbol{x}) + P_\alpha(v_A \geq n+1|s^B, s_{-i}, \boldsymbol{x}).$$

This means we can rewrite $\mathbb{P}_\alpha(A|\boldsymbol{x}, z^A)$ as

$$\begin{aligned}\mathbb{P}_\alpha(A|\boldsymbol{x}, z^A) = &\sum_{s_{-i}} \left(\frac{1}{2} + x_i\right) \mathbb{P}[s_{-i}|x_{-i}, z^A]\alpha_A^i(s{-}i, \boldsymbol{x})\text{piv}_i(s^A, s_{-i}, \boldsymbol{x}) + R_{-i}(\boldsymbol{s}, \boldsymbol{x}, z^A) \\ &+ \sum_{s_{-i}} \left(\frac{1}{2} - x_i\right) \mathbb{P}[s_{-i}|x_{-i}, z^A] \left\{1 - \alpha_B^i(s_{-i}, \boldsymbol{x})\right\} \text{piv}_i(s^B, s_{-i}, \boldsymbol{x}).\end{aligned}$$

where $R_{-i}(\boldsymbol{s}, \boldsymbol{x}, z^A)$, defined next, is a term that does not depend in i's voting strategy:

$$\begin{aligned}R_{-i}(\boldsymbol{s}, \boldsymbol{x}, z^A) \equiv &\sum_{s_{-i}} \left(\frac{1}{2} + x_i\right) \mathbb{P}[s_{-i}|x_{-i}, z^A] P_\alpha(v_A \geq n+1|s^A, s_{-i}, \boldsymbol{x}) \\ &+ \sum_{s_{-i}} \left(\frac{1}{2} - x_i\right) \mathbb{P}[s_{-i}|x_{-i}, z^A] P_\alpha(v_A \geq n+1|s^B, s_{-i}, \boldsymbol{x}).\end{aligned}$$

Following the same steps as above, $P_\alpha(B|x_i, x_{-i}, z_B)$ can be written as

$$\begin{aligned}P_\alpha(B|\boldsymbol{x}, z^B) = &\sum_{s_{-i}} \left(\frac{1}{2} + x_i\right) \mathbb{P}[s_{-i}|x_{-i}, z^B]\alpha_B^i(s{-}i, \boldsymbol{x})\text{piv}_i(s^B, s_{-i}, \boldsymbol{x}) + R_{-i}(\boldsymbol{s}, \boldsymbol{x}, z^B) \\ &+ \sum_{s_{-i}} \left(\frac{1}{2} - x_i\right) \mathbb{P}[s_{-i}|x_{-i}, z^B] \left\{1 - \alpha_A^i(s{-}i, \boldsymbol{x})\right\} \text{piv}_i(s^A, s_{-i}, \boldsymbol{x}).\end{aligned}$$

Electoral accuracy, that is, $(1/2)\left\{P_\alpha(A|\boldsymbol{x}, z^A) + P_\alpha(B|\boldsymbol{x}, z^B)\right\}$, then boils down to

$$\begin{aligned}&\sum_{s_{-i}} \mathbb{P}[s^A, s_{-i}|\boldsymbol{x}] \left\{\text{piv}_i(s^A, s_{-i}, \boldsymbol{x}) \left[\mathbb{P}[z^A|s^A, s_{-i}, \boldsymbol{x}]\alpha_A^i(s_{-i}, \boldsymbol{x}) + \mathbb{P}[z^B|s^A, s_{-i}, \boldsymbol{x}](1 - \alpha_A^i(s_{-i}, \boldsymbol{x}))\right]\right\} + \\ &\sum_{s_{-i}} \mathbb{P}[s^B, s_{-i}|\boldsymbol{x}] \left\{\text{piv}_i(s^B, s_{-i}, \boldsymbol{x}) \left[\mathbb{P}[z^A|s^B, s_{-i}, \boldsymbol{x}](1 - \alpha_B^i(s_{-i}, \boldsymbol{x})) + \mathbb{P}[z^B|s^B, s_{-i}, \boldsymbol{x}]\alpha_B^i(s_{-i}, \boldsymbol{x})\right]\right\} + \\ &\sum_{s_{-i}} \mathbb{P}[s^A, s_{-i}|\boldsymbol{x}] \left\{P_\alpha(v_A \geq n+1|s^A, s_{-i}, \boldsymbol{x})\mathbb{P}[z^A|s^A, s_{-i}, \boldsymbol{x}] + P_\alpha(v_B \geq n+1|s^A, s_{-i}, \boldsymbol{x})\mathbb{P}[z^B|s^A, s_{-i}, \boldsymbol{x}]\right\} + \\ &\sum_{s_{-i}} \mathbb{P}[s^B, s_{-i}|\boldsymbol{x}] \left\{P_\alpha(v_B \geq n+1|s^B, s_{-i}, \boldsymbol{x})\mathbb{P}[z^B|s^B, s_{-i}, \boldsymbol{x}] + P_\alpha(v_A \geq n+1|s^B, s_{-i}, \boldsymbol{x})\mathbb{P}[z^A|s^B, s_{-i}, \boldsymbol{x}]\right\}.\end{aligned}$$

Clearly, at any $(\boldsymbol{s}, \boldsymbol{x})$ for which citizen i is pivotal, voting sincerely is weakly dominant.

Now, define $I \equiv \{s^A, s^B\} \times [0, 1/2]$ as a citizen's private information set, and, for any $i \in N$, $\pi_i : I \times M_{-i} \to \Delta\left(I^{2n}\right)$, citizen i's probability distribution over the private information set of other citizens, given her own private information and the messages observed by the others. The vector $\boldsymbol{\pi} \equiv (\pi_i)_{i\in N}$ is a belief system about the messages that were truly conveyed at the deliberation stage. The estimated electoral accuracy from the perspective of citizen i given (s_i, x_i, m_{-i}) becomes

$$\sum_{s_{-i},x_{-i}} \pi_i(s_{-i}, x_{-i}|s_i, x_i, m_{-i}) \cdot \left\{ \begin{aligned} &\mathrm{piv}_i(s_i, s_{-i}, \boldsymbol{x}) \left[\mathbb{P}[z^A|s_i, s_{-i}, \boldsymbol{x}]\alpha^i_{s_i}(s_{-i}, \boldsymbol{x}) + \mathbb{P}[z^B|s_i, s_{-i}, \boldsymbol{x}](1 - \alpha^i_{s_i}(s_{-i}, \boldsymbol{x}))\right] + \\ &P_\alpha(v_A \geq n+1|s_i, s_{-i}, \boldsymbol{x})\mathbb{P}[z^A|s_i, s_{-i}, \boldsymbol{x}] + P_\alpha(v_B \geq n+1|s_i, s_{-i}, \boldsymbol{x})\mathbb{P}[z^B|s_i, s_{-i}, \boldsymbol{x}] \end{aligned} \right\}.$$

