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The value of perfect information for the problem: a sensitivity analysis

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Abstract

This paper examines problems in decision theory where the number of alternatives and states of nature are finite. Previous studies have defined the concept of "the value of perfect information for the problem" (*VPIP*). This metric allows us to obtain an upper bound on the amount a decision-maker would be willing to pay for perfect information under the specific conditions of a problem. This bound is particularly important when the decision is unrepeatable, providing a more accurately adjusted measure than the one traditionally obtained with "the expected value of perfect information" (*EVPI*). Supported by linear programming, this work proposes a sensitivity analysis of these bounds by seeking to identify the intervals in which the problem values can vary without essentially modifying the structure of the problem. Specifically, the study aims to determine how this variation might affect the *EVPI* and *VPIP* bounds, as well as the difference in the price a decision-maker would be willing to pay for perfect information if any of the problem values were altered. By identifying alternatives and scenarios taking into account the role they play in the problem, this research classifies the data involved in a finite decision problem to ensure the conclusions can be understood as generally as possible. Although the proposed sensitivity analysis is applied to the oil-drilling problem, a classic in decision theory, the conclusions of this work have potential applications in improving environmental decision-making processes.

Keywords Expected value of perfect information · Value of perfect information for the problem · Decision theory · Sensitivity analysis · Finite decision problems · Linear programming

1 Introduction

Decisions in the field of environmental policy are sometimes difficult to repeat. For instance, allocating resources for biodiversity management in a specific protected area (Bolam et al. 2019) or deciding whether to invest in satellites to collect data on air quality or freshwater supply (Laxminarayan and Macauley 2012) are critical choices. Once made, these decisions often have long-term impacts and may not be easily revisited, necessitating thorough consideration to ensure the best possible outcomes. Value of information techniques have proven valuable and are used in environmental decision-making, which often involves uncertainty about both current and future states.

"The concept of the expected value of perfect information" (*EVPI*) oriented to finite decision problems was introduced in the 1960s. A formula was also proposed to calculate this value when the decision criterion used was the maximization of expected utility.

In 2018, in a paper examining the expected value of perfect information in unrepeatable decision-making (Boncompte 2018), the author demonstrated the need to define a concept to identify the amount a decision-maker would be willing to pay to have perfect information in finite decision problems. This would provide a more accurate amount than the one previously provided by *EVPI*. In so doing, and taking into account the specific conditions of each problem, the paper introduced "the value of perfect information for the problem" (*VPIP*) and described a method for calculating it.

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However, the study lacked a sensitivity analysis indicating whether a variation in the problem data implied a change in the *EVPI* and *VPIP* bounds. So far, no sensitivity analysis of theoretical *EVPI* has been proposed in relation to finite decision problems. Therefore, the purpose of this paper is to complete the *VPIP* study by performing a sensitivity analysis of both *EVPI* and *VPIP* values. This sensitivity analysis is conducted with the help of linear programming, which, despite being a widely used technique, is novel in this context and allows for a better understanding of the analysis.

Section 2 presents a synthesis of the works that gave rise to the concept of *EVPI* while indicating the articles that more recently have dealt with sensitivity analysis in decision problems and have related it to *EVPI*. Section 3 restates the prior definitions as used in this paper. Section 4 explains the origin of the present reflection based on a problem for which *EVPI* gives a surprising value that is unacceptable for a sensible decision-maker. Section 5 presents "the value of perfect information for the problem" (*VPIP*) and a method for calculating it. Section 6 constitutes the core of the present paper and reports the development of a sensitivity analysis of the *EVPI* and *VPIP* bounds. Given that modifying the values beyond the intervals presented would originate a different problem requiring a new approach, we present various premises to ensure that the problem keeps its essence. Furthermore, we classify the data according to the role they play in the decision problem, and present tables to summarize the results. We also illustrate the sensitivity analysis by its application to the oil-drilling problem and, finally, discuss the outcomes. Section 7 concludes.

2 Related literature

Studies on the value of information, and specifically on the value of *EVPI*, date back to 1926, when Ramsey (1926) wrote his note "Weight or the value of knowledge." Almost thirty years later, Savage (1954) developed his Bayesian theory in "The Foundations of Statistics," and it was in the 1960s that the concept of *EVPI* appeared. In fact, Chernoff and Moses (1959) spoke of a "crystal ball," although they did not develop the concept of *EVPI*, remaining with "ideal action probabilities" and defining the risk of a strategy as its expected opportunity cost.

Next, Raiffa and Schlaiffer (1961) defined *EVPI* as the price of perfect information and calculated it for some examples in the continuous field. Howard (1966), in his paper "Information Value Theory," coined the term "clairvoyance" and approached the *EVPI* concept eliminating uncertain parameters. A few years later, Howard (1971) related the "clairvoyance" to the risk aversion of the decision-maker.

Furthermore, Szaniawski (1967), in "The Value of Perfect Information," also defined *EVPI* for finite problems and considered it for the different uncertainty criteria (Maximin, Minimax, Laplace, etc.).

An overview of the literature on sensitivity analysis shows that Felli and Hazen (1998) published an analysis concerning the formulation of the expected value of perfect information. This paper, which examines the decision-making process in medical field, studies how the values assigned by the decision-maker to the uncertain input parameters affect the final decision. On the one hand, the authors suggest abandoning the measures previously used; on the other, they recommend using *EVPI* to perform the sensitivity analysis. Felli and Hazen distinguish between value sensitivity and decision sensitivity; the former refers to a modification in the magnitude of a model's optimal value, given a variation in the input parameters, and the latter refers to the change in the preferred alternative identified by the model.

A year later, Hazen and Sounderpandian (1999) observed the behavior of other information measures in the presence of an uncertainty X ("the buying price" BPI_X, "the certainty equivalent" CE_X, "the utility increase" EUI_X, "the certainty equivalent increase" CEI_X, "the selling price" SPI_X, and "the probability price" PPI_X), their possible ordinal equivalence, and their relationships.

More recently, Keisler (2004) identified the conditions for which *EVPI* increases when mean and standard deviation are both linear functions of an exogenous variable. These results provide a generic map of "the value of information space" for a significant class of decisions. Moreover, Keisler et al. (2014) provided an interesting paper that reviews the prevalence of value of information (*VoI*) applications reported in peer-reviewed literature spanning two decades (1990–2011). Keisler (2014) offered some later references in the same special issue of the journal, which focused on "the value of information" (*VoI*), where several other articles applied *VoI* in environmental or ecological contexts; see, for example, Gradowska and Cooke (2014) and Hoang Le et al. (2014).

