Physics Letters B 804 (2020) 135371

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Exclusive hadronic tau decays as probes of non-SM interactions

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ARTICLE INFO

Article history:
Received 20 December 2019
Accepted 15 March 2020
Available online 18 March 2020
Editor: J. Hisano

Keywords:
Effective Field Theories
Beyond Standard Model
Tau decays

ABSTRACT

We perform a global analysis of exclusive hadronic tau decays into one and two mesons using the low-energy limit of the Standard Model Effective Field Theory up to dimension six, assuming left-handed neutrinos. A controlled theoretical input on the Standard Model hadronic form factors, based on chiral symmetry, dispersion relations, data and asymptotic QCD properties, has allowed us to set bounds on the New Physics effective couplings using the present experimental data. Our results highlight the importance of semileptonic τ decays in complementing the traditional low-energy probes, such nuclear β decays or semileptonic pion and kaon decays, and the high-energy measurements at LHC scales. This makes yet another reason for considering hadronic tau decays as golden modes at Belle-II.

1. Introduction

The τ lepton is the only known lepton heavy enough ($m_\tau = 1.77686$ GeV [1]) to decay into hadrons; the $\sim 65\%$ of its partial width contains hadrons in the final state. In the Standard Model (SM), hadronic tau decays proceed through the exchange of $W^\pm$ bosons which couple the τ and the generated neutrino ν_τ together with a quark-antiquark pair that subsequently hadronizes. Such decays thus offer an advantageous laboratory to study low-energy effects of the strong interactions under clean conditions [2] since half of the process is purely electroweak and, therefore, free of uncertainties at the required precision. At the inclusive level, these decays allow to extract fundamental parameters of the SM, most importantly the strong coupling α_S [3,4], but also the CKM quark-mixing matrix element $|V_{us}|$ [5–7] and the mass of the strange quark at high precision [8–14]. On the other hand, exclusive hadronic decays can be used to learn specific properties of the hadrons involved and the interactions among them. These can be classified according to the number of hadrons in the final state. The simple one-meson transitions $\tau^- \to P^-\nu_\tau$ $(P = \pi, K)$ are very well-known due to the precise determinations of the pion and kaon decays constants obtained by the Lattice collaborations [15]. At present, we also have a very good knowledge on the decays into a pair of mesons, the SM input of which is encoded in terms of hadronic form factors. An ideal roadmap to describe meson form factors would require a model-independent approach demanding a full knowledge of QCD in both its perturbative and non-perturbative regimes, knowledge not yet unraveled. An alternative to such enterprise would pursuit a synergy between theoretical calculations and experimental data. In this respect, dispersion relations are a powerful tool to direct oneself towards a model-independent description of meson form factors. For example, the analyses of the decays $\pi^-\pi^0$ [16–19] and $K_S\pi^-$ [20–24], carried out by exploiting the synergy between Resonance Chiral Theory [25] and dispersion theory, are found to be in a nice agreement with the rich data provided by the experiments. Accord with experimental measurements is also found for the K^-K_S [19] and $K^-\eta$ [24,26] decay modes, although higher-quality data on these processes is required to constrain the corresponding theories or models, while the predictions for the isospin-violating $\pi^-\eta^{(\prime)}$ channels [27,28] respect the current experimental upper bounds. The latter are very challenging processes for Belle-II [29]. Higher-multiplicity decay modes involve a richer dynamical structure but accounting for the strong rescattering effects is not an easy task when three or more hadrons are present.

So far, all experimental results with the τ lepton are found to be in accord with the SM, with the exception of the $2.6\sigma(2.4\sigma)$ deviation from lepton flavour universality in $|g_\tau/g_\mu|(|g_\tau/g_e|)$ from $W^- \to \tau^-\bar{\nu}_\tau$ [1,30],[1] of the BaBar measurement of the CP asymmetry in $\tau^- \to K_S\pi^-\nu_\tau$, $A_{CP} = -3.6(2.3)(1.1) \times 10^{-3}$ [32], which

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[1] See also Ref. [31], where the authors show that a NP explanation of this tension is not very plausible.

https://doi.org/10.1016/j.physletb.2020.135371

is 2.8σ off the SM prediction, $A_{CP} = 3.6(1) \times 10^{-3}$ [33], and of the anomalous excess of τ production observed in some B decays. As seen, these effects are not statistically large. However, the increased sensitivities of the most recent experiments yield interesting limits on possible New Physics contributions in the hadronic tau sector.

Several recent works [34–38] have put forward that semileptonic tau decays are not only a clean QCD laboratory but also offer an interesting scenario to set bounds on non-standard weak charged current interactions complementary to the traditional low-energy semileptonic probes such nuclear beta decays, purely leptonic lepton, pion and kaon decays or hyperon decays (see e.g. Refs. [40–47,49,48,50]).

The aim of the present work is to close the circle by extending our previous individual analyses of the decays $\tau^- \to \pi^-\pi^0\nu_\tau$ [35], $\tau^- \to (K\pi)^-\nu_\tau$ [37], $\tau^- \to K^-(K^0, \eta^{(\prime)})\nu_\tau$ [38] and $\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$ [34], carried out using the low-energy limit of the Standard Model Effective Field Theory Lagrangian (SMEFT) [51,52] up to dimension six, to a global analysis of the strangeness-conserving ($\Delta S = 0$) and strangeness-changing ($|\Delta S| = 1$) semileptonic exclusive tau decays into one and two pseudoscalar mesons. The main advantage of this EFT framework is that experimental measurements and their implications for New Physics can be compared unambiguously either at low energies or at the high LHC energies in a model-independent way [36].

