



Research Paper

Entrepreneurship, growth and productivity with bubbles

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ARTICLE INFO

JEL classification:

E22
E44
G12

Keywords:

Bubble
Entrepreneurship
Growth
Productivity

ABSTRACT

Entrepreneurship, growth and total factor productivity are larger when asset prices are high and decline during financial crises. We explain these facts using a growth model with financial bubbles in which individuals have heterogeneous wages and returns on productive investment. Heterogeneity separates individuals between savers and entrepreneurs. Savers buy financial assets, which are deposits or a financial bubble. Entrepreneurs incur in a start-up cost and borrow to invest in productive capital. The bubble provides liquidities to credit-constrained entrepreneurs. These liquidities increase investment, growth and entrepreneurship. Finally, the bubble may increase productivity when the return of each entrepreneur's investment is positively correlated with her previous income.

1. Introduction

After the Great Recession, there has been renewed interest in analyzing the growth effects of financial bubbles. In this paper, we contribute to this literature by studying the effect of financial bubbles on entrepreneurship, growth and productivity. Fig. 1 shows the time series of these variables for the US economy in the period 1995–2009. The different panels show the business cycle component of wealth to income ratio, logarithm of gross domestic product (GDP), total factor productivity (TFP) and logarithm of the number of firms. In panel (a), we observe two periods in which wealth relative to income increases substantially and after declines sharply. These large fluctuations in wealth are driven by asset price fluctuations and, according to Shiller (2016), no fundamental seems to explain the fluctuations of asset prices. As a result, these two periods have been considered as examples of financial bubbles. The rest of panels in Fig. 1 show that these large fluctuations in wealth are closely related to fluctuations in GDP, in TFP and in the number of firms. We observe that these three variables are large when wealth is large and they decrease when wealth declines. In fact, the correlations between the business cycle component of the wealth to income ratio and that of GDP, TFP and the number of firms are, respectively, 0.91, 0.69 and 0.76.² Clearly, correlations are positive, very large and significative.

The literature provides additional empirical evidence for these findings using aggregate and firm-level data for different countries. First, using aggregate data, Campbell (1999) show that asset price volatility is highly procyclical. We follow the literature on financial bubbles and interpret asset price growth as the result of a bubble and the reduction of asset prices as the result of a bubble burst. According to this interpretation, growth is larger when there is a bubble. Caballero et al. (2006) and Martin and Ventura (2012) provide convincing evidence on this relationship. They identify periods during which, according to most of the literature, there is

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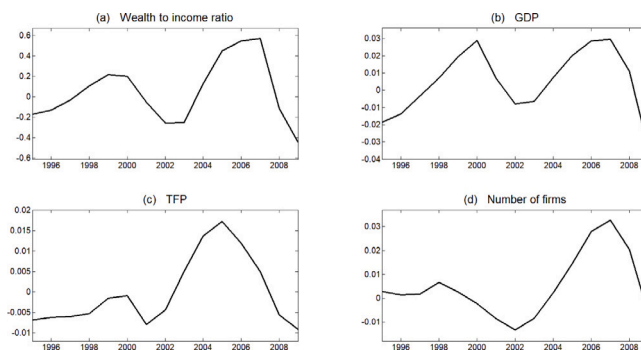


Fig. 1. US time series. Note. The figure plots the cyclical component of the wealth to income ratio, the logarithm of GDP, TFP and the logarithm of the number of firms obtained using the Hodrick-Prescott filter with a smoothing parameter of 100. We use annual data for the US in the period 1995–2009. Wealth to income ratio is households and nonprofit organizations net worth as a percentage of disposable personal income and is obtained from Federal Reserve Bank of St. Louis. GDP is real GDP at chained PPPs and TFP is at constant national prices, both obtained from the Penn World Table 10.0. The number of firms is obtained from the Business Dynamics Statistics from the US Census Bureau.

a bubble and show that growth is large in these periods and it declines when the bubble bursts. Second, [Meza and Quintin \(2005\)](#), [Pratap and Urrutia \(2012\)](#) and [Queraltó \(2011\)](#) find that TFP fell during East Asian, Mexican and Argentine financial crises in the 1990s.³ This well-established pattern for aggregate TFP is compatible with different patterns at the industry level. For instance, [Queirós \(2024a\)](#) show that in some industries TFP declines when firms in the sector become overvalued. [Tang and Zhang \(2022\)](#) and [Queirós \(2024a\)](#) explain this evidence as the result of too many inefficient firm's entering an industry during financial bubbles. Our model will also include the entry of inefficient firms during bubble periods and, therefore, the bubble will increase TFP only when it does not cause a too large increase in the number of firms.

Third, using firm-level data, [Koellinger and Thurik \(2012\)](#) show that entrepreneurship is procyclical.⁴ Since bubbles are also procyclical, this finding suggests that the number of entrepreneurs increases during a bubble and declines when it bursts. This has been confirmed by [Klapper and Love \(2011\)](#) and [Tian \(2018\)](#), who show that the number of entrepreneurs falls during the Great Recession.

We show that these three facts can be explained in an overlapping generations (OLG) model with the following characteristics. First, we assume that individuals live for three periods and can invest in productive capital only in the second period of life, whereas part of the labor income is obtained in the first period. In the first period, individuals are young, work, and save or borrow through two different financial assets: a deposit (or a credit) and a financial bubble, which takes the form of a purely speculative asset. In the second period, individuals are adults and can become entrepreneurs who invest in productive capital and face a borrowing constraint that limits this investment. In the third period, individuals are old and consume their wealth. [Farhi and Tirole \(2012\)](#) show that financial bubbles can be sustained in a model with these characteristics. These bubbles, by increasing the savings devoted to the demand of financial assets, provide the liquidities needed to invest when individuals are credit constrained. This is the liquidity effect that increases investment and growth.⁵ Second, we extend the model by assuming a continuous distribution of abilities in the population, as in [Kunieda and Shibata \(2016\)](#). These abilities are an individual-specific productivity shock that determines productivity in the first two periods of life. In the second period, abilities determine the return of productive investment. Those individuals with a large return become entrepreneurs who invest in productive capital, whereas the rest are savers who accumulate financial assets. In the first period, abilities determine the productivity of workers. As a result, wages in the first period of life are heterogeneous and correlated with the return of productive investment that individuals obtain in the second period. Finally, we assume that individuals incur a start-up cost to be entrepreneurs. The introduction of this cost and wage heterogeneity are the novelties of this model. We show that they introduce new economic mechanisms that are crucial to explain the aforementioned facts.

³ This well-established pattern for aggregate TFP is compatible with different patterns at the industry level. For instance, [Queirós \(2024b\)](#) shows that in some industries TFP declines when firms in the sector become overvalued. [Tang and Zhang \(2022\)](#) and [Queirós \(2024a\)](#) explain this evidence as the result of too many inefficient firms entering an industry during financial bubbles. Our model will also include the entry of inefficient firms during bubble periods and, therefore, the bubble will increase TFP only when it does not cause a too large increase in the number of firms.

⁴ [Koellinger and Thurik \(2012\)](#) show a positive correlation between entrepreneurship and deviations of GDP from trend for a cross-country panel of 22 OECD countries for the period 1972 to 2007. [Bilbiie et al. \(2012\)](#), [Campbell \(1998\)](#), and [Clementi and Palazzo \(2016\)](#) show that, in the US, firms entry is procyclical, while exit is countercyclical.

⁵ The literature distinguishes between two growth-enhancing roles of the bubbles. One is the liquidity role of the bubble: agents hold at the beginning of the period the bubble and sell it to increase their productive investment ([Kocherlakota, 2009](#); [Farhi and Tirole, 2012](#); [Martin and Ventura, 2012](#); [Hirano and Yanagawa, 2017](#); [Miao and Wang, 2018](#)). The other one is the collateral role of the bubble: agents buy the bubble to increase their possibilities to borrow and use these loans to invest in capital ([Kocherlakota, 2009](#); [Martin and Ventura, 2016](#); [Miao and Wang, 2018](#)). In [Clain-Chamosset-Yvrard et al. \(2023\)](#) we show that in the absence of uncertainty both roles are identical. Accordingly, in this paper we do not distinguish them and we simply refer to this growth enhancing effect of the bubble as the liquidity effect.

The equilibrium of this model can converge to two different steady states: a bubbly steady state in which financial assets are deposits and the speculative asset, and a bubbleless steady state in which the only financial assets are deposits. We show the bubbly steady state exists when there is excess supply of savings. We also show that the return of financial assets is larger in the equilibrium with bubbles, which is a consequence of the larger demand of financial assets in this equilibrium. Furthermore, if a bubbly steady state exists, then the return of financial assets equals the growth rate in this steady state and it is lower in the bubbleless steady state. These results coincide with those obtained by [Tirole \(1985\)](#) and [Grossman and Yanagawa \(1993\)](#) in models in which individuals that belong to the same generation are identical.