Many published articles use *EVPI* or some of its variants such as *EVPPI* ("the expected value of partial perfect information") or *EVSI* ("the expected value of sample information") to make decisions in different disciplines, especially in the medical field; see, for example, Yokota and Thompson (2004), and Ekwunife and Lhachimi (2017). Zafari et al. (2016) use *EVPI* to undertake a sensitivity analysis of the input parameters to decide which medical treatment to implement.

Among the peer-reviewed articles that apply value of information study techniques, specifically the concept of *EVPI*, to decision-making in environmental, biodiversity, or ecological contexts, we can cite Bolam et al. (2019) and Canessa et al. (2015), who also work with *EVSI*.

Recently, Haag et al. (2022) employed utility theory and *EVPPI* to measure the expected gain in utility if the optimal alternative was decided based on perfect information about a few uncertain parameters. Since data collection is time-consuming and expensive, they used information value analysis as a form of sensitivity analysis of the choice of new information. In a similar sense, Hazen et al. (2023) introduced a graphical information-based tool for sensitivity analysis, the information density, that conveys both direction and information value, and thereby simultaneously accounts for the direction and importance of input uncertainty.

In the present paper, we study how the *EVPI* and *VPIP* bounds vary in finite decision problems. The focus on such problems and the use of known *a priori* probabilities are what set this article apart from most of those quoted above. Our objective is to assess the difference in the price a decision-maker is willing to pay for perfect information if some value of the problem is altered. Thus, the purpose of this paper is not to analyze the sensitivity of the optimal value nor the optimal decision, but rather to perform a sensitivity analysis of the *EVPI* and *VPIP* bounds themselves, as well as to show that when dealing with monetary values, it is essential to work with the absolute values of the problem.

3 Notation and definitions

First, let us introduce the definitions and notations that we will use throughout the article.

A decision problem is defined (Szaniawski 1967) as the ordered triple $< A, S, u >$, where A is the set of available alternatives (or actions) to the decision-maker, S is the set of states of nature (or scenarios) that determine the consequences of any action, and u is a real function defined on the cartesian product AxS representing the valuation of the consequences. We shall restrict our attention to finite decision problems: A and S have a finite number of elements. Moreover, the information on S will be categorical, i.e., the actual state of nature will simply be a variable X defined on S. It is assumed that exactly one of the statements $X = S_i$ (read: the actual state of nature is S_i) is true, but the decision-maker does not know which one. We do not consider the case in which the information is probabilistic. In that case, the actual state of nature would vary randomly, and X would be a random variable.

According to these premises, the columns represent the different alternatives or actions A_j $(1 \leq j \leq n)$, while the rows represent the scenarios or states of nature S_i $(1 \leq i \leq m)$. The outcome of each alternative will depend on the scenario that occurs. The payoff with A_j in S_i will be denoted as x_{ij}, and the probabilities p_i $(1 \leq i \leq m)$ of occurrence of the different scenarios are known.

It is understood that the results of the alternatives represent the decision-maker's profit and, therefore, the alternative with the highest expected value is the preferred alternative or the optimal action ($A_{\tilde{j}}$). If the preferred alternative offered the best outcome in all scenarios, there would be no decision problem (the solution would be trivial). It is when the optimal action may lead to an undesirable outcome that the decision problem makes sense. This scenario is known as "the worst scenario" since it is the one feared by the decision-maker.

Finally, we represent the best result achieved in each scenario S_i as $\tilde{x}_i$, and the alternative in which this best result is obtained as A_{S_i}.

Table 1 lists the notations that will be used in the present discussion.

4 The need to propose a tighter bound than *EVPI*

Boncompte (2018) examined the expected value of perfect information in unrepeatable decision-making and highlighted the requirement to define a concept to identify the amount a decision-maker would be willing to pay to have perfect information in finite decision problems.

The problem that gave rise to this discussion was a classic one in decision theory: the oil-drilling problem. This problem appeared in Raiffa and Schlaiffer (1961), but we will use the updated version in Hillier and Lieberman (2015), pp. 683–695:

The GOFERBROKE COMPANY owns a tract of land that may contain oil. A consulting geologist has reported to management that he believes there is one-in-four chance of striking oil. Because of this prospect, another oil company has offered to purchase the land for $90, 000. However, Goferbroke is considering holding the land in order to drill for oil itself. The cost of drilling is $100, 000. If oil is found, the resulting expected revenue will be $800, 000, so the company's expected profit (after deducting the cost of drilling) will be $700, 000. A loss of $100, 000 (the drilling cost) will be incurred if the land is dry (no oil).

Table 2 provides a summary of the problem.

To solve the situation, the manager calculates the expected values of the two alternatives: drilling and selling.

- The expected drilling value $= 700,000 * 0.25 - 100,000 * 0.75 = 100,000$
- The expected selling value $= 90,000$

Consequently, the manager will drill because drilling is the alternative with a higher expected value; then, the best expected value provided by alternatives (*BEVA*) is 100,000. So,

- The preferred alternative: Drilling.
- The worst scenario: Dry.

Table 1 Notations

A_j	The alternatives or actions ($1 \leq j \leq n$)
S_i	The scenarios or states of nature ($1 \leq i \leq m$)
p_i	The probability of S_i
$E(A_j)$	The expected value of A_j
x_{ij}	The payoff with A_j in S_i ($1 \leq i \leq m, 1 \leq j \leq n$)
$\tilde{x}_i$	The best value in S_i ($1 \leq i \leq m$)
$A_{\tilde{j}}$	The preferred alternative or the optimal action (The alternative with the best expected value)
Co_{ij}	The opportunity cost of A_j in S_i ($1 \leq i \leq m, 1 \leq j \leq n$)
$S_{\underline{i}}$	The worst scenario (where $A_{\tilde{j}}$ obtains its worst result)
A_{S_i}	The alternative giving the best result in scenario S_i
A_S	The alternative giving the best result in the scenario that has been modified
$x_{\underline{i}\tilde{j}}$	The worst payoff of the preferred alternative $A_{\tilde{j}}$
L	The loss to be avoided
P_c	The payment pertaining to the decision-maker in the current moment in $S_{\underline{i}}$
FPW	The most favorable payoff in the worst scenario
$EVPI$	The expected value of perfect information
$VPIP$	The value of perfect information for the problem
EPI	The expected value when we have perfect information
$BEVA$	The best expected value provided by alternatives (The expected value of $A_{\tilde{j}}$)

Table 2 The oil-drilling problem

	Drilling($)	Selling($)
Oil (0.25)	700,000	90,000
Dry (0.75)	−100,000	90,000
$E(A_j)$	100,000	90,000

However, the manager believes that the geological study is not sufficient and requests a seismological one. The question is, if this study were completely reliable, how much would the manager be willing to pay for it?