We can anticipate that the bounds for the NP couplings that we get in this work (in the $\overline{\text{MS}}$ scheme at scale $\mu = 2$ GeV), obtained from all data available on exclusive τ decays only, are competitive and found to be in line with those of Ref. [36], which were obtained analyzing data including both exclusive and inclusive decays. This agreement represents a good consistency test between exclusive and inclusive determinations.

On the theory side, a controlled theoretical determination, with a robust error band, of the corresponding form factors within the SM is required in order to increase the accuracy of the search for non-standard interactions. At present, we have such a knowledge for the vector and-to a great extent- the scalar form factors, but there are no experimental data that can help us constructing the tensor form factor and, therefore, it has to be built under theoretical considerations only.

The fantastic possibilities offered by the Belle-II experiment [29], and other future Z, tau-charm and B-factories, to study τ physics and low multiplicity final states with high precision make these studies of timely interest.

This letter is organized as follows. The theoretical framework is given in section 2 where we briefly present the effective Lagrangian for weak charge current interactions involving light flavours up to dimension six, assuming left-handed neutrinos. The expressions for the one-and two-meson partial decay width to be used in our fits are also defined in this section. The description of the corresponding form factors is the subject of section 3. In sections 4 and 5 we perform fits to the strangeness-conserving ($\Delta S = 0$) and changing ($|\Delta S| = 1$) transitions, respectively, and set bounds on the New Physics effective couplings. A global fit to both sectors i.e. ($|\Delta S|$ =0 and 1), is performed in section 6. Finally, our conclusions are presented in section 7.

2. SMEFT Lagrangian and decay rate

We start out writing the low-energy limit of the Standard Model Effective Field Theory Lagrangian including dimension six operators that describes semileptonic $\tau^- \to \nu_\tau \bar{u} D$ strangeness-conserving ($D = d$) or strangeness-changing ($D = s$) charged current transitions with left-handed neutrinos. Such Lagrangian reads [40,41]:

$$\begin{aligned}\mathcal{L}_{CC} = -\frac{G_F V_{uD}}{\sqrt{2}}\Big[&(1+\epsilon_L^\tau)\bar{\tau}\gamma_\mu(1-\gamma^5)\nu_\tau \cdot \bar{u}\gamma^\mu(1-\gamma^5)D \\ &+\epsilon_R^\tau\bar{\tau}\gamma_\mu(1-\gamma^5)\nu_\tau \cdot \bar{u}\gamma^\mu(1+\gamma^5)D \\ &+\bar{\tau}(1-\gamma^5)\nu_\tau \cdot \bar{u}(\epsilon_S^\tau - \epsilon_P^\tau\gamma^5)D \\ &+\epsilon_T^\tau\bar{\tau}\sigma_{\mu\nu}(1-\gamma^5)\nu_\tau\,\bar{u}\sigma^{\mu\nu}(1-\gamma^5)D\Big] + h.c.\,, \qquad (1)\end{aligned}$$

where $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$, G_F is the tree-level definition of the Fermi constant and ϵ_i $(i = L, R, S, P, T)$ are effective couplings characterizing NP. These can be complex, although we will assume them real in first approximation since we are only interested in CP conserving quantities.[2] The product $G_F V_{uD}$ in Eq. (1) denotes that its determination from the superallowed nuclear Fermi β decays carries implicitly a dependence on ϵ_L^e and ϵ_R^e that is given by [46]

$$G_F \tilde{V}_{uD}^e = G_F\left(1+\epsilon_L^e+\epsilon_R^e\right)V_{uD}\,, \qquad (2)$$

and that we use for our analysis. Setting the coefficients $\epsilon_i = 0$, one recovers the SM Lagrangian.

The simplest semileptonic decays that can be calculated with the low-energy effective Lagrangian of Eq. (1) are the one-meson decay modes $\tau^- \to P^-\nu_\tau$ $(P = \pi, K)$. The expression for the $\tau^- \to \pi^-\nu_\tau$ decay rate reads

$$\begin{aligned}\Gamma(\tau^- \to \pi^-\nu_\tau) = &\frac{G_F^2|\tilde{V}_{ud}^e|^2 f_\pi^2 m_\tau^3}{16\pi}\left(1-\frac{m_\pi^2}{m_\tau^2}\right)^2 \\ &\times\left(1+\delta_{\text{em}}^{\tau\pi}+2\Delta^{\tau\pi}+\mathcal{O}(\epsilon_i^\tau)^2+\mathcal{O}(\delta_{\text{em}}^{\tau\pi}\epsilon_i^\tau)\right), \qquad (3)\end{aligned}$$

where f_π is the pion decay constant, the quantity $\delta_{\text{em}}^{\tau\pi}$ accounts for the electromagnetic radiative corrections and the term $\Delta^{\tau\pi}$ contains the tree-level NP corrections that arise from the Lagrangian in Eq. (1)[3] that are not absorbed in $\tilde{V}_{ud}^e$. For the channel $\tau^- \to K^-\nu_\tau$, the decay rate is that of Eq. (3) but replacing $\tilde{V}_{ud}^e \to \tilde{V}_{us}^e$, $f_\pi \to f_K$, $m_\pi \to m_K$, and $\delta_{\text{em}}^{\tau\pi}$ and $\Delta^{\tau\pi}$ by $\delta_{\text{em}}^{\tau K}$ and $\Delta^{\tau K}$, respectively.