We compare the two steady states to study the effect of the bubble on the number of entrepreneurs, on growth and on productivity. We first show that two opposite mechanisms determine the effect of the bubble on the composition of the population between entrepreneurs and savers. On the one hand, in a bubbly steady state the return of financial assets is larger, which implies that more individuals choose to be savers, as in [Kunieda and Shibata \(2016\)](#). On the other hand, the liquidities provided by the bubble make adult individuals wealthier, which facilitates that more individuals can afford the start-up cost. Therefore, the number of entrepreneurs is larger in the bubbly steady state when this cost mechanism dominates. This mechanism, that can be interpreted as the extensive margin of the liquidity effect, explains that we observe a larger number of entrepreneurs during bubbly periods. Hence, the inclusion of a start-up cost is essential to explain the rise of entrepreneurship in the midst of financial bubbles.

We also show that the bubble affects growth through two distinct effects: the liquidity and composition effects of the bubble. First, as in [Farhi and Tirole \(2012\)](#), the liquidity effect promotes growth, because the bubble provides the liquidities that credit constrained entrepreneurs need to invest. Second, the composition effect or the extensive margin of the bubble is a contribution of this paper. We show that this effect also promotes growth when the bubble increases the number of entrepreneurs.

We finally show that the previous two effects also modify TFP, which is equal to the average return of productive investment. The composition effect decreases TFP when the number of entrepreneurs increases, since the new entrepreneurs have lower productivities. In contrast, the liquidity effect of the bubble increases TFP when wages are heterogeneous and correlated with the productivity of investment. When this happens, the bubble provides more liquidities to more productive entrepreneurs. As a consequence, it further increases the investment of highly productive entrepreneurs, which explains that the liquidity effect increases TFP. Therefore, wage heterogeneity is necessary to explain the increase in TFP during financial bubbles.

We conclude that the bubble can increase the number of entrepreneurs, growth and TFP. Therefore, by adding the start-up cost and wage heterogeneity, we explain the facts shown in [Fig. 1](#).

This paper is related to two strands of the literature. First, it is related to findings in the literature that studies the effect of financial development on growth and TFP. This literature interprets financial development as access to external financing that, together with self-financing, is used to invest (see [Cooley and Quadrini, 2001](#); [Midrigan and Xu, 2014](#)). In this literature, self-financing substitutes external financing. As a result, [Moll \(2014\)](#) shows that external financing cause a smaller increase in TFP when idiosyncratic shocks are persistent, since more productive entrepreneurs have access to larger self-financing. We also analyze how the interaction between the persistence of idiosyncratic shocks and external financing affects TFP. In our model, the persistence of shocks is measured by the correlation between wages of the young and investment productivity of the adult, self-financing corresponds to the savings of the young individuals who will be entrepreneurs in the following period and we consider two sources of external financing: credit and the bubble. The bubble introduces significant differences. Unlike [Moll \(2014\)](#), we show that when this correlation is large, the external financing, introduced in our framework by the bubble, positively affects TFP. The reason for this different finding is that the bubble increases the returns of the savings of the young individuals and, hence, it enlarges the effect of self-financing on investment.

Second, it is related to the literature on financial bubbles. This literature has studied the growth effects of bubbles since the seminal papers by [Tirole \(1985\)](#) and [Grossman and Yanagawa \(1993\)](#). In these papers, the introduction of a speculative asset without fundamental value, a financial bubble, reduces productive investment and growth. More recent literature has shown that if individuals are heterogeneous and face credit constraints, then bubbles can promote growth, which is more in line with evidence. For instance, [Farhi and Tirole \(2012\)](#) and [Martin and Ventura \(2012\)](#) show that financial bubbles may promote growth when they provide to credit constrained entrepreneurs the liquidities needed to invest.⁶ Some papers have also studied the effect of bubbles on TFP. In particular, [Miao and Wang \(2012\)](#) show that if bubbles increase investment of more productive entrepreneurs relative to less productive ones then TFP is larger with bubbles, which is in line with evidence. [Hirano and Yanagawa \(2017\)](#) obtain a similar conclusion in an endogenous growth model. Finally, very few papers in this literature consider the effect of financial bubbles on the number of entrepreneurs. Two recent examples are the papers by [Queirós \(2024a\)](#) and [Tang and Zhang \(2022\)](#), who examine the impact of bubbles on the value of the firm, directly influencing entry and exit decisions. Consequently, the mechanism relating bubbles to entrepreneurship in these two papers is based on the particular concept of bubble equilibrium assumed and, unlike the aforementioned papers, it does not rely on the liquidity effect of bubbles or other general equilibrium effects that bubbles can generate. Closer to our analysis is the paper by [Kunieda and Shibata \(2016\)](#), who study the effect that, through general equilibrium effects, pure speculative bubbles have on the individuals' decisions between being savers or entrepreneurs.⁷ However, they do not introduce the start-up cost and, hence, the bubble reduces the number of entrepreneurs.

⁶ There are many other examples of models with bubbles and heterogeneous individuals. For instance, [Bengui and Phan \(2018\)](#) and [Graczyk and Phan \(2021\)](#) consider that individuals have different endowments, which separates individuals between borrowers and lenders. This distinction is also in [Basco \(2016\)](#) and in [Kocherlakota \(2009\)](#) in a model of infinitely lived agents. In contrast, [Hillebrand et al. \(2018\)](#) distinguishes between three groups of individuals: savers, entrepreneurs and semi-entrepreneurs.

⁷ [Kunieda \(2008\)](#) considers the same mechanism in the context of an OLG model and [Kunieda \(2014\)](#) also introduces this mechanism to study the effect of bubbles on growth in a model of perpetual youth.

The liquidity effect in our paper, that corresponds to the intensive margin of the bubble, is based on the same mechanisms that the literature on financial bubbles has introduced to explain the effects of bubbles on growth and on productivity. Essentially, entrepreneurs obtain liquidity by selling a purely speculative asset to future generations. We follow [Farhi and Tirole \(2012\)](#) and assume that this asset is bought in an initial period, but we could also have followed [Martin and Ventura \(2012\)](#) and assumed that this asset is received as a bubble shock. Therefore, we contribute to this literature by adding the extensive margin or composition effect of the bubble. Using numerical examples, we demonstrate the importance of adding this margin, as it produces substantial effects on both growth and productivity. This result is consistent with findings in the literature on firm dynamics that has shown that the extensive margin is an important channel through which financial development affects productivity (see [Midrigan and Xu, 2014](#); [Buera et al., 2011](#); [Jeong and Townsend, 2007](#)).

This paper is organized as follows. Section 2 presents the model. Section 3 characterizes the equilibrium. Section 4 analyzes the effect of bubbles on entrepreneurship, growth and productivity, including a numerical illustration and a comparison with the related literature. Concluding remarks are provided in Section 5, while some technical details are relegated to an online appendix.

2. Model

We consider a discrete time overlapping generations model ($t = 1, 2, \dots$) populated by individuals that can be entrepreneurs or savers.

2.1. Production

We consider an aggregate production function that allows for endogenous growth driven by capital accumulation and exhibits a well-defined labor demand. A simple production function that is consistent with these properties is an Ak production function justified through a learning-by-doing externality, as in [Arrow \(1962\)](#) and [Romer \(1986\)](#). We therefore consider an aggregate production function that relates final output, y_t , with aggregate capital, k_t , efficiency units of labor, l_t , and an externality associated to the average capital to labor ratio, \bar{a}_t . Since we assume that workers have heterogeneous efficiency units of labor, the variable l_t measures the aggregate efficiency units of labor supplied by all workers. More precisely, the aggregate production function is

$$y_t = F(k_t, \bar{a}_t l_t).$$

This production function has the usual neoclassical properties, that is, it is a strictly increasing and concave production function satisfying the Inada conditions and is homogeneous of degree one with respect to its two arguments. Profit maximization under perfect competition implies that the wage w_t per efficiency unit and the return of capital q_t are given by

$$w_t = F_2(k_t, \bar{a}_t l_t) \bar{a}_t, \quad (1)$$

$$q_t = F_1(k_t, \bar{a}_t l_t). \quad (2)$$

We will consider only symmetric equilibria for which $\bar{a}_t = a_t$, where $a_t \equiv k_t/l_t$. Using (1) and (2), we deduce that the wage per efficiency unit, the return of capital and aggregate production at an equilibrium satisfy

$$w_t = (1 - s) A a_t, \quad (3)$$

$$q_t = s A, \quad (4)$$

$$y_t = A k_t, \quad (5)$$

where $s \equiv F_1(1, 1)/F(1, 1) \in (0, 1)$ is the capital income share and $A \equiv F(1, 1) > 0$.

2.2. Individuals

The economy is populated by individuals $i \in [0, N]$ that live for three periods: young, adult and old. The mass of individuals in each generation, N , is constant.