Define the belief citizen i assigns to z^ω given (s_i, x_i, m_{-i}), in histories happening with positive probability under π_i, for which she has probability of being pivotal:

$$\mathbb{P}[z^\omega|s_i, x_i, m_{-i}] \equiv \sum_{s_{-i},x_{-i}} \pi_i(s_{-i}, x_{-i}|s_i, x_i, m_{-i})\mathrm{piv}_i(s_i, s_i, x_i, x_{-i})\mathbb{P}[z^\omega|s_i, s_{-i}, x_i, x_{-i}].$$

Then the following threshold strategy for i is optimal

$$\alpha^i_\omega(x_i, m_{-i}) = \begin{cases} 1 & \text{if } \mathbb{P}[z^\omega|s_i, x_i, m_{-i}] > \mathbb{P}[z^{\neg\omega}|s_i, x_i, m_{-i}], \\ [0, 1] & \text{if } \mathbb{P}[z^\omega|s_i, x_i, m_{-i}] = \mathbb{P}[z^{\neg\omega}|s_i, x_i, m_{-i}], \\ 0 & \text{if } \mathbb{P}[z^\omega|s_i, x_i, m_{-i}] < \mathbb{P}[z^{\neg\omega}|s_i, x_i, m_{-i}]. \end{cases}$$

Thus, voting sincerely *given other citizens' messages and her beliefs about such messages* is optimal. Assumption 3 ensures that citizen i assigns probability one to (s_{-i}, x_{-i}) given $m_{-i} = (s_{-i}, x_{-i})$. The threshold strategy boils down to comparing the likelihood of signals under each state, which implies that voting sincerely is weakly dominant. □

Proof of Lemma 2. We start noting that, from Lemma 1, all citizens anticipate that all of them will vote sincerely in Stage 3. Then consider some citizen $i \in N$. Due to Assumption 3, she believes that all the messages she received were truthfully reported. We proceed by contradiction. So suppose that citizen i sends some message m_i in Stage 2 to some citizen j that differs from (x_i, s_i). Now recall that we are assuming that citizen j will vote sincerely in Stage 3, i.e., she will use whatever information she gathered in Stage 2. This implies that there is some (possibly zero) probability that citizen j will vote in Stage 3 for alternative $d \in \{A, B\}$ when, according to the information citizen i holds, i.e., $(x_i, s_i) \times M_i$, alternative $d' \neq d$ should be implemented. Hence, by not truthfully reporting (x_i, z_i) to all other citizens, citizen i expects a utility that is lower than, or equal to, the one she expects if she sends message (x_i, z_i) to all citizens. Therefore, given $(x_i, s_i) \times M_i$, citizen i has no strong incentive to send false information in Stage 3. □

Proof of Proposition 1. Let $(x, \ldots, x) \in [0, 1/2]^{2n+1}$ be a strategy profile of the voting game with deliberation. Without loss of generality, we focus on citizen 1's best response to the remaining $2n$ voters choosing information level $x \geq 0$. We know from Theorem

3 that $x_i = 0$ for all $i \in N$ never constitutes an equilibrium, so we can assume $x > 0$. Moreover, $x = 1/2$ cannot constitute an equilibrium either, since any citizen would strictly prefer to acquire zero information and free ride on the information acquired by the others. Hence, we can assume $x \in (0, 1/2)$.

It is clear that a signal of accuracy $x > 0$ can never compensate two opposite signals of the same accuracy $x > 0$. Therefore, if citizen 1 acquires information level x, her signal will only be followed by all the citizens (including citizen 1) whenever among the remaining citizens there are as many s^A signals as there are s^B signals. But this means that the utility of citizen 1 when $x_1 = x \in [0, \Delta_x^2]$—where Δ_x^2 is defined as in Equation (11)—and all other citizens choose information acquisition level x is

$$b(x) + \left(\frac{1}{2} + x_1\right)\binom{2n}{n}\left(\frac{1}{2} + x\right)^n\left(\frac{1}{2} - x\right)^n - C(x_1),$$

where $b(x)$ is independent of x_1. By differentiating the above expression and equating it to zero, we obtain Equation (4). This means that $x = x^*$, which completes the proof. $\square$

Proof of Theorem 1. From Equation (4), and considering $C(x) = a\widetilde{C}(x)$ with $\widetilde{C}(x)$ satisfying Assumption 1 and $\widetilde{C}'(1/2) < +\infty$, it is clear that x^* converges to $1/2$ as a converges to 0. To make explicit the dependence of a, we write $x^* = x^*(a)$. Because $\widetilde{C}'(\cdot)$ is continuous, for every $\varepsilon > 0$ there exists $\underline{a}(\varepsilon) > 0$ such that,

$$\widetilde{C}'(x^*(a)) \geq \widetilde{C}'(1/2) - \varepsilon \;\text{ for all } a \leq \underline{a}(\varepsilon). \tag{A.1}$$

Now consider any such $\varepsilon > 0$ and any $a \leq \underline{a}(\varepsilon)$. For each $i \in \{1, ..., 2n+1\}$, let Y_i denote the (Bernoulli) random variable such that $Y_i = 1$ if $s_i = s_A$ and $Y_i = 0$ if $s_i = s_B$. Note that $\mathbb{P}(s_i = s_A | z^B) = 1/2 - x^*(a)$. Define as well $S_N := \sum_{i=1}^{2n+1} Y_i$ as the sum of citizens with signal s_A, given the electorate size of $N := 2n + 1$.

On the one hand, the probability of alternative A winning if the state is z^B and every citizen is choosing $x^*(a)$ is bounded from below as follows:

$$\begin{aligned} P(A|\mathbf{x}^*(\mathbf{a}), z^B) &= \mathbb{P}[S_N \geq n+1 | z^B] = \mathbb{P}[S_N = n+1 | z^B] + \mathbb{P}[S_N \geq n+2 | z^B] \\ &> \mathbb{P}[S_N = n+1 | z^B] = \binom{2n+1}{n+1}\left(\frac{1}{2} - x^*(a)\right)^{n+1}\left(\frac{1}{2} + x^*(a)\right)^n. \end{aligned} \tag{A.2}$$

On the other hand, from Equation (4) and Equation (A.1),

$$\left(\frac{1}{2} - x^*(a)\right)^n\left(\frac{1}{2} + x^*(a)\right)^n = \frac{a\widetilde{C}'(x^*(a))}{\binom{2n}{n}} \geq a\left(\frac{\widetilde{C}'(1/2) - \varepsilon}{\binom{2n}{n}}\right). \tag{A.3}$$

Substituting Equation (A.3) into Equation (A.2),

$$\binom{2n+1}{n+1}\left(\frac{1}{2}-x^*(a)\right)^{n+1}\left(\frac{1}{2}+x^*(a)\right)^{n} > \frac{\binom{2n+1}{n+1}}{\binom{2n}{n}}\left(\tilde{C}'(1/2)-\varepsilon\right)\left(\frac{1}{2}-x^*(a)\right)a,$$
$$\equiv \bar{C}(n)\left(\frac{1}{2}-x^*(a)\right)a. \tag{A.4}$$

