

Multi-component dark matter from Minimal Flavor Violation

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ABSTRACT: Minimal Flavor Violation (MFV) offers an appealing framework for exploring physics beyond the Standard Model. Interestingly, within the MFV framework, a new colorless field that transforms non-trivially under a global $SU(3)^3$ quark flavor group can naturally be stable. Such a new field is thus a promising dark matter candidate, provided it is electrically neutral. We extend the MFV framework for dark matter and demonstrate that dark matter can naturally be multi-component across a broad parameter space. For illustration, we consider a gauge singlet, flavor triplet scalar field and identify parameter spaces for multi-component dark matter, where only the lightest flavor component is absolutely stable and heavy flavor components are decaying with lifetimes sufficiently longer than the age of the universe. Phenomenological, cosmological and astrophysical aspects of multi-component flavored dark matter are briefly discussed.

KEYWORDS: Flavour Symmetries, Models for Dark Matter, Particle Nature of Dark Matter, Theories of Flavour

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1 Introduction

Matter content of the Standard Model (SM) comprises five different gauge representations of Weyl fermions, called quarks and leptons. In each representation, there exist three species, or flavors. The SM gauge interactions do not distinguish these three fermion flavors in the same representation, leading to a global $U(3)^5$ flavor symmetry in the gauge sector. This flavor symmetry is explicitly broken by quark and lepton Yukawa interactions to the Higgs doublet field. In particular, the breaking of the $SU(3)^5$ subgroup of $U(3)^5$ governs mixing patterns among different flavors, thereby introducing non-trivial flavor violating processes

at low energies. Flavor violation in the quark sector is characterized by the hierarchical quark masses and the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, whose unique mixing pattern has been confirmed experimentally with a good accuracy. The lepton sector does not exhibit any flavor mixing due to the absence of neutrino masses in the SM. The global $SU(3)_{\ell_R} \times SU(3)_{e_R}$ lepton flavor symmetry is broken down to $U(1)_{L_e-L_\mu} \times U(1)_{L_\mu-L_\tau}$ by the lepton masses.

New interactions from physics beyond the SM can generally provide independent sources of flavor violation. The resulting modifications to flavor violating observables are faced with current precise measurements, if one expects new particles mediating the flavor violation to reside around TeV scales, as motivated by the naturalness problem. This strong flavor constraints can be circumvented by invoking the Minimal Flavor Violation (MFV) hypothesis [1–4], which dictates that new physics interactions also respect the $U(3)^5$ flavor symmetry with the only breaking sources stemming from the quark and lepton Yukawa matrices, $Y_{u,d,e}$. Formally, the MFV interaction structure can be achieved by promoting the Yukawa matrices to spurion fields transforming under the flavor group, $U(3)^5 = U(3)_{q_L} \times U(3)_{u_R} \times U(3)_{d_R} \times U(3)_{\ell_L} \times U(3)_{e_R}$:

$$Y_u \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \quad Y_d \sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}), \quad Y_e \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}). \quad (1.1)$$

This transformation rule assigned to the Yukawa matrices assures the (apparent) flavor invariance of the SM Yukawa interaction Lagrangian,

$$\mathcal{L}_{\text{yuk}} = -\bar{q}_L Y_u \tilde{H} u_R - \bar{q}_L Y_d H d_R - \bar{\ell}_L Y_e H e_R + \text{h.c.}, \quad (1.2)$$

with $\tilde{H} = i\sigma_2 H^*$. Implementing the MFV structure in new physics models is straightforward. For a new interaction operator $\mathcal{O}_{ij\dots}$ (i, j, \dots denote flavor indices), its coupling $C_{ij\dots}$ is parameterized by a series of spurion insertions so that the corresponding interaction is invariant under the flavor transformation. For example, when we consider a flavor violating operator $\mathcal{O}_{ij} = (\bar{u}_{Ri} \gamma^\mu u_{Rj})(X^\dagger i \overleftrightarrow{\partial}_\mu X)$ with a gauge and flavor singlet scalar X , the MFV requires C_{ij} to take the form,

$$C_{ij} = c_0 \delta_{ij} + \epsilon c_1 (Y_u^\dagger Y_u)_{ij} + \epsilon^2 \left[c_2 (Y_u^\dagger Y_u Y_u^\dagger Y_u)_{ij} + c'_2 (Y_u^\dagger Y_d Y_d^\dagger Y_u)_{ij} \right] + \dots, \quad (1.3)$$

where the ellipsis denotes further spurion insertions. Flavor violating effects from this new interaction are suppressed by the power of the quark Yukawa couplings, the CKM off-diagonal elements and a potentially small MFV expansion parameter ϵ .¹

Remarkably, the MFV in new physics models can guarantee the stability of dark matter (DM). It is shown in [6] that within the MFV framework, the lightest state of a new colorless field χ that transforms under the quark flavor subgroup, i.e. $\mathcal{G}_F = SU(3)_{q_L} \times SU(3)_{u_R} \times SU(3)_{d_R}$, is absolutely stable, even if including all higher dimensional operators. That lightest particle is therefore an excellent DM candidate if electrically neutral. This stability discussion relies only on the invariance under the color and flavor groups within the MFV and does not depend on spin and $SU(2)_L \times U(1)_Y$ representation of χ . In [6], they focus on a gauge singlet

¹Here we implicitly assume $\epsilon \ll 1$ and the MFV structure is linearly realized. The MFV implementation can be generalized to $\epsilon \sim \mathcal{O}(1)$ case in which all spurion insertions are equally contributing and have to be resummed appropriately, rendering the MFV non-linearly realized [5].

scalar DM case and study cosmological and phenomenological implications, ranging from the conventional freeze-out production to characteristic collider signals as well as effects on flavor changing neutral current (FCNC) processes. The study of [6] was followed up in detail by [7], where they surveyed the traditional Weakly Interacting Massive Particle (WIMP) DM parameter space for the simplest singlet scalar.

In this paper, we demonstrate that within the MFV framework, DM can naturally be multi-component in certain parameter spaces. Although this possibility was very briefly mentioned in [6], no follow-up studies in this direction have been published thus far. We focus on a model featuring an $SU(2)_L \times U(1)_Y$ singlet flavored scalar field, and evaluate lifetimes of heavy flavor components. We then identify parameter spaces where more than one flavor component has sufficient longevity to serve as DM. Our work is supplemented by studying DM production and direct detection bounds and by giving general comments on phenomenological and cosmological aspects of multi-component flavored DM scenarios.

This paper is organized as follows. We review in section 2 the formulation of the DM stabilization in new physics models based on the MFV principle. An example model we focus on in this paper is explicitly introduced in section 3. Then, in section 4, we evaluate decay widths of heavier states into lighter states and identify parameter spaces where DM can comprise multiple states. In sections 5 and 6, we present our main results and discuss the outlook for phenomenological works in future. In appendices, we provide calculation tools to study multi-body decays and DM production and direct detection.

2 Stability of flavored dark matter

To formulate the DM stability under the MFV, let χ be a singlet of $SU(3)_c$ and a multiplet of $\mathcal{G}_F = SU(3)_{q_L} \times SU(3)_{u_R} \times SU(3)_{d_R}$. The representation of χ under \mathcal{G}_F is specified by the Dynkin coefficients (n_i, m_i) of the corresponding $SU(3)_i$ flavor groups:

$$\chi \sim (n_{q_L}, m_{q_L}) \times (n_{u_R}, m_{u_R}) \times (n_{d_R}, m_{d_R}), \quad (2.1)$$

where we do not specify the spin and $SU(2)_L \times U(1)_Y$ representation of χ , which are irrelevant to the stability discussion. General decay vertices of χ into SM fields formally take the form,

$$\mathcal{O}_{\text{decay}} = \chi \times \underbrace{q_L \dots}_A \underbrace{\bar{q}_L \dots}_{\bar{A}} \underbrace{u_R \dots}_B \underbrace{\bar{u}_R \dots}_{\bar{B}} \underbrace{d_R \dots}_C \underbrace{\bar{d}_R \dots}_{\bar{C}} \underbrace{Y_u \dots}_D \underbrace{Y_u^\dagger \dots}_{\bar{D}} \underbrace{Y_d \dots}_E \underbrace{Y_d^\dagger \dots}_{\bar{E}} \times \mathcal{O}_{\text{weak}}, \quad (2.2)$$

where $\mathcal{O}_{\text{weak}}$ denotes a potential weak operator having no color nor flavor to make $\mathcal{O}_{\text{decay}}$ invariant under the Lorentz and $SU(2)_L \times U(1)_Y$ transformation. The color and quark flavor invariance of $\mathcal{O}_{\text{decay}}$ requires that the triality of each $SU(3)$ group vanishes, i.e.

$$(A + B + C - \bar{A} - \bar{B} - \bar{C}) \bmod 3 = 0, \quad (2.3)$$

$$(n_{q_L} - m_{q_L} + A - \bar{A} + D - \bar{D} + E - \bar{E}) \bmod 3 = 0, \quad (2.4)$$

$$(n_{u_R} - m_{u_R} + B - \bar{B} - D + \bar{D}) \bmod 3 = 0, \quad (2.5)$$

$$(n_{d_R} - m_{d_R} + C - \bar{C} - E + \bar{E}) \bmod 3 = 0, \quad (2.6)$$

which in turn requires the *flavor triality* of χ to vanish: $(n_\chi - m_\chi) \bmod 3 = 0$ where $n_\chi = n_{q_L} + n_{u_R} + n_{d_R}$ and $m_\chi = m_{q_L} + m_{u_R} + m_{d_R}$. In other words, if we choose a flavor representation for χ such that the flavor triality is non-vanishing, i.e.

$$(n_\chi - m_\chi) \bmod 3 \neq 0, \quad (2.7)$$

then $\mathcal{O}_{\text{decay}}$ is forbidden and χ is absolutely stable [6]. If the lightest state of χ is neutral, it is a good DM candidate. It should be noted that we did not restrict the mass dimension of $\mathcal{O}_{\text{decay}}$ and hence this stability discussion can apply for all higher dimensional operators.

After that pioneering work [6], various flavored DM models have been studied. In [7], they scan the conventional WIMP regime of the simplest flavored scalar DM, originally proposed in [6], and evaluate the impact of Higgs portal couplings to the DM phenomenology. In [8], they find out general features of supersymmetric flavored DM models as well as provide a deeper insight into the role of the flavor symmetries in the DM stability. In that paper, it is emphasized that the MFV is sufficient but not necessary for the DM stability, and the most essential is the flavor triality condition eq. (2.7). In fact, [9] shows that in a class of new physics models, even though the MFV is not respected, a new flavored state can have the absolute stability as long as the flavor triality condition is fulfilled. See also [10], where the MFV is crucial for accidental longevity of asymmetric DM. The concept of flavored DM was extended to incorporate a dark flavor symmetry $\text{SU}(3)_\chi$ under which a DM field is charged while all SM fields are not [11]. This extended framework abandons the MFV principle and necessitates an additional global symmetry imposed by hand to guarantee the DM stability, but predicts richer flavor phenomenology. (See [12–14] for related studies on DM carrying a flavor charge, where the DM stability is not necessarily attributed to the MFV.) Currently, the terminology of flavored DM mainly points to the extended framework, but in this paper we build on the original MFV framework in [6].

In general, heavy states of χ can decay into lighter ones. If all heavy states decay quickly, only the lightest one is stable and DM. The simplest case of such a single component flavored DM is studied in [6, 7]. On the other hand, some heavy states can constitute part of cosmological DM if long-lived enough. Lifetimes of heavy states will depend on several factors, such as mass splitting with the lightest state, interaction operators triggering decay, and cutoff scales if heavy states are decaying mostly due to higher dimensional operators. In the following sections, we will take an example model and show that more than one component of a flavored new field can be stable and constitute a significant portion of DM in the universe.

3 Model

We consider a gauge singlet scalar field S , which transforms like $(\mathbf{1}, \mathbf{3}, \mathbf{1})$ under the quark flavor group \mathcal{G}_F . The choice of the flavor representation for S is different from the one studied in [6, 7], but it is irrelevant to our main conclusion. Under the MFV hypothesis, all mass and interaction terms respect the \mathcal{G}_F symmetry with the only breaking sources from the quark Yukawa matrices. The general interaction Lagrangian takes the form

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + (\partial_\mu S_i^*)(\partial_\mu S_i) - V(H, S) + \mathcal{L}_{d>4}, \quad (3.1)$$

where $i = 1, 2, 3$ is the flavor index and $\mathcal{L}_{d>4}$ denotes higher dimensional operators composed of SM fields and S . In this section, we provide renormalizable interactions of the flavored

scalar field S and a set of dimension-6 operators involving two flavored scalar fields, which induce decays of heavy flavor components.

3.1 Renormalizable interactions

The scalar potential takes the form,

$$\begin{aligned} V(H, S) = & m_S^2 S_i^* \left(a_0 \delta_{ij} + \epsilon a_1 (Y_u^\dagger Y_u)_{ij} + \dots \right) S_j \\ & + \lambda S_i^* \left(b_0 \delta_{ij} + \epsilon b_1 (Y_u^\dagger Y_u)_{ij} + \dots \right) S_j (H^\dagger H) \\ & + \left(\lambda_0 \delta_{ij} \delta_{kl} + \epsilon \lambda_1 \delta_{ij} (Y_u^\dagger Y_u)_{kl} + \dots \right) S_i^* S_j S_k^* S_l, \end{aligned} \quad (3.2)$$

where the flavor indices run over $i, j = 1, 2, 3$, λ_i are all real parameters, ϵ is a small MFV expansion parameter, a_0, a_1, b_0, b_1 are $\mathcal{O}(1)$ coefficients, and the ellipsis indicates further MFV spurion insertions involving four or more Yukawa matrices, which we neglect here. Effects of those higher order terms will be discussed in section 4.4.

Without loss of generality, the up-quark Yukawa matrix is expressed as $(Y_u)_{ij} = (V^\dagger \hat{Y}_u)_{ij}$ with $(\hat{Y}_u)_{ij} = y_u^i \delta_{ij}$ and V the CKM matrix. After the electroweak (EW) symmetry breaking, the physical masses and Higgs portal couplings of S_i are expressed by

$$V(H, S) \supset M_i^2 (S_i^* S_i) + \frac{\lambda_{hS_i}}{2} (2vh + h^2) (S_i^* S_i), \quad (3.3)$$

where $v = 246 \text{ GeV}$ and

$$M_i^2 = m_i^2 + \frac{\lambda_{hS_i} v^2}{2}, \quad (3.4)$$

$$m_i^2 = m_S^2 \left(a_0 + \epsilon a_1 (y_u^i)^2 \right), \quad (3.5)$$

$$\lambda_{hS_i} = \lambda \left(b_0 + \epsilon b_1 (y_u^i)^2 \right). \quad (3.6)$$

The mass square difference of the flavored scalars is determined by the up-quark Yukawa couplings,

$$M_j^2 - M_i^2 = \epsilon \left(a_1 m_S^2 + b_1 \frac{\lambda v^2}{2} \right) \left\{ (y_u^j)^2 - (y_u^i)^2 \right\}. \quad (3.7)$$

It follows from this equation that the ratio of the mass square difference is sharply predicted

$$\frac{M_3^2 - M_1^2}{M_2^2 - M_1^2} = \frac{y_t^2 - y_u^2}{y_c^2 - y_u^2} \simeq \frac{y_t^2}{y_c^2}. \quad (3.8)$$

Since the sign of a_1, b_1, λ is arbitrary, the mass ordering of the flavor components S_i is not fixed from the MFV assumption, and the mass spectrum can be either *normal* ($M_1 < M_2 < M_3$) or *inverted* ($M_3 < M_2 < M_1$).

A notable feature of the scalar potential eq. (3.2) is that there is no flavor off-diagonal interaction for S_i , if the MFV expansion is truncated at the order of ϵ . All three scalars are thus individually stable at this order. This threefold stability is broken to the stability of the lightest flavored scalar once including higher order terms in the MFV expansion, see section 4.4 for further details. However, the first flavor off-diagonal vertices appear in the scalar potential at the order of ϵ^2 , and by taking a small ϵ , one can assure sufficient longevity for the heavy scalars to serve as DM.

Before proceeding, we would like to mention theoretical constraints on the scalar potential. We require the potential to have a global minimum at $\langle H \rangle \neq 0$ and $\langle S_i \rangle = 0$, since a non-vanishing vacuum expectation value (VEV) of S_i breaks the flavor symmetry and triggers instability of DM. This requirement also implicitly imposes a bounded-from-below condition, which is read from the quartic terms in the potential,

$$\begin{aligned} V|_{\text{quartic}} = & \lambda_H |H|^4 + \lambda_0 \left(|S_1|^2 + |S_2|^2 \right)^2 + \left(\lambda_0 + \lambda_1 y_t^2 \right) |S_3|^4 \\ & + \lambda_{hS1} |H|^2 \left(|S_1|^2 + |S_2|^2 \right) + \lambda_{hS3} |H|^2 |S_3|^2 \\ & + \left(2\lambda_0 + \lambda_1 y_t^2 \right) \left(|S_1|^2 + |S_2|^2 \right) |S_3|^2, \end{aligned} \quad (3.9)$$

where small yukawa couplings $y_{u,c}$ are ignored. This potential can be written as

$$V|_{\text{quartic}} = \sum_{i,j} \Lambda_{ij} X_i X_j, \quad (3.10)$$

where $X_i \equiv \{|H|^2, |S_1|^2 + |S_2|^2, |S_3|^2\}$ and

$$\Lambda_{ij} \equiv \begin{pmatrix} \lambda_H & \lambda_{hS1}/2 & \lambda_{hS3}/2 \\ \lambda_{hS1}/2 & \lambda_0 & (\lambda_0 + \lambda_t)/2 \\ \lambda_{hS3}/2 & (\lambda_0 + \lambda_t)/2 & \lambda_t \end{pmatrix}, \quad (3.11)$$

with

$$\lambda_t \equiv \lambda_0 + \lambda_1 y_t^2. \quad (3.12)$$

Then, it follows from co-positivity criteria [15] that the bounded-from-below condition is fulfilled if and only if the following inequalities are all satisfied:

$$\lambda_H > 0, \quad \lambda_0 > 0, \quad \lambda_t > 0, \quad (3.13)$$

$$\bar{\Lambda}_{12} := \lambda_{hS1}/2 + \sqrt{\lambda_H \lambda_0} > 0, \quad (3.14)$$

$$\bar{\Lambda}_{13} := \lambda_{hS3}/2 + \sqrt{\lambda_H \lambda_t} > 0, \quad (3.15)$$

$$\bar{\Lambda}_{23} := (\lambda_0 + \lambda_t)/2 + \sqrt{\lambda_0 \lambda_t} > 0, \quad (3.16)$$

and

$$\sqrt{\lambda_H \lambda_0 \lambda_t} + \frac{\lambda_{hS1}}{2} \sqrt{\lambda_t} + \frac{\lambda_{hS3}}{2} \sqrt{\lambda_0} + \frac{\lambda_0 + \lambda_t}{2} \sqrt{\lambda_H} + \sqrt{2\bar{\Lambda}_{12}\bar{\Lambda}_{13}\bar{\Lambda}_{23}} > 0. \quad (3.17)$$

We have confirmed that these conditions are all satisfied in parameter spaces we focus on in this paper.