Young and adult individuals work and obtain wages $w_{y,t}^i$ and $w_{a,t+1}^i$, respectively. Young individuals consume $c_{1,t}^i$ and save using two different financial assets: a speculative asset, $b_{1,t}^i$, with return $R_{1,t+1}$ and a deposit, $d_{1,t}^i$, with return $R_{d,t+1}$. Adult individuals consume $c_{2,t+1}^i$, may invest κ_{t+2}^i in productive capital and save $b'_{2,t+1}^i$ in the speculative asset and $d'_{2,t+1}^i$ in the deposit. We refer to individuals who only save through financial assets as savers and individuals who also invest in productive capital as entrepreneurs. Entrepreneurs obtain a return of productive investment that is individually specific and equal to q_{t+2}^i .⁸ Capital totally depreciates after a period. The return of the speculative asset purchased by adult individuals is $R_{2,t+2}$. These pure speculative assets are the financial

⁸ This individually specific return of investment is consistent with recent evidence that shows that the return of investment increases with the wealth of the investor (see [Fagereng et al., 2020](#)).

bubbles.⁹ Finally, old individuals do not work, obtain the return of the different investments made when adult and consume $c_{3,t+2}^i$. Accordingly, the budget constraints of the young, adult and old individuals are, respectively:

$$c_{1,t}^i + d_{1,t}^i + b_{1,t}^i = w_{y,t}^i, \quad (6)$$

$$c_{2,t+1}^i + \kappa_{t+2}^i + d_{2,t+1}^i + b_{2,t+1}^i = w_{a,t+1}^i + R_{d,t+1} d_{1,t}^i + R_{1,t+1} b_{1,t}^i, \quad (7)$$

$$c_{3,t+2}^i = q_{t+2}^i \kappa_{t+2}^i + R_{d,t+2} d_{2,t+1}^i + R_{2,t+2} b_{2,t+1}^i. \quad (8)$$

Financial assets are used to borrow when they take negative values. A negative value of the speculative asset implies that individuals short sell this asset, whereas a negative deposit is a credit. Therefore, individuals that borrow have access to two different sources of external financing: the credit and the bubble. We assume that adult individuals face the following borrowing constraint that ensures positive wealth in the last period of life¹⁰:

$$d_{2,t+1}^i + b_{2,t+1}^i \geq 0. \quad (9)$$

The returns of the different financial assets have different interpretations. It is an interest factor for deposits, while it is the growth of the price for the speculative asset. Despite this different interpretation, from the budget constraint we observe that the financial assets are perfect substitutes and, therefore, their returns coincide, i.e. $R_{d,t+1} = R_{1,t+1} = R_{2,t+1}$. We denote by R_{t+1} this common return of financial assets.

Preferences of an individual i born in period t are represented by the following utility function:

$$\alpha \ln(c_{1,t}^i) + \beta \ln(c_{2,t+1}^i - h_{t+1}^i) + \gamma \ln(c_{3,t+2}^i), \quad (10)$$

where α , β and γ are positive preference parameters that satisfy $\alpha + \beta + \gamma = 1$, and h_{t+1}^i is a start-up cost that individuals must pay to be entrepreneurs.

Adult individuals that decide to be entrepreneurs spend time searching for productive investment opportunities, which introduces a start-up cost that takes the form of a time cost. We interpret this cost as an opportunity cost. In our model, we assume that it reduces leisure time and, therefore, causes a utility loss. Since it is a time cost, we assume that it is proportional to the wage of adult individuals, $w_{a,t+1}^i$. More specifically, the start-up cost is $h_{t+1}^i = \xi w_{a,t+1}^i > 0$ if the individual is an entrepreneur, whereas $h_{t+1}^i = 0$ if she is not.¹¹ The parameter $\xi \in (0, 1)$ is the start-up cost rate that measures the fraction of time spent searching for investment opportunities. The additive form of the utility function used to introduce the start-up cost has the advantage that the cost can also be interpreted as a reduction of the adult individuals' employment or as a cost in terms of consumption goods. In fact, the solution of the individuals problem and the mechanism that determines the number of entrepreneurs are identical under these different interpretations.¹² Finally, it is important to underline that the start-up cost does not depend on the amount of entrepreneurs' investment. As a result, this cost introduces a discontinuity in the utility function.

We finish the description of the model by introducing heterogeneity. We assume that individuals are heterogeneous in their innate abilities, δ^i . In every generation, δ^i follows a time invariant and continuously differentiable cumulative distribution function $F(\delta^i)$ with support $\delta^i \in (0, \delta_{\max})$. These abilities are an individual-specific productivity shock that determines an individual's productivity as an entrepreneur and also as a worker. Therefore, they determine the return of productive investment and wages. On the one hand, an individual i that invests κ_{t+2}^i units when adult obtains $\delta^i \kappa_{t+2}^i$ units of productive capital when old. Therefore, the return of investment is $q_{t+2}^i = q_{t+2} \delta^i$, where $q_{t+2} = sA$ is the constant return of capital. It follows that the return of investment is perfectly correlated with abilities.

On the other hand, wages of the young individuals satisfy: $w_{y,t}^i = (\delta^i)^v w_t$, where w_t is the wage per efficiency unit, $(\delta^i)^v$ measures the efficiency units of a young individual i and $v \geq 0$ determines the correlation between wages of young individuals and abilities. There is a positive correlation when $v > 0$ and no correlation when $v = 0$. Since abilities are perfectly correlated with the return of productive investment, v also determines the correlation between this return and the wages of young individuals. Finally, for the sake of simplicity, we assume that all adult individuals receive $\phi \geq 1$ efficiency units of labor. Therefore, adult's wage satisfies $w_{a,t+1}^i = \phi w_{t+1}$.¹³

⁹ This model could be extended to introduce the creation of bubbles, as in [Martin and Ventura \(2012\)](#). If we assume that young individuals receive these new bubbles, then they will also be a source of liquidity for adult individuals. Moreover, this liquidity effect will remain in the long-run if we assume that bubble shocks grow at the same rate than gross domestic product. Therefore, bubble creation could be a complementary explanation of the liquidity effect.

¹⁰ In [Clain-Chamosset-Yvrard et al. \(2021\)](#), the working paper version of this paper, we consider that adult individuals can borrow by using productive investment as collateral. We show that the main results obtained in this paper still hold when adult individuals can borrow.

¹¹ [Poschke \(2013\)](#), among many others, has also assumed that the cost of becoming an entrepreneur is an opportunity cost in terms of forgone wages. In [Chatterjee et al. \(1993\)](#) and many others, this opportunity cost directly causes a utility loss.

¹² We have interpreted the start-up cost as a reduction in the time devoted to leisure. However, we could have also interpreted it as a reduction in the time available to work or as an expenditure. To see that the solution of the consumers' problem is identical under these different interpretations of the cost, it is enough to define consumption when adult as $\tilde{c}_{2,t+1}^i = c_{2,t+1}^i - h_{t+1}^i$. Rewriting the adults' budget constraint using $\tilde{c}_{2,t+1}^i$, it is immediate to see that adults' labor income is $(1 - \xi) w_{a,t+1}^i$, which is consistent with the cost implying a reduction in the time devoted to work. It is also consistent with an increase in the expenditures of $\xi w_{a,t+1}^i$.

¹³ In [Clain-Chamosset-Yvrard et al. \(2021\)](#), we assume that the wages of adult individuals are an increasing function of abilities satisfying: $w_{a,t+1}^i = \phi (\delta^i)^\varphi w_{t+1}$, where $\phi (\delta^i)^\varphi$ measures the efficiency units of labor of an adult individual i . The parameter $\varphi \geq 0$ measures the correlation between the wages of adult individuals and the return on investment. We show that the results of this paper remain valid when $v \geq \varphi$.

2.3. Individuals' decisions

To characterize individual's decisions on both consumption and investment, we must take into account that the start-up cost introduces a discontinuity in the utility function. Therefore, we solve the individuals' problem by backward induction following a two-step procedure. First, we obtain the individuals' optimal demands of both consumption and assets that maximize (10) subject to the budget constraints (6)–(8), the credit constraint (9) and a non-negativity constraint on investment, $\kappa_{t+2}^i \geq 0$. By solving this maximization problem, we distinguish between two groups of individuals: savers and entrepreneurs. Savers are those individuals that only invest in financial assets, whereas entrepreneurs are those individuals that pay the start-up cost and invest in productive capital. In a second step, we use the individuals' consumption demands to obtain the indirect utility function of both savers and entrepreneurs. We compare these indirect utility functions to determine the amount of entrepreneurs. Details are given in the online appendix.