Therefore, the probability of alternative B willing if the state is z^B is bounded above as

$$P(B|\mathbf{x}^*(\mathbf{a}),z^B) = 1 - P(A|\mathbf{x}^*(\mathbf{a}),z^B) < 1-\bar{C}(n)\left(\frac{1}{2}-x^*(a)\right)a.$$

By the symmetry of the information structure and the prior distribution, the same bound is achieved for $P(A|\mathbf{x}^*(\mathbf{a}),z^A)$. It then follows from Equation (3) that citizen i's payoff from choosing $x^*(a)$ while others do the same is bounded from above as follows:

$$G(\mathbf{x}^*(a)) < 1-\bar{C}(n)\left(\frac{1}{2}-x^*(a)\right)a - a\widetilde{C}(x^*(a)). \tag{A.5}$$

If citizen i deviated to $x_i = 1/2$, her payoff would be $1 - C(1/2)$. Hence, this deviation is profitable if Equation (A.5) is no greater than $1 - a\widetilde{C}(1/2)$ or, equivalently, if

$$\frac{\tilde{C}(1/2)-\tilde{C}(x^*(a))}{1/2-x^*(a)} \le \bar{C}(n).$$

Because $\widetilde{C}(\cdot)$ is convex from Assumption 1, and using Equation (A.1), the above condition is implied by either of the following two equivalent conditions

$$\widetilde{C}'(1/2) \le \bar{C}(n) \quad \Leftrightarrow \quad \varepsilon \le \widetilde{C}'(1/2)\left(\frac{n}{2n+1}\right) := \underline{\varepsilon}(n). \tag{A.6}$$

Therefore, it suffices to consider $\varepsilon = \underline{\varepsilon}(n)$ and $\underline{a}(n) := \underline{a}(\underline{\varepsilon}(n))$.

Finally, we prove that $\underline{a}(n) < 1/\widetilde{C}(1/2)$. From Equation (4) and Equation (A.1), at $\varepsilon = \underline{\varepsilon}(n)$ and $a = \underline{a}(n)$,

$$\underline{a}(n) = \frac{\binom{2n}{n}\left(\frac{1}{4}-(x^*(\underline{a}(n)))^2\right)^n}{\widetilde{C}'(1/2)\frac{n+1}{2n+1}} = \frac{\binom{2n+1}{n+1}\left(\frac{1}{4}-(x^*(\underline{a}(n)))^2\right)^n}{\widetilde{C}'(1/2)} < \frac{\binom{2n+1}{n+1}\left(\frac{1}{4}\right)^n}{\widetilde{C}'(1/2)}.$$

It then follows that $\binom{2n+1}{n+1} \le 4^n$, because $\binom{2n+1}{n+1} \le \sum_{k=n+1}^{2n+1}\binom{2n+1}{k} = 4^n$. □

Lemma A.1. *Let C be an information acquisition function satisfying Assumption 5 and let b_1, b_2, c_1, c_2 be some positive constants satisfying $b_2 > b_1$, $c_1 > c_2$, and*

$$\hat{x} = \frac{c_1 - c_2}{b_2 - b_1} \in (0, 1/2). \tag{A.7}$$

Define functions $y_1(x) := c_1 + b_1 x - C(x)$ and $y_2(x) := c_2 + b_2 x - C(x)$, which cross at

$\hat{x}$. Then assume that

$$y_1'(\hat{x}) < 0 \tag{A.8}$$

and

$$y_2'(\hat{x}) < -y_1'(\hat{x}). \tag{A.9}$$

If there exists $\overline{x} \in [\hat{x}, 1/2]$ such that

$$y_2'(\overline{x}) < 0, \tag{A.10}$$

then it must be that

$$\max_{[0,\hat{x}]} y_1(x) \geq \max_{[0,\overline{x}]} y_2(x). \tag{A.11}$$

Proof. Note that for $k \in \{1,2\}$, Assumption 5 guarantees that $y_k'(0) = b_k - C'(0) = b_k > 0$ and $y_k''(x) = -C'''(x) \leq 0$. Together with Conditions (A.8) and (A.10), we obtain $y_1'(x) < 0$ for all $x \geq \hat{x}$ and $y_2'(x) < 0$ for all $x \geq \overline{x}$. Therefore, for $k \in \{1,2\}$,

$$x_k := \arg \max_{x \in [0,1/2]} y_k(x),$$

is well-defined and satisfies $b_k = C'(x_k)$. Conditions (A.8) and (A.10) imply that $x_1 \in (0, \hat{x})$ and $x_2 \in (0, \overline{x})$, while $C'(\cdot)$ being increasing for $(0, 1/2)$ together with $b_2 > b_1$ and $b_k = C'(x_k)$ imply that $x_1 < x_2$. Finally, note that for all $x \in [0, 1/2]$,

$$y_2'(x) - y_1'(x) = b_2 - b_1 > 0. \tag{A.12}$$

Next, we distinguish two cases.

Case I: $y_2'(\hat{x}) \leq 0$. In this case,

$$\max_{[0,\overline{x}]} y_2(x) = \max_{[0,\hat{x}]} y_2(x) < \max_{[0,\hat{x}]} y_1(x),$$

where the equality follows from $y_2'(\hat{x}) \leq 0$ and $y_k''(x) = -C'''(x) \leq 0$, both ensuring $x_2 \leq \hat{x}$. The inequality is explained as follows. Since $y_2(x)$ is maximized at $x = x_2$, it suffices that $y_1(x_2) \geq y_2(x_2)$. Indeed,

$$y_1(x_2) = y_1(\hat{x}) + \int_{\hat{x}}^{x_2} y_1'(x)dx = y_2(\hat{x}) - \int_{x_2}^{\hat{x}} y_1'(x)dx > y_2(\hat{x}) - \int_{x_2}^{\hat{x}} y_2'(x)dx = y_2(x_2),$$

where the first and third equality follow from the fundamental theorem of calculus, the second equality holds since $y_1(x)$ and $y_2(x)$ cross at $\hat{x}$, and the inequality is due to Condition (A.12).