The amplitude for two-meson decays $\tau^- \to (PP')^-\nu_\tau$ that arises from the Lagrangian in Eq. (1) contains a vector, an scalar and a tensor contribution. The structure of the amplitude, including a detailed definition of the corresponding hadronic matrix element, can be found in our previous works i.e. in Ref. [35] for $\pi^-\pi^0$, in Ref. [37] for the $(K\pi)^-$ system, and in Ref. [38] for the cases $K^-(K^0, \eta^{(\prime)})$, and we therefore have decided not repeat it here once again.

The resulting partial decay width for two-meson decays is given by (the variable s is the invariant mass of the corresponding two-meson system):

$$\begin{aligned}\frac{d\Gamma}{ds} = &\frac{G_F^2|\tilde{V}_{uD}^e|^2 m_\tau^3 S_{EW}}{384\pi^3 s}\left(1-\frac{s}{m_\tau^2}\right)^2\lambda^{1/2}(s, m_P^2, m_{P'}^2) \\ &\times\Big[\left(1+2(\epsilon_L^\tau-\epsilon_L^e+\epsilon_R^\tau-\epsilon_R^e)\right)X_{VA} \\ &+\epsilon_S^\tau X_S+\epsilon_T^\tau X_T+(\epsilon_S^\tau)^2 X_{S^2}+(\epsilon_T^\tau)^2 X_{T^2}\Big], \qquad (4)\end{aligned}$$

where

[2] The only coupling sensitive to an imaginary part is ϵ_S^τ from the decay $\tau^- \to \pi^-\eta\nu_\tau$ [36] that we do not consider in this work for lack of data.

[3] In Eq. (3) we have expanded up to linear order on the ϵ_i^τ couplings.

$$X_{VA} = \frac{1}{2s^2}\left\{3\left(C^S_{PP'}\right)^2 |F_0^{PP'}(s)|^2\Delta^2_{PP'} + \left(C^V_{PP'}\right)^2 |F_+^{PP'}(s)|^2\left(1+\frac{2s}{m_\tau^2}\right)\lambda(s,m_P^2,m_{P'}^2)\right\},$$
$$X_S = \frac{3}{s m_\tau}\left(C^S_{PP'}\right)^2 |F_0^{PP'}(s)|^2\frac{\Delta^2_{PP'}}{m_d-m_u},$$
$$X_T = \frac{6}{s m_\tau} C^V_{PP'}\,\mathrm{Re}\left[F_T^{PP'}(s)\left(F_+^{PP'}(s)\right)^*\right]\lambda(s,m_P^2,m_{P'}^2),$$
$$X_{S^2} = \frac{3}{2m_\tau^2}\left(C^S_{PP'}\right)^2 |F_0^{PP'}(s)|^2\frac{\Delta^2_{PP'}}{(m_d-m_u)^2},$$
$$X_{T^2} = \frac{4}{s}|F_T^{PP'}(s)|^2\left(1+\frac{s}{2m_\tau^2}\right)\lambda(s,m_P^2,m_{P'}^2), \tag{5}$$

with $C^V_{PP'}$ and $C^S_{PP'}$ being the corresponding Clebsch-Gordan coefficients and where we have defined $\Delta_{PP'} = m_P^2 - m_{P'}^2$. In Eq. (4), $S_{EW} = 1.0201$ [53] resums the short-distance electroweak corrections and the function $\lambda(x,y,z) = x^2+y^2+z^2-2xy-2xz-2yz$ is the usual Kallen function.

The functions $F_0^{PP'}(s)$, $F_+^{PP'}(s)$ and $F_T^{PP'}(s)$ in Eq. (5) are, respectively, the scalar, the vector and the tensor form factors, and their respective parametrizations is the subject of the next section.

3. Two-meson form factors

In this section, we provide a brief overview of the description of the scalar, vector and tensor form factors that we employ in our analysis. It is fundamental to have good control over them since they are used as SM inputs for binding the non-standard interactions. We will not discuss them here at length but rather provide a compilation of the main formulae to make this work self-contained.

To describe the pion vector form factor we follow the representation outlined in Ref. [19], and briefly summarized below for the convenience of the reader, and write a thrice subtracted dispersion relation

$$F_+^{\pi\pi}(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}}\frac{ds'}{(s')^3}\frac{\phi(s')}{(s'-s-i0)}\right], \tag{6}$$

where α_1 and α_2 are two subtraction constants that can be related to the slope and curvature appearing in the low-energy expansion of the form factor. The use of a three-times subtracted dispersion relation reduces the high-energy contribution of the integral where the phase is less well-known. In Eq. (6), $s_{\rm cut}$ is a cut-off whose value is fixed from the requirement that the fitted parameters are compatible within errors with the case $s_{\rm cut}\to\infty$. The value of $s_{\rm cut} = 4$ GeV^2 was found to satisfy this criterion [19], and variations of $s_{\rm cut}$ were used to estimate the associated systematic error. For the input phase $\phi(s)$ we use [19]

$$\phi(s) = \begin{cases} \delta_1^1(s) & 4m_\pi^2 \le s < 1\,\mathrm{GeV}^2, \\ \psi(s) & 1\,\mathrm{GeV}^2 \le s < m_\tau^2, \\ \psi_\infty(s) & m_\tau^2 \le s. \end{cases} \tag{7}$$