We consider first the optimal decisions of savers. They are not credit constrained and their consumption decisions are determined by the following first order conditions:

$$c_{2,t+1}^{i,S} = (\beta/\alpha) R_{t+1} c_{1,t}^{i,S}, \quad (11)$$

$$c_{3,t+2}^{i,S} = (\gamma/\beta) R_{t+2} c_{2,t+1}^{i,S}, \quad (12)$$

where $c_{1,t}^{i,S}$, $c_{2,t+1}^{i,S}$ and $c_{3,t+2}^{i,S}$ denote, respectively, the consumption of young, adult and old savers.¹⁴ Savers only invest in financial assets, the deposit and the bubble. We denote the value of financial assets owned by young and adult savers as $x_{1,t}^{i,S} = b_{1,t}^{i,S} + d_{1,t}^{i,S}$ and $x_{2,t+1}^{i,S} = b_{2,t+1}^{i,S} + d_{2,t+1}^{i,S}$. We obtain, from the budget constraints (6)–(8) and from the first order conditions (11) and (12), that the demands of financial assets satisfy:

$$x_{1,t}^{i,S} = (1 - \alpha) w_{y,t}^i - \alpha \frac{w_{a,t+1}^i}{R_{t+1}}, \quad (13)$$

$$x_{2,t+1}^{i,S} = \gamma(R_{t+1} w_{y,t}^i + w_{a,t+1}^i). \quad (14)$$

We next consider the optimal decisions of entrepreneurs. They are credit constrained and their consumption and investment decisions are determined by the following first order conditions:

$$c_{2,t+1}^{i,E} - h_{t+1}^i = (\beta/\alpha) R_{t+1} c_{1,t}^{i,E}, \quad (15)$$

$$c_{3,t+2}^{i,E} = (\gamma/\beta) q_{t+2}^i (c_{2,t+1}^{i,E} - h_{t+1}^i), \quad (16)$$

where $c_{1,t}^{i,E}$, $c_{2,t+1}^{i,E}$ and $c_{3,t+2}^{i,E}$ denote, respectively, the consumption of young, adult and old entrepreneurs.

We define the value of financial assets owned by young and adult entrepreneurs as $x_{1,t}^{i,E} = b_{1,t}^{i,E} + d_{1,t}^{i,E}$ and $x_{2,t+1}^{i,E} = b_{2,t+1}^{i,E} + d_{2,t+1}^{i,E}$ and we use the binding credit constraint, the budget constraints (6)–(8) and the first order conditions (15) and (16) to obtain

$$x_{1,t}^{i,E} = (1 - \alpha) w_{y,t}^i - \alpha \frac{w_{a,t+1}^i - h_{t+1}^i}{R_{t+1}}, \quad (17)$$

$$x_{2,t+1}^{i,E} = 0, \quad (18)$$

and the amount of productive investment

$$\kappa_{t+2}^i = \gamma \left(R_{t+1} w_{y,t}^i + w_{a,t+1}^i - h_{t+1}^i \right). \quad (19)$$

Entrepreneurs are not credit constrained when young and, therefore, they use the financial assets to smooth consumption between their first two periods of life. Instead, they are credit constrained when adult. Finally, from (19) we observe that investment in productive capital increases with the adults' wealth, defined as the present value of labor income net of start-up costs. Since entrepreneurs with larger innate abilities benefit from larger wealth, they invest more.

We next consider the individual decision between being a saver or an entrepreneur. In the online appendix, we show that an individual that is adult in period t obtains a larger utility as entrepreneur when $q_{t+1}^i > R_{t+1}/\omega_t^i$, where

$$\omega_t^i = \left(1 - \frac{h_t^i}{w_{a,t}^i + R_t w_{y,t-1}^i} \right)^{\frac{1}{\gamma}}. \quad (20)$$

Note that $\omega_t^i \in (0, 1)$ when $h_t^i > 0$. Therefore, an individual becomes an entrepreneur when the return of productive investment is strictly larger than the return of financial assets; that is, $q_{t+1}^i > R_{t+1}$. In contrast, in the absence of a start-up cost, $\omega_t^i = 1$ and, therefore, an individual with $q_{t+1}^i = R_{t+1}$ is indifferent between being an entrepreneur or a saver. It follows that a larger start-up cost increases the minimum return of productive investment necessary to be an entrepreneur and, hence, reduces the number of entrepreneurs.

¹⁴ The superscript S identifies the optimal decisions of savers, whereas the superscript E identifies the optimal decisions of entrepreneurs.

Using (3) and (20), we rewrite ω_t^i as the following increasing function of both δ^i and the ratio between the interest factor and the growth factor, $z_t \equiv R_t/g_t$, where $g_t \equiv a_t/a_{t-1}$:

$$\omega_t^i \equiv \omega(\delta^i, z_t) = \left(1 - \frac{\xi \phi}{\phi + (\delta^i)^v z_t} \right)^{\frac{1}{\gamma}}. \quad (21)$$

As shown in (20), the effect of the start-up cost on ω_t^i is determined by the ratio $h_t^i / (w_{a,t}^i + R_t w_{y,t-1}^i)$. This ratio measures the cost as a fraction of the present value of labor income in the second period of life. Since young individuals obtain wages in the first period and $h_t^i = \xi w_{a,t}^i$, this ratio decreases with the interest factor and increases with the growth rate of wages, which in this model coincides with the growth rate of a_t . As a result, the ratio between the start-up cost and the present value of labor income decreases with the ratio z_t . This explains that ω_t^i increases with z_t . In other words, the ratio z_t determines the effect of first period wages on adult's wealth. Hence, when z_t increases, adult's wealth increases and the cost as a fraction of this wealth decreases.

We next determine the number of entrepreneurs. To this end, we define by $\bar{\delta}_t$ the ability of the marginal individual that in period t is indifferent between investing in financial assets or in productive capital. This individual satisfies that $q_{t+1}^i = R_{t+1}/\omega_t^i$ and, since $q_{t+1}^i = \delta^i sA$, we obtain that $\bar{\delta}_t$ is the solution of the following equation:

$$\bar{\delta}_t = \frac{R_{t+1}}{\omega(\bar{\delta}_t, z_t) sA}. \quad (22)$$

Note that those adult individuals with $\delta^i > \bar{\delta}_t$ satisfy $q_{t+1}^i > R_{t+1}/\omega_t^i$ and, hence, are entrepreneurs. The rest are savers. Therefore, the fraction of adult entrepreneurs in t , λ_t , satisfies $\lambda_t = 1 - F(\bar{\delta}_t)$.

The solution to Eq. (22) is a function $\bar{\delta}_t = \bar{\delta}(R_{t+1}, z_t)$. Using (21), we obtain that this function is increasing in R_{t+1} and decreasing in the ratio z_t when $\xi > 0$. Since the fraction of entrepreneurs decreases with $\bar{\delta}_t$, we obtain that this fraction decreases with R_{t+1} and increases with z_t . The intuition is as follows. A larger return of financial assets, R_{t+1} , decreases the number of entrepreneurs, since more individuals obtain a larger utility when they only invest in financial assets. In contrast, an increase in the ratio z_t makes adult individuals wealthier and, as a consequence, more individuals find affordable the start-up cost and become entrepreneurs.

3. Equilibrium

In this section, we first define the equations that determine the intertemporal equilibrium and after we characterize the steady states. We show that results regarding existence of the steady states and the effect of the bubble on the steady states values of R and z are identical to those obtained by the literature in other settings.

3.1. Intertemporal equilibrium

We determine the equilibrium using the market clearing conditions for productive capital and financial assets. We proceed to obtain these two market clearing conditions as equations relating $\bar{\delta}_t$, z_t and R_t . The first one implies that the firms' aggregate demand of productive capital in period $t+1$, k_{t+1} , equals the aggregate supply of productive capital that is obtained from the aggregation of the product between investment productivity, δ^i , and investments, κ_{t+1}^i , of each entrepreneur. Therefore, the market clearing condition in period $t+1$ is

$$k_{t+1} = \int_{\bar{\delta}_t}^{\delta_{\max}} \delta^i \kappa_{t+1}^i N f(\delta^i) d\delta^i, \quad (23)$$

where $f(\delta^i)$ is the density function of the distribution of abilities. We rewrite the market clearing condition for productive capital as

$$k_{t+1} = \gamma N w_t \tau_t, \quad (24)$$

where $\tau_t = \int_{\bar{\delta}_t}^{\delta_{\max}} (\delta^i \kappa_{t+1}^i / \gamma w_t) f(\delta^i) d\delta^i$. Using (19) and after some computations, we obtain

$$\tau_t \equiv \bar{\tau}(\bar{\delta}_t, z_t) = \int_{\bar{\delta}_t}^{\delta_{\max}} \delta^i [z_t (\delta^i)^v + (1 - \xi) \phi] f(\delta^i) d\delta^i. \quad (25)$$

Eq. (24) indicates that capital depends on the product between the wage per efficiency unit and τ_t , which is a measure of the aggregate efficiency units of labor of entrepreneurs that takes into account the productivity of investment. The term of τ_t inside the square brackets amounts for the efficiency units of labor at young and adult ages. Young individuals efficiency units are multiplied by the ratio z_t , because young individuals obtain labor income one period before individuals invest in capital, and adult individuals efficiency units are multiplied by $1 - \xi$ to subtract the start-up cost. From (25), it is immediate to see that τ_t decreases with $\bar{\delta}_t$ and increases with z_t . The intuition is quite immediate. First, an increase in $\bar{\delta}_t$ reduces the number of entrepreneurs and, as a result, capital accumulation decreases. Second, adult individuals are wealthier when z_t increases, which explains the positive effect of this ratio on capital accumulation.