The manager will be willing to pay *EVPI*, defined (Raiffa and Schlaiffer 1961) as the price of perfect information. It is calculated as usual (Szaniawski 1967): as the difference between the expected value when we have perfect information (*EPI*) and the best expected value without perfect information (*BEVA*). Note that the best expected value without perfect information is just the expected value of the best alternative.

$$EVPI = EPI - BEVA = \sum_{i=1}^{m} \tilde{x}_i p_i - E(A_{\tilde{j}}) = \sum_{i=1}^{m} \tilde{x}_i p_i - \sum_{i=1}^{m} x_{i\tilde{j}} p_i \quad (1)$$

In our problem, if we knew we would strike oil, then we would drill; an outcome that will occur in 25% of cases. In other contingencies, we would sell (75%). Then, if we had perfect information, our expected value *EPI* would be

$$EPI = 700,000 * 0.25 + 90,000 * 0.75 = 242,500$$

If we had no perfect information, our expected value would be the expected value of drilling (our preferred alternative). Then, we obtain

$$EVPI = (700,000 * 0.25 + 90,000 * 0.75) - 100,000 = 142,500$$

Consequently, considering *EVPI*, the answer is as follows: the manager would be willing to pay up to \$142,500 for perfect information. However, he would be unlikely to pay \$142,500 for a study that provides perfect information, as this value exceeds the cost of drilling, which supplies perfect information *par excellence*. If the decision-maker resorts to decision theory, it is due to the cost of drilling; otherwise, if drilling had no cost, the company would drill and no decision problem would arise.

It is true that this high value has its origin in the opportunity cost of the drilling alternative. As usual, we refer to the opportunity cost (Co_{ij}) of the alternative A_j in the scenario S_i as the difference between the best payoff in the scenario S_i and the payoff that takes place with alternative A_j in the scenario S_i:

$$Co_{ij} = \tilde{x}_i - x_{ij}$$

Thus, we obtain that *EVPI* can also be interpreted as "the expected value of the opportunity cost of the preferred alternative," that is, the expected value of the amount that we lose with the preferred alternative compared to the best option in each scenario (Szaniawski 1967):

$$EVPI = \sum_{i=1}^{m} \tilde{x}_i p_i - \sum_{i=1}^{m} x_{i\tilde{j}} p_i = \sum_{i=1}^{m} (\tilde{x}_i - x_{i\tilde{j}}) p_i \tag{2}$$

Table 3 presents the opportunity costs array associated with the problem.

It is verified that the opportunity cost of the drilling alternative is the same as the value of *EVPI* of the problem.

$$EVPI = 190,000 * 0.75 = 142,500$$

The value of *EVPI* is explained by the fact that, if the decision-maker drills and the land is dry, he loses not only the drilling expenses but also what could have been obtained from an hypothetical sale. However, even with this justification, does it make sense to pay \$142,500 for the report? Boncompte (2018) highlighted that the simple calculation of *EVPI* could sometimes lead to unacceptable results for a decision-maker when it involves unrepeatable decisions. In that paper, she questioned the role of probability theory when applied to unrepeatable decision-making problems, since probability theory is based precisely on the study of the results obtained by repeating the same experiment a significant number of times. For this reason, she emphasized that the decision-maker has to use probability theory to initially discard the alternatives that probabilistically give the worst results, but, from then on, the decision-maker enters a second phase in which he must take into consideration the absolute values of the problem to accept an alternative and make a consistent decision.

5 The value of perfect information for the problem *VPIP*

In Boncompte (2018), a new bound called "the value of perfect information for the problem" (*VPIP*) is proposed. *VPIP* is defined as an upper bound of the set of all possible amounts that a decision-maker would be willing to pay in order to obtain perfect information considering the specific conditions of a problem. To ensure that the cost of the study is covered in all cases, *VPIP* takes into account not only *EVPI* but also the loss to be avoided (in the case of the oil-drilling problem, it will be, nominally at least, the drilling cost) and the most favorable payoff in the worst scenario (in the example problem, the selling price). So, to calculate *VPIP*, three parameters should be considered: *EVPI*, "the loss to be avoided" (L), and "the most favorable payoff in the worst scenario" (*FPW*):

$$VPIP = \min\{EVPI,\ L,\ FPW\} \tag{3}$$

The first value to consider has to be *EVPI*, that has already been discussed in Section (4).

The second value in the equation is "the loss to be avoided" (L). Indeed, the decision-maker is willing to pay for perfect information because the alternative that is probabilistically the best option could give an undesirable result in a certain scenario. It is precisely this outcome that leads the decision-maker to pay to know in what scenario he is really in. Logically, in no circumstances will the decision-maker pay an amount higher than the loss he wants to avoid. Boncompte (2018) provided a formula to determine the value of L relying not only on the worst payment associated with the preferred alternative but also on the payment corresponding to the decision-maker in the *status quo* (Pc). Note that, although the current payout could vary depending on the scenarios, the current payment in the worst-case scenario is the only one relevant for calculating the loss to be avoided.

$$L = \max\left\{P_c - \min_i x_{i\tilde{j}},\ 0\right\} = \max\left\{P_c - x_{\tilde{i}\tilde{j}},\ 0\right\} \tag{4}$$

In the oil-drilling problem, note that, according to Eq. 4, the problem of determining L is actually that of determining P_c. If the decision-maker understands that at the current moment his payment is \$0, then "the loss to be avoided" will be the drilling cost. Only if the decision-maker reckons that the sale of the land is a fact and considers that he has the \$90,000 in his pocket, will "the loss to be avoided" be the \$190,000 indicated by the opportunity cost, and, consequently, the decision-maker might be willing to pay \$142,500 for perfect information.