Case II: $y_2'(\hat{x}) > 0$. In this case, $y_2'(\hat{x}) > 0$ and Condition (A.10) imply that $x_2 \in$

$(\hat{x}, \overline{x})$. We claim that $|x_1 - \hat{x}| \geq |x_2 - \hat{x}|$. Indeed, note that

$$C'(\hat{x}) > \frac{b_1 + b_2}{2} = \frac{C'(x_1) + C'(x_2)}{2} \geq C'\left(\frac{x_1 + x_2}{2}\right),$$

where the first inequality can be derived from Condition (A.9), the equality follows from $b_k = C'(x_k)$, and the last inequality holds since $C'(\cdot)$ is convex in $(0, 1/2)$. Since $C'(\cdot)$ is also strictly increasing in $(0, 1/2)$, it follows that $\hat{x} > (x_1 + x_2)/2$. This inequality together with $x_1 < x_2$ show that the claim holds. Let $x \in [\hat{x}, x_2]$. Then $y_2'(x) \geq 0$, and from the proved claim, we obtain $x_1 < \hat{x} - (x - \hat{x}) \;\; \Rightarrow y_1'(\hat{x} - (x - \hat{x})) < 0$. Moreover,

$$\begin{aligned} -y_1'(\hat{x} - (x - \hat{x})) &= -y_1'(\hat{x}) - \int_{\hat{x}-(x-\hat{x})}^{\hat{x}} C''(t)dt > y_2'(\hat{x}) - \int_{\hat{x}}^{x} C''(t)dt, \\ &\geq y_2'(\hat{x}) - \int_{\hat{x}}^{x} C''(t)dt = y_2'(x), \end{aligned} \tag{A.13}$$

where the two equalities follow from the fundamental theorem of calculus, the first inequality is implied by $x_2 \in (\hat{x}, \overline{x})$, and the second inequality is due to *(i)* $x \geq \hat{x}$; *(ii)* $\hat{x} - (\hat{x} - (x - \hat{x})) = x - \hat{x}$; and *(iii)* $C''(x)$ is non-decreasing for $x \in (0, 1/2)$.

Finally, we claim that

$$y_1(\hat{x} - (x_2 - \hat{x})) \geq y_2(x_2), \tag{A.14}$$

which implies Condition (A.11) and finishes the proof. To see that this is true, we start noting that if we use the fundamental theorem of calculus we can write

$$y_1(\hat{x} - (x_2 - \hat{x})) = y_1(\hat{x}) + \int_{\hat{x}}^{\hat{x}-(x_2-\hat{x})} y'(x)dx = y_1(\hat{x}) + \int_{\hat{x}-(x_2-\hat{x})}^{\hat{x}} -y_1'(x)dx, \tag{A.15}$$

and

$$y_2(x_2) = y_2(\hat{x}) + \int_{\hat{x}}^{x_2} y_2'(x)dx. \tag{A.16}$$

Using (A.13) for all $x \in [\hat{x}, x_2]$ and noting that $\hat{x} - (\hat{x} - (x_2 - \hat{x})) = x_2 - \hat{x}$,

$$\int_{\hat{x}-(x_2-\hat{x})}^{\hat{x}} -y_1'(x)dx > \int_{\hat{x}}^{x_2} y_2'(x)dx.$$

This last inequality, together with Equations (A.7) and (A.15)–(A.16) imply claim (A.14). □

Lemma A.2. *Let C be an information acquisition function satisfying Assumption 5 and let $x^* \in (0, 1/2]$. For all $x \geq x^*$,*

$$\frac{C'(x)}{C'(x^*)} \geq \frac{x}{x^*}. \tag{A.17}$$

Proof. The above inequality is equivalent to $x^*C'(x) \geq xC'(x^*)$. For all $x \in (0, 1/2]$, define $h(x) := x^*C'(x) - xC'(x^*)$. Note that $h'(x) = x^*C''(x) - C'(x^*) \geq x^*C''(x)$ and, from Assumption 5, $h''(x) = x^*C'''(x) \geq 0$. By definition of function h and since $C'(0) = 0$, $h(0) = h(x^*) = 0$. We claim that

$$h'(x^*) = x^*C''(x^*) - C'(x^*) \geq 0, \tag{A.18}$$

which together with $h''(x) \geq 0$ implies that $h'(x) \geq 0$ for all $x \geq x^*$. Then, for all $x \geq x^*$,

$$h(x) = h(x^*) + \int_x^{x^*} h'(x)dx = \int_x^{x^*} h'(x)dx \geq 0,$$

where the first equality follows the fundamental theorem of calculus, the second equality follows from $h(0) = 0$, and the inequality follows from $h'(x) \geq 0$ for all $x \geq x^*$. Indeed,

$$C'(x^*) = C'(0) + \int_0^{x^*} C''(x)dx = \int_0^{x^*} C''(x)dx \leq \int_0^{x^*} C''(x^*)dx = x^*C''(x^*),$$

where the first equality follows from the fundamental theorem of calculus, the second equality follows from the assumption that $C'(0) = 0$, and the inequality holds since $C'''(x) \geq 0$ for all $x \in [0, 1/2]$ due to Assumption 5. $\square$

Lemma A.3. *Let* $k \geq 1$. *Then,* $\lim_{x \to 0} \frac{\Delta_x^k}{x} = k$.

Proof. We have a 0/0 indeterminacy. However, using L'Hôpital's rule, for $k \geq 1$,

$$\lim_{x \to 0} \frac{\Delta_x^k}{x} = \lim_{x \to 0} \frac{k\left(0.25 - 1.x^2\right)^{k-1}}{\left((0.5 - x)^k + (x + 0.5)^k\right)^2} = k.$$

$\square$

Proof of Theorem 2. We assume that $2n$ citizens choose to acquire information level x^* and analyze the best response of the remaining citizen, who we consider to be citizen 1 without loss of generality. Citizen 1 chooses x_1 to maximize

$$G(x_1, x^*, \ldots, x^*) = \frac{1}{2}\left\{P_\alpha(A|x_1, x^*, \ldots, x^*, z^A) + P_\alpha(B|x_1, x^*, \ldots, x^*, z^B)\right\} - C(x_1).$$

We divide the proof in five steps.

Step 1. We derive an explicit expression for $G(x_1, x^*, \ldots, x^*)$ for all $x_1 \in [0, 1/2]$ and show that it is continuous in x_1 in the entire interval. Function $G(x_1, x^*, \ldots, x^*)$ is defined piecewise. Indeed, for $k \in \{0, \ldots, n\}$, the restriction of $G(x_1, x^*, \ldots, x^*)$ to the interval $(\Delta_{x^*}^{2k}, \Delta_{x^*}^{2k+2}]$ coincides in this interval with the following function:

$$G^k(x_1, x^*, \dots, x^*) = -C(x_1) + \left(\frac{1}{2} + x_1\right) \binom{2n}{n} \left(\frac{1}{2} + x^*\right)^n \left(\frac{1}{2} - x^*\right)^n + \sum_{i=k+1}^{n} \binom{2n}{n+i} \left(\frac{1}{2} + x^*\right)^{n+i} \left(\frac{1}{2} - x^*\right)^{n-i}$$
$$+ \left(\frac{1}{2} + x_1\right) \cdot \left[\sum_{i=1}^{k} \binom{2n}{n+i} \left(\left(\frac{1}{2} + x^*\right)^{n+i} \left(\frac{1}{2} - x^*\right)^{n-i} + \left(\frac{1}{2} + x^*\right)^{n-i} \left(\frac{1}{2} - x^*\right)^{n+i}\right)\right]. \tag{A.19}$$

Function $G^k(x_1, x^*, \dots, x^*)$ is citizen 1's expected utility when the remaining citizens choose to acquire accuracy x^* and citizen 1 chooses an information that is accurate enough to offset $2k$ opposite signals of accuracy x^*.