It is convenient to rewrite (24) and (25) in terms of R_{t+1} and z_t . To this end, we use the definition of a_t to obtain that $a_t = k_t/l_t$, where l_t is the total efficiency units of employment that satisfy $l_t = N(\chi + \phi)$, and

$$\chi = \int_0^{\delta_{\max}} (\delta^i)^v f(\delta^i) d\delta^i$$

are the average efficiency units of employment of young individuals. We use the definitions of a_t and z_t and Eq. (3) to rewrite the market clearing condition for productive capital, (24), as

$$R_{t+1} = \frac{A\gamma(1-s)}{\chi + \phi} z_{t+1} \tau_t. \quad (26)$$

We next characterize the market clearing condition for financial assets. To this end, we obtain the aggregate value of financial assets. We first use (3), (13) and (14) to deduce that the aggregate value of the financial assets owned by young and adult savers is

$$x_{1t}^S = (1-s)ANa_t \left[(1-\alpha)\eta_{t+1} - \frac{\alpha\phi}{z_{t+1}} F(\bar{\delta}_{t+1}) \right], \quad (27)$$

$$x_{2t+1}^S = \gamma(1-s)ANa_{t+1} \left[z_{t+1}\eta_{t+1} + \phi F(\bar{\delta}_{t+1}) \right], \quad (28)$$

where

$$\eta_{t+1} \equiv \tilde{\eta}(\bar{\delta}_{t+1}) = \int_0^{\bar{\delta}_{t+1}} (\delta^i)^v f(\delta^i) d\delta^i,$$

measures the aggregate efficiency units of labor of young savers and it is an increasing function of $\bar{\delta}_{t+1}$.

Using (3), (17) and (18), we obtain that the aggregate value of the financial assets owned by young and adult entrepreneurs is

$$x_{1t}^E = (1-s)ANa_t \left((1-\alpha)\pi_{t+1} - \alpha \frac{(1-\xi)\phi \left[1 - F(\bar{\delta}_{t+1}) \right]}{z_{t+1}} \right), \quad (29)$$

$$x_{2t+1}^E = 0, \quad (30)$$

where

$$\pi_{t+1} \equiv \tilde{\pi}(\bar{\delta}_{t+1}) = \int_{\bar{\delta}_{t+1}}^{\delta_{\max}} (\delta^i)^v f(\delta^i) d\delta^i,$$

measures the aggregate efficiency units of labor of young entrepreneurs, which are a decreasing function of $\bar{\delta}_{t+1}$. Observe that $\tilde{\pi}(\bar{\delta}_{t+1}) + \tilde{\eta}(\bar{\delta}_{t+1}) = \chi$.

We define the aggregate value of financial assets owned by individuals at period t as $\Psi_t = x_{1t}^E + x_{1t}^S + x_{2t}^E + x_{2t}^S$. Using (27)–(30), we obtain

$$\Psi_t = (1-s)ANa_t \Delta_t,$$

where

$$\Delta_t = (1-\alpha)\chi - \alpha\phi \frac{1-\xi \left[1 - F(\bar{\delta}_{t+1}) \right]}{z_{t+1}} + \gamma \left[z_t \eta_t + \phi F(\bar{\delta}_t) \right]. \quad (31)$$

The market clearing condition for financial assets depends on the type of financial assets. In a bubbleless equilibrium, $b_{1,t}^i = b_{2,t}^i = 0$ and the financial assets are only deposits and credits. Since the aggregate value of deposits equals the aggregate value of credits in every period, the market clearing condition implies that the aggregate value of the financial assets owned by individuals is zero in every period; that is $\Psi_t = \int_0^{\delta_{\max}} d_{1,t}^i N f(\delta^i) d\delta^i + \int_0^{\delta_{\max}} d_{2,t}^i N f(\delta^i) d\delta^i = 0$ or, equivalently, $\Delta_t = 0$. In contrast, in a bubbly equilibrium, financial assets include the bubble and also deposits and credits. The equality between the aggregate values of deposits and credits implies that the aggregate value of the financial assets equals the value of the bubble, which is positive in a bubbly equilibrium; that is $\Psi_t = \int_0^{\delta_{\max}} b_{1,t}^i N f(\delta^i) d\delta^i + \int_0^{\delta_{\max}} b_{2,t}^i N f(\delta^i) d\delta^i > 0$. We assume that the supply of speculative assets is fixed. In this case, the market clearing condition states that the value of the bubble purchased at $t+1$ by young and adult individuals equals the value of the bubble that in period $t+1$ adult and old individuals sell. These individuals sell the bubble purchased in the previous period, Ψ_t , multiplied by the growth of the price, R_{t+1} . Therefore, the market clearing condition is

$$\Psi_{t+1} = R_{t+1} \Psi_t. \quad (32)$$

Using the definition of the ratio z_t , this market clearing condition can be rewritten as

$$\Delta_{t+1} = z_{t+1} \Delta_t. \quad (33)$$

The aggregate value of financial assets shows the difference between the two sources of external financing. When there is no bubble, the aggregate value of credit limits the amount of savings through financial assets, since $\Psi_t = 0$. In other words, the supply of savings is limited by the demand of savings. The existence of a financial bubble overcomes this limitation, since next generation

purchases of the bubble, Ψ_{t+1} , are an additional source of demand of assets that provides liquidities in equilibrium. To gain more intuition, we can provide an interpretation of (32) in terms of demand and supply of financial assets. Ψ_t can be interpreted as the net supply of assets and it is increasing in R_{t+1} , as follows from (31). Following this interpretation, the net demand of financial assets is Ψ_{t+1}/R_{t+1} when there is a bubble and zero otherwise. The larger demand of financial assets implies a larger return of these assets in the equilibrium with bubbles. We prove this result in the following section.

We use below the two market clearing conditions to define an equilibrium.

Definition 1. An equilibrium of this economy is a path of $\left\{R_t, z_t, \tau_t, \bar{\delta}_t, \Delta_t\right\}_{t=1}^{\infty}$ that, given z_1 , solves the two market clearing conditions (26) and (33), satisfies (22), (25) and (31), and along which the value of the bubble is non-negative, $\Delta_t \geq 0$.

In the online Appendix, we show that the equations characterizing the dynamic equilibrium can be reduced to a system of three equations that determine the transition of a state variable, z_t , and of two control variables, $\bar{\delta}_t$ and Δ_t . In the Appendix, we use this reduced system to determine the stability properties of the different steady states.

At this point, we introduce constraints on the domain of the distribution in order to ensure that at the equilibrium there is a positive number of both savers and entrepreneurs. In particular, we assume that $\bar{\delta}_t \in (0, \delta_{\max})$ so that $\lambda_t \in (0, 1)$. Using (22), these conditions are rewritten as constraints on the return of the financial assets in the following condition:

Condition A. $R_{t+1} \in (0, \bar{R})$ with $\bar{R} = \omega(\delta_{\max}, z) s A \delta_{\max}$.

In the following section, we show that the equilibrium can converge to two different steady states: a bubbly steady state with $\Delta_t > 0$ and a bubbleless one with $\Delta_t = 0$. We show that these steady states satisfy Condition A.

3.2. Steady states

We denote by g^* , R^* , $\bar{\delta}^*$ and z^* the constant growth factor, interest factor, ability of the marginal individual and ratio z at the bubbly steady state and we denote by g^o , R^o , $\bar{\delta}^o$ and z^o the corresponding values of these variables at the bubbleless steady state. In this subsection, we obtain conditions that ensure existence of these two steady states. To this end, we rewrite the two market clearing conditions when $R_t = R$ and $z_t = z$ for all t as two functions relating z with R .

We combine (22) and (26) to obtain the function $z = \varphi(R)$ that describes the market clearing condition for productive capital. The following proposition characterizes this function.

Proposition 1. The pairs R and z for which the market for productive capital clears satisfy the following increasing and continuous function: $z = \varphi(R)$. This function is defined in the domain $R \in (0, \bar{R})$ and satisfies $\varphi(0) = 0$ and $\varphi(\bar{R}) = \infty$.

Proof. See the online appendix.