The third value to consider is "the most favorable payoff in the worst scenario" (*FPW*). This value guarantees that, if the worst scenario were to occur, the amount the decision-maker would pay for perfect information would not exceed the best payoff he could obtain in that worst scenario. In other words, it is a way of ensuring that the cost of the study is covered in all scenarios.

In the oil-drilling problem, $FPW = \$90,000$ because if the land was dry and the decision-maker had perfect information, the chosen alternative would be to sell, and the price obtained from the sale would have to cover the costs of the study.

Then,

$$\begin{aligned} VPIP &= min\{EVPI,\ L,\ FPW\} \\ &= min\{142,500; 100,000; 90,000\} \\ &= 90,000 \end{aligned}$$

Table 3 The oil-drilling problem. Opportunity costs

Opp. costs	Drilling	Selling
Oil (0.25)	0	610,000
Dry (0.75)	190,000	0

It could be argued that, if the decision-maker's utility function is correctly fitted with a sufficiently concave function to reflect his risk-averse attitude, there is no need to resort to the new *VPIP* bound; but even if we dispense with *FPW* as a more fine-tuned consideration, the absolute value of *L* should always be taken into account to verify that the value of *EVPI* obtained does not exceed *L*, with the chosen utility function. This is the same idea as in the previous section, which the authors wish to emphasize: when decision-making does not admit repetition, the absolute values of a problem cannot be relegated to second place.

Be aware that, of course, *VPIP* is still only an upper bound, since the decision-maker could identify other restrictions related to the problem that might further reduce the amount to pay for perfect information.

6 Sensitivity analysis

6.1 Approach

This section conducts a sensitivity analysis of the *EVPI* and *VPIP* bounds. By definition, this analysis involves "small" modifications that do not alter the core of the problem, as substantial changes would necessitate a re-evaluation from the start. Therefore, the following premises are suggested to maintain the problem's identity.

1. The preferred alternative should remain the same ($A_{\bar{j}}$), even though its expected value could vary.
2. The worst scenario should not vary ($S_{\underline{i}}$). This is the scenario that the decision-maker fears the most, and it is the origin of the decision problem.
3. The alternative with the best result in the modified scenario should not change (A_S). In each scenario, the alternative that is originally the best should be preserved.

First, we examine how *EVPI* is affected by a variation in the problem data. Second, we analyze the effect of the modification on *VPIP* and the interval within which the data can vary while still fulfilling the established assumptions using linear programming.

Linear programming is a branch of optimization theory that studies problems in which all the functions involved are linear. It consists of a variety of methods and procedures aimed at identifying which points maximize or minimize the objective function value within a set of points that meet all the established constraints.

For this purpose, and given the Eq. 3, we formulate a maximization linear programming problem with two variables (x,y), where x variable represents the value of *VPIP* and y variable is the value modified.

Table 4 Linear programming problem

Max x		
$x \le EVPI$	$EVPI$	(1)
$x \le L$	L	(2)
$x \le FPW$	FPW	(3)
The preferred alternative should remain the same (y)	$A_{\bar{j}}$	(4)
The alternative with the best result in the modified scenario should not vary (y)	A_S	(5)
The worst scenario should not vary (y)	$S_{\underline{i}}$	(6)

In this way, a linear programming problem will initially arise with six inequality constraints (see Table 4). The first three are related to the x variable and are a consequence of how the *VPIP* is defined (see Eq. 3). The last three use the y variable and result from imposing the above-mentioned three premises. In some cases, an additional restriction must be imposed on the y variable to prevent a loss from becoming a gain or *vice versa* (this constraint is indicated with *Neg* or *Pos* as required).

By adopting this approach, a set of feasible solutions of two variables in $\mathbb{R}^2$ is obtained. This outcome allows for the graphical representation of the maximum value of x for each value of y.

Sometimes, the maximum value of x is bounded by *EVPI*, while at other times it is constrained by *L* or *FPW*.

Bear in mind that the set of feasible solutions (x,y) corresponds to all possible variations of y in which the premises outlined above are respected. However, for each y_0, the set of feasible solutions is represented as a horizontal ray where the y-coordinate is y_0, and the objective is to maximize the x-coordinate.

This maximum value of x will be the *VPIP* amount, and therefore, also the minimum between the *EVPI*, *L*, and *FPW* values. The graphs illustrate how, for each value of y-coordinate y_0, the x-coordinate can increase until it is constrained by the first intersecting line, whether it corresponds to *EVPI*, *L*, or *FPW*. At this point, the value of x-coordinate indicates *VPIP*.

6.2 Classification of alternatives and scenarios

Given a decision problem with a finite number of alternatives and states of nature, the values displayed in terms of alternatives correspond to

1. The alternative with the best expected value; thus, the preferred alternative.
2. Another alternative different from the preferred alternative.

Among the columns representing the different alternatives, there is one that represents the preferred alternative. According to the established premises in 6.1, when a data point in this column is changed, it must be considered that it is the preferred alternative and must remain so.

In the case of states of nature or scenarios, two options arise:

1. The worst scenario.
2. A scenario other than the worst scenario.

Among the rows representing the different scenarios, there is one showing the worst scenario. When the value being modified belongs to this row, it should be considered that the preferred alternative must still obtain its worst result in that scenario.

Another case arises when the modified value is the best in its scenario. In this situation, whether it corresponds to the preferred alternative or not, the alternative in which this best result is obtained must remain the same.

6.3 Sensitivity analysis of *EVPI*

Recall that *EVPI* is the difference between "the expected value with perfect information" (*EPI*) and "the expected value of the preferred alternative" (*BEVA*)(see Eq. 1). Therefore, note that *EVPI* does not vary when the modified y value meets the following:

1. It belongs to the preferred alternative and is the best outcome of its scenario. This is because the y value will intervene equally in both *EPI* and *BEVA*, and consequently, will not affect the final result.
2. It neither belongs to the preferred alternative nor is the best result of its scenario. This is because y value will not be relevant in the *EVPI* calculation.

In all other cases, the amended y value is only involved in a single summand of *EVPI*. So, if it corresponds to an alternative that is not the preferred one but is the best result for its scenario, then

$$\Delta EVPI = \Delta y * p_i$$

because the variation occurs in *EPI*.

However, if it corresponds to the preferred alternative but is not the best result for its scenario, then

$$\Delta EVPI = -\Delta y * p_i$$

for the reason that the variation occurs in *BEVA*, which is subtracted.