3.2 Dimension-6 operators

Three flavored scalars $S_{1,2,3}$ are individually stable with the scalar potential eq. (3.2) unless taking into account higher order terms in the ϵ expansion. However, inclusion of higher dimensional operators causes the heavy scalars to decay into the lighter ones even at the

leading order of ϵ . Of the most relevance are dimension-6 operators involving two quarks and two flavored scalar fields S_i , given by

$$\mathcal{L}_{d=6} = \frac{1}{\Lambda^2} \sum_I c_{ijkl}^I \mathcal{O}_{ijkl}^I, \quad (3.18)$$

where

$$\begin{aligned} \mathcal{O}_{ijkl}^1 &= (\bar{q}_{Li} \gamma^\mu q_{Lj}) (S_k^* i \overleftrightarrow{\partial}_\mu S_l), & \mathcal{O}_{ijkl}^2 &= (\bar{u}_{Ri} \gamma^\mu u_{Rj}) (S_k^* i \overleftrightarrow{\partial}_\mu S_l), \\ \mathcal{O}_{ijkl}^3 &= (\bar{d}_{Ri} \gamma^\mu d_{Rj}) (S_k^* i \overleftrightarrow{\partial}_\mu S_l), & \mathcal{O}_{ijkl}^4 &= (\bar{q}_{Li} \tilde{H} u_{Rj}) (S_k^* S_l), \\ \mathcal{O}_{ijkl}^5 &= (\bar{q}_{Li} H d_{Rj}) (S_k^* S_l). \end{aligned} \quad (3.19)$$

The coefficients c_{ijkl}^I are expanded with respect to the quark Yukawa matrices following the MFV, and for example, we have for \mathcal{O}_{ijkl}^4

$$\begin{aligned} c_{ijkl}^4 &= c_1 (Y_u)_{il} \delta_{kj} + c_2 (Y_u)_{ij} \delta_{kl} \\ &+ \epsilon \left[c_3 (Y_u Y_u^\dagger Y_u)_{ij} \delta_{kl} + c_4 (Y_u Y_u^\dagger Y_u)_{il} \delta_{kj} + c_5 (Y_u)_{ij} (Y_u^\dagger Y_u)_{kl} + c_6 (Y_u)_{il} (Y_u^\dagger Y_u)_{jl} \right] \\ &+ \mathcal{O}(\epsilon^2). \end{aligned} \quad (3.20)$$

Here, we expect $\epsilon \ll 1$ and ignore higher order terms until section 4.4.

Let us focus on the \mathcal{O}_{ijkl}^4 operator at the order of ϵ^0 ,

$$\mathcal{L}_{d=6} = \frac{1}{\Lambda^2} \left[c_1 \left(\bar{q}_{Li} (V^\dagger \hat{Y}_u)_{ij} S_j \right) \tilde{H} (S_k^* \delta_{kl} u_{Rl}) + c_2 \left(\bar{q}_{Li} (V^\dagger \hat{Y}_u)_{ij} \tilde{H} u_{Rj} \right) (S_k^* \delta_{kl} S_l) \right] + \text{h.c.} \quad (3.21)$$

After the EW symmetry breaking and taking the up-type quark mass basis (i.e. $u_L \rightarrow V^\dagger u_L$), this Lagrangian reduces to

$$\mathcal{L}_{d=6} = \frac{1}{\Lambda^2} \left[c_1 \bar{u}_i (m_i P_R + m_j P_L) u_j (S_j^* S_i) + c_2 (m_i \bar{u}_i u_i) (S_j^* S_j) \right], \quad (3.22)$$

where $u_{i,j}$ denotes the up-quark fields in the mass basis. It is easy to see that the c_2 term does not induce decay of heavy scalars, since it only produces flavor diagonal interactions like $(\bar{u}_i u_i) (S_j^* S_j)$. In contrast, the c_1 interactions cause heavy scalar decays, whose partial decay width scales as

$$\Gamma(S_i \rightarrow S_j u_{Li} \bar{u}_{Rj}) \sim \left(\frac{c_1}{\Lambda^2} \right)^2 \left\{ (m_u^i)^2 + (m_u^j)^2 \right\} \times \int d\Phi_3, \quad (3.23)$$

where $d\Phi_3$ denotes three-body phase space. In the case of the normal spectrum, S_2 and S_3 are unstable and decaying. S_2 is expected to decay into S_1 with a very suppressed rate because of the mass degeneracy between S_2 and S_1 , whereas S_3 has a moderately large mass splitting and relatively easily decays into S_1 or S_2 . Thus, there will be a parameter space where both S_1 and S_2 are stable on the cosmological time scale, while S_3 decays away in the early universe. In other case, S_3 can also be stable if the mass splitting is small (e.g. by taking $\epsilon \rightarrow 0$) or the cutoff scale Λ is high enough, resulting in all three components being DM. The same argument can apply for the inverted spectrum, and some or all of the

flavored scalars can form DM, depending on their mass splittings and the magnitude of the cutoff scale. In either case, if the heavy scalars are unstable, they have to decay prior to ~ 1 sec to avoid Big Bang Nucleosynthesis (BBN) bounds, and if they are long-lived, their lifetimes must be longer than the age of the universe, $t_U \simeq 13.8 \times 10^9$ yrs. These bounds on the heavy scalar lifetimes constrain the cutoff scale Λ .

4 Decay of heavy states

In this section, we investigate decay of the heavy scalars. For illustration, we focus on the normal ordering spectrum $M_1 < M_2 < M_3$ and study decay of the heaviest scalar S_3 induced only from the \mathcal{O}_{ijkl}^4 operator eq. (3.22) and renormalizable scalar interactions. Decay of the second heaviest scalar S_2 can be easily translated from that of S_3 , thanks to the flavor symmetry. We cover only the leading order terms in the ϵ expansion until section 4.4, where effects from higher order terms are discussed.

In the following subsections, we will make various approximations to evaluate lifetimes, and pay a particular attention to scaling of decay widths in terms of model parameters, rather than to accuracy of calculations. Hence lifetime calculations given in this section should be regarded as estimates. We add that in this MFV framework there are a lot of UV model-dependent $\mathcal{O}(1)$ coefficients, which absorb calculation uncertainties to some extent. Thus our conclusion will not largely be changed by performing precise calculations. Incidentally, we have confirmed that our calculation provides an order-of-magnitude estimate, compared with a numerical calculation using MadGraph [16].

The dominant decay mode depends on the mass splitting $\Delta M \equiv M_3 - M_1$. Given that the leading decay vertex eq. (3.22) is necessarily accompanied by the top quark due to the flavor symmetry, S_3 can decay predominantly into a pair of the top quark and a lighter quark if $\Delta M > m_t$. If $\Delta M < m_t$, however, it cannot decay into the on-shell top quark and the dominant decay mode should be four or five-body processes through the off-shell top-quark propagator. From section 4.1 to 4.3, we elaborate on such multi-body decays produced at the leading order of ϵ . Although these multi-body decays are the leading order in the ϵ expansion, we will find that three-body processes induced at higher orders of ϵ can surpass the leading order ones for $\Delta M \ll m_t$, due in part to strong phase-space suppression in the latter. We will assess the higher order processes in section 4.4, then identify parameter spaces for multi-component DM in section 5.

4.1 $S_3 \rightarrow S_1 t \bar{u}$

If ΔM is larger than the top quark mass, S_3 decays into the on-shell top quark (figure 1). Since the decay vertex is parameterized by the top Yukawa coupling, this decay mode is naturally dominant if kinematically allowed.

The squared amplitude for this decay process is given by

$$\sum_{\text{spin, color}} |\mathcal{M}(S_3 \rightarrow S_1 t \bar{u})|^2 = N_c \left(\frac{c_1 m_t}{\Lambda^2} \right)^2 (m_{12}^2 - m_t^2), \quad (4.1)$$

where we ignored the up-quark mass and $N_c = 3$ denotes the number of quark colors. The invariant masses, m_{12}^2 and m_{23}^2 , are defined in terms of the outgoing four-momenta of the

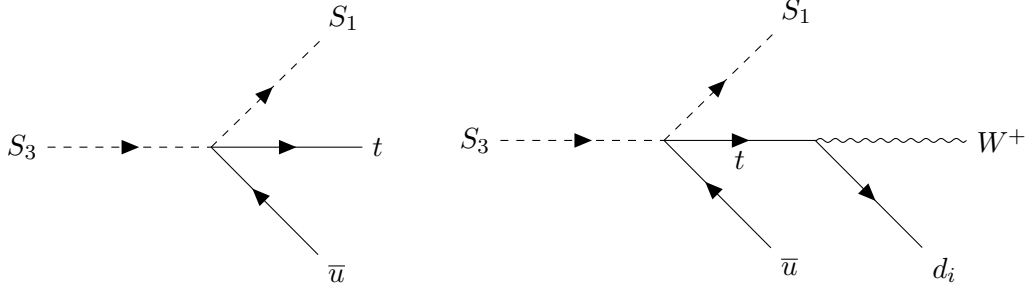


Figure 1. Feynman diagrams for $S_3 \rightarrow S_1 t \bar{u}$ (left) and $S_3 \rightarrow S_1 d_i \bar{u} W^+$ (right).

decay products,

$$m_{12}^2 = (p_1 + p_t)^2, \quad m_{23}^2 = (p_t + p_u)^2. \quad (4.2)$$

The partial decay width is evaluated by

$$\Gamma(S_3 \rightarrow S_1 t \bar{u}) = \frac{N_c}{256\pi^3 M_3^3} \left(\frac{c_1 m_t}{\Lambda^2} \right)^2 \int_{m_t^2}^{(\Delta M)^2} dm_{23}^2 \frac{(m_{23}^2 - m_t^2)^2}{m_{23}^2} \sqrt{\lambda(M_3^2, M_1^2, m_{23}^2)}, \quad (4.3)$$

with $\lambda(\alpha, \beta, \gamma) = \alpha^2 + \beta^2 + \gamma^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$. While this integral is performed numerically in our analysis, it is useful to provide an approximate width for $m_t \ll \Delta M \ll M_3 + M_1$. In this limit, we have

$$\Gamma(S_3 \rightarrow S_1 t \bar{u}) \simeq \frac{N_c (M_1 + M_3) (\Delta M)^5}{960\pi^3 M_3^3} \left(\frac{c_1 m_t}{\Lambda^2} \right)^2 =: \Gamma_0. \quad (4.4)$$

This is a baseline decay width for S_3 and we define it as Γ_0 for later convenience.

4.2 $S_3 \rightarrow S_1 d_i \bar{u} W$

Below the top threshold $\Delta M \leq m_t$, S_3 can only decay into the off-shell top quark. Then, four-body processes $S_3 \rightarrow S_1 d_i \bar{u} W^+$, which are allowed for $m_W + m_{d_i} \leq \Delta M$, take the place of the dominant decay mode (figure 1). We estimate these decay widths in this subsection.

For $\Delta M \ll m_t$, we find the squared decay amplitudes,

$$|\overline{\mathcal{M}}|^2 := \sum_{\text{spin, color}} |\mathcal{M}(S_3 \rightarrow S_1 d_i \bar{u} W^+)|^2 \simeq N_c \left(\frac{y_t m_W |V_{ti}|}{\Lambda^2 m_t^2} \right)^2 \times \frac{8(p_{d_i} \cdot p_W)^2 (p_u \cdot p_{d_i})}{m_W^2}, \quad (4.5)$$

where we take $p_u^2 = p_{d_i}^2 = p_W^2 = 0$ and keep the leading term in the limit where $m_W^2 \ll (p_u \cdot p_{d_i}), (p_{d_i} \cdot p_W)$. The partial decay widths are obtained by integrating the squared amplitudes over four-body phase space. Such a integral can be performed numerically or using a public calculation package, such as MadGraph [16], but we instead estimate the four-body decay widths as

$$\Gamma(S_3 \rightarrow S_1 d_i \bar{u} W^+) \sim \frac{(2\pi)^4}{2M_3} |\overline{\mathcal{M}}|^2 \times \Phi_4(M_3; M_1, 0), \quad (4.6)$$

where $\Phi_4(M_3; M_1, 0)$ is the four-body phase space for only S_1 massive and the others massless. Using two-cluster decomposition (see appendix A for the detail), we have $\Phi_4(M_3; M_1, 0)$ in the form,

$$\Phi_4(M_3; M_1, 0) = \frac{M_3^4}{393216\pi^9} f_3(M_1^2/M_3^2), \quad (4.7)$$

where

$$f_3(v) \simeq \frac{1}{10}(1-v)^5 \quad \text{for } v \simeq 1. \quad (4.8)$$

For $\Delta M \ll M_1 + M_3$, we find

$$\Gamma(S_3 \rightarrow S_1 d_i \bar{u} W^+) \sim \frac{N_c (\Delta M)^{11}}{414720\pi^5 M_3^2} \left(\frac{y_t |V_{ti}|}{\Lambda^2 m_t^2} \right)^2. \quad (4.9)$$

Here, we replaced the scalar products of the final-state momenta in $|\overline{\mathcal{M}}|^2$ with their mean values. Concretely, using the energy-momentum conservation,

$$(\Delta M)^2 \simeq (p_3 - p_1)^2 = (p_{d_i} + p_W + p_u)^2 \simeq 2(p_{d_i} \cdot p_W + p_{d_i} \cdot p_u + p_u \cdot p_W), \quad (4.10)$$

and symmetry among $p_{d_i, u, W}$, we approximate²

$$p_{d_i} \cdot p_W \sim p_{d_i} \cdot p_u \sim p_u \cdot p_W \sim \frac{(\Delta M)^2}{6}. \quad (4.11)$$

The ratios of these four-body decay widths (or equally the branching ratios) are determined only by the CKM matrix elements:

$$\Gamma(S_3 \rightarrow S_1 b \bar{u} W^+) \simeq \left| \frac{V_{tb}}{V_{ts}} \right|^2 \Gamma(S_3 \rightarrow S_1 s \bar{u} W^+) \simeq \left| \frac{V_{tb}}{V_{td}} \right|^2 \Gamma(S_3 \rightarrow S_1 d \bar{u} W^+). \quad (4.12)$$

This relation is robust and independent of whether we evaluate the phase-space integral numerically or make just an estimate like above.

4.3 $S_3 \rightarrow S_1 d_i \bar{u} f \bar{f}'$

As the mass splitting ΔM gets smaller than the W boson mass, even $S_3 \rightarrow S_1 d_i \bar{u} W$ decays are kinematically forbidden. In this case, five-body processes via the off-shell W exchange (figure 2) start to dominate the S_3 decay. Here, we estimate these decay widths assuming the mass splitting is larger than 1 GeV so that we can evaluate the widths by parton-level calculation.

For $\Delta M \ll m_W$, the squared decay amplitudes are given by

$$\begin{aligned} |\overline{\mathcal{M}}|^2 &= \sum_{\text{spin, color}} |\mathcal{M}(S_3 \rightarrow S_1 d_i \bar{u} f \bar{f}')|^2 \\ &= \left(\frac{2|V_{ti}||U_{ff'}|}{\Lambda^2 m_t v^2} \right)^2 \times 32 N_c N_{c,f} (p_b \cdot p_f) \times \\ &\quad \{ (p_{d_i} \cdot p_{f'} + p_f \cdot p_{f'}) (p_{d_i} \cdot p_u + p_u \cdot p_f) - (p_{d_i} \cdot p_f) (p_u \cdot p_{f'}) \}, \end{aligned} \quad (4.13)$$

²If we evaluate the decay width for $S_3 \rightarrow S_1 t \bar{u}$ in a similar way, the width is overestimated by a factor of 2.5.

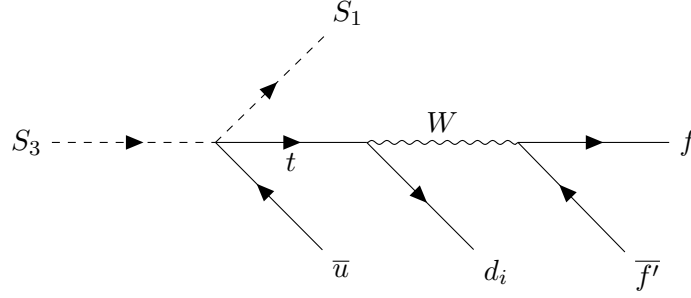


Figure 2. Feynman diagram for five-body decay processes $S_3 \rightarrow S_1 d_i \bar{u} f \bar{f}'$.

where $U_{ff'} = V_{ff'}$ for quarks and $U_{ff'} = \delta_{ff'}$ for leptons, and $N_{c,f}$ is the number of colors for a fermion f . The partial decay widths are estimated in a similar way to the four-body case by making an approximation,

$$\Gamma(S_3 \rightarrow S_1 d_i \bar{u} f \bar{f}') \sim \frac{(2\pi)^4}{2M_3} |\overline{\mathcal{M}}|^2 \times \Phi_5(M_3; M_1, 0), \quad (4.14)$$

where $\Phi_5(M_3; M_1, 0)$ is the five-body phase space for only S_1 massive and the others massless and given explicitly by

$$\Phi_5(M_3; M_1, 0) = \frac{M_3^6}{75497472\pi^{11}} f_4(M_1^2/M_3^2), \quad (4.15)$$

with

$$f_4(v) \simeq \frac{1}{35} (1-v)^7 \quad \text{for } v \simeq 1. \quad (4.16)$$

Then, we find the decay widths for $\Delta M \ll M_1 + M_3$,

$$\Gamma(S_3 \rightarrow S_1 d_i \bar{u} f \bar{f}') \sim \frac{N_c N_{c,f} (\Delta M)^{13}}{11612160\pi^7 M_3^2} \left(\frac{|V_{ti}| |U_{ff'}|}{\Lambda^2 m_t v^2} \right)^2. \quad (4.17)$$

Here, we applied a similar approximation to the four-momentum products for the light final-state particles, i.e.

$$p_A \cdot p_B \sim \frac{(\Delta M)^2}{12}, \quad (4.18)$$

for $A, B = d_i, u, f, f'$. In addition to a small numerical factor $1/(11612160\pi^7)$, the decay widths are proportional to the power of huge scale differences originating from $\Delta M \ll v, M_3, \Lambda$. These suppression factors rapidly reduce the widths as ΔM becomes small. It is easy to find again a close relation among these five-body decay widths,

$$\Gamma(S_3 \rightarrow S_1 b \bar{u} f \bar{f}') \simeq \left| \frac{V_{tb}}{V_{ts}} \right|^2 \Gamma(S_3 \rightarrow S_1 s \bar{u} f \bar{f}') \simeq \left| \frac{V_{tb}}{V_{td}} \right|^2 \Gamma(S_3 \rightarrow S_1 d \bar{u} f \bar{f}'), \quad (4.19)$$

if these decay processes are kinematically allowed.

4.4 Higher-order contributions in the MFV expansion

We have studied the leading order effects in the MFV expansion up to here, and found that those contribution to the heavy scalar decay receives strong phase-space suppression for a small mass splitting. That encourages us to evaluate higher order contribution that has extra suppression from ϵ but evades strong phase-space suppression. In this section, we study the impact of higher order terms on the heavy scalar decay.