We next rewrite the market clearing condition for financial assets, (33), as

$$\Delta(1 - z) = 0, \quad (34)$$

and we combine (22) and (31) to obtain that $\Delta = \tilde{\Delta}(R, z)$ with

$$\tilde{\Delta}(R, z) \equiv (1 - \alpha)\chi - \alpha\phi \frac{1 - \xi \left[1 - F(\bar{\delta}(R, z))\right]}{z} + \gamma \left[z\tilde{\eta}(\bar{\delta}(R, z)) + \phi F(\bar{\delta}(R, z))\right]. \quad (35)$$

We use Proposition 1 and (34) to characterize the two steady states. As for the bubbly steady state, since $\Delta > 0$, we deduce, from (34), that $z^* = 1$, which implies that $R^* = g^*$. Therefore, the growth of the bubble equals the growth of wages, which is a well-known result since Tirole (1985) and Grossman and Yanagawa (1993).¹⁵ We next use the market clearing condition for productive capital to obtain that R^* is such that $1 = \varphi(R^*)$. Since $\varphi(R)$ is a continuous and increasing function, $\varphi(0) = 0$ and $\varphi(\bar{R}) = \infty$, there exists a unique R^* that clears the market of productive capital and satisfies Condition A. $R_t = R^*$ and $z_t = 1$ define a bubbly steady state when the aggregate value of the speculative assets owned by individuals is positive; i.e. $\Delta > 0$ when $R_t = R^*$ and $z_t = 1$, which occurs when:

$$\tilde{\Delta}(R^*, 1) > 0. \quad (36)$$

We conclude that there exists a unique bubbly steady state when (36) is satisfied. This condition implies that the supply of savings exceeds the demand of investment in the bubbly steady state.

Regarding the steady state without bubbles, we deduce from (34) that $\tilde{\Delta}(R, z) = 0$. This equation implicitly defines a continuous function $z = \zeta(R)$, along which the value of financial assets is zero. The steady state without bubbles is the solution to equations $z = \zeta(R)$ and $z = \varphi(R)$. In the online Appendix, we use these two functions to study existence and uniqueness of this steady state and we show that this steady state satisfies Condition A. The following proposition summarizes the properties of the two types of steady state.

¹⁵ If $R^* > g^*$ the bubble is not sustainable because the price grows faster than the wage and if $R^* < g^*$ the bubble asymptotically vanishes, implying that the equilibrium converges to the bubbleless steady state.

Proposition 2.

(i) There exists a unique bubbly steady state that satisfies $R^* = g^*$ when condition (36) is satisfied. It is either saddle path stable or locally stable when ξ is sufficiently small.

(ii) There exists a unique bubbleless steady state when ξ is sufficiently small. This steady state is saddle path stable and satisfies $R^o < g^o$ and $R^* > R^o$.

Proof. Results for the bubbly steady state follow from the previous arguments. Existence and uniqueness of the bubbleless steady state are shown in the online appendix. This appendix also studies stability of the two steady states.

While the existence of a steady state without bubbles does not depend on the shape of the function $\zeta(R)$, uniqueness does. In the online appendix, we show that the shape of the function $\zeta(R)$ depends on the value of the start-up cost rate. The function $\zeta(R)$ is downward slopping when ξ is small, which ensures uniqueness of the bubbleless steady state because the function $\varphi(R)$ is upward slopping.

In Proposition 2, we also compare the two steady states when we assume that ξ is small enough so that there is a unique bubbleless steady state. In the online appendix, we show that a small value ξ also implies that $\tilde{A}(R, z)$ increases with z . Since $\tilde{A}(R, z)$ is increasing in z , the value of the financial assets is positive when $z > \zeta(R)$ and zero when $z = \zeta(R)$. Condition (36) implies that at the bubbly steady state $1 > \zeta(R^*)$ and at the bubbleless steady state $z^o = \zeta(R^o)$. These conditions and $\varphi(R)$ being increasing imply that $R^* > R^o$ and $z^o < 1$ when there is a unique bubbleless steady state. This comparison between steady states is shown graphically in Figure 2 of the online appendix.

The results in Proposition 2 imply that the bubble increases both R and z . On the one hand, the larger return of financial assets in the bubbly steady state is a well-known result that is explained because the bubble increases the savings devoted to the demand of financial assets (see, for instance, Farhi and Tirole, 2012). On the other hand, the smaller value of z at the bubbleless steady state implies that $R^o < g^o$. This is also a well-known relation that the bubbleless steady state must satisfy to ensure the existence of a bubbly steady state (see Grossman and Yanagawa, 1993).

Finally, we show that when ξ is sufficiently small, the bubbleless steady state is saddle path stable and the bubbly steady state is either locally stable or saddle path stable. These results imply that the dynamic path can converge to both steady states and, therefore, the particular steady state toward which the economy converges depends on individuals' expectations.

4. Entrepreneurs, growth and productivity

In this section, we investigate the effects of the bubble on the number of entrepreneurs, economic growth, and TFP. We begin by examining the different mechanisms through which the presence of a bubble influences these variables. In the subsequent subsection, we adopt an exponential distribution for innate abilities and calibrate for the US economy to show the importance of the extensive margin of the bubble. Finally, we compare our findings with the results obtained in related studies.

4.1. Extensive and intensive margins of the bubble

We first study the effect of the bubble on the composition of the population between entrepreneurs and savers. This composition effect is determined by the ability of the marginal individual, $\bar{\delta}_t$. As follows from (22), this ability increases with R_{t+1} and decreases with the ratio z_t . Since the bubble increases both R_{t+1} and z_t , the composition effect of the bubble is ambiguous. This ambiguity is the consequence of two opposite mechanisms. On the one hand, in a bubbly steady state the return on financial assets is larger, which implies that more individuals choose to be savers. On the other hand, adult individuals are wealthier with the bubble. As a result, more individuals find affordable the start-up cost. Therefore, the number of entrepreneurs is larger at the bubbly steady state when this cost mechanism dominates. In the following section, we show that this occurs when the start-up cost rate is sufficiently large.

The bubble increases the number of entrepreneurs by reducing the ability of the marginal individual. This implies that new entrepreneurs are less productive than existing ones. Moreover, these new entrepreneurs benefit from a lower labor income and, hence, they invest less. Therefore, new entrepreneurs are less productive and smaller.

We next analyze the effect of the bubble on growth. To this end, we use (26) and the definition of z_t to obtain

$$g_{t+1} = \frac{A\gamma(1-s)}{\chi + \phi} \tilde{\tau}(\bar{\delta}_t, z_t). \quad (37)$$

Eq. (37) shows that the bubble may only increase growth if it enlarges τ_t . In Section 3.1, we have shown that τ_t decreases with $\bar{\delta}_t$ and increases with z_t . Each of these two variables introduces a distinct effect of the bubble. The first variable, $\bar{\delta}_t$, measures the composition effect of the bubble. A larger $\bar{\delta}_t$ reduces the number of entrepreneurs and, as a consequence, capital accumulation and growth decrease. Since the bubble may either increase or decrease the number of entrepreneurs, the composition effect of the bubble on growth is ambiguous. It is positive when the start-up cost rate is large, since the number of entrepreneurs increases in this case.

The second variable, z_t , measures the liquidity effect of the bubble, which has been introduced by Farhi and Tirole (2012). Since part of the labor income is obtained in the first period of life, but investment can only be done in the second, the bubble provides liquidities that increase adult's wealth. As a consequence, adult individuals increase capital accumulation and growth.

The intuition on the liquidity effect is as follows. Adult individuals save, as the aggregate financial assets of adult savers and entrepreneurs satisfy $x_{2,t}^S > 0$ and $x_{2,t}^E = 0$. Since in the bubbleless steady state the value of the aggregate financial assets equals zero, the aggregate financial assets of young individuals must be negative, i.e. $x_{1,t}^S + x_{1,t}^E < 0$. In other words, young individuals borrow from the deposits accumulated by adult savers. These loans are paid back when adult, which limits productive investment. In contrast, in the bubbly steady state, aggregate financial assets are positive. That is, young individuals can hold the bubble and sell it in the following period, even if adult individuals also buy the bubble to postpone consumption. As a consequence, the amount borrowed when young and the amount adult individuals must pay for the credit decline with the bubble. Adult individuals then are wealthier and can invest more in productive capital. In this way, the bubble provides liquidities to adult credit constrained entrepreneurs.

We have seen that the bubble makes adult individuals wealthier through the liquidity effect. As a result, entrepreneurs increase investment and more individuals find affordable the start-up cost and become entrepreneurs. Thus, the aforementioned cost mechanism of the bubble can be interpreted as the extensive margin of the liquidity effect. Following this interpretation, the increase in investment of each entrepreneur corresponds to the intensive margin. We conclude that through these two margins the bubble can increase growth.

Finally, we consider the effect of the bubble on TFP. To obtain TFP, we use (5) and (23) to deduce that $y_{t+1} = TFP_{t+1} \int_{\bar{\delta}_t}^{\delta_{\max}} \kappa_{t+1}^i N f(\delta^i) d\delta^i$, where

$$TFP_{t+1} = A \frac{\int_{\bar{\delta}_t}^{\delta_{\max}} \delta^i \kappa_{t+1}^i f(\delta^i) d\delta^i}{\int_{\bar{\delta}_t}^{\delta_{\max}} \kappa_{t+1}^i f(\delta^i) d\delta^i}.$$

Notice that TFP equals the average return that entrepreneurs obtain per unit invested. It depends on two margins: extensive, related to the number of entrepreneurs, and intensive, which is determined by the amounts invested by each entrepreneur. In what follows, we show how these two margins are affected by the bubble. To this end, we use (3), (19), and (25) to rewrite TFP as

$$TFP_{t+1} = A \frac{\tau_t(\bar{\delta}_t, z_t)}{v_t(\bar{\delta}_t, z_t)}, \quad (38)$$

where

$$v_t = \int_{\bar{\delta}_t}^{\delta_{\max}} [z_t(\delta^i)^v + (1 - \xi)\phi] f(\delta^i) d\delta^i.$$

In the online appendix, we prove that TFP increases with both $\bar{\delta}_t$ and z_t . These two variables measure, respectively, the composition and liquidity effects of the bubble. The composition effect is the extensive margin of the bubble. An increase in $\bar{\delta}_t$ reduces the number of entrepreneurs and, since the remaining ones are more productive, TFP increases. Following the same argument, we assert that the composition effect reduces TFP when the bubble increases the number of entrepreneurs.