Let us also recall that the variation has to respect the established premises in Sect. 6.1.

6.4 Sensitivity analysis of *VPIP*

Section 6.3 has shown the cases in which *EVPI* changes. However, it remains to be determined in which situations L and *FPW* can vary.

In accordance with Eq. 4, the data that might cause a variation in L are

- The minimum payment associated with the preferred alternative, i. e., the payment associated with the preferred alternative in the worst scenario.
- The data belonging to alternatives other than the preferred alternative, representing the current payment in the worst scenario (note that this may or may not be the best result for its scenario).

The data that might cause a variation in *FPW* are

- The data belonging to alternatives other than the preferred alternative, which are also the best result for the worst scenario.

Note that the data that may cause L and *FPW* to vary are the worst scenario's data.

Table 5 summarizes the situation of the data that may lead to a variation in *EVPI*, L, *FPW*, and *VPIP*. The modified data point is labeled as y. Note that this table constitutes the core of this work and contains its main results.

Respecting the established premises, the problem data can swing between the intervals shown in Table 6.

6.5 Examples

Below, by way of illustration, we apply the sensitivity analysis developed in the present article to the oil-drilling problem. Let us recall, according to Sect. 4 and Table 2:

- Preferred alternative: Drilling.
- The worst scenario: Dry.
- $VPIP = min\{EVPI, L, FPW\} = min\{142,500;100,000;90,000\} = 90,000$

6.5.1 Variation in the value of the preferred alternative belonging to the worst scenario

Let us consider the drilling cost. In what range can this cost vary so that the problem does not change in essence? How would this affect *VPIP*? In this case, the proposed amendment corresponds to "the preferred alternative" in "the worst scenario." Table 7 shows the location of the y data point that will be modified.

Table 5 Variation of *EVPI*, *L*, *FPW*, and *VPIP* according to the situation of the *y* data point that is modified

	y is a data point that belongs to the preferred alternative	y is a data point that belongs to an alternative other than the preferred alternative
y is the scenario's best result	*EVPI* does not vary	$\Delta EVPI = \Delta y * p_i$
	L and *FPW* do not vary: *VPIP* does not vary	L^1 and FPW^1 do not vary
y is not the scenario's best result (other scenario values)	$\Delta EVPI = -\Delta y * p_i$ L^2 and *FPW* do not vary	*EVPI* does not vary L^3 and *FPW* do not vary: *VPIP* does not vary

1- If y corresponds to the worst scenario, *FPW* varies and *L* may vary (only when $P_c = FPW$).
2- If y corresponds to the worst scenario, *L* varies, except in the case where the original value of $x_{ij}$is positive and the modified value is also positive, since in both cases *L* would be 0.
3- If y corresponds to the worst scenario and $P_c \neq 0$ and $P_c \neq FPW$, *L* could vary, and thus, *VPIP* too.

Note: The worst scenario is defined as the scenario in which the preferred alternative gives its worst result. However, this does not rule out the possibility that it could, in fact, be the best result for that scenario. Nevertheless, in this case, there would not be any decision problem, since the preferred alternative would also offer the best result even in the most feared scenario for the decision-maker. For this reason, the possibility of being in the worst scenario is not considered when modifying a data point in the preferred alternative that is also the best result for that scenario

Table 6 Intervals in which the modified *y* data point can swing respecting the established premises

	y is a data point that belongs to the preferred alternative	y is a data point that belongs to an alternative other than the preferred alternative
y is the scenario's best result	y can increase indefinitely	y can increase as long as it does not become the preferred alternative
	y can fall as long as it remains • the preferred alternative • the scenario's best result	y can fall as long as it remains • the scenario's best result
y is not the scenario's best result	y^1 can increase as long as it does not become the scenario's best result	y can increase as long as it does not become the preferred alternative or the scenario's best result
	y can fall as long as it remains • the preferred alternative	y can fall indefinitely

1- If y belongs to the worst scenario, it can increase as long as it remains the worst scenario.

Table 7 The oil-drilling problem. Variation in the value of the preferred alternative (Drilling) belonging to the worst scenario (Dry)

	Drilling($)	Selling($)
Oil (0.25)	700,000	90,000
Dry (0.75)	y	90,000
$E(A_j)$	175,000+0.75y	90,000

In such circumstances, apart from respecting the premises of Section 6.1, the problem as stated does not allow replacing the −\$100, 000, which represents the drilling cost, with a positive amount. Consequently, as indicated there, a new constraint (Neg) must be added to maintain y as a negative value. However, if for some reason y were to take a positive value, consider also that the loss to be avoided would be 0 according to Eq. 4.

Thus, the linear programming problem emerging would be the one presented in Table 8 and represented graphically in Fig. 1.

Table 8 Linear programming problem that determines the value of *VPIP* when the drilling cost varies

Max x		
$x + 0.75y \le 67,500$	*EVPI*	(1)
$x + y \le 0$	*L*	(2)
$x \le 90,000$	*FPW*	(3)
$y \ge -113,300$	$A_{\bar{j}}$	(4)
$y \le 90,000$	A_S	(5)
$y \le 0$	Neg	(6)

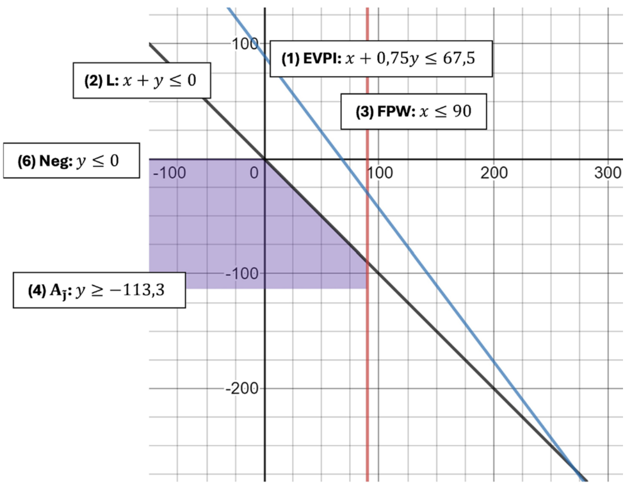

Fig. 1 Graphical representation of the linear programming problem that determines the value of *VPIP* when the drilling cost varies. Amounts are expressed in thousands of dollars

The shaded area indicates the set of feasible solutions (x, y) formed by all points whose coordinates satisfy all the constraints. Therefore, for each value y_0 that satisfies the premises of Section 6.1, the maximum value of x that is less than or equal to *EVPI*, *L*, and *FPW* is the x-coordinate of the point (x, y_0) that lies on the boundary of the feasible solution set and maximizes the x-coordinate values.