For the scalar mass and Higgs portal coupling, higher order corrections are taken into account by

$$m_S^2 \mathbf{1} \rightarrow m_S^2 \left[a_0 \mathbf{1} + \epsilon a_1 (Y_u^\dagger Y_u) + \epsilon^2 (a_2 Y_u^\dagger Y_d Y_d^\dagger Y_u + a'_2 (Y_u^\dagger Y_u)^2) + \mathcal{O}(\epsilon^3) \right], \quad (4.20)$$

$$\lambda \mathbf{1} \rightarrow \lambda \left[b_0 \mathbf{1} + \epsilon b_1 (Y_u^\dagger Y_u) + \epsilon^2 (b_2 Y_u^\dagger Y_d Y_d^\dagger Y_u + b'_2 (Y_u^\dagger Y_u)^2) + \mathcal{O}(\epsilon^3) \right], \quad (4.21)$$

where a 's and b 's are $\mathcal{O}(1)$ coefficients. The first flavor off-diagonal elements arise from the ϵ^2 terms. Given that $(Y_u)_{ij} = (V^\dagger \hat{Y}_u)_{ij}$ and $(Y_d)_{ij} = (\hat{Y}_d)_{ij} = y_d^i \delta_{ij}$, the scalar mass terms are expressed by

$$\mathcal{L}_{\text{mass}} = -S_i^* (M_S^2)_{ij} S_j, \quad M_S^2 = \begin{pmatrix} M_1^2 & \Delta M_{12}^2 & \Delta M_{13}^2 \\ \Delta M_{21}^2 & M_2^2 & \Delta M_{23}^2 \\ \Delta M_{31}^2 & \Delta M_{32}^2 & M_3^2 \end{pmatrix}, \quad (4.22)$$

where $\Delta M_{ij}^2 = \epsilon^2 (a_2 m_S^2 + b_2 \lambda v^2 / 2) y_u^i V_{ik} (y_d^k)^2 V_{jk}^* y_u^j$. The mass matrix is diagonalized by a unitary matrix U_S , which is given for $\epsilon \ll 1$ by

$$S_i \rightarrow (U_S)_{ij} S_j, \quad U_S \simeq \begin{pmatrix} 1 & -\theta_{12} & -\theta_{13} \\ \theta_{12} & 1 & -\theta_{23} \\ \theta_{13} & \theta_{23} & 1 \end{pmatrix}, \quad (4.23)$$

where we approximate $|\theta_{ij}| \ll 1$. The mixing angle between S_i and S_j is given by

$$\theta_{ij} \simeq \frac{\Delta M_{ij}^2}{M_i^2 - M_j^2} = \epsilon R \frac{y_u^i V_{ik} (y_d^k)^2 V_{jk}^* y_u^j}{(y_u^i)^2 - (y_u^j)^2}, \quad (4.24)$$

with $M_i^2 - M_j^2 = \epsilon [(y_u^i)^2 - (y_u^j)^2] (a_1 m_S^2 + b_1 \lambda v^2 / 2)$ and

$$R \equiv \frac{a_2 m_S^2 + b_2 \lambda v^2 / 2}{a_1 m_S^2 + b_1 \lambda v^2 / 2} = \mathcal{O}(1). \quad (4.25)$$

Note that although the off-diagonal elements in the mass matrix appear at the order of ϵ^2 , the scalar mixing angle is of the order of ϵ because the mass splitting is generated at the order of ϵ .

4.4.1 Scalar mixing

The scalar mixing induces new decay modes that do not appear at the order of ϵ^0 . At the order of ϵ , only the c_1 term generates such new decay modes via the scalar mixing.³ In the

³Flavor off-diagonal interactions like $\bar{u}u S_1^* S_3$ do not stem from the c_2 term at the order of ϵ , even if the scalar mixing is present. This is understood from the fact that $(S_i^* S_i)$ is invariant under the mass diagonalization $S_i \rightarrow (U_S)_{ij} S_j$.

mass basis, the pertinent interaction Lagrangian is given by

$$\mathcal{L}_{d=6} = \frac{c_1}{\Lambda^2} [\bar{u}_k(m_k P_R + m_l P_L) u_l (S_i^* (U_S^*)_{li} (U_S)_{kj} S_j)] . \quad (4.26)$$

We are particularly interested in S_3 decay into u, c quarks. Taking $j = 3$ and $k, l \neq 3$, the largest contribution comes from $i = l$, yielding

$$\Gamma(S_3 \rightarrow S_i u_k \bar{u}_i) \simeq \Gamma_0 \times |\theta_{i3}|^2 \frac{(m_u^i)^2 + (m_u^k)^2}{m_t^2} . \quad (4.27)$$

Here, we used for a small mixing angle (or equally a small ϵ)

$$(U_S)_{ij} \simeq \begin{cases} 1 & (i = j) \\ -\theta_{ij} \simeq -(U_S^*)_{ji} & (i \neq j) \end{cases} . \quad (4.28)$$

Since $\theta_{i3} \propto \epsilon$, the decay width is suppressed by ϵ^2 but without extra phase-space suppression, compared with the reference three-body width Γ_0 , eq. (4.4). Thus, this process might be as large as the four- or five-body decays discussed in sections 4.2 and 4.3, depending on the values of ϵ and ΔM .

By closing the quark lines for the $k = l$ interaction vertices in eq. (4.26), we have $S_j \rightarrow S_i \gamma \gamma$ decay at one-loop level (figure 3). The decay amplitude is given by

$$i\mathcal{M}(S_j \rightarrow S_i \gamma \gamma) = i \frac{c_1}{\Lambda^2} (\mathcal{A}_{\gamma\gamma})_{ij} \epsilon_\mu^*(p) \epsilon_\nu^*(q) [(p \cdot q) g^{\mu\nu} - p^\nu q^\mu] , \quad (4.29)$$

where p and q denote outgoing four-momenta of two final-state photons and

$$(\mathcal{A}_{\gamma\gamma})_{ij} = \frac{N_c \alpha Q_u^2}{2\pi} \sum_{k=1,2,3} (U_S^*)_{ki} (U_S)_{kj} F_{1/2}(\tau_k) , \quad (4.30)$$

with $\tau_k = 4(m_u^k)^2/(2p \cdot q)$. The loop function is well-known in the context of the Higgs diphoton decay (see e.g. [17]), and given by

$$F_{1/2}(\tau) = -2\tau [1 + (1 - \tau) f(\tau)] , \quad (4.31)$$

where

$$f(\tau) = \begin{cases} \arcsin^2(\sqrt{1/\tau}) & (\tau \geq 1) \\ -\frac{1}{4} [\ln(\eta_+/\eta_-) - i\pi]^2 & (\tau < 1) \end{cases} , \quad (4.32)$$

with $\eta_\pm = 1 \pm \sqrt{1 - \tau}$. It is useful to show two limits of $F_{1/2}$: $F_{1/2}(\tau \rightarrow \infty) = -4/3$ and $F_{1/2}(\tau \rightarrow 0) = 0$. It follows from these two limits that the top loop contribution dominates $S_3 \rightarrow S_1 \gamma \gamma$ decay for $m_u \ll \Delta M \ll m_t$. The charm loop contribution to $(\mathcal{A}_{\gamma\gamma})_{13}$ is proportional to $\theta_{12}^* \theta_{23} \propto \epsilon^2$ and thus negligible. For $m_u \ll \Delta M \ll m_t$, therefore, the decay width approximates to

$$\Gamma(S_3 \rightarrow S_1 \gamma \gamma)_{\text{loop}} \simeq \frac{M_1 + M_3}{256\pi^3 M_3^3} |\theta_{13}|^2 \left(\frac{c_1 N_c \alpha Q_u^2}{3\pi \Lambda^2} \right)^2 \times \frac{16}{105} (\Delta M)^7 . \quad (4.33)$$

Note that because of unitarity of the scalar mixing matrix, the up- and top-quark contributions cancelled each other $(\mathcal{A}_{\gamma\gamma})_{13} \simeq 0$ for $\Delta M \ll m_u$, where the up-quark contribution is also saturated with its asymptotic value $F_{1/2}(\tau_u) \simeq -4/3$. However, the up-quark contribution is highly sensitive to how we treat the up-quark mass, resulting in a large calculation uncertainty. If we use current quark mass, $(\mathcal{A}_{\gamma\gamma})_{13} \simeq 0$ below $\mathcal{O}(\text{MeV})$, while if we take it as constituent quark mass, $(\mathcal{A}_{\gamma\gamma})_{13} \simeq 0$ below $\mathcal{O}(100 \text{ MeV})$.

As the mass splitting becomes below $\sim 1 \text{ GeV}$, we have to consider decay into hadrons rather than partons, eq. (4.27). To evaluate such hadronic decays, we first derive effective Lagrangian at the QCD scale by taking $k = l$ in eq. (4.26) and integrating out charm and top quarks. The relevant effective interactions are given by

$$\mathcal{L}_{\text{eff, mixing}} = \frac{c_1}{\Lambda^2} \left[m_u (U_S^*)_{1i} (U_S)_{1j} (\bar{u}u) - \sum_{k=2,3} (U_S^*)_{ki} (U_S)_{kj} \frac{\alpha_s}{12\pi} G_{\mu\nu}^a G^{\mu\nu a} \right] (S_i^* S_j), \quad (4.34)$$

where the second term arises from charm and top loops. Using unitarity of the scalar mixing matrix, we can rewrite the second term in the square bracket with

$$\sum_{k=2,3} (U_S^*)_{ki} (U_S)_{kj} \frac{\alpha_s}{12\pi} G_{\mu\nu}^a G^{\mu\nu a} = [\delta_{ij} - (U_S^*)_{1i} (U_S)_{1j}] \frac{\alpha_s}{12\pi} G_{\mu\nu}^a G^{\mu\nu a}, \quad (4.35)$$

which yields for $i \neq j$

$$\mathcal{L}_{\text{eff, mixing}} = \frac{c_1}{\Lambda^2} \left(m_u \bar{u}u + \frac{\alpha_s}{12\pi} G_{\mu\nu}^a G^{\mu\nu a} \right) (U_S^*)_{1i} (U_S)_{1j} (S_i^* S_j). \quad (4.36)$$

In the leading order chiral perturbation, we find hadronic matrix elements for quarks and gluon [18–22],

$$\langle \pi^a(p) \pi^b(q) | m_u \bar{u}u | 0 \rangle = \frac{1}{2} \delta^{ab} m_\pi^2, \quad (4.37)$$

$$\langle \pi^a(p) \pi^b(q) | \frac{9\alpha_s}{8\pi} G_{\mu\nu}^a G^{\mu\nu a} | 0 \rangle = -\delta^{ab} (s + m_\pi^2), \quad (4.38)$$

where we use the isospin symmetry. For $m_\pi \ll \Delta M$, the decay widths are approximately evaluated as

$$\Gamma(S_j \rightarrow S_i \pi^a \pi^b) \simeq \frac{\delta^{ab} (M_j + M_i)}{512\pi^3 M_j^3} \left(\frac{2c_1}{27\Lambda^2} \right)^2 |(U_S^*)_{1i} (U_S)_{1j}|^2 \times \frac{16}{105} (M_j - M_i)^7. \quad (4.39)$$

Using eq. (4.28), we notice $\Gamma(S_3 \rightarrow S_2 \pi^a \pi^b) \propto |\theta_{12}^* \theta_{13}|^2 \propto \epsilon^4$ and it has a minor effect in our order-counting. Note that there should be large calculation uncertainties due to significant final-state interactions of pions [22–25]. For $2m_\pi < \Delta M < 1 \text{ GeV}$, the decay widths could receive an enhancement by as much as a factor of 10. Nonetheless, these calculation uncertainties would not change our basic conclusion.

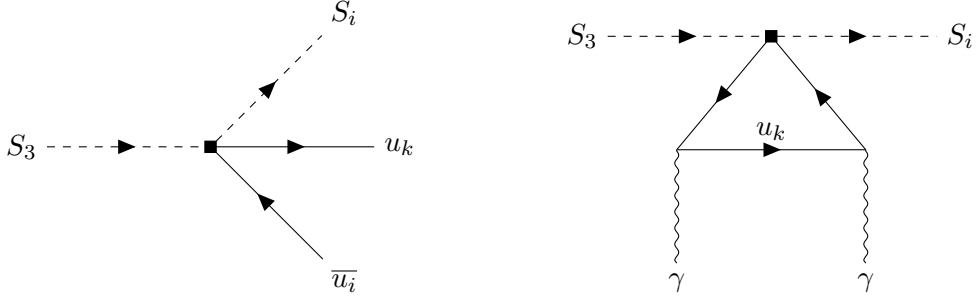


Figure 3. (Left) Feynman diagram for $S_3 \rightarrow S_i u_k \bar{u}_i$ decay, which arises from the scalar mixing at the order of ϵ . The black square dot means the \mathcal{O}_{ijkl}^4 vertex at the order of ϵ . (Right) Feynman diagram for $S_3 \rightarrow S_i \gamma \gamma$ decay through the same interaction vertex at the order of ϵ .

4.4.2 Higgs portal contribution

Higher order terms of the Higgs portal coupling also induces other new decay modes. In the mass basis of S_i , flavor off-diagonal interactions to the Higgs boson stem from the ϵ^2 terms,

$$\begin{aligned} V(H, S) &\supset \lambda v \left\{ \epsilon b_1 \sum_k (y_u^k)^2 [(\theta^*)_{ki} \delta_{kj} + \delta_{ki} (\theta)_{kj}] + \epsilon^2 b_2 y_u^i y_u^j \sum_k (y_d^k)^2 V_{ik} V_{jk}^* \right\} h S_i^* S_j, \\ &\simeq \epsilon^2 \lambda v A \left(y_u^i y_u^j \sum_k (y_d^k)^2 V_{ik} V_{jk}^* \right) h S_i^* S_j =: \epsilon^2 \lambda_{ij} v h S_i^* S_j, \end{aligned} \quad (4.40)$$

with

$$\lambda_{ij} \equiv \lambda A \left(y_u^i y_u^j \sum_k (y_d^k)^2 V_{ik} V_{jk}^* \right), \quad (4.41)$$

where A denotes a model-dependent $\mathcal{O}(1)$ coefficient defined by

$$A \equiv -b_1 \frac{a_2 m_S^2 + b_2 \lambda v^2 / 2}{a_1 m_S^2 + b_1 \lambda v^2 / 2} + b_2. \quad (4.42)$$

The first term in A comes from a combination of the scalar mixing and the flavor diagonal couplings, both at the order of ϵ . Meanwhile, the second term stems purely from the flavor off-diagonal elements of the Higgs portal coupling. These two terms equally contribute to new decay modes.

First, eq. (4.40) induces two-body decay $S_3 \rightarrow S_1 h$, whose width is given by

$$\Gamma(S_3 \rightarrow S_1 h) \simeq \frac{\epsilon^4 |\lambda_{13}|^2 v^2 \beta}{16\pi M_3}, \quad (4.43)$$

with

$$\beta = \sqrt{1 - \frac{(M_1 + m_h)^2}{M_3^2}} \sqrt{1 - \frac{(M_1 - m_h)^2}{M_3^2}}. \quad (4.44)$$

This decay is possible only for $\Delta M > m_h$. Compared with $S_3 \rightarrow S_1 t \bar{u}$, the decay width for $S_3 \rightarrow S_1 h$ is suppressed by ϵ^4 and seems not to be dominant. This is, however, two-body decay and has no suppression from the UV cutoff scale Λ , so it can have some impact for a large Λ .

For $\Delta M < m_h$, $S_3 \rightarrow S_1 h$ is forbidden and off-shell Higgs-mediated processes begin to dominate. For $\Delta M \lesssim \mathcal{O}(v)$, effective interactions to the SM fields are given by

$$\mathcal{L}_{\text{eff, higgs}} = \frac{\epsilon^2 \lambda_{13}}{m_h^2} (S_1^* S_3) \left(\sum_f m_f \bar{f} f + \frac{\alpha_s F_g}{16\pi} G_{\mu\nu}^a G^{\mu\nu a} + \frac{\alpha F_\gamma}{8\pi} F_{\mu\nu} F^{\mu\nu} \right), \quad (4.45)$$

where f runs over all charged leptons and quarks that are lighter than $\Delta M/2$. The effective couplings to gluon and photon fields are given by

$$F_g = \sum_q F_{1/2}(\tau_q) \Theta(2m_q - \Delta M), \quad (4.46)$$

$$F_\gamma = F_1(\tau_W) + \sum_f N_{c,f} q_f^2 F_{1/2}(\tau_f) \Theta(2m_f - \Delta M), \quad (4.47)$$

with $N_{c,f}$ and q_f being the color and electric charge for a fermion f and $\tau_i = 4m_i^2/(2p \cdot q)$ for a particle i in the loop. The loop functions are given by eq. (4.31) and

$$F_1(\tau) = 2 + 3\tau + 2\tau(2 - \tau)f(\tau) \quad \text{with } F_1(\tau \rightarrow \infty) = 7, \quad (4.48)$$

where $f(\tau)$ is in eq. (4.32). S_3 can decay into S_1 plus a pair of fermions, gluons or photons. We approximately obtain the partial widths for those decay processes,

$$\Gamma(S_3 \rightarrow S_1 f \bar{f}) \simeq \frac{N_{c,f} (M_1 + M_3) (\Delta M)^5}{480\pi^3 M_3^3} \left(\frac{\epsilon^2 |\lambda_{13}| m_f}{m_h^2} \right)^2, \quad (4.49)$$

$$\Gamma(S_3 \rightarrow S_1 g g) \simeq \frac{M_1 + M_3}{512\pi^3 M_3^3} \left(\frac{\epsilon^2 |\lambda_{13}|}{m_h^2} \right)^2 \left(\frac{\alpha_s}{2\pi} \frac{4}{3} N_h \right)^2 \times \frac{15}{106} (\Delta M)^7, \quad (4.50)$$

$$\Gamma(S_3 \rightarrow S_1 \gamma \gamma) \simeq \frac{M_1 + M_3}{1024\pi^3 M_3^3} \left(\frac{\epsilon^2 |\lambda_{13}|}{m_h^2} \right)^2 \left(\frac{\alpha}{2\pi} f_\gamma \right)^2 \times \frac{15}{106} (\Delta M)^7, \quad (4.51)$$

where N_h denotes the number of quarks heavier than $\Delta M/2$ and

$$f_\gamma = 7 - \frac{4}{3} \sum_f N_{c,f} q_f^2 \Theta(2m_f - \Delta M). \quad (4.52)$$

To obtain these approximate widths, we assumed $m_f \ll \Delta M \ll M_3 + M_1$ and used the corresponding values of $F_1(\tau \rightarrow \infty)$ and $F_{1/2}(\tau \rightarrow \infty)$. We also ignored the interference with the c_1 -induced terms, obtained in section 4.4.1. The cutoff scale Λ is replaced with the Higgs boson mass in these processes. This means that ϵ or λ must be small enough to make S_3 long-lived. We will see how small ϵ and λ should be in the next section.