This phase consists in matching smoothly at 1 GeV the phase $\psi(s)$, that we will explain in the following, to the phase-shift $\delta_1^1(s)$ solution of the Roy equations of Ref. [54]. We thus exploit Watson's theorem [55].[4] The phase $\delta_1^1(s)$ encodes the physics of the ρ-meson, it is totally general and provides a phase which perfectly agrees with the P-wave $\pi\pi$ experimental data within the elastic region. For $\psi(s)$, we use a physically motivated parametrization that contains the physics of the inelastic regime until m_τ^2. This phase can be extracted from the relation

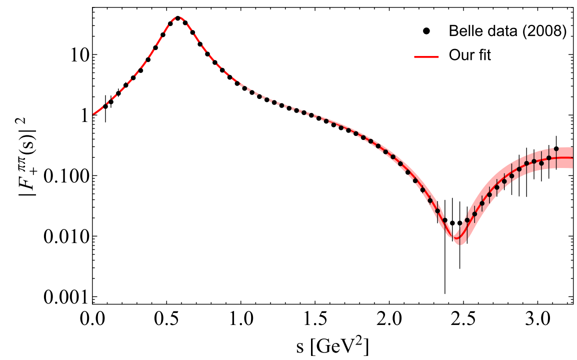

Fig. 1. Belle measurement of the modulus squared of the pion vector form factor [57] as compared to our fits [19].

$$\tan\psi(s) = \frac{\mathrm{Im} f_+^{\pi\pi}(s)}{\mathrm{Re} f_+^{\pi\pi}(s)}, \tag{8}$$

where $f_+^{\pi\pi}(s)$ includes the contributions from the excited resonances ρ' and ρ'' that cannot be neglected. The expression of $f_+^{\pi\pi}(s)$ that we use for our study is given by Eq. (17) of Ref. [19]. Finally, for the high-energy region, we guide smoothly the phase to π at m_τ^2 ($\psi_\infty(s)$) to ensure the correct asymptotic $1/s$ fall-off of the form factor [56].[5] Armed with this parametrization, in [19] we have analyzed the high-statistics Belle data [57] on the pion vector form factor. The outcome that better illustrates the resulting analysis, and that we use for this work, is displayed in Fig. 1, where the red error band denotes the statistical fit uncertainty.[6]

The corresponding vector form factors for the $(K\pi)^-$, K^-K^0 and $K^-\eta^{(\prime)}$ systems can be obtained following a similar dispersive procedure. We do not show here the explicit expressions that we use for our analysis but rather provide a graphical account of their applications (of some) against the Belle $\tau^-\to K_S\pi^-\nu_\tau$ (red solid circles) [58] and $\tau^-\to K^-\eta\nu_\tau$ (green solid squares) [59] experimental data (Fig. 2) and refer the interested reader to Refs. [19,22,24,26], where they are derived and explained in detail. As seen, the $K_S\pi^-$ spectrum is dominated by the $K^*(892)$ resonance, whose peak is neatly visible, followed by a mild shoulder due to the heavier $K^*(1410)$. There is no such a clear peak structure for the $K^-\eta$ channel as a consequence of the interplay between both K^* resonances. In all, satisfactory agreement with data is seen for all data points.

Regarding the scalar form factors we take: the phase dispersive representation of the $\pi^-\pi^0$ scalar form factor from Ref. [28] while for the K^-K^0 ones, we use the results of Refs. [60–62].[7] These were obtained after the unitarization, based on the method of N/D, of the complete one-loop calculation of the strangeness

[4] Watson's theorem applied to the pion vector form factor tells us that the form factor phase equals that of the two-pion scattering within the elastic region.

[5] In fact, this behaviour it is not guaranteed because the subtraction constants in Eq. (6) are fixed from a fit to data. However, we have checked that our form factor is indeed a decreasing function of s (apart from the resonance peak structures) within the entire range where we apply it.

[6] In [19], we have also estimated potential systematic uncertainties.

[7] We thank very much Zhi-Hui Guo for providing us tables with the unitarized $\pi\eta$, $\pi\eta'$ and $K^0\bar{K}^0$ scalar form factors. We translate the result of $K^0\bar{K}^0$ to the K^-K^0 concerning us through $F_0^{K^-K^0}(s) = -F_0^{K^0\bar{K}^0}(s)/\sqrt{2}$.

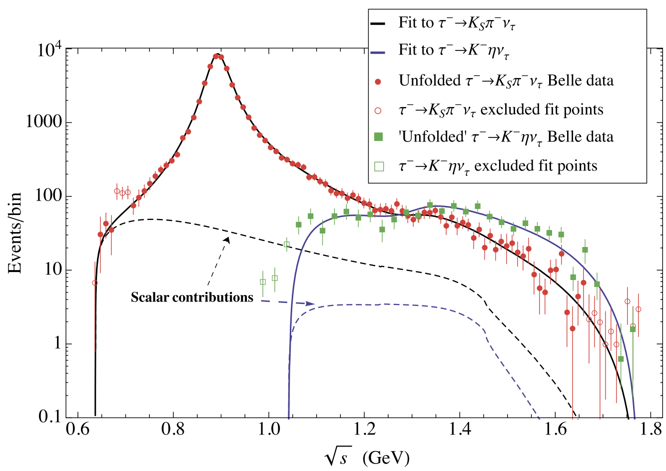

Fig. 2. Belle $\tau^- \to K_S\pi^-\nu_\tau$ (red solid circles) [58] and $\tau^- \to K^-\eta\nu_\tau$ (green solid squares) [59] measurements as compared to our best fit results in [24] (solid black and blue lines, respectively) obtained from a combined fit to both data sets. The small scalar contributions are represented by black and blue dashed lines.

conserving scalar form factors within $U(3)$ ChPT. Finally, for the $K\pi$ and $K\eta^{(\prime)}$ scalar form factors, we employ the well-established results of Ref. [63] derived from a dispersive analysis with three coupled channels $(K\pi, K\eta, K\eta')$.[8] the $K\pi$ scalar form factor contribution, although small, is important to describe the data immediately above threshold, while the $K\eta$ one is irrelevant for the decay distribution.