Figure 1 shows that as long as the drilling cost is less than \$90, 000 (the land selling price), *VPIP* will be given by the loss to be avoided, which will match the drilling cost:

$$VPIP = L = -y \qquad y \in [-90,000;0]$$

Also, when the drilling cost is between \$90, 000 and \$113, 300, *VPIP* will indicate that the maximum amount that can be spent on perfect information cannot exceed \$90, 000 (*FPW*), which is the amount that could be obtained from the sale.

$$VPIP = FPW = 90,000 \qquad y \in [-113,300; -90,000]$$

A drilling cost higher than \$113, 300 would not respect the premises of Sect. 6.1, since drilling would no longer be the preferred alternative. Therefore, considering *VPIP* as a function of the drilling cost (y), which is the value we modify, we can write

$$VPIP(y) = \begin{cases} FPW = 90,000 & y \in [-113,300; -90,000] \\ L = -y & y \in [-90,000;0] \end{cases}$$

Figure 2 explains that the horizontal ray represents all the points in the feasible solution set where the drilling cost is \$100, 000. These are points of the form $(x, -100)$, where the y-coordinate is fixed at the cost of interest for us. The objective of the linear problem is to maximize the x-coordinate within the feasible set. The arrows indicate the direction in which the x-coordinate grows. The point at which it reaches the maximum value is $(90, -100)$. At this point, the feasible set is bounded by the constraint *FPW*. Hence, the value of the x-coordinate gives us *VPIP* for the drilling cost of \$100, 000 (in the graphical representation, it is denoted by *VPIP**). It also represents how much could the x-coordinate grow if we ignore the *FPW* limit and it continues moving forward until reaching the *EVPI* limit at the point (142.5, -100). Again, the x-coordinate gives us the value of *EVPI* for the drilling cost of \$100, 000 (in the graphical representation, it is denoted by *EVPI**).

By varying the drilling costs within the feasible set, all the values of the *VPIP* bound will be found. The whole *VPIP* bound is represented by a line of large dots. Observe that the specific values taken by *VPIP* and *EVPI* when the cost of drilling is \$100, 000 are located on the x-axis.

Also, note that in the graphical representation the *VPIP* function appears rotated (symmetric with respect to the y-axis) since y is the independent variable.

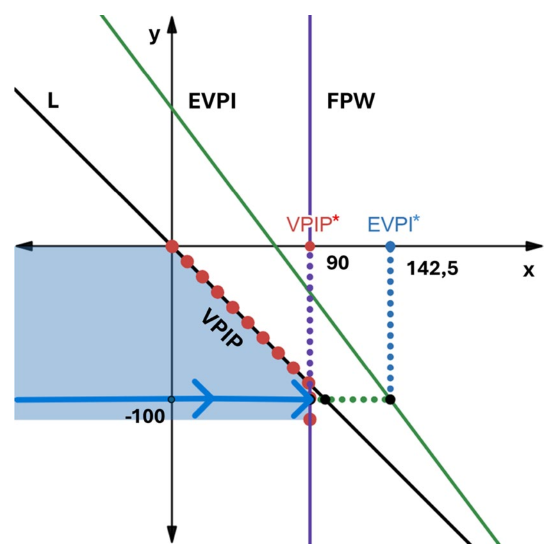

Fig. 2 Graphical representation of the *VPIP* and *EVPI* values when the drilling cost is \$100, 000. These values are denoted by *VPIP** and *EVPI**, respectively. Amounts are expressed in thousands of dollars

6.5.2 Variation of the value that gives the best result in the worst scenario (and, therefore, belongs to a non-preferred alternative).

Now, it is proposed to vary the price of the possible sale. In this case, it represents the best outcome of the worst scenario and the change affects both scenarios.

Table 9 shows the location of the y data point that will be modified.

Similar to the previous case and as indicated in Sect. 6.1, a constraint (Pos) must be added to ensure that the selling price is positive. The corresponding linear programming problem is presented in Table 10 and represented graphically in Fig. 3.

It is shown how the selling price could increase up to \$100,000, as above this value the preferred option would be to sell. Also, *VPIP* will be given by *FPW*, which will grow with the value of the sale:

$$VPIP = FPW = y \qquad y \in [0;100,000]$$

Analogous to the previous subsection, the horizontal ray represents all the points in the feasible solution set where the selling price is $\$90,000$. These are points of the form (x, 90) at which the y-coordinate is fixed at the selling price of interest for us. The objective of the linear problem is to maximize the x-coordinate within the feasible set. The arrows indicate the direction in which the x-coordinate grows. The point at which it reaches the maximum value is (90, 90). At that point the feasible set is bounded by the constraint *FPW*. Therefore, the value of the x-coordinate gives us *VPIP* for the selling price of $\$90,000$ (in the graphical representation, it is denoted by $VPIP^*$). It also represents how much the x-coordinate could grow if we ignore the *FPW* limit and it continues moving forward until reaching the *EVPI* limit at the point (142.5, 90). Again, the x-coordinate gives us the value of *EVPI* for the selling price of $\$90,000$ (in the graphical representation, it is denoted by $EVPI^*$).

Table 9 The oil-drilling problem. Variation of the value that gives the best result in the worst scenario (Dry) and, therefore, belongs to a non-preferred alternative (Selling)

	Drilling(\$)	Selling(\$)
Oil (0.25)	700,000	y
Dry (0.75)	−100,000	y
$E(A_j)$	100,000	y

Table 10 Linear programming problem that determines the value of *VPIP* when the selling price varies

Max x		
$x - 0.75y \le 75,000$	*EVPI*	(1)
$x \le 100,000$	*L*	(2)
$x \le y$	*FPW*	(3)
$y \le 100,000$	Not $A_{\bar{j}}$	(4)
$y \ge -100,000$	A_S	(5)
$y \ge 0$	Pos	(6)

6.5.3 Variation in the value of the preferred alternative yielding the scenario's best result

We only need to consider the interval in which the profit of $\$700,000$ might oscillate and how this would affect *VPIP*. This is the case of "the preferred alternative" and "the scenario's best result." Table 11 shows the location of the y data point that will be modified.