Below 1 GeV, we have to consider hadronic decay. The effective Lagrangian at 1 GeV consists of light quarks and gluon,

$$\mathcal{L}_{\text{eff, higgs}} = \frac{\epsilon^2 \lambda_{ij}}{m_h^2} (S_i^* S_j) \left(\sum_{q=u,d,s} m_q \bar{q} q - N_h \frac{\alpha_s}{12\pi} G_{\mu\nu}^a G^{\mu\nu a} \right), \quad (4.53)$$

where $N_h = 3$ counts c, b, t quarks. Using the matrix elements for the light quarks and gluon, evaluated in the leading order chiral perturbation,

$$\langle \pi^a(p) \pi^b(q) | m_u \bar{u} u + m_d \bar{d} d | 0 \rangle = \delta^{ab} m_\pi^2, \quad \langle \pi^a(p) \pi^b(q) | m_s \bar{s} s | 0 \rangle = 0, \quad (4.54)$$

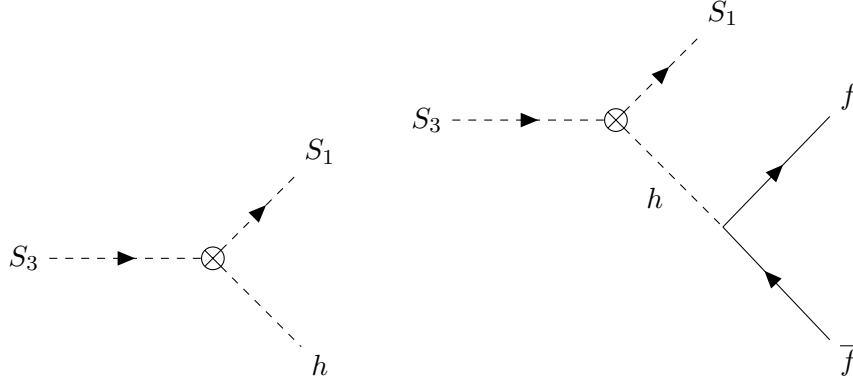


Figure 4. Feynman diagrams for $S_3 \rightarrow S_1 h$ (left) and $S_3 \rightarrow S_1 h^* \rightarrow S_1 f \bar{f}$ (right). The crossed dot means the Higgs portal vertex appearing at the order of ϵ^2 .

we obtain an approximate form of the partial width for $S_3 \rightarrow S_1 \pi^a \pi^b$,

$$\Gamma(S_3 \rightarrow S_1 \pi^a \pi^b) \simeq \frac{\delta^{ab} (M_1 + M_3)}{512 \pi^3 M_3^3} \left(\frac{2\epsilon^2 |\lambda_{13}|}{9m_h^2} \right)^2 \times \frac{15}{106} (\Delta M)^7. \quad (4.55)$$

4.4.3 Dimension-6 operators

For the \mathcal{O}_{ijkl}^4 operator, higher order corrections correspond to taking

$$c_{ijkl}^4 \sim \epsilon^2 \left[c (Y_u Y_u^\dagger Y_d Y_d^\dagger Y_u)_{il} \delta_{kj} + c' (Y_u)_{ij} (Y_u^\dagger Y_d Y_d^\dagger Y_u)_{kl} + c'' (Y_u)_{il} (Y_u^\dagger Y_d Y_d^\dagger Y_u)_{kj} \right], \quad (4.56)$$

where we suppress irrelevant terms that appear at the same order but do not lead to new decay modes. There also exists flavor off-diagonal contribution through a combination of the scalar mass mixing and $c_{ijkl}^4 \sim \mathcal{O}(\epsilon)$ terms, but we ignore it here since it has the same coupling scaling. From eq. (4.56), we have

$$\begin{aligned} \mathcal{L}_{d=6} \sim & \frac{\epsilon^2}{\Lambda^2} \sum_{i,j,k,l} \left(y_u^i y_u^j (y_d^k)^2 V_{ik} V_{jk}^* \right) \left\{ c m_u^i (\bar{u}_{Li} u_{Rl}) (S_l^* S_j) \right. \\ & \left. + c' m_u^l (\bar{u}_{Ll} u_{Rl}) (S_i^* S_j) + c'' m_u^l (\bar{u}_{Ll} u_{Rj}) (S_i^* S_l) \right\} + \text{h.c.}, \end{aligned} \quad (4.57)$$

where all scalar and quark fields are in the mass basis. These interactions enable new three-body decay modes that exclude the top quark in the final states. The partial widths for such decay processes is evaluated using eqs. (4.4) and (4.57) as

$$\Gamma(S_3 \rightarrow S_1 u_i \bar{u}_j) \simeq \Gamma_0 \times \epsilon^4 \left| c \delta_{1j} (y_u^i)^2 \sum_k (y_d^k)^2 V_{ik} V_{tk}^* + c' \delta_{ij} y_u^i y_u \sum_k (y_d^k)^2 V_{uk} V_{tk}^* \right|^2, \quad (4.58)$$

where $i, j \neq 3$. This is suppressed by ϵ^4 and the light quark Yukawa couplings as well as the CKM matrix elements, compared with the baseline $S_3 \rightarrow S_1 t \bar{u}$ decay width Γ_0 . Therefore, these are expected not to surpass the other processes already discussed above, unless there is a significant accidental cancellation among the $\mathcal{O}(1)$ coefficients in the other decay modes.

5 Parameter spaces for multi-component dark matter

We are ready to survey model parameter spaces and identify where the heavy components are also DM. Our benchmark model is characterized mostly by four parameters,

$$M_1, \quad \Lambda, \quad \lambda, \quad \epsilon \quad (5.1)$$

which control not only the heavy scalar decays but DM physics. Magnitude of these four parameters is arbitrary as long as they respect perturbative unitarity ($\lambda \leq 4\pi$) and validity of the EFT ($M_1 \leq \Lambda$). As for ϵ , one can take any value in principle, but we assume $\epsilon \ll 1$ in order to justify the MFV expansion. Note that even if taking $\epsilon = 0$ at the beginning, loop diagrams with the weak interactions necessarily generate higher order terms of Y_u and Y_d . This suggests that there is a minimum (or natural) value for ϵ , which is obtained by identifying ϵ as a loop factor: $\epsilon \sim 1/(4\pi)^2 \sim 10^{-2}-10^{-3}$. We therefore take $\epsilon = 10^{-2}$ as our benchmark value.

The mass differences among S_i are generated by ϵ . We compute two mass differences, $\Delta M = M_3 - M_1$ and $\delta M = M_2 - M_1$, by numerically solving eq. (3.7). For a small ϵ , these are approximately given by

$$\epsilon \simeq \frac{2\Delta M}{y_t^2 M_1} \simeq \frac{2\delta M}{y_c^2 M_1}, \quad (5.2)$$

which means that the mass splitting between S_3 and S_1 is around 0.5 % for $\epsilon = 10^{-2}$. We ignore flavor-diagonal corrections to the Higgs portal coupling λ , and take $\lambda_{hSi} = \lambda$ in the following analysis. This choice does not influence our results, since the leading Higgs-mediated decays are already suppressed by ϵ^4 and further corrections are insignificant. We also set all UV-model dependent $\mathcal{O}(1)$ coefficients to unity: $c_1 = c_2 = A = R = 1$.

In figure 5, we show constraints from lifetimes of the heavy scalars $S_{2,3}$ with $\lambda = 0$ (left) and $\lambda = 10^{-11}$ (right). In the blue region, the lifetime of S_2 is shorter than the age of the universe t_U , while in the orange region the lifetime of S_3 lies in $1 \text{ sec} < \tau_{S_3} < t_U$. Thus, two-component DM consisting of S_1 and S_2 is realized between the upper boundary of the blue region and the lower boundary of the orange region. Above the upper boundary of the orange region, all three scalars are stable and three-component DM scenario is realized. The blue (orange) dashed and dot-dashed lines respectively stand for contours of τ_{S_2} (τ_{S_3}) = 10^{24} sec and 10^{28} sec, as a reference of constraints from indirect DM searches and cosmological observations. See discussion in section 6 for further details. On the right panel, we set the Higgs portal coupling to $\lambda = 10^{-11}$. Since it is very weak, the exclusion regions, filled with colors, are the same in two plots. The only difference is seen in the lifetime contours of 10^{24} sec and 10^{28} sec for high cutoff scales Λ .

The lifetime bounds in figure 5 exhibit several kinks, which appear at kinematical thresholds of decay processes. Figure 6 shows partial decay widths for two heavy scalars, S_2 and S_3 , that are induced solely by the dim-6 interactions. We take $\Lambda = 10^3 \text{ GeV}$ and $\epsilon = 10^{-2}$ there. The widths for different Λ are obtained by an overall scaling $\Gamma \propto 1/\Lambda^4$. For S_3 decay, each threshold appears when the mass splitting ΔM is around the top quark mass (at $M_1 \simeq 35 \text{ TeV}$), the W boson mass (at $M_1 \simeq 16 \text{ TeV}$), the bottom quark mass (at $M_1 \simeq 860 \text{ GeV}$), the charm quark mass (at $M_1 \simeq 260 \text{ GeV}$), the QCD scale (at $M_1 \simeq 200 \text{ GeV}$), which we take 1 GeV , and the pion mass (at $M_1 \simeq 28 \text{ GeV}$). The corresponding thresholds are shown by the vertical gray dotted lines. The processes induced at the higher order

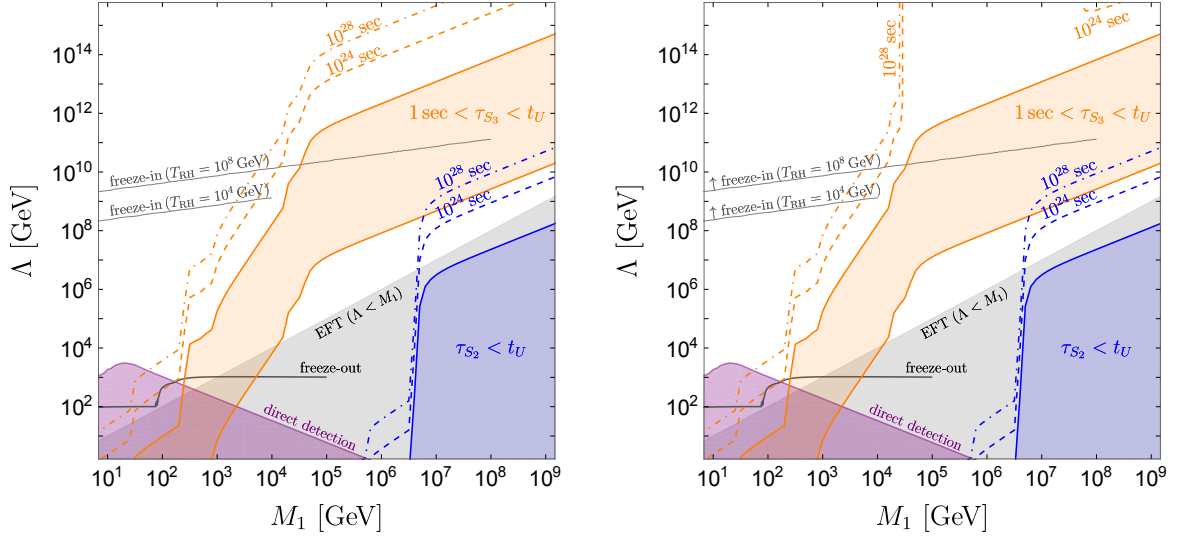


Figure 5. The constraints from the heavy scalar lifetimes for $\lambda = 0$ (left) and $\lambda = 10^{-11}$ (right). The MFV expansion parameter is fixed to $\epsilon = 10^{-2}$, which is related to the mass splitting as $\epsilon \simeq 2\Delta M/(y_t^2 M_1) \simeq 2\delta M/(y_c^2 M_1)$. The orange region, where $1 \text{ sec} \leq \tau_{S_3} \leq t_U$, is excluded from the S_3 stability and the BBN bound. In the blue region, S_2 is unstable and cannot be DM. The blue (orange) dashed and dot-dashed lines respectively stand for contours of τ_{S_2} (τ_{S_3}) = 10^{24} sec and 10^{28} sec. In the gray shaded region, we have $\Lambda \leq M_1$ and the EFT description is not justified. On the black (gray) line, the DM abundance is correctly produced by the freeze-out (freeze-in) mechanism.

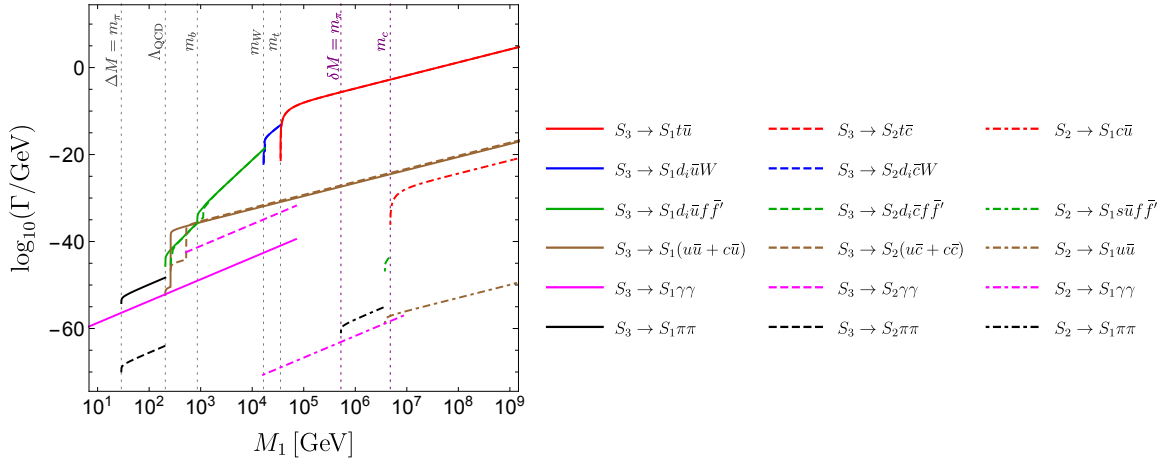


Figure 6. Partial decay widths of the heavy scalars, S_2 and S_3 , induced from the dim-6 interactions. We take $\Lambda = 10^3$ GeV and $\epsilon = 10^{-2}$. The MFV expansion parameter ϵ is related to the mass splitting as $\epsilon \simeq 2\Delta M/(y_t^2 M_1) \simeq 2\delta M/(y_c^2 M_1)$, where $\Delta M = M_3 - M_1$ and $\delta M = M_2 - M_1$. The solid lines indicate the decay widths of S_3 into S_1 , while the dashed lines represent those into S_2 . The dot-dashed lines show the decay widths of S_2 into S_1 . The vertical gray (purple) dotted lines show the representative kinematical thresholds for the S_3 (S_2) decay processes.

of ϵ surpass the leading-order five-body processes only below the bottom threshold. For S_2 decay, kinks are visible at $M_1 \sim 5$ PeV and 0.5 PeV in figure 5, which correspond to the charm quark and pion thresholds, respectively. See the vertical purple dotted lines in figure 6. We add that our width calculation suffers from a large hadronic uncertainty around ΔM , $\delta M \sim 1$ GeV, because of a complication of QCD dynamics. A special care to evaluate hadronic contributions is necessary in that region.

We also show other theoretical and experimental constraints in figure 5. In the gray region, we have $\Lambda < M_1$ and the EFT description is not justified. The purple region is excluded by direct DM detection bound, see appendix C for the detail. On the black and gray lines, total relic abundance of stable flavored scalars can account for the observed DM abundance, $\Omega h^2 = 0.12$. We consider two production mechanisms: one is the conventional thermal freeze-out production in the radiation dominated universe, and the other is the freeze-in production (see appendix B). In the freeze-out case, only the dim-6 interactions are responsible for the production, since the Higgs portal coupling is vanishing or too weak to contribute. The observed DM abundance is explained for $\Lambda \simeq 10^2$ – 10^3 GeV, although only a limited mass range $M_1 \simeq 180$ – 210 GeV is compatible with the bounds from the S_3 lifetime and direct detection. Note that the freeze-out production does not work for $M_1 \gtrsim 100$ TeV due to unitarity limit [26, 27], so the black line ends at $M_1 = 100$ TeV. In the freeze-in case, the DM production crucially depends on the Higgs portal coupling. If that coupling is much weaker than $\lambda = 10^{-11}$, the freeze-in production proceeds mostly through the dim-6 interactions with negligible Higgs portal contribution. The correct abundance is accommodated in $\Lambda \simeq 10^7$ – 10^{10} GeV, see the gray lines in figure 5 (left). The production rate with the dim-6 interactions is larger at higher temperatures. The DM abundance is thus sensitive to how the universe is reheated after inflation. We assume instantaneous reheating at a temperature T_{RH} in our freeze-in calculation and integrate Boltzmann equations from $T = T_{\text{RH}}$ to $T = T_0$ with zero initial DM abundance. In contrast, if the Higgs portal coupling λ amounts to 10^{-11} or larger, it can significantly contribute to the freeze-in production via $h \rightarrow S_i S_i^*$ and $hh \rightarrow S_i S_i^*$. The required coupling for the correct abundance is $\lambda \simeq 2.2 \times 10^{-11}$ for $m_h < M_i$ and $\lambda \simeq 1.2 \times 10^{-11} \sqrt{\frac{\text{GeV}}{M_i}}$ for $m_h \gg M_i$ in a pure Higgs portal DM case [28]. In this regime, the production depends insensitively on the reheating temperature if $T_{\text{RH}} \gg M_i$. Instead, one has to tame thermalization and overproduction via the dim-6 interactions. These restrictions are avoided above the gray lines for a given reheating temperature, see figure 5 (right). It would be worth mentioning that in our benchmark model, two-component parameter spaces are not consistent with either the standard freeze-out or freeze-in production. Other production mechanisms or non-standard cosmological history should be considered there.

In figure 7, we show the lifetime constraints in the (M_1, λ) plane. The cutoff scale is fixed to $\Lambda = 10^4$ GeV (left) and $\Lambda = 10^{13}$ GeV (right). Color coding of each constraint is the same as in figure 5, except for the green region which is excluded by the Higgs invisible decay bounds [29, 30]. On the black line, the DM abundance is correctly produced by the freeze-out mechanism with the Higgs portal coupling, albeit only in the Higgs resonance region $M_1 \simeq m_h/2$. The dim-6 interactions do not make significant contribution to the freeze-out. For $\Lambda = 10^{13}$ GeV, the flavored scalars are not thermalized via the dim-6 interactions unless the reheating temperature is extremely high. In this case, the freeze-in production via the Higgs portal processes succeeds for $\lambda \sim 10^{-11}$ [28].

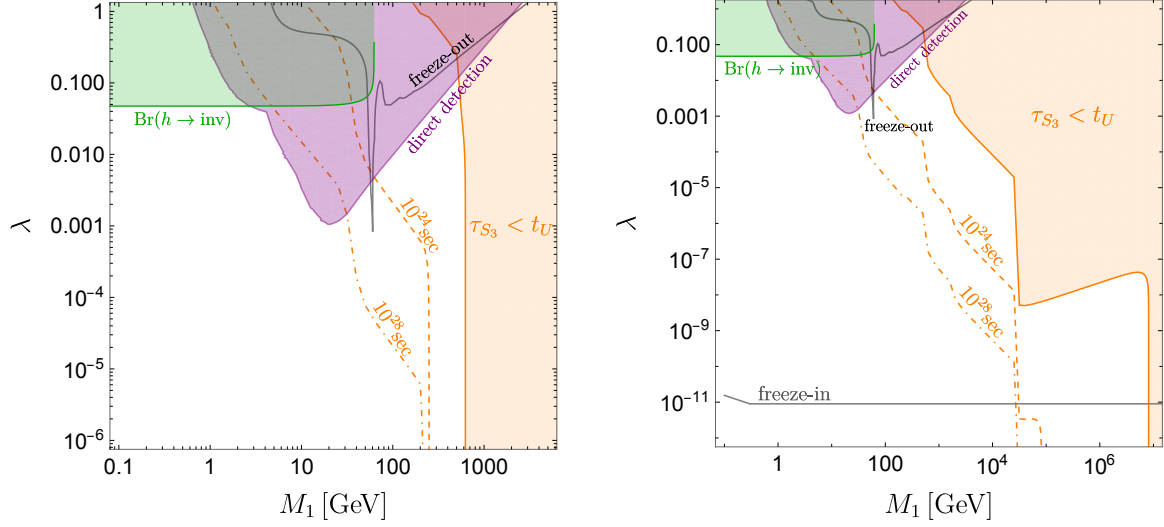


Figure 7. The lifetime constraints and experimental bounds for $\Lambda = 10^4$ GeV (left) and $\Lambda = 10^{13}$ GeV (right). The MFV expansion parameter is fixed to $\epsilon = 10^{-2}$, which is related to the mass splitting as $\epsilon \simeq 2\Delta M/(y_t^2 M_1)$. Color coding of each constraint is the same as in figure 5, except for the green region which is excluded by the Higgs invisible decay bound. The lifetime of S_2 is longer than the age of the universe in the entire region. The orange dashed and dot-dashed lines correspond to the lifetime contours of $\tau_{S_3} = 10^{24}$ sec and 10^{28} sec, respectively. On the black (gray) line, the DM abundance is correctly produced by the freeze-out (freeze-in) mechanism.

6 Discussion

We saw that within the MFV framework, DM can comprise multiple components that originate in one flavor multiplet. More than one component of those flavored states has sufficient longevity to serve as DM across a broad parameter space, while we have not studied their detailed phenomenology. In this section, we enumerate some brief comments on phenomenological implications of multi-component flavored DM, which are left for future works.