The liquidity effect of the bubble affects TFP through the intensive margin. The effect of this margin on TFP has been studied by Miao and Wang (2012) and Hirano and Yanagawa (2017). These authors argue that the liquidity effect of the bubble increases TFP when it increases the investment of more productive entrepreneurs to a larger extent. This is also the case in our model, where the liquidity effect arises because the wealth of entrepreneurs increases with z_t . To understand how the liquidity effect affects TFP, remember that z_t determines the effect that the wages of the young individuals have on the wealth of adult individuals. Therefore, the liquidity effect of the bubble causes a larger increase in investment when the wages of the young individuals are larger. As a result, when $v > 0$ and the wages of the young individuals are correlated with the productivity of investment, the bubble increases to a larger extent the investment of more productive entrepreneurs. This explains that the liquidity effect increases TFP when $v > 0$.

In summary, we have shown that the number of entrepreneurs, growth and productivity can be larger in the steady state with a bubble only when we introduce the start-up cost and wages are correlated with investment productivity. We conclude that these two features are necessary to explain the different facts mentioned in the introduction. To better illustrate the validity of our model in capturing these facts, we present numerical examples in the next section.

4.2. Numerical examples

In this section, we use numerical examples to show that the bubble can increase growth, TFP and the number of entrepreneurs. Although the model is too simple to perform a serious quantitative exercise and the goal of these examples is illustrative only, we introduce some discipline and we set the parameters so that the bubbly steady state of the benchmark economy matches several targets of the US economy in the period 2010–2020. Table 1 summarizes the parameters of the calibration.

In the numerical examples, we first assume that abilities follow an exponential distribution function with density function: $f(\delta) = \sigma e^{-\sigma\delta}$, with $\delta_{\max} = \infty$ and where the parameter $\sigma > 0$ equals the inverse of the mean of abilities. This parameter is set to have in the bubbly steady state an annual growth rate equal to 2.5%. Second, technological parameters, s and A , are set so that the labor income share equals 60%. Third, preference parameters, α , β and γ , are set so that the ratio of consumption expenditure to income equals 84% in the first period of life and to 85% in the second period. Fourth, the labor earnings parameters ϕ and v are set to match labor income differences between the first two period of life and the ratio between average labor income in the third

Table 1
Calibration

Parameters	Values	Targets	Data	Model
A	1	Normalization	–	–
s	0.4	Labor income share ^a	60%	60%
σ	0.064	Annual growth rate ^a	2.5%	2.5%
α	0.3984	Consumption to Inc. ratio of the young ^b	84%	84%
β	0.4441	Consumption to Inc. ratio of the adult ^b	85%	85%
γ	0.1575	Normalization	–	–
ϕ	603.03	Adult to young labor income ^c	1.12	1.12
v	2	Third to first labor income quartile ^c	1.97	1.41
ξ	0.8735	Fraction of entrepreneurs in labor force ^d	10.57%	10.57%

Notes:

^a We obtain the labor income share and the annual growth rate from the PWT 9.1.^b We obtain the consumption expenditure to income ratio from the 2019 US consumption expenditure survey. It is defined as the average value of the ratio between annual expenditures and after tax income for households whose reference person is aged between 25 and 44 years for the young and the reference person is aged between 45 and 64 for the adult.^c Data on the distribution of labor income across age groups and income quartiles is obtained from the US Bureau of Labor Statistics, first quartile of 2020. The income of the age groups is defined as the median income of individuals aged between 25 and 44 for the young and between 45 and 64 for the adult.^d The fraction of entrepreneurs is obtained from the OECD as the ratio between self-employed (both own-account workers and also self-employed who are employers) and total employment. This ratio changes substantially among OECD countries. In the US, this ratio is 10.6 per cent.**Table 2**
Steady state values

Bubbly	Benchmark	Economy 1	Economy 2
$R = g$	1.64	2.00	1.54
λ	10.57%	18.34%	9.11%
TFP	61.93	54.28	64.03
Value bubble	190.61	174.68	194.72
Cost/GDP	28.96%	16.58%	33.15%
Bubbleless			
R	0.51	0.73	0.46
g	0.92	1.28	0.83
λ	10.43%	22.50%	8.50%
TFP	61.85	50.56	64.86
Comparison			
g^*/g^o	178.63%	156.12%	185.19%
λ^*/λ^o	101.30%	81.50%	107.07%
TFP^*/TFP^o	100.13%	107.36%	98.71%

Note. Benchmark: $\xi = 0.87$; Economy 1: $\xi = 0.5$; Economy 2: $\xi = 1$.

and first quartiles. Notice that the model does not generate the large inequalities observed in labor income. Finally, ξ is set to have in the bubbly steady state a fraction of the entrepreneurs in the labor force of 10.57%. The implied start-up cost is 29% of the GDP per capita.¹⁶

Table 2 shows the values of the return of financial assets, growth rate, fraction of entrepreneurs and productivity in the bubbly and bubbleless steady states. From the comparison between the two steady states of the benchmark economy, it follows that growth and the return of financial assets are clearly larger with bubbles and both the number of entrepreneurs and TFP are slightly larger in the bubbly steady state. Thus, in the benchmark economy, bubbles increase the number of entrepreneurs, growth and TFP, which is consistent with the empirical findings mentioned in the introduction. In addition, the large change in the growth rate compared to the small changes in TFP and number of entrepreneurs is also consistent with evidence. For instance, the US economy between 2007 and 2009 suffers a reduction in GDP growth, TFP and number of firms of 94%, 0.54% and 3%, respectively.¹⁷ The benchmark economy generates a change between the two steady states of 79%, 0.13% and 1.3% in these same variables. Clearly, the change in the variables is of similar magnitude.

In Table 2, we compare the benchmark economy with two different counterfactual economies to show the importance of the composition effect or extensive margin of the bubble. First, Economy 1 differs from the benchmark economy in the value of the start-up cost rate, which is substantially smaller. As a result, compared to the benchmark economy, there are more entrepreneurs at both steady states. This larger amount of entrepreneurs explains the larger growth and the smaller TFP in both steady states.

¹⁶ Start-up cost is measured as a bureaucratic cost by Klapper et al. (2006). They report huge differences between countries ranging from 0.5% to 81% of per capita GDP.

¹⁷ These growth rates are obtained using the variables defined in Fig. 1.

Moreover, the reduction in the start-up cost rate weakens the cost mechanism, which explains that in Economy 1 there are more entrepreneurs in the bubbleless steady state than in the bubbly one. As a consequence, the composition effect of the bubble reduces the differences between the growth rates of the bubbly and the bubbleless steady states, whereas it increases substantially the differences in TFP. In Economy 2, the start-up cost rate is larger than in the benchmark economy. This larger cost rate explains the smaller fraction of entrepreneurs, which reduces growth and increases TFP in both steady states. In this economy, the cost mechanism is more intense than in the benchmark and, as a consequence, the fraction of entrepreneurs is substantially larger in the bubbly steady state than in the bubbleless one. This implies that the composition effect of the bubble is large and, as a result, TFP is smaller in the bubbly steady state. These numerical examples show that the composition effect may be sizeable and determine the overall effect that bubbles have on growth and productivity.

4.3. Comparison with the literature

We next compare the findings in this paper with the results obtained by the literature that studies the effects of financial bubbles in models with heterogeneous individuals. To organize this comparison, we distinguish three groups of models: (i) models with two exogenous groups of individuals: savers and entrepreneurs; (ii) with two exogenous groups of entrepreneurs: high and low ability entrepreneurs; and (iii) with endogenous composition of the population between savers and entrepreneurs.

(i) Savers and entrepreneurs

Martin and Ventura (2012), Farhi and Tirole (2012) and Raurich and Seegmuller (2019) among many others have considered models in which the population is divided in two constant groups of individuals. The model of Section 2 can be adapted to this setting by assuming that the distribution function of δ is discrete and has the following properties: a constant fraction λ of individuals has a high ability, δ^H , and the rest, $1 - \lambda$, has low ability, δ^L . Moreover, we assume that the support of the distribution satisfies $\delta^L < R_{t+2}/(\omega sA) < \delta^H$. This assumption implies that in equilibrium individuals with low ability will be savers and individuals with high ability will be entrepreneurs. Therefore, the fraction of entrepreneurs in the population is constant, it is equal to the parameter λ , and it is not affected by the bubble. In other words, the bubble does not generate the composition effect.