According to Tables 5 and 6, this profit can rise indefinitely without changing *EVPI* or *VPIP*. However, it can only fall to $\$660,000$ because at this point the expected value of the drilling alternative is equal to the expected value of the selling alternative, and below it, drilling would cease to be the preferred alternative. Additionally, $\$660,000$ remains the best result if oil is struck (Fig. 4).

The linear programming problem that arises is described in Table 12 and represented graphically in Fig. 5.

In this case, note that for any value of y higher than $\$660,000$, *VPIP* is given by the vertical $FPW = \$90,000$. The profit obtained by striking oil may vary in the interval [660,000; $+\infty$]. Then, the premises will be met, and the value of *VPIP* will be maintained at $\$90,000$.

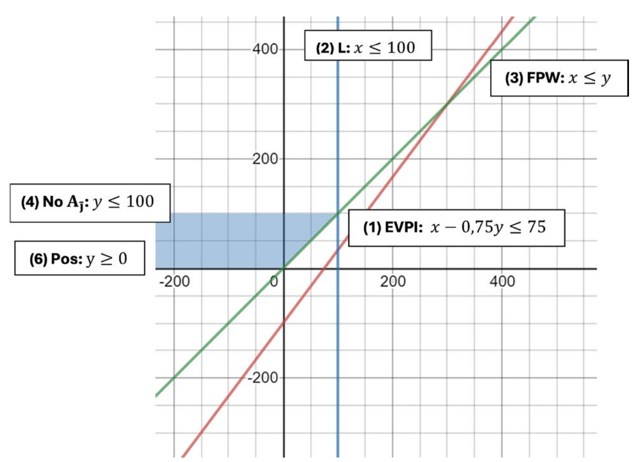

Fig. 3 Graphical representation of the linear programming problem that determines the value of *VPIP* when the selling price varies. Amounts are expressed in thousands of dollars

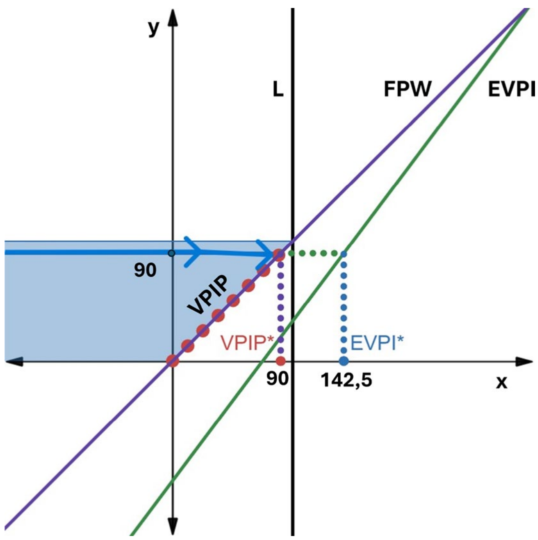

Fig. 4 Graphical representation of the *VPIP* and *EVPI* values when the selling price is $90,000. These values are denoted by *VPIP** and *EVPI**, respectively. Amounts are expressed in thousands of dollars

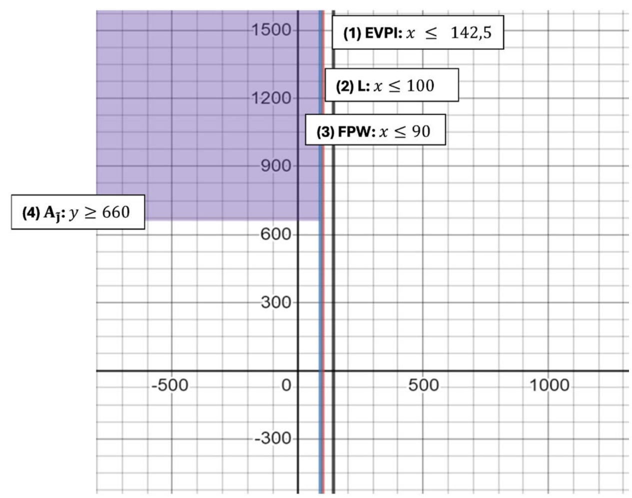

Fig. 5 Graphical representation of the linear programming problem that determines the value of *VPIP* when the profits from striking oil vary. Amounts are expressed in thousands of dollars

Table 11 The oil-drilling problem. Variation in the value of the preferred alternative (Drilling), which is also the best result in its scenario (Oil)

	Drilling($)	Selling($)
Oil (0.25)	y	90,000
Dry (0.75)	−100,000	90,000
$E(A_j)$	0.25y-75,000	90,000

Table 12 Linear programming problem that determines the value of *VPIP* when the profits from striking oil vary

Max x		
$x \leq 142,500$	*EVPI*	(1)
$x \leq 100,000$	*L*	(2)
$x \leq 90,000$	*FPW*	(3)
$y \geq 660,000$	$A_{\bar{j}}$	(4)
$y \geq 90,000$	A_S	(5)

So,

$$VPIP = FPW = 90,000 \quad y \in [660,000; +\infty]$$

6.5.4 Sensitivity analysis in linear programming

Since linear programming is a widespread technique with many applications, there are plenty of refined tools that develop it. One of them is the Solver add-in in Excel, which solves linear programming problems and provides sensitivity analysis tables. Table 13 is obtained by Solver for the linear programming problem in Table 8 with the difference that the drilling cost has been set at $100,000 and the last three constraints have been eliminated. We use $1,000 as monetary unit.

In this table, it can be noticed that there is only one saturated constraint (the one corresponding to *FPW*) and that its shadow price is 1. This means that if the selling price increases by 1 unit, *VPIP* also increases by 1 unit. These results are in agreement with what we have already seen in Sect. 6.5.1, since we argued that in the interval $[-113,300; -90]$, *VPIP* meets *FPW*. In the "allowable increase" column, the table also indicates that the increase margin for the independent term of the third constraint (*FPW*) is 10 units. Indeed, it is clear from Fig. 2 that keeping the horizontal $y_0 = -100$, the distance between *FPW* and *L* is 10 units. If the increase was more than 10 units, the constraint that would become saturated would be *L*.