We next turn to the tensor form factor. This is the most difficult input to be reliably estimated since there are no experimental data that can help its construction. Therefore, we shall rely on theoretical considerations only. The key observation is that the tensor form factor admits an Omnès dispersive representation [35,37,38,64,65]. We thus write the general two-meson (PP') tensor form factor as

$$F_T^{PP'}(s) = F_T^{PP'}(0)\exp\left[\frac{s}{\pi}\int_{s_{\rm th}}^{s_{\rm cut}}\frac{ds'}{s'}\frac{\delta_T^{PP'}(s')}{(s'-s-i0)}\right], \tag{9}$$

where $s_{\rm th} = (m_P + m_{P'})^2$ is the corresponding two-meson production threshold, and where in the elastic region, the phase of the tensor form factor equals the P-wave phase of the corresponding vector one i.e. $\delta_T^{PP'}(s) = \delta_+^{PP'}(s)$. We will assume the previous relations also hold above the onset of inelasticities until m_τ^2 where we guide smoothly the tensor phase to π as in Ref. [19] to ensure the asymptotic $1/s$ behaviour dictated by perturbative QCD [56]. Lacking of precise low-energy information, we do not increase the number of subtractions, which, in turn, would reduce the importance of the higher-energy part of the integral, but rather cut the integral at different values of $s_{\rm cut}$ e.g. $s_{\rm cut} = 4, 9$ GeV^2, and consider the differing with respect to the case $s_{\rm cut} \to \infty$, that we take as a baseline hypothesis, as an estimate of our (uncontroled) theoretical systematic uncertainty for the results presented in the following sections. For the required normalization $F_T^{PP'}(0)$, we take the corresponding ChPT based results derived in [35,37,38] obtained with the use of the corresponding determination on the lattice [39]. In these references, a graphical account of the energy-dependence of the tensor form factors is also shown.

[8] We are very grateful to Matthias Jamin and Jose Antonio Oller for providing us their solutions in tables.

4. New physics bounds from $\Delta S = 0$ decays

We start with the individual analysis of the decay mode with lowest multiplicity, $\tau^- \to \pi^-\nu_\tau$. Taking the decay rate given in Eq. (3) and using $f_\pi = 130.2(8)$ MeV from the lattice[9] [15] together with $\delta_{\rm em}^{\tau\pi} = 1.92(24)\%$, obtained from a combination of the values given in Refs. [66–68], and the PDG reported values [1] for: $|\tilde{V}_{ud}^e| = 0.97420(21)$ from nuclear β decays, the measured branching ratio $BR(\tau^- \to \pi^-\nu_\tau) = 10.82(5)\%$, $m_\pi = 0.13957061(24)$ GeV, $m_\tau = 1.77686(12)$ GeV, $\Gamma_\tau = 2.265\times10^{-12}$ GeV and $G_F = 1.16637(1)\times10^{-5}$ GeV^{-2}, we get the constraint:

$$\epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_\pi^2}{m_\tau(m_u+m_d)}\epsilon_P^\tau = (-0.12 \pm 0.68)\times10^{-2}, \tag{10}$$

where the uncertainty is dominated by f_π, followed by the error of branching ratio and the radiative corrections uncertainty. The central value in Eq. (10) shows a slight difference with respect to the result of [36], $(-0.15 \pm 0.67)\times10^{-2}$, that we may attribute to a different numerical input.

We next perform a simultaneous fit to one and two meson strangeness-conserving exclusive hadronic tau decays. For our analysis, we consider the following observables: the high-statistics $\tau^- \to \pi^-\pi^0\nu_\tau$ experimental data reported by the Belle collaboration [57], including both the normalized unfolded spectrum and the branching ratio, and the branching ratios of the decay $\tau^- \to K^-K^0\nu_\tau$ and of the one-meson $\tau^- \to \pi^-\nu_\tau$ transition. The χ^2 function to be minimized in our fits is

$$\chi^2 = \sum_k\left(\frac{\bar{N}_k^{\rm th} - \bar{N}_k^{\rm exp}}{\sigma_{\bar{N}_k^{\rm exp}}}\right)^2 + \left(\frac{BR_{\pi\pi}^{\rm th} - BR_{\pi\pi}^{\rm exp}}{\sigma_{BR_{\pi\pi}^{\rm exp}}}\right)^2 + \left(\frac{BR_{KK}^{\rm th} - BR_{KK}^{\rm exp}}{\sigma_{BR_{KK}^{\rm exp}}}\right)^2 + \left(\frac{BR_{\tau\pi}^{\rm th} - BR_{\tau\pi}^{\rm exp}}{\sigma_{BR_{\tau\pi}^{\rm exp}}}\right)^2, \tag{11}$$