Using (38), we obtain that $TFP = A\delta^H$. Therefore, since all entrepreneurs are identical, TFP is constant and it is not affected by the bubble. Moreover, using (37), we obtain that the ratio between the growth rate in the bubbly and in the bubbleless steady states equals

$$\frac{g^*}{g^o} = \frac{(\delta^H)^v + \phi(1 - \xi)}{z^o (\delta^H)^v + \phi(1 - \xi)} > 1.$$

The ratio of growth rates is larger than one because $z^o < 1$. This positive growth effect of the bubble is due to the liquidity effect, which is reinforced by a high value of $(\delta^H)^v$. Therefore, in models that divide the population into two constant and homogeneous groups of individuals, the bubble increases growth. However, these models do not explain that TFP and entrepreneurship increase with the bubble.

(ii) High and low ability entrepreneurs

Miao and Wang (2012) and Hirano and Yanagawa (2017) consider two constant groups of entrepreneurs with different ability to account for the effects that the bubble may have on TFP through the intensive margin.¹⁸ The model of Section 2 can also be adapted to this context. To this end, we assume again that the distribution function of δ is discrete and has the following properties: a constant fraction λ^L of individuals has low ability, δ^L , a constant fraction λ^M has a middle ability, δ^M , and the rest, λ^H , has high ability, δ^H . Obviously, $\lambda^L + \lambda^M + \lambda^H = 1$. We also assume that $\delta^L < R_{t+2}/(\omega sA) < \delta^M$, which implies that low ability individuals are savers and the rest, middle and high ability individuals, are entrepreneurs.

As occurs in models with identical entrepreneurs, the bubble increases growth and the fraction of entrepreneurs in the population is constant, equal to $\lambda^M + \lambda^H$, and it is not affected by the bubble. However, since there are two different groups of entrepreneurs, the bubble increases TFP through the intensive margin. To see this, we use (38) to obtain that TFP equals

$$TFP_{t+1} = A \frac{\lambda^M \delta^M [z_t (\delta^M)^v + (1 - \xi)\phi] + \lambda^H \delta^H [z_t (\delta^H)^v + (1 - \xi)\phi]}{\lambda^M [z_t (\delta^M)^v + (1 - \xi)\phi] + \lambda^H [z_t (\delta^H)^v + (1 - \xi)\phi]}.$$

It is immediate to see that TFP increases with z_t when $v > 0$ and it does not depend on z_t when $v = 0$. Therefore, the bubble increases TFP through the liquidity effect when $v > 0$.

Models with two groups of entrepreneurs can explain that growth and productivity are larger with bubbles. However, they do not consider the composition effect of the bubble, which can be sizeable, according to the numerical examples.

(iii) Endogenous composition of the population

Kunieda and Shibata (2016) consider a model with a continuous distribution function of abilities to study the effect of the bubble on growth when the composition of the population between savers and entrepreneurs is endogenous. The model of Section 2 is adapted to their setting when $\xi = 0$ and $v = 0$.

¹⁸ In an appendix, Hirano and Yanagawa (2017) also consider a continuous distribution of abilities to show that their findings about the relation between growth and the interest rate remain when the distribution function is continuous.

A first difference with this paper is that they do not introduce the start-up cost. As a consequence, the number of entrepreneurs is smaller in the bubbly steady state. Therefore, as follows from (25) and (37), the bubble causes two opposite effects on growth. First, the smaller number of entrepreneurs reduces capital accumulation and growth. This is the composition effect of the bubble. Second, the bubble still has a growth enhancing liquidity effect. Thus, the bubble promotes growth when the reduction in the number of entrepreneurs is not too large. Finally, TFP simplifies as follows

$$TFP_{t+1}(\bar{\delta}_t) = A \frac{\int_{\bar{\delta}_t}^{\delta_{\max}} \delta^i f(\delta^i) d\delta^i}{\int_{\bar{\delta}_t}^{\delta_{\max}} f(\delta^i) d\delta^i}.$$

Since $v = 0$, the liquidity effect of the bubble does not modify TFP. Therefore, TFP only depends on the composition effect. Given that in this economy the bubble reduces the number of entrepreneurs, TFP increases.

Our contribution lies in showing that the bubble can effectively increase the number of entrepreneurs when we introduce the start-up cost. By incorporating this factor, we are able to explore the composition effect of the bubble, which introduces a novel channel through which the bubble influences growth and productivity.

5. Concluding remarks

Entrepreneurship, growth and TFP increase when there is a financial bubble and decline during financial crises. We explain these facts as the result of a transition between two steady states: a steady state without bubbles, in which financial assets consists only of deposits, and another one with bubbles, in which financial assets also include a pure speculative asset.

We show that the aforementioned facts can be explained in an overlapping generations growth model populated by heterogeneous individuals that live for three periods. We distinguish three sources of heterogeneity. First, individuals are heterogeneous in the return of productive investment. This heterogeneity separates individuals in two groups: savers and entrepreneurs. Savers only invest in financial assets, whereas entrepreneurs invest in productive capital. A novelty of this paper is the introduction of a start-up cost that entrepreneurs must pay. The bubble changes the composition of the population between savers and entrepreneurs. On the one hand, the bubble increases the return of financial assets, which increases the amount of savers. On the other hand, the bubble makes adult individuals wealthier. As a result, more individuals are willing to pay the cost and become entrepreneurs. We show that the bubble increases the number of entrepreneurs when this cost mechanism dominates.

Second, workers are heterogeneous because individuals work both when young and adult. Since investment can only be done when adult and part of the labor income is obtained when young, the bubble provides liquidities to credit constrained entrepreneurs. This is the liquidity effect of the bubble that increases growth. It operates through two channels: the intensive margin, which involves increasing the investment made by each entrepreneur, and the extensive margin, which involves expanding the number of entrepreneurs. The extensive margin of the liquidity effect represents a significant contribution of this paper, shedding new light on the effect of bubbles on economic growth.

Third, wages are heterogeneous among workers of the same generation. This is another novelty of this paper. We show that this heterogeneity is necessary to explain that TFP is larger with bubbles. In particular, we show that the liquidity effect of the bubble increases TFP when the productivity of investment is correlated with the wages of the young workers. In this case, the bubble further increases the investment of more productive entrepreneurs, which causes the increase in TFP.

We conclude that the model explains the aforementioned facts when we introduce two assumptions: a large start-up cost rate and a positive correlation between the productivity of investment and the wages of the young workers. The first assumption introduces a wealth effect, which is needed to explain that entrepreneurship is larger with the bubble. Hence, we claim that the positive effect of bubbles on entrepreneurship does not depend on the particular functional form of the start-up cost, but on the introduction of a wealth effect.

The second assumption is needed to explain a larger TFP with the bubble. This assumption ensures that more productive entrepreneurs obtain a larger income in the first period. In other words, it ensures that the idiosyncratic shock on abilities is persistent. This suggests that similar results could be obtained in other settings in which shock persistence can be introduced. As an example, we could consider a version of the model with bubble shocks introduced by [Martin and Ventura \(2012\)](#) and assume that the most productive entrepreneurs receive a large shock. Since bubble shocks provide liquidities, these more productive entrepreneurs would benefit more of the liquidity provided by the bubble. As a result, the bubble would further increase the investment of more productive entrepreneurs. This example could be an alternative model that, using the same mechanisms, explains the positive effects of bubbles on TFP.

The bubble affects growth by introducing outside liquidity. There are other assets that also introduce outside liquidity. An important example is public debt, which introduces additional demand for savings and outside liquidity enabling young individuals to save even when adults are also saving. These similarities with the bubble suggest that public debt may have similar effects on the number of entrepreneurs, growth and productivity. However, there are significant differences between public debt and bubbles. First, public debt is a stock, whereas a bubble is a forward-looking variable. In addition, public debt raises several relevant issues concerning the government decisions on financing and utilization of public debt, which may alter its effects in this model.

Data availability

Data will be made available on request.

Acknowledgments

Xavier Raurich thanks financial support from the Spanish and Catalan Governments through grants PID2021-126549NB-I00 and SGR-00862. Thomas Seegmuller thanks the financial support of the french government under the “France 2030” investment plan managed by the French National Research Agency Grant ANR-17-EURE-0020 and by the Excellence Initiative of Aix-Marseille University - A*MIDEX, and the financial support of the French National Research Agency Grant ANR-15-CE33-0001-01. This paper benefits from the comments of Anpeng Lee, of the participants of seminars at the Dongbei University, the University of Guanajuato, the University of Porto and of the participants in the Financial and Real Interdependencies 2021 workshop, the Africa 2021 meeting of the Econometric Society, the LAGV 2021 conference, the SAET 2021 meeting, the ASSET 2021 conference and the International Conference on Macroeconomic Analysis and International Finance 2022.

Appendix A. Online appendix

The appendix to this article can be found online at <https://doi.org/10.1016/j.jmacro.2024.103622>.

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