- **Indirect searches for heavy decaying DM components:** if heavy flavor components are DM, their present-time decay in galaxies produces a large number of energetic photons, positrons and neutrinos, which contribute to photon and cosmic-ray fluxes in space. These additional fluxes are constrained by astrophysical observations of gamma-rays [31–51], X-rays [52–60], radio-waves [39, 61], positrons [60, 62] and neutrinos [63–67]. See also a comprehensive review [68] and references therein. For a DM particle decaying only into SM particles, the current best lower limits on DM lifetimes reach $\tau_{\text{DM}} \sim 10^{24}–10^{28}$ sec [68], depending on mass range and decay modes of DM. These astrophysical bounds suggest that lifetimes of decaying DM have to be much longer than the age of the universe and, therefore, some of multi-component parameter spaces (figures 5 and 7) might be excluded by comparing with photon and cosmic-ray observations.
- **Cosmological bounds:** decay of heavy states into SM particles in the early universe leaves observable imprints on cosmology, even if their lifetimes are longer than the age

of the universe. For instance, exotic energy injection due to DM decay into SM particles has a significant impact on the ionization and thermal history of the universe, and distorts anisotropy spectra of Cosmic Microwave Background (CMB) [69–79]. The CMB data currently impose lower bounds on lifetimes, $\tau_{\text{DM}} \gtrsim 10^{24}\text{--}10^{25}$ sec [78–81], which are comparable with constraints from indirect DM searches. There are also relevant constraints from Lyman- α [80, 82], 21-cm [83–93] and heating of a gas-rich dwarf galaxy Leo T [94]. While the current best cosmological bounds ($\tau_{\text{DM}} \gtrsim 10^{25}$ sec) are derived from the CMB and Lyman- α , it is remarkable that future HERA measurements of the 21-cm power spectrum can surpass the CMB and Lyman- α sensitivity and reach lifetimes of $10^{27}\text{--}10^{28}$ sec [92, 93]. That encourages us to pursue cosmological searches in addition to indirect DM searches.

- **Flavor physics:** although the quark flavor symmetry is naturally conserving in this framework, the intrinsic flavor violation from the CKM matrix can still accommodate additional contribution to flavor violating observables on the top of the SM contribution [4, 6]. The new physics effects can be analyzed on a model-by-model basis or in a general way by matching with the Standard Model Effective Field Theory (SMEFT) [95–97] with MFV Wilson coefficients [98–108] if the new physics scales, Λ or M_i , are high enough. Besides, one potentially interesting phenomenon might be an apparent flavor violation from a natural-flavor-conserving new physics sector, which can occur due to the fact that DM particles carry quark flavor charges. Such a process might have some implication for a recent Belle-II excess in the $B \rightarrow K\nu\nu$ process [109]. As discussed in earlier works [110, 111], a new three-body decay channel $B \rightarrow K\chi\chi$ with χ being an invisible particle provides a good fit to that excess (see also [112–115]). This three-body process naturally appears in our framework via $b \rightarrow s\chi_3\bar{\chi}_2$.
- **Inelastic scattering:** multiple states of DM with a small mass splitting leaves a unique signal at DM direct detection experiments through inelastic scattering, e.g. $\chi_i N \rightarrow \chi_j N$ [116–122]. In general, both up-scattering and down-scattering off a nucleus are possible. Such processes are known in the context of (endothermic) inelastic DM ($M_i < M_j$) [123–125] and of exothermic DM ($M_i > M_j$) [126–128]. The MFV framework would offer natural UV prescriptions for those inelastic DM scenarios.
- **Detection of boosted lighter components:** annihilation or decay of heavier components can produce lighter DM components with a velocity larger than their virial velocities in halos. Such boosted DM components are detected at terrestrial experiments.

In addition to the above-mentioned subjects, one can pursue model building of flavored DM in the MFV framework. In this paper, we only considered a gauge singlet scalar DM, while the stability discussion in section 2 is applied for any spin and EW representations. Different choices of those representations, such as fermionic fields or EW multiplets, would result in different phenomenology. Additionally, one can include other new particles that reside around DM mass scales and mediate interactions between flavored DM and the SM fields. Such an extension would not spoil the DM stability unless the MFV ansatz and the flavor triality condition are violated. It could expand viable parameter spaces, making it

compatible with the freeze-out and freeze-in production. Two-component DM parameter spaces might be enabled by this extension.

As a final remark, it should be noticed that the flavor triality condition is a sufficient condition for the DM stability, but not necessary. Thus, it is possible in general that one finds a specific combination of flavor, EW and Lorentz representations that does not satisfy the triality condition, but leads to an accidentally long-lived or absolutely stable neutral state. Such a new candidate could be systematically explored by employing the Hilbert series [129].

7 Summary

The MFV hypothesis provides a robust framework for studying new physics models that include additional sources of flavor violation. As application of this framework to DM, it is established that the lightest component of a new flavored field can be naturally stabilized [6].

In this paper, we investigated a possibility that, under the MFV framework, the heavy components of such a new flavored field are also stable over cosmological timescales and constitute a significant portion of DM. For illustration, we consider a gauge singlet, $SU(3)_{u_R}$ triplet scalar field, which is one of the simplest candidates for flavored DM. All relevant interactions and the mass spectrum of the flavored scalars are governed by the quark Yukawa couplings, the CKM matrix and the MFV expansion parameter ϵ , up to UV-model dependent $\mathcal{O}(1)$ coefficients. We evaluate the lifetimes of the heavy components as they decay into the lighter ones and SM particles. The decay processes are driven by the couplings to the Higgs boson and the dim-6 operators. The Higgs-mediated decay does not occur at the leading order of the MFV expansion and is significantly suppressed by the small expansion parameter ϵ and the light-quark Yukawa couplings. Conversely, the dim-6 operators induce the heavy scalar decay even in the $\epsilon \rightarrow 0$ limit. Meanwhile, such decay processes are suppressed by the cutoff scale Λ , which can be extremely high.

We identified parameter spaces where two or three components of the flavored scalar field have sufficient longevity to serve as DM. In the analysis, we adopt $\epsilon = 10^{-2}$, which is a minimum value induced from radiative corrections through the weak interactions. The parameter spaces for multi-component DM are derived by requiring that the lifetimes of the heavy states are longer than the age of the universe. These parameter spaces are compatible with the DM production in the conventional freeze-out and freeze-in mechanisms and the current direct detection bounds. See figures 5 and 7 for our main results. In conclusion, multi-component flavored DM we proposed in this paper would provide a rich phenomenology and cosmology. Several implications are briefly mentioned in section 6. These subjects will be addressed in future.

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A N -body phase space with cluster decomposition

A.1 Methodology

Lorentz-invariant N -body phase space with invariant mass M is defined by

$$\Phi_N(M^2; m_i^2) = \int \delta^4(Q - \sum_i^N p_i) \prod_i^N \frac{d^3 p_i}{(2\pi)^3 2E_i}. \quad (\text{A.1})$$

Here, we define $Q^2 = M^2$ and $p_i^2 = m_i^2$. The phase space Φ_N is a function only of M^2 and m_i^2 and useful to evaluate partial width for an N -body decay process $A(Q) \rightarrow \sum_i^N a_i(p_i)$,

$$\Gamma = \int \frac{(2\pi)^4}{2M} |\mathcal{M}|^2 d\Phi_N. \quad (\text{A.2})$$

Using two-cluster decomposition [130], eq. (A.1) can be decomposed into two clusters of m and n particles with $N = m + n$ (i.e. one being a cluster of the m particles with invariant mass M_1 and the other a cluster for the remaining n particles with invariant mass M_2),

$$\Phi_N(M^2; \mu_i^2, \mu_j^2) = \frac{\pi}{2} \int dM_1^2 dM_2^2 \Phi_m(M_1^2; \mu_i^2) F_1(M_1^2/M^2, M_2^2/M^2) \Phi_n(M_2^2; \mu_j^2), \quad (\text{A.3})$$

where μ_i denote masses of the particles in the m -cluster and μ_j in the n -cluster, and

$$F_1(x, y) = \sqrt{1 - 2(x + y) + (x - y)^2}. \quad (\text{A.4})$$

In some case, it is convenient to introduce normalized masses,

$$x = \frac{M_1^2}{M^2}, \quad u_i = \frac{\mu_i^2}{M^2}, \quad (\text{A.5})$$

$$y = \frac{M_2^2}{M^2}, \quad v_j = \frac{\mu_j^2}{M^2}, \quad (\text{A.6})$$

and normalized phase space,

$$\Phi_N(1; u_i, v_j) = \frac{\pi}{2} \int dx dy \Phi_m(x; u_i) F_1(x, y) \Phi_n(y; v_j), \quad (\text{A.7})$$

which is related to eq. (A.1) as

$$\Phi_N(1; u_i, v_j) = \frac{\Phi_N(M^2; \mu_i^2, \mu_j^2)}{(M^2)^{N-2}}. \quad (\text{A.8})$$

The integrand of eq. (A.7) can be understood as a joint distribution in x, y . Note that once applying eq. (A.7) for $m = 1$ (with its normalized mass being x) and $N = n + 1$ clusters, we obtain

$$\Phi_{n+1}(1; x, v_j) = \frac{1}{(2\pi)^3} \frac{\pi}{2} \int dy F_1(x, y) \Phi_n(y; v_j), \quad (\text{A.9})$$

which leads to another integral form of $\Phi_N(1; u_i, v_j)$,

$$\Phi_N(1; u_i, v_j) = (2\pi)^3 \int dx \Phi_m(x; u_i) \Phi_{n+1}(1; x, v_j). \quad (\text{A.10})$$

The latter expression is useful in some case.

For $N \geq 4$, in general, the N -body phase space eq. (A.1) is not expressed in closed form except for two special situations: (i) all particles massless, (ii) one particle massive and the others massless. In addition, we have closed form phase space in a general case for $N = 1, 2$ and in a case with two particles massive and the other massless for $N = 3$. We explicitly show those phase space expressions in the following subsections.

A.2 For $N = 1, 2$

The 1-body and 2-body phase spaces are trivial and found to be

$$\Phi_1(M^2; m_1^2) = \frac{1}{(2\pi)^3} \delta(m_1^2 - M^2), \quad (\text{A.11})$$

$$\Phi_2(M^2; m_1^2, m_2^2) = \frac{1}{128\pi^5} F_1(m_1^2/M^2, m_2^2/M^2). \quad (\text{A.12})$$

A.3 For $N = 3$

The 3-body phase space with two particles massive and one massless is given by

$$\Phi_3(1; u_1, u_2, 0) = 2\Phi_3(1; 0) \int_{(\sqrt{u_1} + \sqrt{u_2})^2}^1 dy (1-y) F_1(u_1/y, u_2/y), \quad (\text{A.13})$$

which is expressed explicitly in terms of the elementary functions,

$$\begin{aligned} \Phi_3(1; u_1, u_2, 0) = \Phi_3(1; 0) & \left\{ (1 + u_1 + u_2) F_1(u_1, u_2) \right. \\ & + (u_1 + u_2 + |u_1 - u_2| - u_1 u_2) \ln(4u_1 u_2) \\ & + 2(2u_1 u_2 - u_1 - u_2) \ln |F_1(u_1, u_2) + 1 - u_1 - u_2| \\ & \left. - 2|u_1 - u_2| \ln \left| (u_1 - u_2)^2 - (u_1 + u_2) + |u_1 - u_2| F_1(u_1, u_2) \right| \right\}. \quad (\text{A.14}) \end{aligned}$$

A.4 For any N with all particles massless

In a case with all particles massless, we have the N -body phase space in closed form for an arbitrary N . It is

$$\Phi_N(M^2; 0) = \frac{8(M^2)^{N-2}}{(4\pi)^{2N+1} (N-1)! (N-2)!}. \quad (\text{A.15})$$

This is consistent with the result in [131].

We prove eq. (A.15) here. To this end, we first relate $\Phi_N(1;0)$ to $\Phi_{N-1}(1;0)$. Taking $m = N - 1$ and $n = 1$ in eq. (A.10) for all particles massless, $\Phi_N(1;0)$ is written by

$$\begin{aligned}\Phi_N(1;0) &= (2\pi)^3 \int_0^1 dx \Phi_{N-1}(x;0) \Phi_2(1;x,0) \\ &= \frac{1}{(4\pi)^2} \int_0^1 dx (1-x) \Phi_{N-1}(x;0) \\ &= \frac{1}{(4\pi)^2} \int_0^1 dx (1-x) x^{(N-1)-2} \Phi_{N-1}(1;0) \\ &= \frac{\Phi_{N-1}(1;0)}{(4\pi)^2} \int_0^1 dx (1-x) x^{N-3},\end{aligned}\tag{A.16}$$

where $\Phi_N(x;0) = x^{N-2} \Phi_N(1;0)$ is used in the third equality. Using the mathematical equality in the Γ functions,

$$\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} = \int_0^1 dt t^{x-1} (1-t)^{y-1},\tag{A.17}$$

$\Phi_N(1;0)$ is related to $\Phi_{N-1}(1;0)$ as

$$\begin{aligned}\Phi_N(1;0) &= \frac{\Phi_{N-1}(1;0)}{(4\pi)^2} \frac{\Gamma(N-2)\Gamma(2)}{\Gamma(N)} \\ &= \frac{\Phi_{N-1}(1;0)}{(4\pi)^2 (N-1)(N-2)}.\end{aligned}\tag{A.18}$$

Using this relation recursively, we get

$$\begin{aligned}\Phi_N(1;0) &= \frac{1}{(4\pi)^2 (N-1)(N-2)} \times \cdots \times \frac{1}{(4\pi)^2 (3-1)(3-2)} \Phi_2(1;0) \\ &= \frac{\Phi_2(1;0)}{(4\pi)^{2(N-2)} (N-1)!(N-2)!} \\ &= \frac{8}{(4\pi)^{2N+1} (N-1)!(N-2)!}.\end{aligned}\tag{A.19}$$

In the end, eq. (A.15) is easily obtained using $\Phi_N(M;0) = M^{2N-4} \Phi_N(1;0)$.

A.5 For any N with one particle massive and the others massless

In a case with only one massive (its mass μ) and the others massless, we also have a closed form phase space for any N . It is given by

$$\Phi_N(1;v) = \frac{8(N-1)(N-2)}{(4\pi)^{2N+1} (N-1)!(N-2)!} \int_0^{x_{\max}} dx x^{N-3} F_1(x,v),\tag{A.20}$$

where $v = \mu^2/M^2$ and $x_{\max} = (1 - \mu/M)^2$. It is easy to see that $\Phi_N(1;v)$ can be expressed in terms of $\Phi_N(1;0)$ (the N -body phase space with all massless particles),

$$\Phi_N(1;v) = \Phi_N(1;0) f_{N-1}(v),\tag{A.21}$$

where we define $f_N(v)$ as

$$f_1(v) := F_1(v, 0) = 1 - v, \quad (\text{A.22})$$

$$f_N(v) := N(N-1) \int_0^{x_{\max}} dx x^{N-2} F_1(x, v). \quad (N \geq 2) \quad (\text{A.23})$$

Below, we list explicit forms of $f_N(v)$ for $N = 2, 3, 4$ for a practical purpose

$$f_2(v) = 1 - v^2 + 2v \ln v, \quad (\text{A.24})$$

$$f_3(v) = 1 + 9v - 9v^2 - v^3 + 6v(1+v) \ln v, \quad (\text{A.25})$$

$$f_4(v) = 1 + 28v - 28v^3 - v^4 + 12v(1+3v+v^2) \ln v, \quad (\text{A.26})$$

leading to

$$\Phi_2(1; v) = \frac{1-v}{128\pi^5}, \quad \Phi_3(1; v) = \frac{f_2(v)}{4096\pi^7}, \quad (\text{A.27})$$

$$\Phi_4(1; v) = \frac{f_3(v)}{393216\pi^9}, \quad \Phi_5(1; v) = \frac{f_4(v)}{75497472\pi^{11}}. \quad (\text{A.28})$$

One finds that $\Phi_2(1; v)$ above is consistent with eq. (A.12). In some case, it is useful to expand $f_N(v)$ around $v = 1$. We have in the leading order

$$f_2(v) \simeq \frac{1}{3}(1-v)^3, \quad f_3(v) \simeq \frac{1}{10}(1-v)^5, \quad f_4(v) \simeq \frac{1}{35}(1-v)^7. \quad (\text{A.29})$$

These expressions are used to evaluate the approximate decay widths in section 4.

B Dark matter production

Regardless of whether DM is composed of a single component or multi-component, they have to be produced with the correct cosmological abundance in the early universe. In our model, DM can be produced from the SM plasma through the Higgs portal interactions and dim-6 operators. In this appendix, we evaluate DM relic abundance by taking the conventional thermal freeze-out [132–135] and freeze-in [136–146]⁴ as their production mechanisms. Other production mechanisms can succeed, depending on parameter choice and cosmological history.

In both production mechanisms, the time evolution of number densities n_i for S_i is governed by Boltzmann equations,

$$\frac{dn_i}{dt} + 3Hn_i = 2 \int \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i} \mathcal{C}[f_i], \quad (\text{B.1})$$

where H is the expansion rate,

$$H = \sqrt{\frac{8\pi G_N}{3}\rho}, \quad \rho = \frac{\pi^2}{30} g_*(T) T^4, \quad (\text{B.2})$$

with T being the temperature of the SM plasma and $g_*(T) = g_{*,\text{SM}}(T) + \sum_i g_{*,S_i}(T)$ the effective relativistic degrees of freedom. The collision term $\mathcal{C}[f_i]$ encodes all of DM

⁴Freeze-in production is also discussed in the case of DM being axino [147, 148], sneutrino [149–151] and sterile neutrino [152–154].

number changing reactions induced from microphysical interactions. Focusing only on $2 \rightarrow 2$ processes, the pertinent contribution in our model comes from flavored scalar (co)annihilation $S_i S_j^* \rightarrow u_k \bar{u}_l$ and its inverse process, which are induced by the interactions in eqs. (3.2) and (3.22). For simplicity, the Higgs portal interactions $\lambda_{h S_i}$ are ignored here.⁵ Then, the collision term takes the form,

$$\begin{aligned} \mathcal{C}[f_i] = & -\frac{1}{2} \int \frac{d^3 \mathbf{p}_j}{(2\pi)^3 2E_j} \frac{d^3 \mathbf{p}_k}{(2\pi)^3 2E_k} \frac{d^3 \mathbf{p}_l}{(2\pi)^3 2E_l} (2\pi)^4 \delta^{(4)}(p_i + p_j - p_k - p_l) \left| \mathcal{M}(S_i S_j^* \rightarrow u_k \bar{u}_l) \right|^2 \\ & \times \{ f_i(\mathbf{p}_i) f_j(\mathbf{p}_j) [1 - f_k(\mathbf{p}_k)] [1 - f_l(\mathbf{p}_l)] - f_k(\mathbf{p}_k) f_l(\mathbf{p}_l) [1 + f_i(\mathbf{p}_i)] [1 + f_j(\mathbf{p}_j)] \} , \end{aligned} \quad (\text{B.3})$$

where $f(\mathbf{p})$ denotes momentum distribution for a particle species with four-momentum $p^\mu = (E, \mathbf{p})$ and $E = (m^2 + \mathbf{p}^2)^{1/2}$, and we assume the time reversal is respected in the processes, i.e. $|\mathcal{M}(S_i S_j^* \rightarrow u_k \bar{u}_l)| = |\mathcal{M}(u_k \bar{u}_l \rightarrow S_i S_j^*)|$. The spin-summed squared amplitude is given by

$$\begin{aligned} \sum_{\text{spin}} |\mathcal{M}(S_i S_j^* \rightarrow u_k \bar{u}_l)|^2 = & \frac{N_c}{\Lambda^4} \left[\left(s - (m_u^k + m_u^l)^2 \right) \left\{ (c_1)^2 \left((m_u^k)^2 + (m_u^l)^2 \right) \delta_{ik} \delta_{jl} \right. \right. \\ & + 2c_1 c_2 m_u^k \left(m_u^k + m_u^l \right) \delta_{ij} \delta_{kl} \delta_{ik} + 2(c_2)^2 (m_u^k)^2 \delta_{ij} \delta_{kl} \Big\} \\ & \left. + 2(c_1)^2 m_k m_l (m_k - m_l)^2 \delta_{ik} \delta_{jl} \right] . \end{aligned} \quad (\text{B.4})$$

In the freeze-out scenario, the annihilation evaluated at $s \simeq (M_i + M_j)^2$ determines the DM relic abundance, whereas the freeze-in production is most effective at $s \simeq T_{\text{RH}}^2$, where T_{RH} is reheating temperature.

B.1 Freeze-out

In the freeze-out scenario, DM particles are assumed to be in thermal equilibrium with the SM plasma at high temperatures, when DM annihilation and creation reactions are balanced. As the universe expands and cools down, the rate of the reactions decreases and in the end, when the temperature cools down to $T \sim m_{\text{DM}}/20$, the DM number changing processes are frozen and the DM abundance is fixed.