where $\bar{N}_k^{\rm th}$ relates the decay rate of Eq. (4) for $\tau^- \to \pi^-\pi^0\nu_\tau$ to the normalized distribution of the measured number of events through

$$\frac{1}{N_{\rm events}}\frac{dN_{\rm events}}{ds} = \frac{1}{\Gamma(\epsilon_i^\tau, \epsilon_j^e)}\frac{d\Gamma(s, \epsilon_i^\tau, \epsilon_j^e)}{ds}\Delta^{\rm bin} \tag{12}$$

where $N_{\rm events}$ is the total number of measured events and $\Delta^{\rm bin}$ is the bin width. $\bar{N}_k^{\rm exp}$ and $\sigma_{\bar{N}_k^{\rm exp}}$ in Eq. (11) are, respectively, the experimental number of events and the corresponding uncertainties in the k-th bin. The unfolded distribution measured by Belle is available in 62 equally distributed bins with bin width of 0.05 GeV^2. The second, third and fourth terms in the χ^2 function Eq. (11) are data points that are used as a constraint of the branching ratios of $\tau^- \to \pi^-\pi^0\nu_\tau$ ($BR_{\pi\pi}^{\rm exp} = 25.49(9)\%$), of $\tau^- \to K^-K^0\nu_\tau$ ($BR_{KK}^{\rm exp} = 1.486(34)\times10^{-3}$) and of $\tau^- \to \pi^-\nu_\tau$ ($BR_{\tau\pi}^{\rm exp} = 10.82(5)\%$) [1].

The bounds for the non-SM effective couplings resulting from the global fit are found to be (in the $\overline{\rm MS}$ scheme at scale $\mu = 2$ GeV)

[9] The pion decay constant determined from data cannot be employed as it may be contaminated with NP effects.

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau + \frac{m_\pi^2}{2m_\tau(m_u+m_d)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.5 \pm 0.6^{+2.3}_{-1.8}{}^{+0.2}_{-0.1} \pm 0.4 \\ 0.3 \pm 0.5^{+1.1}_{-0.9}{}^{+0.1}_{-0.0} \pm 0.2 \\ 9.7^{+0.5}_{-0.6} \pm 21.5^{+0.0}_{-0.1} \pm 0.2 \\ -0.1 \pm 0.2^{+1.1}_{-1.4}{}^{+0.0}_{-0.1} \pm 0.2 \end{pmatrix} \times 10^{-2}, \tag{13}$$

with χ^2/d.o.f.$\sim$ 0.6, and where the first error is the statistical fit uncertainty while the associated (statistical) correlation matrix (ρ_{ij}) is

$$\rho_{ij} = \begin{pmatrix} 1 & 0.684 & -0.493 & -0.545 \\ & 1 & -0.337 & -0.372 \\ & & 1 & 0.463 \\ & & & 1 \end{pmatrix}. \tag{14}$$

The second error in Eq. (13) is the dominant one and comes from the theoretical uncertainty associated to the pion vector form factor (cf. Fig. 1), while the third and fourth ones are systematic uncertainties coming, respectively, from the error of the quark masses and from the uncertainty associated to the corresponding tensor form factors. The systematic errors, here and hereafter, have been obtained by taking the difference of the central values that are obtained while varying the corresponding inputs with respect to the reported central fit values.

Comparing our limits[10] in Eq. (13) with the bounds, $\epsilon_S^\mu = (-0.039 \pm 0.049) \times 10^{-2}$ and $\epsilon_T^\mu = (0.05 \pm 0.52) \times 10^{-2}$ [46], obtained from semileptonic kaon decays involving muons, and with those from hyperon decays [44], where $|\epsilon_S| < 4 \times 10^{-2}$ and $|\epsilon_T| < 5 \times 10^{-2}$ are found at a 90% C.L., we conclude that while it is impossible to compete with the limits on ϵ_S coming from $K_{\ell 3}$ decays, our analysis yields a very competitive constraint on the coupling ϵ_T.

Our results are in accord with those of [36],[11] which were obtained through a combination of inclusive and exclusive (strangeness-conserving) tau decays, but for the limit on the coefficient ϵ_S^τ. Ours is much weaker, but still compatible within errors with, the bounds set in [34,36], since we are not using the $\tau^- \to \pi^-\eta\nu_\tau$ decay in the global fit for lack of experimental measurements. The differing bound on ϵ_S obtained with and without the $\pi\eta$ mode increases the interest of its measurement and demands improved theoretical understanding accordingly.

5. New physics bounds from $|\Delta S| = 1$ decays

The lowest multiplicity strangeness-changing tau decay is $\tau^- \to K^-\nu_\tau$, which can be used to restrict the combination of the couplings of the left-hand side of Eq. (10), but replacing $m_d \to m_s$ and $m_\pi \to m_K$ and with the ϵ's corresponding to $u \to s$ transitions.[12] Using the lattice calculation of $f_K = 155.7(7)$ MeV [15], the radiative corrections $\delta_{\text{em}}^{\tau K} = 1.98(31)\%$ from Refs. [66–68] and $|\tilde{V}_{us}^e| = 0.2231(7)$, $BR(\tau^- \to K^-\nu_\tau) = 6.96(10) \times 10^{-3}$ and $m_K = 0.493677(16)$ GeV from the PDG [1] as numerical inputs, we obtain the constraint:

$$\epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_K^2}{m_\tau(m_u+m_s)}\epsilon_P^\tau = (-0.41 \pm 0.93) \times 10^{-2}, \tag{15}$$

where the error is dominated by f_K and $|V_{us}|$ followed by the branching ratio and the radiative corrections uncertainty.