To show the continuity of function $G(x_1, x^*, \dots, x^*)$ for all $x^1 \in [0, 1/2]$, it suffices to show that for all $k \in \{0, \dots, n-1\}$,

$$G^k(\Delta_{x^*}^{2k+2}, x^*, \dots, x^*) = G^{k+1}(\Delta_{x^*}^{2k+2}, x^*, \dots, x^*). \tag{A.20}$$

To show the above equality, note that

$$G^{k+1}(x_1, x^*, \dots, x^*) - G^k(x_1, x^*, \dots, x^*) = -\binom{2n}{n+k+1} \left(\frac{1}{2} + x^*\right)^{n+k+1} \left(\frac{1}{2} - x^*\right)^{n-k-1}$$
$$+ \left(\frac{1}{2} + x_1\right) \binom{2n}{n+k+1} \left(\frac{1}{2} + x^*\right)^{n+k+1} \left(\frac{1}{2} - x^*\right)^{n-k-1}$$
$$+ \left(\frac{1}{2} + x_1\right) \binom{2n}{n+k+1} \left(\frac{1}{2} + x^*\right)^{n-k-1} \left(\frac{1}{2} - x^*\right)^{n+k+1},$$

and $G^{k+1}(x_1, x^*, \dots, x^*) - G^k(x_1, x^*, \dots, x^*) = 0$ is a linear equation in x_1. Solving for x_1 leads to $x_1 = \Delta_{x^*}^{2k+2}$. This completes the proof of Step 1.

Step 2. We show that for $k \in \{1, \dots, n\}$, there exists $a_1^*(n, k)$ such that if $a \geq a_1^*(n, k)$,

$$\left.\frac{\partial G^{k-1}(x_1, x^*, \dots, x^*)}{\partial x_1}\right|_{x_1 = \Delta_{x^*}^{2k}} < 0.$$

Define the following function:

$$f_k^n(x^*) := (1 - 2k) \binom{2n}{n} \left(\frac{1}{2} + x^*\right)^n \left(\frac{1}{2} - x^*\right)^n$$
$$+ \sum_{i=1}^{k-1} \binom{2n}{n+i} \left(\left(\frac{1}{2} + x^*\right)^{n+i} \left(\frac{1}{2} - x^*\right)^{n-i} + \left(\frac{1}{2} + x^*\right)^{n-i} \left(\frac{1}{2} - x^*\right)^{n+i}\right).$$

Then

$$f_k^n(0) = (1 - 2k)\binom{2n}{n} \left(\frac{1}{2}\right)^{2n} + \sum_{i=1}^{k-1} \binom{2n}{n+i} \left(2\left(\frac{1}{2}\right)^{2n}\right) \tag{A.21}$$
$$\leq (1 - 2k)\binom{2n}{n} \left(\frac{1}{2}\right)^{2n} + 2(k-1)\binom{2n}{n+1} \left(\frac{1}{2}\right)^{2n} = \left(\frac{1}{2}\right)^{2n} \left(-(2k-1)\binom{2n}{n} + 2(k-1)\binom{2n}{n+1}\right) < 0.$$

Next, note that

$$\left.\frac{\partial G^{k-1}(x_1,x^*,\ldots,x^*)}{\partial x_1}\right|_{x_1=\Delta^{2k}_{x^*}} = -aC'(\Delta^{2k}_{x^*}) + \binom{2n}{n}\left(\frac{1}{2}+x^*\right)^n\left(\frac{1}{2}-x^*\right)^n \tag{A.22}$$
$$+\sum_{i=1}^{k-1}\binom{2n}{n+i}\left[\left(\frac{1}{2}+x^*\right)^{n+i}\left(\frac{1}{2}-x^*\right)^{n-i}+\left(\frac{1}{2}+x^*\right)^{n-i}\left(\frac{1}{2}-x^*\right)^{n+i}\right],$$

where, from Equation (4)

$$a=\frac{\binom{2n}{n}\left(\frac{1}{2}+x^*\right)^n\left(\frac{1}{2}-x^*\right)^n}{C'(x^*)}. \tag{A.23}$$

If we substitute (A.23) into (A.22), we obtain

$$\left.\frac{\partial G^{k-1}(x_1,x^*,\ldots,x^*)}{\partial x_1}\right|_{x_1=\Delta^{2k}_{x^*}} = \left(1-\frac{C'(\Delta^{2k}_{x^*})}{C'(x^*)}\right)\binom{2n}{n}\left(\frac{1}{2}+x^*\right)^n\left(\frac{1}{2}-x^*\right)^n$$
$$+\sum_{i=1}^{k-1}\binom{2n}{n+i}\left[\left(\frac{1}{2}+x^*\right)^{n+i}\left(\frac{1}{2}-x^*\right)^{n-i}+\left(\frac{1}{2}+x^*\right)^{n-i}\left(\frac{1}{2}-x^*\right)^{n+i}\right].$$

From Lemma A.2, $\frac{C'(\Delta^{2k}_{x^*})}{C'(x^*)} \geq \frac{\Delta^{2k}_{x^*}}{x^*}$. Therefore,

$$\left.\frac{\partial G^{k-1}(x_1,x^*,\ldots,x^*)}{\partial x_1}\right|_{x_1=\Delta^{2k}_{x^*}} \leq g^n_k(x^*) := \left(1-\frac{\Delta^{2k}_{x^*}}{x^*}\right)\binom{2n}{n}\left(\frac{1}{2}+x^*\right)^n\left(\frac{1}{2}-x^*\right)^n$$
$$+\sum_{i=1}^{k-1}\binom{2n}{n+i}\left[\left(\frac{1}{2}+x^*\right)^{n+i}\left(\frac{1}{2}-x^*\right)^{n-i}+\left(\frac{1}{2}+x^*\right)^{n-i}\left(\frac{1}{2}-x^*\right)^{n+i}\right].$$

Since $g^n_k(x^*)$ is continuous in x^*, Lemma A.3 and Equation (A.21) guarantee that $\lim_{x^*\to 0} g^n_k(x^*) = f^n_k(0) < 0$. Hence, there exists $\overline{x}_1(k,n) > 0$ such that,

$$x^* \leq \overline{x}_1(k,n) \Rightarrow g^n_k(x^*) < 0. \tag{A.24}$$

Finally, from Remark 1 we obtain that *(a)* x^* is decreasing in a, and *(b)* $\lim_{a\to\infty} x^* = 0$. Using *(a)* and *(b)* together with Equation (A.24) imply that there is $a^*_1(n,k)$ such that $a \geq a^*_1(n,k) \Rightarrow x^* \leq \overline{x}_1(k,n)$. This completes the proof of Step 2.