Ignoring the quantum statistical factors and assuming that S_i are in kinetic equilibrium with the thermal plasma, eq. (B.3) is simplified to [155, 156]

$$\frac{dn_i}{dt} + 3Hn_i = - \sum_{j,k,l} \langle \sigma v_r \rangle_{ij \rightarrow kl} \left(n_i n_j - n_i^{\text{eq}} n_j^{\text{eq}} \right) , \quad (\text{B.5})$$

where n_i^{eq} is the equilibrium number density of S_i ,

$$n_i^{\text{eq}}(T) = \frac{m_i^2 T}{2\pi^2} K_2(m/T) , \quad (\text{B.6})$$

⁵See [7] for the freeze-out production with the Higgs portal coupling in a single-component DM scenario.

with $K_2(x)$ the modified Bessel function of second kind of order 2. Here, we assumed that DM is symmetric relic (i.e. $n_{S_i} = n_{S_i^*} = n_i$) and all SM particles follow the thermal distribution. The thermal averaged cross section is defined by

$$\langle \sigma v_r \rangle_{ij \rightarrow kl} \equiv \frac{T}{32\pi^4} \frac{1}{n_i^{\text{eq}}(T) n_j^{\text{eq}}(T)} \int_{s_{\min}}^{\infty} ds \sigma_{ij \rightarrow kl} \frac{\lambda(s, M_i^2, M_j^2)}{\sqrt{s}} K_1(\sqrt{s}/T), \quad (\text{B.7})$$

where $s_{\min} = \text{Max}[(M_i + M_j)^2, (m_u^k + m_u^l)^2]$ and $\sigma_{ij \rightarrow kl}$ is the cross section for the $S_i S_j^* \rightarrow u_k \bar{u}_l$ process.

In parameter spaces where the freeze-out production succeeds, the cutoff scale will be around the EW scale. In order for the heavy flavored scalars to be stable and DM, they have to be highly degenerate with S_1 . If unstable, instead, they have to decay into S_1 prior to the BBN. In either case, the total energy density of DM is given to a good approximation by $\rho_{\text{DM}} = 2 \sum_i M_i n_i \simeq M_1 n_{\text{DM}}$, where $n_{\text{DM}} = 2 \sum_i n_i$ and the prefactor of 2 counts DM and anti-DM. Moreover, conversion reactions, such as $S_1 t \leftrightarrow S_3 u$, occur more frequently than the annihilation reactions at the freeze-out time, because the number density of the SM particles is many orders of magnitude larger than that of the DM particles. Then, the fraction of the number densities of S_i follows that of the equilibrium distributions, that is,

$$\frac{n_i}{n_{\text{DM}}} \simeq \frac{n_i^{\text{eq}}}{n_{\text{DM}}^{\text{eq}}}. \quad (\text{B.8})$$

As a result, we obtain just a single equation for the time evolution of the total DM number density,

$$\frac{dn_{\text{DM}}}{dt} + 3H n_{\text{DM}} = - \langle \sigma v_r \rangle_{\text{eff}} \left[(n_{\text{DM}})^2 - (n_{\text{DM}}^{\text{eq}})^2 \right], \quad (\text{B.9})$$

where the effective cross section is defined by

$$\langle \sigma v_r \rangle_{\text{eff}} = \sum_{i,j} \langle \sigma v_r \rangle_{ij} \frac{2n_i^{\text{eq}} n_j^{\text{eq}}}{(n_{\text{DM}}^{\text{eq}})^2}, \quad (\text{B.10})$$

with

$$\langle \sigma v_r \rangle_{ij} = \sum_{k,l} \langle \sigma v_r \rangle_{ij \rightarrow k,l}. \quad (\text{B.11})$$

It is well known that $\langle \sigma v_r \rangle_{\text{eff}} \simeq 3 \times 10^{-26} \text{cm}^3/\text{sec}$ at $T \simeq m_{\text{DM}}/20$ provides the canonical cross section to produce the observed DM abundance in the freeze-out scenario.

It is illuminating to estimate the thermal relic abundance in the case of $M_1 \simeq M_2 \simeq M_3$. In this case, the flavored scalars have comparable equilibrium densities, $n_1^{\text{eq}} \simeq n_2^{\text{eq}} \simeq n_3^{\text{eq}} \simeq n_{\text{DM}}^{\text{eq}}/6$. Then, the effective cross section approximates to

$$\langle \sigma v_r \rangle_{\text{eff}} \simeq \frac{1}{18} \sum_{i,j} \langle \sigma v_r \rangle_{ij}. \quad (\text{B.12})$$

For $M_i \geq m_t$, the (co)annihilation processes involving top quarks in the final states dominate the production. The cross section for those processes in the non-relativistic limit $s \simeq (M_i + M_j)^2$ is given by

$$\langle \sigma v_r \rangle_{ij} \simeq \frac{N_c m_t^2}{4\pi\Lambda^4} \times \left\{ \frac{(c_1)^2 (\delta_{i3} + \delta_{j3})}{2} + 2c_1 c_2 \delta_{i3} \delta_{j3} + (c_2)^2 \delta_{ij} \right\}, \quad (\text{B.13})$$

leading to

$$\langle \sigma v_r \rangle_{\text{eff}} \simeq \frac{N_c c^2 m_t^2}{9\pi \Lambda^4} \simeq 3.3 \times 10^{-26} \text{cm}^3/\text{sec} \times c^2 \left(\frac{1 \text{ TeV}}{\Lambda} \right)^4, \quad (\text{B.14})$$

with

$$c^2 \equiv \frac{3(c_1)^2 + 2c_1 c_2 + 3(c_2)^2}{8}. \quad (\text{B.15})$$

When the (co)annihilation into top quarks are kinematically forbidden, we need to take into account the (co)annihilation into lighter quarks. Taking $m_c \leq M_i \leq m_t/2$ for concreteness,⁶ we find the effective cross section to be

$$\langle \sigma v_r \rangle_{\text{eff}} \simeq \frac{N_c c^2 m_c^2}{9\pi \Lambda^4} \simeq 2.0 \times 10^{-26} \text{cm}^3/\text{sec} \times c^2 \left(\frac{100 \text{ GeV}}{\Lambda} \right)^4. \quad (\text{B.16})$$

The cutoff scale takes $\Lambda \simeq 100 \text{ GeV} - 1 \text{ TeV}$ as anticipated.

B.2 Freeze-in

In the freeze-in scenario, it is assumed that DM particles never reach equilibrium with the SM plasma during the cosmological history. The time evolution of the flavored scalar number densities is governed by Boltzmann equations implementing only the one-way processes $u_k \bar{u}_l \rightarrow S_i S_j^*$. Assuming the Boltzmann distribution $f_{k,l} = e^{-E_{k,l}/T}$ for initial-state up-type quarks and ignoring unimportant quantum statistical factors for S_i , the collision integral takes the form,

$$\mathcal{N}(kl \rightarrow ij) \equiv \int \frac{d^3 \mathbf{p}_i}{(2\pi)^3 E_i} \mathcal{C}[f_i] = \frac{T}{32\pi^4} \int_{s_{\min}}^{\infty} ds \sigma_{kl \rightarrow ij} \frac{\lambda(s, (m_u^k)^2, (m_u^l)^2)}{\sqrt{s}} K_1(\sqrt{s}/T), \quad (\text{B.17})$$

where $\sigma_{kl \rightarrow ij}$ denotes the cross section for $u_k \bar{u}_l \rightarrow S_i S_j^*$,

$$\sigma_{kl \rightarrow ij} = \frac{1}{4v_{\text{Mø}} E_k E_l} \int \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} \frac{d^3 \mathbf{p}_j}{(2\pi)^3 2E_j} (2\pi)^4 \delta^{(4)}(p_k + p_l - p_i - p_j) |\mathcal{M}(u_k \bar{u}_l \rightarrow S_i S_j^*)|^2, \quad (\text{B.18})$$

with $E_{k(l)}$ being the energy of $u_k(\bar{u}_l)$ in a reference frame, in which the Møller velocity is defined by

$$v_{\text{Mø}} = \sqrt{(\mathbf{v}_k - \mathbf{v}_l)^2 - (\mathbf{v}_k \times \mathbf{v}_l)^2}. \quad (\text{B.19})$$

Compared with eq. (B.7), one realizes that the collision integral is expressed by the thermal averaged cross section,

$$\mathcal{N}(kl \rightarrow ij) = \langle \sigma v_r \rangle_{kl \rightarrow ij} n_k^{\text{eq}}(T) n_l^{\text{eq}}(T). \quad (\text{B.20})$$

It is convenient to rewrite the Boltzmann equations using $Y_i = n_i/s$, where s is the entropy density of the universe. Given $\frac{d}{dt} = -\tilde{H}T \frac{d}{dT}$ in the radiation dominant universe, we find

$$\frac{dY_i}{dT} = -\frac{1}{s\tilde{H}T} \sum_{j,k,l} \mathcal{N}(kl \rightarrow ij), \quad (\text{B.21})$$

⁶This assumption is justified in the freeze-out case, since thermal relic DM with a lighter mass is excluded by the CMB measurements.

where $i = 1, 2, 3$ and

$$\tilde{H} \equiv H \left(1 + \frac{1}{3} \frac{d \ln g_{*s}}{d \ln T} \right)^{-1}. \quad (\text{B.22})$$

The present yield $Y_i(T_0)$ of each scalar is obtained by solving eq. (B.21) from $T = T_{\text{RH}}$ to $T = T_0$ with zero initial abundance $Y_i(T_{\text{RH}}) = 0$ as a boundary condition. The total DM abundance is calculated by

$$\Omega_{\text{DM}} = \frac{s_0}{\rho_{\text{crit}}} \times 2 \sum_i M_i Y_i(T_0). \quad (\text{B.23})$$

With the values of the present entropy density s_0 and critical density ρ_{crit} [157],

$$s_0 = 2891.2 \text{ cm}^{-3}, \quad \rho_{\text{crit}} = 1.053672(24) \times 10^{-5} h^2 \text{ GeV/cm}^3, \quad (\text{B.24})$$

we obtain

$$\Omega_{\text{DM}} h^2 \simeq 0.12 \times \sum_i \frac{M_i}{100 \text{ GeV}} \frac{Y_i(T_0)}{2.2 \times 10^{-12}}. \quad (\text{B.25})$$

Let us estimate the total DM abundance for $M_i, m_u^i \ll T_{\text{RH}} \leq \Lambda$. Assuming instantaneous reheating and $Y_i(T_{\text{RH}}) \simeq 0$, the solution of the Boltzmann equations is given by

$$Y_i(T_0) \simeq \int_{T_0}^{T_{\text{RH}}} d \ln T \frac{1}{s \tilde{H}} \sum_{k,l,j} \mathcal{N}(kl \rightarrow ij). \quad (\text{B.26})$$

The integrand is proportional to T^1 , since $\mathcal{N}(kl \rightarrow ij) \propto T^6$, $s(T) \propto T^3$ and $\tilde{H} \simeq H \propto T^2$ at high enough temperatures. This means that the DM production occurs most efficiently at $T \simeq T_{\text{RH}}$, and eq. (B.26) approximates to

$$Y_i(T_0) \simeq \gamma_i T_{\text{RH}}, \quad \gamma_i \simeq \frac{1}{s H T} \sum_{k,l,j} \mathcal{N}(kl \rightarrow ij) \Big|_{T=T_{\text{RH}}}. \quad (\text{B.27})$$

Given that at high temperatures the cross section approximates to

$$\sum_{k,l} \langle \sigma v_r \rangle_{kl \rightarrow ij} \simeq \frac{N_c m_t^2}{8\pi \Lambda^4} \times \left\{ \frac{(c_1)^2 (\delta_{i3} + \delta_{j3})}{2} + 2c_1 c_2 \delta_{i3} \delta_{j3} + (c_2)^2 \delta_{ij} \right\}, \quad (\text{B.28})$$

we find

$$\gamma_i \simeq 0.5 \times 10^{-14} c_i^2 \left(\frac{10^{10} \text{ GeV}}{\Lambda} \right)^4 \left(\frac{1}{10^8 \text{ GeV}} \right), \quad (\text{B.29})$$

where we take $m_t = 162 \text{ GeV}$ and $g_* = g_{*,s} = 106.75$ and

$$c_i^2 := \sum_j \left[\frac{(c_1)^2 (\delta_{i3} + \delta_{j3})}{2} + 2c_1 c_2 \delta_{i3} \delta_{j3} + (c_2)^2 \delta_{ij} \right]. \quad (\text{B.30})$$

The freeze-in abundance is given by

$$\Omega_{\text{DM}} h^2 \simeq 0.12 \times \left(\frac{10^{10} \text{ GeV}}{\Lambda} \right)^4 \left(\frac{T_{\text{RH}}}{10^8 \text{ GeV}} \right) \sum_i c_i^2 \left(\frac{M_i}{45 \text{ TeV}} \right). \quad (\text{B.31})$$

We have confirmed that this abundance estimate agrees with numerical results calculated by micrOMEGAs_5_2_4 [158] in an appropriate limit.

$f_{n,u}$	0.0110	$f_{p,u}$	0.0153
$f_{n,d}$	0.0273	$f_{p,d}$	0.0191
$f_{n,s}$	0.0447	$f_{p,s}$	0.0447

Table 1. The nucleon matrix elements for the light quarks. The values correspond to those of the micrOMEGAs default [159].

C Direct detection with nuclear recoils

DM can be directly detected in terrestrial experiments through scattering off nucleons and electrons in a target material. This direct detection approach provides strong constraints on DM candidates produced by the thermal freeze-out mechanism.

In our case, DM can (in)elastically scatter off nucleons in a target nucleus, $S_i N \rightarrow S_j N$, with the dim-6 operators and the Higgs portal interactions. For a while, we ignore the latter interactions. Since in the case of inelastic scattering, terrestrial direct detection experiments can only probe a small mass splitting $\Delta M_S \lesssim \mathcal{O}(100)$ keV which is out of our scope, we focus on the elastic scattering case here.⁷ After the EW symmetry breaking and in the leading MFV expansion, the relevant interaction Lagrangian is given by eq. (3.22). From these interactions, we find spin-independent cross section for S_i -nucleon elastic scattering,

$$\sigma_{\text{SI},i} = \frac{\mu^2 m_N^2}{4\pi M_i^2} \frac{1}{\Lambda^4} \left(\frac{Z C_{p,i} + (A - Z) C_{n,i}}{A} \right)^2. \quad (\text{C.1})$$

Here, $\mu = m_N M_i / (m_N + M_i)$ is DM-nucleon reduced mass and Z and A are atomic number and mass of a target nucleus, and

$$C_{N,i} = c_1 f_{N,u,i} + c_2 (f_{N,u} + f_{N,c} + f_{N,t}), \quad \text{for } N = p, n \quad (\text{C.2})$$

with nucleon matrix elements $f_{N,q}$ for quarks q ,

$$\langle N | m_q \bar{q} q | N \rangle = m_N f_{N,q}. \quad (\text{C.3})$$

Using the QCD trace anomaly matching, the nucleon matrix elements for heavy quarks Q are related to the one for gluon. At the leading order, we find

$$\langle N | m_Q \bar{Q} Q | N \rangle \simeq -\langle N | \frac{\alpha_s}{12\pi} G_{\mu\nu} G^{\mu\nu} | N \rangle = \frac{2}{27} m_N f_{N,g}, \quad (\text{C.4})$$

where

$$m_N f_{N,g} = -\langle N | \frac{9\alpha_s}{8\pi} G_{\mu\nu} G^{\mu\nu} | N \rangle \quad (\text{C.5})$$

and $f_{N,g} = 1 - \sum_{q=u,d,s} f_{N,q}$. See table 1 for the values of $f_{N,q}$ for light quarks, which we use in our numerical analysis. Note that effects of non-vanishing Higgs portal couplings can be easily included by modifying in eq. (C.1) as

$$C_{N,i} \rightarrow C_{N,i} + \lambda_{hSi} \frac{\Lambda^2}{m_h^2} \left(\frac{2}{9} + \frac{7}{9} \sum_{q=u,d,s} f_{N,q} \right). \quad (\text{C.6})$$

⁷A larger mass splitting up to $\Delta M_S \lesssim 100$ MeV might be tested with future neutron star surface temperature observations [160–165], although recent studies discuss a possibility that built-in heating mechanisms of neutron stars would conceal extra heating through DM scattering and annihilation [166–169].

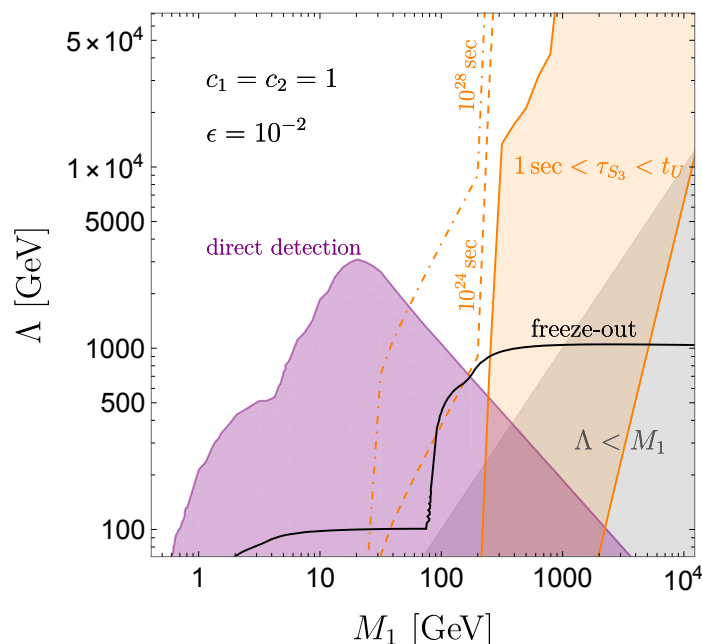


Figure 8. Direct detection bound (purple) with $c_1 = c_2 = 1$. The MFV expansion parameter is set to $\epsilon = 10^{-2}$, which is related to the mass splitting as $\epsilon \simeq \Delta M/(y_t^2 M_1)$. The black line corresponds to the total abundance being equal to the observed value, i.e. $\sum_{i=1}^3 \Omega_{S_i+S_i^*} h^2 = 0.12$, where we assume the freeze-out production. The orange region is excluded by the constraint on the S_3 lifetime, where $1 \text{ sec} < \tau_{S_3} < t_U$. The orange dashed and dot-dashed lines correspond to contours of $\tau_{S_3} = 10^{24} \text{ sec}$ and 10^{28} sec .

In figure 8, we show the current direct detection bound with $c_1 = c_2 = 1$ and $\lambda = 0$, which excludes the purple shaded region. The MFV expansion parameter is set to $\epsilon = 10^{-2}$, which is related to the mass splitting as $\epsilon \simeq \Delta M/(y_t^2 M_1)$, see also eq. (3.7). The black line corresponds to the total abundance being equal to the observed value, i.e. $\sum_{i=1}^3 \Omega_{S_i+S_i^*} h^2 = 0.12$, where we assume the freeze-out production. The orange region is excluded by the constraint on the S_3 lifetime, where $1 \text{ sec} < \tau_{S_3} < t_U$. The orange dashed and dot-dashed lines correspond to contours of $\tau_{S_3} = 10^{24} \text{ sec}$ and 10^{28} sec . In the gray shaded region, we have $\Lambda \leq M_1$, where the EFT description is not justified.

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References

- [1] R.S. Chivukula and H. Georgi, *Composite Technicolor Standard Model*, *Phys. Lett. B* **188** (1987) 99 [[INSPIRE](#)].
- [2] L.J. Hall and L. Randall, *Weak scale effective supersymmetry*, *Phys. Rev. Lett.* **65** (1990) 2939 [[INSPIRE](#)].
- [3] A.J. Buras et al., *Universal unitarity triangle and physics beyond the standard model*, *Phys. Lett. B* **500** (2001) 161 [[hep-ph/0007085](#)] [[INSPIRE](#)].