Analogously to the previous section, we next analyze strangeness-changing exclusive transitions with one and two mesons in the final state simultaneously. In particular, we fit the $\tau^- \to K_S\pi^-\nu_\tau$ Belle spectrum [58][13] including the measured branching ratio, $BR_{K\pi}^{\text{exp}} = 0.404(2)(13)\%$, as experimental datum to constrain the fit. The PDG branching ratio [1] of the decays $\tau^- \to K^-\eta\nu_\tau$ ($BR_{K\eta}^{\text{exp}} = 1.55(8) \times 10^{-4}$)[14] and $\tau^- \to K^-\nu_\tau$ ($BR_{\tau K}^{\text{exp}} = 6.96(10) \times 10^{-3}$) are also added as external restrictions to the fit. The decay $\tau^- \to K^-\eta'\nu_\tau$ has not been detected yet, there is only an upper limit at the 90% confidence level placed by BaBar [69] and we therefore have decided to not include it in our analysis. Hence, the χ^2 function to be minimized in this case is chosen to be

$$\chi^2 = \sum_k \left(\frac{\bar{N}_k^{\text{th}} - \bar{N}_k^{\text{exp}}}{\sigma_{\bar{N}_k^{\text{exp}}}}\right)^2 + \left(\frac{BR_{K\pi}^{\text{th}} - BR_{K\pi}^{\text{exp}}}{\sigma_{BR_{K\pi}^{\text{exp}}}}\right)^2 + \left(\frac{BR_{K\eta}^{\text{th}} - BR_{K\eta}^{\text{exp}}}{\sigma_{BR_{K\eta}^{\text{exp}}}}\right)^2 + \left(\frac{BR_{\tau K}^{\text{th}} - BR_{\tau K}^{\text{exp}}}{\sigma_{BR_{\tau K}^{\text{exp}}}}\right)^2, \tag{16}$$

where now $\bar{N}_k^{\text{th}}$ refers to the $K_S\pi^-$ decay mode and its expression is given by

$$\frac{dN_{\text{events}}}{d\sqrt{s}} = \frac{N_{\text{events}}}{\Gamma(\epsilon_i^\tau, \epsilon_j^e)} \frac{d\Gamma(\sqrt{s}, \epsilon_i^\tau, \epsilon_j^e)}{d\sqrt{s}} \Delta^{\text{bin}}. \tag{17}$$

The number of events is $N_{\text{events}} = 53113.21$, the bin width is $\Delta^{\text{bin}} = 11.5$ MeV [58] and the number of fitted data points is 86 for the spectrum,[15] together with the respective branching ratios used as a constraint: thus 89 data points in total.

In this case, the limits for the NP effective couplings are found to be (in the $\overline{\text{MS}}$ scheme at scale $\mu = 2$ GeV)

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau + \frac{m_K^2}{2m_\tau(m_u+m_s)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.5 \pm 1.5 \pm 0.3 \\ 0.4 \pm 0.9 \pm 0.2 \\ 0.8^{+0.8}_{-0.9} \pm 0.3 \\ 0.9 \pm 0.7 \pm 0.4 \end{pmatrix} \times 10^{-2}, \tag{18}$$

where the first error is the statistical fit uncertainty while the second one is a systematic uncertainty due to the tensor form factor. Different to Eq. (18), the uncertainty associated to the kaon vector form factor and to the quark masses is negligible.

The (statistical) correlation matrix associated to the results of Eq. (18) is

$$\rho_{ij} = \begin{pmatrix} 1 & 0.854 & -0.147 & 0.437 \\ & 1 & -0.125 & 0.373 \\ & & 1 & -0.055 \\ & & & 1 \end{pmatrix}, \tag{19}$$

[10] For the comparison, here and throughout the rest of the paper, we need to assume lepton universality because our study involves the tau lepton, while theirs electrons and muons. Given the smallness of possible lepton universality violations, this is enough for current precision. We have also assumed that the corresponding CKM matrix elements do not change under NP interactions, which is the case if $\epsilon(lud) = \epsilon(lus)$ [70].

[11] We would like to notice that our fit to $\Delta S = 0$ processes is not sensitive to the coefficients ϵ_P^τ and ϵ_R^τ individually but rather to a combination of them (given by the second row in Eq. (13)). However, as we will see in section 6, one can still fit them separately if one performs a global fit including strangeness-changing decays. This is also the case in the next section.

[12] In the chiral limit ϵ_P^τ is the same as in Eq. (10).

[13] We thank the Belle collaboration, in particular S. Eidelman, D. Epifanov and B. Shwartz, for providing their data and for useful discussions.

[14] While the $\tau^- \to K^-\eta\nu_\tau$ decay spectrum has been measured by Belle [59], unfolding detector effects has not been performed and we therefore have decided to include only the branching ratio in our study.

[15] The points corresponding to bins 5,6 and 7 are difficult to bring into accord with theoretical parametrizations, even when non-standard interactions are considered [37], and have been excluded from the minimization. The first point has not been included either, since the centre of the bin lies below the $K_S\pi^-$ production threshold. We have furthermore excluded data corresponding to bin numbers larger than 90 following a suggestion from the experimentalists.

with χ^2/d.o.f.~ 0.9.