Step 3. We show that for $k \in \{1,\ldots,n\}$, there is $a^*_2(n,k)$ so that if $a \geq a^*_1(n,k)$,

$$-\left.\frac{\partial G^{k-1}(x_1,x^*,\ldots,x^*)}{\partial x_1}\right|_{x_1=\Delta^{2k}_{x^*}} > \left.\frac{\partial G^{k}(x_1,x^*,\ldots,x^*)}{\partial x_1}\right|_{x_1=\Delta^{2k}_{x^*}}.$$

Define the following function:

$$h_k^n(x) := (4k-2)\binom{2n}{n}\left(\frac{1}{2}+x\right)^n\left(\frac{1}{2}-x\right)^n - \binom{2n}{n+k}\left(\left(\frac{1}{2}+x\right)^{n+k}\left(\frac{1}{2}-x\right)^{n-k} + \left(\frac{1}{2}+x\right)^{n-k}\left(\frac{1}{2}-x\right)^{n+k}\right)$$
$$-2\sum_{i=1}^{k-1}\binom{2n}{n+i}\left(\left(\frac{1}{2}+x\right)^{n+i}\left(\frac{1}{2}-x\right)^{n-i} + \left(\frac{1}{2}+x\right)^{n-i}\left(\frac{1}{2}-x\right)^{n+i}\right).$$

From standard algebraic manipulations we obtain

$$h_k^n(0) = (4k-2)\binom{2n}{n}\left(\frac{1}{2}\right)^{2n} - 2\binom{2n}{n+k}\left(\frac{1}{2}\right)^{2n} - 4\sum_{i=1}^{k-1}\binom{2n}{n+i}\left(\frac{1}{2}\right)^{2n}$$
$$= \left(\frac{1}{2}\right)^{2n}\left((4k-2)\binom{2n}{n} - 2\binom{2n}{n+k} - 4\sum_{i=1}^{k-1}\binom{2n}{n+i}\right)$$
$$\geq \left(\frac{1}{2}\right)^{2n}\left((4k-2)\binom{2n}{n} - 2\binom{2n}{n+k} - 4(k-1)\binom{2n}{n+1}\right) \geq \left(\frac{1}{2}\right)^{2n}\left((4k-4)\binom{2n}{n} - (4k-4)\binom{2n}{n+1}\right) > 0.$$

Next, note that

$$-\left.\frac{\partial G^{k-1}(x_1,x^*,\ldots,x^*)}{\partial x_1}\right|_{x_1=\Delta_{x^*}^{2k}} + \left.\frac{\partial G^{k}(x_1,x^*,\ldots,x^*)}{\partial x_1}\right|_{x_1=\Delta_{x^*}^{2k}} = \left(2\frac{C'(\Delta_{x^*}^{2k})}{C'(x^*)} - 2\right)\binom{2n}{n}\left(\frac{1}{2}+x^*\right)^n\left(\frac{1}{2}-x^*\right)^n$$
$$-\binom{2n}{n+k}\left(\left(\frac{1}{2}+x^*\right)^{n+k}\left(\frac{1}{2}-x^*\right)^{n-k} + \left(\frac{1}{2}+x^*\right)^{n-k}\left(\frac{1}{2}-x^*\right)^{n+k}\right)$$
$$-2\sum_{i=1}^{k-1}\binom{2n}{n+i}\left(\left(\frac{1}{2}+x^*\right)^{n+i}\left(\frac{1}{2}-x^*\right)^{n-i} + \left(\frac{1}{2}+x^*\right)^{n-i}\left(\frac{1}{2}-x^*\right)^{n+i}\right)$$
$$\geq -2\sum_{i=1}^{k-1}\binom{2n}{n+i}\left(\left(\frac{1}{2}+x^*\right)^{n+i}\left(\frac{1}{2}-x^*\right)^{n-i} + \left(\frac{1}{2}+x^*\right)^{n-i}\left(\frac{1}{2}-x^*\right)^{n+i}\right)$$
$$-\binom{2n}{n+k}\left(\left(\frac{1}{2}+x^*\right)^{n+k}\left(\frac{1}{2}-x^*\right)^{n-k} + \left(\frac{1}{2}+x^*\right)^{n-k}\left(\frac{1}{2}-x^*\right)^{n+k}\right)$$
$$+\left(2\frac{\Delta_{2k}^{x^*}}{x^*} - 2\right)\binom{2n}{n}\left(\frac{1}{2}+x^*\right)^n\left(\frac{1}{2}-x^*\right)^n := l_k^n(x^*),$$

where to derive the equality we used Equation (A.23) to substitute for a and the inequality follows from Lemma A.2. Since $l_k^n(x^*)$ is continuous in x^*, Lemma A.3 guarantees that $\lim_{x^*\to 0} l_k^n(x^*) = h_k^n(0)$. Hence, there exists $\overline{x}_2(k,n) > 0$ such that,

$$x^* \leq \overline{x}_2(k,n) \implies l_k^n(x^*) > 0. \tag{A.25}$$

Finally, from Equation (4), we know the following two properties: *(i)* x^* is decreasing in a, and *(ii)* $\lim_{a\to\infty} x^* = 0$. Using *(i)*–*(ii)* together with Equation (A.25) implies that there exists $a_2^*(n,k)$ such that

$$a \geq a_2^*(n,k) \Rightarrow x^* \leq \overline{x}_2(k,n).$$

This completes the proof of Step 3.

Step 4. We prove that given n, there exists $a_3^*(n)$ such that if $a > a_3^*(n)$, then

$$\left.\frac{\partial G^n(x_1, x^*, \ldots, x^*)}{\partial x_1}\right|_{x_1=1/2} < 0.$$

We know that $G^n(1/2, x^*, \ldots, x^*) = \left(\frac{1}{2} + x_1\right) - aC(x_1)$, so

$$\left.\frac{\partial G^n(x_1, x^*, \ldots, x^*)}{\partial x_1}\right|_{x_1=1/2} = 1 - aC'(1/2),$$

and choosing $a_3^*(n) = 1/C'(1/2)$ suffices.

Step 5. We now use Lemma A.1 to prove the statement of the theorem. Given n, consider $\bar{a}(n) \coloneqq \max\{a_1(n,k), a_2(n,k), a_3(n)\}_{k\in\{1,\ldots,n\}}$ and $a > \bar{a}(n)$. Then, the following properties hold.

1. For all $k \in \{1, \ldots, n\}$, $\frac{\partial G^{k-1}(\Delta^{x^*}_{2k}, x^*, \ldots, x^*)}{\partial x_1} < 0$.
2. For all $k \in \{1, \ldots, n\}$, $-\frac{\partial G^{k-1}(\Delta^{x^*}_{2k}, x^*, \ldots, x^*)}{\partial x_1} > \frac{\partial G^{k}(\Delta^{x^*}_{2k}, x^*, \ldots, x^*)}{\partial x_1}$.
3. $\frac{\partial G^n(1/2, x^*, \ldots, x^*)}{\partial x_1} < 0$.