Therefore, the manager of Goferbroke, taking into account Sect. 6.5.1 and Table 13, will know that as long as the drilling cost exceeds the offered price to purchase the land, the optimal value of the linear programming problem (*VPIP*) will coincide with the selling price (*FPW*), which is the only saturated constraint. That is, he will understand that the purchase price of the land indicates the maximum amount he would be willing to pay for perfect information. However, he will note that if the drilling cost is lower than the selling price, *VPIP* will coincide with the drilling cost, and thus, the manager will never pay an amount higher than the drilling cost for perfect information.

The "allowable increases" indicate to the manager the margin between one situation and another. In this case, since the drilling cost (*L*) has been set at $100,000 and the selling price (*FPW*) at $90,000, it is only allowed to increase the selling price by the difference ($10,000), which is the value

Table 13 Sensitivity analysis obtained with the Solver add-in in Excel for the linear programming problem in Table 8 setting the drilling cost at $100, 000

Variable cells						Allowable
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
*C*15	*x*	90	0	1	1E+30	1
*D*15	*y*	−100	0	0	1E+30	0
Constraints						Allowable
Cell	Name	Final Value	Shadow Price	Constraint R.H.Side	Allowable Increase	Allowable Decrease
EVPI	≤	15	0	67.5	1E+30	52.5
L	≤	-10	0	0	1E+30	10
FPW	≤	90	1	90	10	1E+30

indicated in the "allowable increase." In other words, if the selling price increases by more than $10, 000, the optimal *VPIP* would no longer be given by the selling price (*FPW*) but by the drilling cost (*L*). The shadow price allows us to calculate how much *VPIP* will vary if the selling price changes within this interval:

$\Delta VPIP=$ Shadow price * Δ Selling price

In this case, the shadow price equal to 1 indicates that if the selling price increases by *x* units, *VPIP* will also increase by *x* units.

The sensitivity analysis we offer applied to the oil-drilling problem could be reproduced with respect to politically challenging environmental problems, such as those related to the so-called "creative destruction" (Kivimaa and Kern 2016) within the scope of the phase-out to achieve decarbonisation (Trencher et al. 2022). If we manage to reduce these problems, which involve decisions that are difficult to repeat, to their basic structure, we facilitate their understanding and decision-making.

6.5.5 Distance between *EVPI* and *VPIP* bounds in the oil-drilling problem

Finally, the following table shows the values *EPI*, *BEVA*, *EVPI*, and *VPIP* for different drilling costs (keeping the original values of profit and selling price fixed). Two columns are added to indicate the difference between *EVPI* and *VPIP*, as well as the ratio between *EVPI* and the drilling cost (Table 14).

It should be noted that from a drilling cost of $90, 000, the lower the cost, the greater the difference between *EVPI* and *VPIP*. This is clearly seen in Figs. 1 and 2. If the price of drilling were $10, 000, would the decision-maker be willing to pay for perfect information 7.5 times the drilling cost? And if drilling were free, would the decision-maker be willing to pay just anything for perfect information? Certainly not up to three-quarters of the land's offer price.

EVPI is only an indicator of the value of information, but cannot be considered the maximum value that a decision-maker would be willing to pay for perfect information. The primary objective of our study was to highlight the importance of absolute values when making difficult to repeat decisions.

7 Conclusions

The concept of *VPIP* defined in Boncompte (2018) has been expanded with a sensitivity analysis of the *EVPI* and *VPIP* bounds in the field of finite decision problems using known *a priori* probabilities.

This paper has consolidated the terms defined in Boncompte (2018) to identify decision alternatives and scenarios

Table 14 Comparison between the values obtained for *EVPI* and *VPIP* in the oil-drilling problem (Table 2) when the drilling cost is varied

Drilling cost	EPI	BEVA	EVPI	VPIP	Difference	EVPI/ drilling cost
100,000	242,500	100,000	142,500	90,000	52.500	1.425
90,000	242,500	107,500	135,000	90,000	45,000	1.5
40,000	242,500	145,000	97,500	40,000	57,500	2.44
10,000	242,500	167,500	75,000	10,000	65,000	7.5
0	242,500	175,000	67,500	0	67,500	

which allow us to classify the data of a finite decision problem according to the roles played in it. At the same time, the sensitivity analysis has been presented as generally as possible through this classification.

First of all, some premises have been established to prevent the problem from losing its essence when proposing modifications that exceed the limits associated with a sensitivity analysis. Within these conditions, we have studied how data variations can influence *EVPI*. It is also shown that *EVPI* does not change when a data point that belongs to the preferred alternative and is the best result of its scenario is modified within the allowed range. Nor does *EVPI* change when the variation affects a data point belonging to a non-preferred alternative that is also not the scenario's best result. In all other cases, the variation in the absolute value of *EVPI* is directly proportional to the probability of the scenario to which the modified data point corresponds.

Next, to observe how varying a data point in the problem changes *VPIP*, we have addressed a linear programming problem with just two variables (x, y). The x variable represents the *VPIP* value and the y variable is the data point to be modified. Moreover, it has been explained that a proper reading of the graphical resolution of this problem will indicate the behavior of *VPIP* in each case. It is worth noting the novelty of using linear programming to study the behavior of these bounds.

Furthermore, the presentation of the results has been completed with two tables, whose entries depend on the role played in the problem by the data point to be modified. In the first table, it is displayed how the variation of this data point affects both *EVPI* and *VPIP*. In the second one, the boundaries of the interval within which the data point can oscillate are fixed while respecting the established premises.

Finally, all of the foregoing has been illustrated by applying the sensitivity analysis to the oil-drilling problem. The comparison between the values obtained for *EVPI* and *VPIP* in this example is particularly interesting. This comparison underscores the need to use a more adjusted value than *EVPI* when attempting to determine how much the decision-maker would be willing to pay for perfect information. In the specific case of the oil-drilling problem, it is noted that if the cost of drilling were only $\$10,000$, *EVPI* would be $\$75,000$. Therefore, *EVPI* would give a value 7.5 times higher than the loss that the decision-maker wants to avoid by resorting to decision theory. Thus, the aforementioned reiterates the requirement to consider the absolute values of the problem when decisions are difficult to repeat. This is especially important in the environmental context, where many factors come into play and data are often provided in the form of indices and percentages. We believe that reducing the problem to its basic structure, properly presenting alternatives and scenarios, taking absolute values into account, and considering the reflections on the concept of *VPIP* as discussed in this article can enhance the acquisition of appropriate information and, ultimately, facilitate decision-making.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethics approval Not applicable.

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