- [4] G. D'Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, *Minimal flavor violation: An effective field theory approach*, *Nucl. Phys. B* **645** (2002) 155 [[hep-ph/0207036](#)] [[INSPIRE](#)].
- [5] A.L. Kagan, G. Perez, T. Volansky and J. Zupan, *General Minimal Flavor Violation*, *Phys. Rev. D* **80** (2009) 076002 [[arXiv:0903.1794](#)] [[INSPIRE](#)].
- [6] B. Batell, J. Pradler and M. Spannowsky, *Dark Matter from Minimal Flavor Violation*, *JHEP* **08** (2011) 038 [[arXiv:1105.1781](#)] [[INSPIRE](#)].
- [7] L. Lopez-Honorez and L. Merlo, *Dark matter within the minimal flavour violation ansatz*, *Phys. Lett. B* **722** (2013) 135 [[arXiv:1303.1087](#)] [[INSPIRE](#)].
- [8] B. Batell, T. Lin and L.-T. Wang, *Flavored Dark Matter and R-Parity Violation*, *JHEP* **01** (2014) 075 [[arXiv:1309.4462](#)] [[INSPIRE](#)].
- [9] F. Bishara et al., *Dark Matter and Gauged Flavor Symmetries*, *JHEP* **12** (2015) 130 [[arXiv:1505.03862](#)] [[INSPIRE](#)].
- [10] F. Bishara and J. Zupan, *Continuous Flavor Symmetries and the Stability of Asymmetric Dark Matter*, *JHEP* **01** (2015) 089 [[arXiv:1408.3852](#)] [[INSPIRE](#)].
- [11] P. Agrawal, M. Blanke and K. Gemmler, *Flavored dark matter beyond Minimal Flavor Violation*, *JHEP* **10** (2014) 072 [[arXiv:1405.6709](#)] [[INSPIRE](#)].
- [12] J. Kile and A. Soni, *Flavored Dark Matter in Direct Detection Experiments and at LHC*, *Phys. Rev. D* **84** (2011) 035016 [[arXiv:1104.5239](#)] [[INSPIRE](#)].
- [13] J.F. Kamenik and J. Zupan, *Discovering Dark Matter Through Flavor Violation at the LHC*, *Phys. Rev. D* **84** (2011) 111502 [[arXiv:1107.0623](#)] [[INSPIRE](#)].
- [14] P. Agrawal, S. Blanchet, Z. Chacko and C. Kilic, *Flavored Dark Matter, and Its Implications for Direct Detection and Colliders*, *Phys. Rev. D* **86** (2012) 055002 [[arXiv:1109.3516](#)] [[INSPIRE](#)].
- [15] K. Kannike, *Vacuum Stability Conditions From Copositivity Criteria*, *Eur. Phys. J. C* **72** (2012) 2093 [[arXiv:1205.3781](#)] [[INSPIRE](#)].
- [16] J. Alwall et al., *The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations*, *JHEP* **07** (2014) 079 [[arXiv:1405.0301](#)] [[INSPIRE](#)].
- [17] J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *The Higgs Hunter's Guide*, CRC Press (2000) [[DOI:10.1201/9780429496448](#)] [[INSPIRE](#)].
- [18] A.I. Vainshtein, V.I. Zakharov and M.A. Shifman, *Higgs Particles*, *Sov. Phys. Usp.* **23** (1980) 429 [[INSPIRE](#)].
- [19] M.B. Voloshin and V.I. Zakharov, *Measuring QCD Anomalies in Hadronic Transitions Between Onium States*, *Phys. Rev. Lett.* **45** (1980) 688 [[INSPIRE](#)].
- [20] V.A. Novikov and M.A. Shifman, *Comment on the ψ -prime $\rightarrow J/\psi \pi\pi$ Decay*, *Z. Phys. C* **8** (1981) 43 [[INSPIRE](#)].
- [21] M.B. Voloshin, *Once Again About the Role of Gluonic Mechanism in Interaction of Light Higgs Boson with Hadrons*, *Sov. J. Nucl. Phys.* **44** (1986) 478 [[INSPIRE](#)].
- [22] B. Grinstein, L.J. Hall and L. Randall, *Do B meson decays exclude a light Higgs?*, *Phys. Lett. B* **211** (1988) 363 [[INSPIRE](#)].
- [23] S. Raby and G.B. West, *The Branching Ratio for a Light Higgs to Decay Into $\mu^+\mu^-$ Pairs*, *Phys. Rev. D* **38** (1988) 3488 [[INSPIRE](#)].

- [24] T.N. Truong and R.S. Willey, *Branching Ratios for Decays of Light Higgs Bosons*, *Phys. Rev. D* **40** (1989) 3635 [[INSPIRE](#)].
- [25] R.S. Chivukula et al., *Higgs decay into Goldstone bosons*, *Annals Phys.* **192** (1989) 93 [[INSPIRE](#)].
- [26] K. Griest and M. Kamionkowski, *Unitarity Limits on the Mass and Radius of Dark Matter Particles*, *Phys. Rev. Lett.* **64** (1990) 615 [[INSPIRE](#)].
- [27] J. Smirnov and J.F. Beacom, *TeV-Scale Thermal WIMPs: Unitarity and its Consequences*, *Phys. Rev. D* **100** (2019) 043029 [[arXiv:1904.11503](#)] [[INSPIRE](#)].
- [28] O. Lebedev and T. Toma, *Relativistic Freeze-in*, *Phys. Lett. B* **798** (2019) 134961 [[arXiv:1908.05491](#)] [[INSPIRE](#)].
- [29] CMS collaboration, *Search for invisible decays of a Higgs boson produced through vector boson fusion in proton-proton collisions at $\sqrt{s} = 13$ TeV*, *Phys. Lett. B* **793** (2019) 520 [[arXiv:1809.05937](#)] [[INSPIRE](#)].
- [30] ATLAS Collaboration, *Search for invisible Higgs boson decays with vector boson fusion signatures with the ATLAS detector using an integrated luminosity of 139 fb^{-1}* , [INSPIRE](#).
- [31] M. Cirelli et al., *Gamma ray constraints on Decaying Dark Matter*, *Phys. Rev. D* **86** (2012) 083506 [[arXiv:1205.5283](#)] [[INSPIRE](#)].
- [32] A. Massari et al., *Strong Optimized Conservative Fermi-LAT Constraints on Dark Matter Models from the Inclusive Photon Spectrum*, *Phys. Rev. D* **91** (2015) 083539 [[arXiv:1503.07169](#)] [[INSPIRE](#)].
- [33] T. Cohen et al., *γ -ray Constraints on Decaying Dark Matter and Implications for IceCube*, *Phys. Rev. Lett.* **119** (2017) 021102 [[arXiv:1612.05638](#)] [[INSPIRE](#)].
- [34] M. Chianese et al., *Decaying dark matter at IceCube and its signature on High Energy gamma experiments*, *JCAP* **11** (2019) 046 [[arXiv:1907.11222](#)] [[INSPIRE](#)].
- [35] A. Esmaili and P.D. Serpico, *First implications of Tibet AS γ data for heavy dark matter*, *Phys. Rev. D* **104** (2021) L021301 [[arXiv:2105.01826](#)] [[INSPIRE](#)].
- [36] T.N. Maity, A.K. Saha, A. Dubey and R. Laha, *Search for dark matter using sub-PeV γ -rays observed by Tibet AS γ* , *Phys. Rev. D* **105** (2022) L041301 [[arXiv:2105.05680](#)] [[INSPIRE](#)].
- [37] M. Chianese et al., *Constraints on heavy decaying dark matter with current gamma-ray measurements*, *JCAP* **11** (2021) 035 [[arXiv:2108.01678](#)] [[INSPIRE](#)].
- [38] D. Song, K. Murase and A. Kheirandish, *Constraining decaying very heavy dark matter from galaxy clusters with 14 year Fermi-LAT data*, *JCAP* **03** (2024) 024 [[arXiv:2308.00589](#)] [[INSPIRE](#)].
- [39] K. Dutta, A. Ghosh, A. Kar and B. Mukhopadhyaya, *A general study of decaying scalar dark matter: existing limits and projected radio signals at the SKA*, *JCAP* **09** (2022) 005 [[arXiv:2204.06024](#)] [[INSPIRE](#)].
- [40] FERMI-LAT collaboration, *Constraints on the Galactic Halo Dark Matter from Fermi-LAT Diffuse Measurements*, *Astrophys. J.* **761** (2012) 91 [[arXiv:1205.6474](#)] [[INSPIRE](#)].
- [41] VERITAS collaboration, *VERITAS Deep Observations of the Dwarf Spheroidal Galaxy Segue 1*, *Phys. Rev. D* **85** (2012) 062001 [Erratum *ibid.* **91** (2015) 129903] [[arXiv:1202.2144](#)] [[INSPIRE](#)].
- [42] J. Aleksić et al., *Optimized dark matter searches in deep observations of Segue 1 with MAGIC*, *JCAP* **02** (2014) 008 [[arXiv:1312.1535](#)] [[INSPIRE](#)].

- [43] MAGIC collaboration, *Constraining Dark Matter lifetime with a deep gamma-ray survey of the Perseus Galaxy Cluster with MAGIC*, *Phys. Dark Univ.* **22** (2018) 38 [[arXiv:1806.11063](#)] [[INSPIRE](#)].
- [44] HAWC collaboration, *Dark Matter Limits From Dwarf Spheroidal Galaxies with The HAWC Gamma-Ray Observatory*, *Astrophys. J.* **853** (2018) 154 [[arXiv:1706.01277](#)] [[INSPIRE](#)].
- [45] HAWC collaboration, *A Search for Dark Matter in the Galactic Halo with HAWC*, *JCAP* **02** (2018) 049 [[arXiv:1710.10288](#)] [[INSPIRE](#)].
- [46] HAWC collaboration, *An optimized search for dark matter in the galactic halo with HAWC*, *JCAP* **12** (2023) 038 [[arXiv:2305.09861](#)] [[INSPIRE](#)].
- [47] HAWC collaboration, *Search for decaying dark matter in the Virgo cluster of galaxies with HAWC*, *Phys. Rev. D* **109** (2024) 043034 [[arXiv:2309.03973](#)] [[INSPIRE](#)].
- [48] LHAASO collaboration, *Constraints on Heavy Decaying Dark Matter from 570 Days of LHAASO Observations*, *Phys. Rev. Lett.* **129** (2022) 261103 [[arXiv:2210.15989](#)] [[INSPIRE](#)].
- [49] FERMI-LAT collaboration, *Updated search for spectral lines from Galactic dark matter interactions with pass 8 data from the Fermi Large Area Telescope*, *Phys. Rev. D* **91** (2015) 122002 [[arXiv:1506.00013](#)] [[INSPIRE](#)].
- [50] J.W. Foster et al., *Search for dark matter lines at the Galactic Center with 14 years of Fermi data*, *Phys. Rev. D* **107** (2023) 103047 [[arXiv:2212.07435](#)] [[INSPIRE](#)].
- [51] P. De La Torre Luque, J. Smirnov and T. Linden, *Gamma-ray lines in 15 years of Fermi-LAT data: New constraints on Higgs portal dark matter*, *Phys. Rev. D* **109** (2024) L041301 [[arXiv:2309.03281](#)] [[INSPIRE](#)].
- [52] R. Essig et al., *Constraining Light Dark Matter with Diffuse X-Ray and Gamma-Ray Observations*, *JHEP* **11** (2013) 193 [[arXiv:1309.4091](#)] [[INSPIRE](#)].
- [53] K.K. Boddy and J. Kumar, *Indirect Detection of Dark Matter Using MeV-Range Gamma-Ray Telescopes*, *Phys. Rev. D* **92** (2015) 023533 [[arXiv:1504.04024](#)] [[INSPIRE](#)].
- [54] K.C.Y. Ng et al., *New Constraints on Sterile Neutrino Dark Matter from NuSTAR M31 Observations*, *Phys. Rev. D* **99** (2019) 083005 [[arXiv:1901.01262](#)] [[INSPIRE](#)].
- [55] R. Laha, J.B. Muñoz and T.R. Slatyer, *INTEGRAL constraints on primordial black holes and particle dark matter*, *Phys. Rev. D* **101** (2020) 123514 [[arXiv:2004.00627](#)] [[INSPIRE](#)].
- [56] M. Cirelli, N. Fornengo, B.J. Kavanagh and E. Pinetti, *Integral X-ray constraints on sub-GeV Dark Matter*, *Phys. Rev. D* **103** (2021) 063022 [[arXiv:2007.11493](#)] [[INSPIRE](#)].
- [57] B.M. Roach et al., *Long-exposure NuSTAR constraints on decaying dark matter in the Galactic halo*, *Phys. Rev. D* **107** (2023) 023009 [[arXiv:2207.04572](#)] [[INSPIRE](#)].
- [58] F. Calore, A. Dekker, P.D. Serpico and T. Siebert, *Constraints on light decaying dark matter candidates from 16 yr of INTEGRAL/SPI observations*, *Mon. Not. Roy. Astron. Soc.* **520** (2023) 4167 [[arXiv:2209.06299](#)] [[INSPIRE](#)].
- [59] M. Cirelli et al., *Putting all the X in one basket: Updated X-ray constraints on sub-GeV Dark Matter*, *JCAP* **07** (2023) 026 [[arXiv:2303.08854](#)] [[INSPIRE](#)].
- [60] P. De la Torre Luque, S. Balaji and J. Koechler, *Importance of Cosmic-Ray Propagation on Sub-GeV Dark Matter Constraints*, *Astrophys. J.* **968** (2024) 46 [[arXiv:2311.04979](#)] [[INSPIRE](#)].
- [61] M. Regis et al., *The EMU view of the Large Magellanic Cloud: troubles for sub-TeV WIMPs*, *JCAP* **11** (2021) 046 [[arXiv:2106.08025](#)] [[INSPIRE](#)].

- [62] M. Boudaud, J. Lavalle and P. Salati, *Novel cosmic-ray electron and positron constraints on MeV dark matter particles*, *Phys. Rev. Lett.* **119** (2017) 021103 [[arXiv:1612.07698](#)] [[INSPIRE](#)].
- [63] ICECUBE collaboration, *Search for dark matter from the Galactic halo with the IceCube Neutrino Telescope*, *Phys. Rev. D* **84** (2011) 022004 [[arXiv:1101.3349](#)] [[INSPIRE](#)].
- [64] ICECUBE collaboration, *Search for neutrinos from decaying dark matter with IceCube*, *Eur. Phys. J. C* **78** (2018) 831 [[arXiv:1804.03848](#)] [[INSPIRE](#)].
- [65] ICECUBE collaboration, *Search for neutrino lines from dark matter annihilation and decay with IceCube*, *Phys. Rev. D* **108** (2023) 102004 [[arXiv:2303.13663](#)] [[INSPIRE](#)].
- [66] C.A. Argüelles et al., *Dark matter decay to neutrinos*, *Phys. Rev. D* **108** (2023) 123021 [[arXiv:2210.01303](#)] [[INSPIRE](#)].
- [67] K. Akita and M. Niibo, *Updated constraints and future prospects on majoron dark matter*, *JHEP* **07** (2023) 132 [[arXiv:2304.04430](#)] [[INSPIRE](#)].
- [68] M. Cirelli, A. Strumia and J. Zupan, *Dark Matter*, [arXiv:2406.01705](#) [[INSPIRE](#)].
- [69] J. Silk and A. Stebbins, *Decay of longlived particles in the early Universe*, *Astrophys. J.* **269** (1983) 1 [[INSPIRE](#)].
- [70] M. Kawasaki and K. Sato, *The Effect of Radiative Decay of Massive Particles on the Spectrum of the Microwave Background Radiation*, *Phys. Lett. B* **169** (1986) 280 [[INSPIRE](#)].
- [71] W. Hu and J. Silk, *Thermalization constraints and spectral distortions for massive unstable relic particles*, *Phys. Rev. Lett.* **70** (1993) 2661 [[INSPIRE](#)].
- [72] J.A. Adams, S. Sarkar and D.W. Sciama, *CMB anisotropy in the decaying neutrino cosmology*, *Mon. Not. Roy. Astron. Soc.* **301** (1998) 210 [[astro-ph/9805108](#)] [[INSPIRE](#)].
- [73] X.-L. Chen and M. Kamionkowski, *Particle decays during the cosmic dark ages*, *Phys. Rev. D* **70** (2004) 043502 [[astro-ph/0310473](#)] [[INSPIRE](#)].
- [74] N. Padmanabhan and D.P. Finkbeiner, *Detecting dark matter annihilation with CMB polarization: Signatures and experimental prospects*, *Phys. Rev. D* **72** (2005) 023508 [[astro-ph/0503486](#)] [[INSPIRE](#)].
- [75] T. Kanzaki, M. Kawasaki, K. Kohri and T. Moroi, *Cosmological Constraints on Neutrino Injection*, *Phys. Rev. D* **76** (2007) 105017 [[arXiv:0705.1200](#)] [[INSPIRE](#)].
- [76] J. Chluba and R.A. Sunyaev, *The evolution of CMB spectral distortions in the early Universe*, *Mon. Not. Roy. Astron. Soc.* **419** (2012) 1294 [[arXiv:1109.6552](#)] [[INSPIRE](#)].
- [77] R. Diamanti et al., *Constraining Dark Matter Late-Time Energy Injection: Decays and P-Wave Annihilations*, *JCAP* **02** (2014) 017 [[arXiv:1308.2578](#)] [[INSPIRE](#)].
- [78] H. Liu, T.R. Slatyer and J. Zavala, *Contributions to cosmic reionization from dark matter annihilation and decay*, *Phys. Rev. D* **94** (2016) 063507 [[arXiv:1604.02457](#)] [[INSPIRE](#)].
- [79] T.R. Slatyer and C.-L. Wu, *General Constraints on Dark Matter Decay from the Cosmic Microwave Background*, *Phys. Rev. D* **95** (2017) 023010 [[arXiv:1610.06933](#)] [[INSPIRE](#)].
- [80] F. Capozzi, R.Z. Ferreira, L. Lopez-Honorez and O. Mena, *CMB and Lyman- α constraints on dark matter decays to photons*, *JCAP* **06** (2023) 060 [[arXiv:2303.07426](#)] [[INSPIRE](#)].
- [81] H. Liu, W. Qin, G.W. Ridgway and T.R. Slatyer, *Exotic energy injection in the early Universe. II. CMB spectral distortions and constraints on light dark matter*, *Phys. Rev. D* **108** (2023) 043531 [[arXiv:2303.07370](#)] [[INSPIRE](#)].