From Equation (A.19), it is clear that we can write

$$G^k(x_1, x^*, \ldots, x^*) = c_k(x^*) + b_k(x^*)x_1 - C(x_1),$$

with $c_k(x^*) < c_{k-1}(x^*)$ and $b_k(x^*) > b_{k-1}(x^*)$ for $k \in \{1, \ldots, n\}$. This, along with the three properties enumerated above, allows us to apply Lemma A.1 to functions $G^k(x_1, x^*, \ldots, x^*)$ and $G^{k+1}(x_1, x^*, \ldots, x^*)$. For all $k \in \{0, \ldots, n-1\}$, we therefore obtain

$$\max_{[0,\Delta^{x^*}_{2k}]} G^{k-1}(x_1, x^*, \ldots, x^*) \geq \max_{[0,\Delta^{x^*}_{2k+2}]} G^{k}(x_1, x^*, \ldots, x^*),$$

which implies that for all $k \in \{1, \ldots, n\}$,

$$\max_{[0,\Delta^{x^*}_{2}]} G^{0}(x_1, x^*, \ldots, x^*) \geq \max_{[0,\Delta^{x^*}_{2k+2}]} G^{k}(x_1, x^*, \ldots, x^*).$$

Accordingly, it only remains to be shown that $x^* = \max_{[0,\Delta^{x^*}_{2}]} G^0(x_1, x^*, \ldots, x^*)$, but we know this from the proof of Proposition 1. This completes the proof of the theorem. $\square$

Proof of Theorem 3. We want to show that $(x_i^*, 0, \ldots, 0)$ is an equilibrium for some value of x_i^* and any citizen $i \in N$. That is, any $j \in N\backslash\{i\}$ is a citizen for which $x_j^* = 0$. We start by analyzing citizen i's best response when no other citizen acquires information. If citizen i chooses $x_i > 0$, then whichever signal she receives is the alternative that will be chosen. The reason for this is two-fold: on the one hand, all signals and information acquisition levels are public; on the other, all citizens vote sincerely. Accordingly, if $x_i > 0$, voter i's *ex-ante* expected payoff given x^*_{-i} is $G(x_i, x^*_{-i}) = \left(\frac{1}{2} + x_i\right) - C(x_i)$. It is easy to verify that $G(0, x^*_{-i}) = 1/2$, since in such a case no citizen acquires any information. Since $C'(0) = 0$, we obtain $G'(0, x^*_{-i}) = 1 > 0$, so $x_i = 0$ cannot be a best reply to x^*_{-i}. Hence, citizen i's best response to x^*_{-i} is either interior if $C'(1/2) > 1$, or is $x^*_i = 1/2$ if $C'(1/2) \leq 1$. In the former case, the interior solution x^*_i corresponds to the information acquisition level that solves the following equation:

$$1 = C'(x^*_i). \tag{A.26}$$

Next, we take as given the optimal choice of $x^*_i > 0$, and verify that no citizen $j \in N \setminus \{i\}$ wishes to deviate from $x_j = 0$ to $x_j > 0$, taking also as given that $x_k = 0$ for all $k \in N \setminus \{i, j\}$. Acquiring $x_j \in (0, x^*_i)$ cannot be optimal as $x_j = 0$ is a profitable deviation. This follows from the fact that *(a)* $C(x)$ is strictly increasing for $x \in (0, 1/2)$, and that *(b)* if citizen j acquires an information level lower than x^*_i, the alternative chosen by all citizens (including citizen j) will continue to be the one that matches citizen i's signal. We split the remainder of the proof in two cases, depending on the value of the derivative of the cost function at 1/2.

Case I: $C'(1/2) \leq 1$. In this case, citizen i acquires full information, i.e., $x^*_i = 1/2$. Since the correct alternative is therefore chosen with probability one and C is strictly increasing, it is clear that citizen j strictly prefers to not acquire any information. Therefore, $x^*_i = 1/2$ and $x^*_j = 0$ for all $j \in N \setminus \{i\}$ is a Nash equilibrium.

Case II: $C'(1/2) > 1$. In this case, citizen i acquires an interior level of information, i.e., $0 < x^*_i < \frac{1}{2}$, which solves Equation (A.26). If citizen j deviates from choosing $x_j = 0$ to $x_j \in (x^*_i, 1/2]$, then whichever alternative matching citizen j's signal will be chosen by all citizens. The reason for this is the same as in Case I. Accordingly, citizen j's *ex-ante* expected payoff for $x_j \in (x^*_i, 1/2]$ is $G(x_j, x^*_{-j}) = \left(\frac{1}{2} + x_j\right) - C(x_j)$. It then suffices to note that for $x_j \in (x^*_i, 1/2]$,

$$\frac{\partial G(x_j, x^*_{-j})}{\partial x_j} = 1 - C'(x_j) < 1 - C'(x^*_i) = 0,$$

where the inequality follows from the fact that C is strictly convex and the last equality is equivalent to Equation (A.26). Hence, for citizen j, no deviation from choosing $x_j = 0$ to $x_j \in (x^*_i, 1/2]$ is profitable.

It remains to verify the case where citizen j deviates from $x_j = 0$ to $x_j = x^*_i$. There are two cases. First, if $s_i = s_j$, then all citizens vote according to citizen i's and citizen j's signals. Therefore, the alternative corresponding to such signals is chosen with probability one. Second, if $s_i \neq s_j$, citizens vote according to their own signal due to Assumption 4. In particular, citizen i votes for alternative s_i and citizen j votes for alternative s_j. This means that both votes cancel each other out, so the election outcome depends on the remaining $2n-1$ citizens. Hence, the probability of alternative A (B) winning under state z^A (z^B) is the probability of at least n over $2n-1$ voters obtaining signals s^A (s^B). This leads to

$$\begin{aligned}
G(x_j; x^*_{-j}) &= \left(\frac{1}{2}+x^*_i\right)^2 + 2\left(\frac{1}{2}+x^*_i\right)\left(\frac{1}{2}-x^*_i\right)\sum_{k=n}^{2n-1}\binom{2n-1}{k}\left(\frac{1}{2}\right)^{2n-1} - C(x^*_i) \\
&= \left(\frac{1}{2}+x^*_i\right)^2 + 2\left(\frac{1}{2}+x^*_i\right)\left(\frac{1}{2}-x^*_i\right)\left(\frac{1}{2}\right)^{2n-1}\sum_{k=n}^{2n-1}\binom{2n-1}{k} - C(x^*_i) \\
&= \left(\frac{1}{2}+x^*_i\right)^2 + 2\left(\frac{1}{2}+x^*_i\right)\left(\frac{1}{2}-x^*_j\right)\left(\frac{1}{2}\right)^{2n-1} 2^{2(n-1)} - C(x^*_i) \\
&= \left(\frac{1}{2}+x^*_i\right)^2 + \left(\frac{1}{2}+x^*_i\right)\left(\frac{1}{2}-x^*_i\right) - C(x^*_i) = \left(\frac{1}{2}+x^*_i\right) - C(x^*_i).
\end{aligned}$$