- [82] H. Liu, W. Qin, G.W. Ridgway and T.R. Slatyer, *Lyman- α constraints on cosmic heating from dark matter annihilation and decay*, *Phys. Rev. D* **104** (2021) 043514 [[arXiv:2008.01084](#)] [[INSPIRE](#)].
- [83] Y.A. Shchekinov and E.O. Vasiliev, *Particle decay in the early universe: predictions for 21 cm*, *Mon. Not. Roy. Astron. Soc.* **379** (2007) 1003 [[astro-ph/0604231](#)] [[INSPIRE](#)].
- [84] S.R. Furlanetto, S.P. Oh and E. Pierpaoli, *The Effects of Dark Matter Decay and Annihilation on the High-Redshift 21 cm Background*, *Phys. Rev. D* **74** (2006) 103502 [[astro-ph/0608385](#)] [[INSPIRE](#)].
- [85] M. Valdes, A. Ferrara, M. Mapelli and E. Ripamonti, *Constraining DM through 21 cm observations*, *Mon. Not. Roy. Astron. Soc.* **377** (2007) 245 [[astro-ph/0701301](#)] [[INSPIRE](#)].
- [86] D.T. Cumberbatch et al., *Signatures of clumpy dark matter in the global 21 cm Background Signal*, *Phys. Rev. D* **82** (2010) 103508 [[arXiv:0808.0881](#)] [[INSPIRE](#)].
- [87] G. D’Amico, P. Panci and A. Strumia, *Bounds on Dark Matter annihilations from 21 cm data*, *Phys. Rev. Lett.* **121** (2018) 011103 [[arXiv:1803.03629](#)] [[INSPIRE](#)].
- [88] S. Clark et al., *21 cm limits on decaying dark matter and primordial black holes*, *Phys. Rev. D* **98** (2018) 043006 [[arXiv:1803.09390](#)] [[INSPIRE](#)].
- [89] K. Cheung, J.-L. Kuo, K.-W. Ng and Y.-L.S. Tsai, *The impact of EDGES 21-cm data on dark matter interactions*, *Phys. Lett. B* **789** (2019) 137 [[arXiv:1803.09398](#)] [[INSPIRE](#)].
- [90] H. Liu and T.R. Slatyer, *Implications of a 21-cm signal for dark matter annihilation and decay*, *Phys. Rev. D* **98** (2018) 023501 [[arXiv:1803.09739](#)] [[INSPIRE](#)].
- [91] A. Mitridate and A. Podo, *Bounds on Dark Matter decay from 21 cm line*, *JCAP* **05** (2018) 069 [[arXiv:1803.11169](#)] [[INSPIRE](#)].
- [92] G. Facchinetti, L. Lopez-Honorez, Y. Qin and A. Mesinger, *21cm signal sensitivity to dark matter decay*, *JCAP* **01** (2024) 005 [[arXiv:2308.16656](#)] [[INSPIRE](#)].
- [93] Y. Sun et al., *Inhomogeneous Energy Injection in the 21-cm Power Spectrum: Sensitivity to Dark Matter Decay*, [arXiv:2312.11608](#) [[INSPIRE](#)].
- [94] D. Wadekar and Z. Wang, *Strong constraints on decay and annihilation of dark matter from heating of gas-rich dwarf galaxies*, *Phys. Rev. D* **106** (2022) 075007 [[arXiv:2111.08025](#)] [[INSPIRE](#)].
- [95] W. Buchmuller and D. Wyler, *Effective Lagrangian Analysis of New Interactions and Flavor Conservation*, *Nucl. Phys. B* **268** (1986) 621 [[INSPIRE](#)].
- [96] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian*, *JHEP* **10** (2010) 085 [[arXiv:1008.4884](#)] [[INSPIRE](#)].
- [97] C.N. Leung, S.T. Love and S. Rao, *Low-Energy Manifestations of a New Interaction Scale: Operator Analysis*, *Z. Phys. C* **31** (1986) 433 [[INSPIRE](#)].
- [98] R. Alonso, E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology*, *JHEP* **04** (2014) 159 [[arXiv:1312.2014](#)] [[INSPIRE](#)].
- [99] I. Brivio, Y. Jiang and M. Trott, *The SMEFTsim package, theory and tools*, *JHEP* **12** (2017) 070 [[arXiv:1709.06492](#)] [[INSPIRE](#)].
- [100] L. Silvestrini and M. Valli, *Model-independent Bounds on the Standard Model Effective Theory from Flavour Physics*, *Phys. Lett. B* **799** (2019) 135062 [[arXiv:1812.10913](#)] [[INSPIRE](#)].

- [101] T. Hurth, S. Renner and W. Shepherd, *Matching for FCNC effects in the flavour-symmetric SMEFT*, *JHEP* **06** (2019) 029 [[arXiv:1903.00500](#)] [[INSPIRE](#)].
- [102] R. Aoude, T. Hurth, S. Renner and W. Shepherd, *The impact of flavour data on global fits of the MFV SMEFT*, *JHEP* **12** (2020) 113 [[arXiv:2003.05432](#)] [[INSPIRE](#)].
- [103] D.A. Faroughy, G. Isidori, F. Wilsch and K. Yamamoto, *Flavour symmetries in the SMEFT*, *JHEP* **08** (2020) 166 [[arXiv:2005.05366](#)] [[INSPIRE](#)].
- [104] A. Greljo, A. Palavrić and A.E. Thomsen, *Adding Flavor to the SMEFT*, *JHEP* **10** (2022) 005 [[arXiv:2203.09561](#)] [[INSPIRE](#)].
- [105] S. Bruggisser, D. van Dyk and S. Westhoff, *Resolving the flavor structure in the MFV-SMEFT*, *JHEP* **02** (2023) 225 [[arXiv:2212.02532](#)] [[INSPIRE](#)].
- [106] A. Greljo and A. Palavrić, *Leading directions in the SMEFT*, *JHEP* **09** (2023) 009 [[arXiv:2305.08898](#)] [[INSPIRE](#)].
- [107] R. Bartocci, A. Biekötter and T. Hurth, *A global analysis of the SMEFT under the minimal MFV assumption*, *JHEP* **05** (2024) 074 [[arXiv:2311.04963](#)] [[INSPIRE](#)].
- [108] A. Greljo, A. Palavrić and A. Smolkovič, *Leading directions in the SMEFT: Renormalization effects*, *Phys. Rev. D* **109** (2024) 075033 [[arXiv:2312.09179](#)] [[INSPIRE](#)].
- [109] BELLE-II collaboration, *Evidence for $B^+ \rightarrow K^+ \nu \bar{\nu}$ decays*, *Phys. Rev. D* **109** (2024) 112006 [[arXiv:2311.14647](#)] [[INSPIRE](#)].
- [110] K. Fridell, M. Ghosh, T. Okui and K. Tobioka, *Decoding the $B \rightarrow K \nu \nu$ excess at Belle II: Kinematics, operators, and masses*, *Phys. Rev. D* **109** (2024) 115006 [[arXiv:2312.12507](#)] [[INSPIRE](#)].
- [111] P.D. Bolton, S. Fajfer, J.F. Kamenik and M. Novoa-Brunet, *Signatures of light new particles in $B \rightarrow K(^*) \text{Emiss}$* , *Phys. Rev. D* **110** (2024) 055001 [[arXiv:2403.13887](#)] [[INSPIRE](#)].
- [112] R. Bause, H. Gisbert and G. Hiller, *Implications of an enhanced $B \rightarrow K \nu \bar{\nu}$ branching ratio*, *Phys. Rev. D* **109** (2024) 015006 [[arXiv:2309.00075](#)] [[INSPIRE](#)].
- [113] A.J. Buras, J. Harz and M.A. Mojahed, *Disentangling new physics in $K \rightarrow \pi \nu \bar{\nu}$ and $B \rightarrow K(K^*) \nu \bar{\nu}$ observables*, *JHEP* **10** (2024) 087 [[arXiv:2405.06742](#)] [[INSPIRE](#)].
- [114] T. Felkl, A. Giri, R. Mohanta and M.A. Schmidt, *When energy goes missing: new physics in $b \rightarrow s \nu \nu$ with sterile neutrinos*, *Eur. Phys. J. C* **83** (2023) 1135 [[arXiv:2309.02940](#)] [[INSPIRE](#)].
- [115] X.-G. He, X.-D. Ma and G. Valencia, *Revisiting models that enhance $B^+ \rightarrow K^+ \nu \bar{\nu}$ in light of the new Belle II measurement*, *Phys. Rev. D* **109** (2024) 075019 [[arXiv:2309.12741](#)] [[INSPIRE](#)].
- [116] XENON10 collaboration, *Constraints on inelastic dark matter from XENON10*, *Phys. Rev. D* **80** (2009) 115005 [[arXiv:0910.3698](#)] [[INSPIRE](#)].
- [117] ZEPLIN-III collaboration, *Limits on inelastic dark matter from ZEPLIN-III*, *Phys. Lett. B* **692** (2010) 180 [[arXiv:1003.5626](#)] [[INSPIRE](#)].
- [118] CDMS-II and CDMS collaborations, *Search for inelastic dark matter with the CDMS II experiment*, *Phys. Rev. D* **83** (2011) 112002 [[arXiv:1012.5078](#)] [[INSPIRE](#)].
- [119] XENON100 collaboration, *Implications on Inelastic Dark Matter from 100 Live Days of XENON100 Data*, *Phys. Rev. D* **84** (2011) 061101 [[arXiv:1104.3121](#)] [[INSPIRE](#)].
- [120] J. Bramante, P.J. Fox, G.D. Kribs and A. Martin, *Inelastic frontier: Discovering dark matter at high recoil energy*, *Phys. Rev. D* **94** (2016) 115026 [[arXiv:1608.02662](#)] [[INSPIRE](#)].

- [121] PANDAX-II collaboration, *Exploring the dark matter inelastic frontier with 79.6 days of PandaX-II data*, *Phys. Rev. D* **96** (2017) 102007 [[arXiv:1708.05825](#)] [[INSPIRE](#)].
- [122] PANDAX collaboration, *A search for two-component Majorana dark matter in a simplified model using the full exposure data of PandaX-II experiment*, *Phys. Lett. B* **832** (2022) 137254 [[arXiv:2205.08066](#)] [[INSPIRE](#)].
- [123] D. Tucker-Smith and N. Weiner, *Inelastic dark matter*, *Phys. Rev. D* **64** (2001) 043502 [[hep-ph/0101138](#)] [[INSPIRE](#)].
- [124] D. Tucker-Smith and N. Weiner, *The status of inelastic dark matter*, *Phys. Rev. D* **72** (2005) 063509 [[hep-ph/0402065](#)] [[INSPIRE](#)].
- [125] S. Chang, G.D. Kribs, D. Tucker-Smith and N. Weiner, *Inelastic Dark Matter in Light of DAMA/LIBRA*, *Phys. Rev. D* **79** (2009) 043513 [[arXiv:0807.2250](#)] [[INSPIRE](#)].
- [126] D.P. Finkbeiner, T.R. Slatyer, N. Weiner and I. Yavin, *PAMELA, DAMA, INTEGRAL and Signatures of Metastable Excited WIMPs*, *JCAP* **09** (2009) 037 [[arXiv:0903.1037](#)] [[INSPIRE](#)].
- [127] R. Essig, J. Kaplan, P. Schuster and N. Toro, *On the Origin of Light Dark Matter Species*, [arXiv:1004.0691](#) [[INSPIRE](#)].
- [128] P.W. Graham, R. Harnik, S. Rajendran and P. Saraswat, *Exothermic Dark Matter*, *Phys. Rev. D* **82** (2010) 063512 [[arXiv:1004.0937](#)] [[INSPIRE](#)].
- [129] B. Grinstein, X. Lu, L. Merlo and P. Quil ez, *Hilbert series for covariants and their applications to minimal flavor violation*, *JHEP* **06** (2024) 154 [[arXiv:2312.13349](#)] [[INSPIRE](#)].
- [130] M.M. Block and J.D. Jackson, *Applications of a cluster decomposition of many dimensional phase space to high-energy collisions*, *Z. Phys. C* **3** (1980) 255 [[INSPIRE](#)].
- [131] L. Di Luzio, R. Gr ober, J.F. Kamenik and M. Nardecchia, *Accidental matter at the LHC*, *JHEP* **07** (2015) 074 [[arXiv:1504.00359](#)] [[INSPIRE](#)].
- [132] B.W. Lee and S. Weinberg, *Cosmological Lower Bound on Heavy Neutrino Masses*, *Phys. Rev. Lett.* **39** (1977) 165 [[INSPIRE](#)].
- [133] P. Hut, *Limits on Masses and Number of Neutral Weakly Interacting Particles*, *Phys. Lett. B* **69** (1977) 85 [[INSPIRE](#)].
- [134] K. Sato and M. Kobayashi, *Cosmological Constraints on the Mass and the Number of Heavy Lepton Neutrinos*, *Prog. Theor. Phys.* **58** (1977) 1775 [[INSPIRE](#)].
- [135] M.I. Vysotsky, A.D. Dolgov and Y.B. Zeldovich, *Cosmological Restriction on Neutral Lepton Masses*, *JETP Lett.* **26** (1977) 188 [[INSPIRE](#)].
- [136] J.R. Ellis, A.D. Linde and D.V. Nanopoulos, *Inflation Can Save the Gravitino*, *Phys. Lett. B* **118** (1982) 59 [[INSPIRE](#)].
- [137] D.V. Nanopoulos, K.A. Olive and M. Srednicki, *After Primordial Inflation*, *Phys. Lett. B* **127** (1983) 30 [[INSPIRE](#)].
- [138] J.R. Ellis, J.E. Kim and D.V. Nanopoulos, *Cosmological Gravitino Regeneration and Decay*, *Phys. Lett. B* **145** (1984) 181 [[INSPIRE](#)].
- [139] V.S. Berezinsky, *Cosmology of gravitino as the lightest supersymmetric particle*, *Phys. Lett. B* **261** (1991) 71 [[INSPIRE](#)].
- [140] T. Moroi, H. Murayama and M. Yamaguchi, *Cosmological constraints on the light stable gravitino*, *Phys. Lett. B* **303** (1993) 289 [[INSPIRE](#)].

- [141] M. Kawasaki and T. Moroi, *Gravitino production in the inflationary universe and the effects on big bang nucleosynthesis*, *Prog. Theor. Phys.* **93** (1995) 879 [[hep-ph/9403364](#)] [[INSPIRE](#)].
- [142] M. Bolz, W. Buchmuller and M. Plumacher, *Baryon asymmetry and dark matter*, *Phys. Lett. B* **443** (1998) 209 [[hep-ph/9809381](#)] [[INSPIRE](#)].
- [143] M. Bolz, A. Brandenburg and W. Buchmuller, *Thermal production of gravitinos*, *Nucl. Phys. B* **606** (2001) 518 [[hep-ph/0012052](#)] [[INSPIRE](#)].
- [144] J. McDonald, *Thermally generated gauge singlet scalars as selfinteracting dark matter*, *Phys. Rev. Lett.* **88** (2002) 091304 [[hep-ph/0106249](#)] [[INSPIRE](#)].
- [145] L.J. Hall, K. Jedamzik, J. March-Russell and S.M. West, *Freeze-In Production of FIMP Dark Matter*, *JHEP* **03** (2010) 080 [[arXiv:0911.1120](#)] [[INSPIRE](#)].
- [146] F. Elahi, C. Kolda and J. Unwin, *UltraViolet Freeze-in*, *JHEP* **03** (2015) 048 [[arXiv:1410.6157](#)] [[INSPIRE](#)].
- [147] L. Covi, H.-B. Kim, J.E. Kim and L. Roszkowski, *Axinos as dark matter*, *JHEP* **05** (2001) 033 [[hep-ph/0101009](#)] [[INSPIRE](#)].
- [148] L. Covi, L. Roszkowski and M. Small, *Effects of squark processes on the axino CDM abundance*, *JHEP* **07** (2002) 023 [[hep-ph/0206119](#)] [[INSPIRE](#)].
- [149] T. Asaka, K. Ishiwata and T. Moroi, *Right-handed sneutrino as cold dark matter*, *Phys. Rev. D* **73** (2006) 051301 [[hep-ph/0512118](#)] [[INSPIRE](#)].
- [150] T. Asaka, K. Ishiwata and T. Moroi, *Right-handed sneutrino as cold dark matter of the universe*, *Phys. Rev. D* **75** (2007) 065001 [[hep-ph/0612211](#)] [[INSPIRE](#)].
- [151] V. Page, *Non-thermal right-handed sneutrino dark matter and the Ω_{DM}/Ω_b problem*, *JHEP* **04** (2007) 021 [[hep-ph/0701266](#)] [[INSPIRE](#)].
- [152] M. Shaposhnikov and I. Tkachev, *The nuMSM, inflation, and dark matter*, *Phys. Lett. B* **639** (2006) 414 [[hep-ph/0604236](#)] [[INSPIRE](#)].
- [153] A. Kusenko, *Sterile neutrinos, dark matter, and the pulsar velocities in models with a Higgs singlet*, *Phys. Rev. Lett.* **97** (2006) 241301 [[hep-ph/0609081](#)] [[INSPIRE](#)].
- [154] K. Petraki and A. Kusenko, *Dark-matter sterile neutrinos in models with a gauge singlet in the Higgs sector*, *Phys. Rev. D* **77** (2008) 065014 [[arXiv:0711.4646](#)] [[INSPIRE](#)].
- [155] P. Gondolo and G. Gelmini, *Cosmic abundances of stable particles: Improved analysis*, *Nucl. Phys. B* **360** (1991) 145 [[INSPIRE](#)].
- [156] J. Edsjo and P. Gondolo, *Neutralino relic density including coannihilations*, *Phys. Rev. D* **56** (1997) 1879 [[hep-ph/9704361](#)] [[INSPIRE](#)].
- [157] PARTICLE DATA GROUP collaboration, *Review of Particle Physics*, *PTEP* **2022** (2022) 083C01 [[INSPIRE](#)].
- [158] G. Bélanger et al., *micrOMEGAs5.0: Freeze-in*, *Comput. Phys. Commun.* **231** (2018) 173 [[arXiv:1801.03509](#)] [[INSPIRE](#)].
- [159] G. Bélanger, F. Boudjema, A. Pukhov and A. Semenov, *Dark matter direct detection rate in a generic model with micrOMEGAs 2.2*, *Comput. Phys. Commun.* **180** (2009) 747 [[arXiv:0803.2360](#)] [[INSPIRE](#)].
- [160] M. Baryakhtar et al., *Dark Kinetic Heating of Neutron Stars and An Infrared Window On WIMPs, SIMPs, and Pure Higgsinos*, *Phys. Rev. Lett.* **119** (2017) 131801 [[arXiv:1704.01577](#)] [[INSPIRE](#)].

- [161] N. Raj, P. Tanedo and H.-B. Yu, *Neutron stars at the dark matter direct detection frontier*, *Phys. Rev. D* **97** (2018) 043006 [[arXiv:1707.09442](#)] [[INSPIRE](#)].
- [162] N.F. Bell, G. Busoni and S. Robles, *Heating up Neutron Stars with Inelastic Dark Matter*, *JCAP* **09** (2018) 018 [[arXiv:1807.02840](#)] [[INSPIRE](#)].
- [163] J.F. Acevedo, J. Bramante, R.K. Leane and N. Raj, *Warming Nuclear Pasta with Dark Matter: Kinetic and Annihilation Heating of Neutron Star Crusts*, *JCAP* **03** (2020) 038 [[arXiv:1911.06334](#)] [[INSPIRE](#)].
- [164] M. Fujiwara, K. Hamaguchi, N. Nagata and J. Zheng, *Capture of electroweak multiplet dark matter in neutron stars*, *Phys. Rev. D* **106** (2022) 055031 [[arXiv:2204.02238](#)] [[INSPIRE](#)].
- [165] G. Alvarez, A. Joglekar, M. Phoroutan-Mehr and H.-B. Yu, *Heating neutron stars with inelastic dark matter and relativistic targets*, *Phys. Rev. D* **107** (2023) 103024 [[arXiv:2301.08767](#)] [[INSPIRE](#)].
- [166] K. Yanagi, N. Nagata and K. Hamaguchi, *Cooling Theory Faced with Old Warm Neutron Stars: Role of Non-Equilibrium Processes with Proton and Neutron Gaps*, *Mon. Not. Roy. Astron. Soc.* **492** (2020) 5508 [[arXiv:1904.04667](#)] [[INSPIRE](#)].
- [167] K. Hamaguchi, N. Nagata and K. Yanagi, *Dark Matter Heating vs. Rotochemical Heating in Old Neutron Stars*, *Phys. Lett. B* **795** (2019) 484 [[arXiv:1905.02991](#)] [[INSPIRE](#)].
- [168] M. Fujiwara, K. Hamaguchi, N. Nagata and M.E. Ramirez-Quezada, *Vortex creep heating vs. dark matter heating in neutron stars*, *Phys. Lett. B* **848** (2024) 138341 [[arXiv:2309.02633](#)] [[INSPIRE](#)].
- [169] M. Fujiwara, K. Hamaguchi, N. Nagata and M.E. Ramirez-Quezada, *Vortex creep heating in neutron stars*, *JCAP* **03** (2024) 051 [[arXiv:2308.16066](#)] [[INSPIRE](#)].