The above expression increases if j chooses any $x'_j < x^*_i$ instead of $x_j = x^*_i$: the probability of choosing the right alternative will still be $\left(\frac{1}{2}+x^*_i\right)$, but the cost incurred will be smaller since C is strictly increasing for $x \in (0, 1/2)$. Thus, it is a best response for $j \in N \setminus \{i\}$ to choose $x^*_j = 0$. □

Proof of Proposition 2. Suppose that citizen i chooses $x_i \in (0, 1/2]$ and the remainder citizens acquire no information at all, i.e., citizens $j \in N \setminus \{i\}$ choose $x_j = 0$. We inquire now if for some citizen $j \in N \setminus \{i\}$, $x_j = 0$ can be the best response to $(x_k)_{k \in N\setminus\{j\}}$. Citizen k's vote only matters when there is a tie among the other voters, so for $x_j = 0$ to be a best response it is necessary that

$$\begin{aligned}
C'(0) &\geq \left(\frac{1}{2}+x_i\right)\binom{2n-1}{n}\left(\frac{1}{2}\right)^{2n} + \left(\frac{1}{2}-x_i\right)\binom{2n-1}{n}\left(\frac{1}{2}\right)^{2n} \\
&= \binom{2n-1}{n}\left(\frac{1}{2}\right)^{2n} > 0.
\end{aligned}$$

To derive the right-hand side of the first inequality we have used Assumption 4, but other tie-breaking rules would lead to the same result. However, the assumption that $C'(0) = 0$ leads to a contradiction with the above inequality. □

Proof of Proposition 3. If $C'(1/2) \leq 1$, then in the equilibria with a political expert, the expert acquires perfectly accurate information, leading to the highest electoral accuracy. This result does not depend on the size of the electorate. If $C''(0) > 0$ in addition, it follows from Remark 3 that the electoral accuracy is bounded away from one, even as n goes to infinity. □

Proof of Proposition 4. On the one hand, if $\tilde{C}'(1/2) > 1$ then in any equilibrium with a political expert, the expert does not acquire perfect information, no matter the size of the electorate. Hence, the electoral accuracy from an equilibrium with a political expert is bounded away from one; say, it equals $1-\varepsilon$, where $\varepsilon \in (0,1)$ On the other hand, if $\tilde{C}''(0) = 0$, it follows from Remark 3 that the electoral accuracy converges to one as n goes to infinity.

Combining these results with Theorem 2, we can choose $n^* := n^*(\varepsilon) \in \mathbb{N}$ to ensure that, for all $n \geq n^*$, there exists $\bar{a}(n)$ such that, for all $a \geq \bar{a}(n)$, (i) the symmetric equilibrium also exists and (ii) the electoral accuracy under the symmetric equilibrium is higher than the electoral accuracy under any equilibrium with a political expert:

$$\forall\, d \in \{A, B\},\ \ \mathbb{P}_\alpha(d|x^*(n), z^d) > 1-\varepsilon = \mathbb{P}_\alpha(d|x^{**}, z^d),$$

where x^{**} is the chosen level of accuracy by the political expert in equilibrium. $\square$

Appendix B

Formal derivations for results in Example 1. **Case 1:** $a > 1$. We prove that there is a symmetric equilibrium. To do so, it suffices to compare citizen 1's expected payoff if $x_1 = x^*$ to her payoff if $x_1 = 1/(2a)$. Given Expressions (7) and (9), we have that

$$G^0(x^*, x^*, x^*) = 0.5a^3 - 0.5a^2\sqrt{a^2+1} + 0.5\sqrt{a^2+1} - 0.25a + 0.5, \tag{A.27}$$

and

$$G^1\left(\frac{1}{2a}, x^*, x^*\right) = 0.5 + \frac{1}{4a}. \tag{A.28}$$

We claim, that if $a > 1$, then $G^0(x^*, x^*, x^*) \geq G^1(1/(2a), x^*, x^*)$. Using Equations (A.27) and (A.28), the above-claimed inequality can be arranged (if we multiply it by $1/4a$) as $f(a) := 2a^4 - 2a^3\sqrt{a^2+1} + 2a\sqrt{a^2+1} - a^2 > 1$. It is immediate to verify that $f(1) = 1$. Hence, the claim holds if we show that if $a > 1$, then $f'(a) > 0$. To show this, we note that $f'(a) = 0$ is equivalent to $8a^3\sqrt{a^2+1} - 2a\sqrt{a^2+1} = 8a^4 + 2a^2 - 2$. Next, we apply some non-injective transformations to the above equation and we obtain

$$\left(8a^3\sqrt{a^2+1} - 2a\sqrt{a^2+1}\right)^2 = \left(8a^4 + 2a^2 - 2\right)^2 \ \Leftrightarrow\ a = \pm\frac{\sqrt{3}}{3}.$$

It is straightforward to verify that $a = -\frac{\sqrt{3}}{3}$ is the only solution to $f'(a) = 0$. Finally, $f'(0) > 0$ implies $f'(a) > 0$.

Case 2: $a \leq 1$. We prove that there is not a symmetric equilibrium. To do so, we compare citizen 1's expected payoff if $x_1 = x^*$ to her payoff if $x_1 = 1/2$. As before, $G^0(x^*, x^*, x^*)$ is given by Equation (A.27). $G^0(1/2, , x^*, x^*)$ is

$$G(1/2, x^*, x^*) = 1 - \frac{a}{4}. \tag{A.29}$$

We claim that if $a \in (0,1]$, then $G(1/2, x^*, x^*) > G(x^*, x^*, x^*)$. Using Equations (A.27) and (A.29), the claimed inequality can be rearranged as

$$g(a) := 0.5a^3 - 0.5a^2\sqrt{a^2+1} + 0.5\sqrt{a^2+1} + 0.5 < 1.$$

It is immediate to check that $g(0) = g(1) = 1$. Hence, it suffices to prove that $g'(a) = 0$ has only one solution in $[0,1]$ and that it corresponds to a minimum of $g(a)$. Note that

$$g'(a) = \frac{a\left(-1.5a^2 + 1.5a\sqrt{a^2+1} - 0.5\right)}{\sqrt{a^2+1}}.$$

Therefore, $g'(a) = 0$ if and only if $-1.5a^2 + 1.5a\sqrt{a^2+1} - 0.5 = 0$. this equation can be solved using arguments analogous as those we use in the previous case. If we do so, we obtain that $a = \sqrt{3}/3$ is the only solution to $g'(a) = 0$ in the interval $[0,1]$. It is then straightforward to check that $g''(\sqrt{3}/3) > 0$. Accordingly, we have proved $g(a) < 1$, which means that $G(1/2, x^*, x^*) > G(x^*, x^*, x^*)$. □

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