Notice that ρ_{12} in Eq. (19) is large (it was also the largest element in Eq. (14)). As we will see in section 6, where we will perform a global fit to both $\Delta S = 0$ and $|\Delta S| = 1$ sectors and obtain both ϵ_R^τ and ϵ_P^τ independently, this is due to the strong correlation between ϵ_R^τ and ϵ_P^τ.

The limits obtained from the $|\Delta S| = 1$ transitions in Eq. (18) serve as a consistency check upon comparison with those of Eq. (13) from the $\Delta S = 0$ ones. As one can observe, the results of the first and second lines in Eq. (18) are found to be in line with those from Eq. (13). As for the central value of the coefficient ϵ_S^τ (ϵ_T^τ) from the $|\Delta S| = 1$ sector, it has decreased (increased) by about one order of magnitude with respect to the $\Delta S = 0$ one; the ϵ_S^τ coupling is now more competitive while ϵ_T^τ has changed sign. We can anticipate, however, that the global fit in section 6 benefits from ϵ_T from the $\Delta S = 0$ decays and from ϵ_S from the $|\Delta S| = 1$ ones.

6. New physics bounds from a global fit to both $\Delta S = 0$ and $|\Delta S| = 1$ sectors

In this section, we close our exploratory analysis by performing a global fit to both $\Delta S = 0$ and $|\Delta S| = 1$ sectors simultaneously. The participant $|V_{ud}|$ and $|V_{us}|$ elements of the CKM matrix to be used in this case are not independent but rather correlated according to [15]

$$\frac{|V_{us}|}{|V_{ud}|} = 0.2313(5)\,. \tag{20}$$

For our analysis, we take $|V_{us}| = 0.2231(7)$ [1] and extract $|V_{ud}|$ through Eq. (20).

The χ^2 function to be minimized in the global fit includes all the quantities in Eqs. (11) and (16) that were used for the individual analysis of the $\Delta S = 0$ and $|\Delta S| = 1$ transitions, respectively. The resulting limits for the NP effective couplings are (in the $\overline{\text{MS}}$ scheme at scale $\mu = 2$ GeV)

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 2.9 & \pm 0.6 & {}^{+1.0}_{-0.9} & \pm 0.6 & \pm 0.0 & \pm 0.4 & {}^{+0.2}_{-0.3} \\ 7.1 & \pm 4.9 & {}^{+0.5}_{-0.4} & {}^{+1.3}_{-1.5} & {}^{+1.2}_{-1.3} & \pm 0.2 & {}^{+40.9}_{-14.1} \\ -7.6 & \pm 6.3 & \pm 0.0 & {}^{+1.9}_{-1.6} & {}^{+1.7}_{-1.6} & \pm 0.0 & {}^{+19.0}_{-53.6} \\ 5.0 & {}^{+0.7}_{-0.8} & {}^{+0.8}_{-1.3} & {}^{+0.2}_{-0.1} & \pm 0.0 & \pm 0.2 & {}^{+1.1}_{-0.6} \\ -0.5 & \pm 0.2 & {}^{+0.8}_{-1.0} & \pm 0.0 & \pm 0.0 & \pm 0.6 & \pm 0.1 \end{pmatrix} \times 10^{-2}\,, \tag{21}$$

where the first error is the statistical error resulting from the fit, the second one comes from the uncertainty on the pion vector form factor, the third error corresponds to the CKM elements $|V_{ud}|$ and $|V_{us}|$, the fourth one is due to the radiative corrections $\delta_{\text{em}}^{\tau\pi}$ and $\delta_{\text{em}}^{\tau K}$, the fifth estimates the (uncontrolled) systematic uncertainty associated to the tensor form factor, while the sixth, and last error, is due to the errors of the quark masses.

The (statistical) correlation matrix associated to the limits of Eq. (21) is

$$\mathcal{A} = \begin{pmatrix} 1 & 0.055 & 0.000 & -0.279 & -0.394 \\ & 1 & -0.997 & -0.015 & -0.022 \\ & & 1 & 0.000 & 0.000 \\ & & & 1 & 0.243 \\ & & & & 1 \end{pmatrix}\,, \tag{22}$$

with χ^2/d.o.f.~ 1.38.

As anticipated in the previous section, the combined fit yields an independent determination of the couplings ϵ_R^τ and ϵ_P^τ which, in turn, carry a large statistical (and systematic) error. This originates in the fact that these parameters are almost 100% correlated (cf. Eq. (22)). For the combination of the couplings of the first line in Eq. (21), our limits are competitive and within errors with [36]. Regarding ϵ_S^τ, our limit is not competitive and disagrees with the values of Refs. [36,34], where a constraint for ϵ_S^τ was placed from the isospin-violating decay $\tau^- \to \pi^- \eta \nu_\tau$. We do not take into account this channel here since it has not been measured yet; only an upper bound exists. Finally, our bound for ϵ_T^τ is competitive and found to be in agreement with [36,35]. We would like to notice that the uncertainty associated to the CKM elements dominates the error of those coefficients in Eq. (21) for what we get competitive bounds. Therefore, future lattice results can result in tighter constraints.

Our limits on the NP effective couplings Eq. (21) can be translated into bounds on the corresponding NP scale Λ through the relation

$$\Lambda \sim v\,(V_{uD}\epsilon_i)^{-1/2}\,, \tag{23}$$

where $v = (\sqrt{2}G_F)^{-1/2} \sim 246$ GeV. Our bounds can probe scales as high as $\sim \mathcal{O}(5)$ TeV, which are quite restricted compared to the energy scale probed in semileptonic kaon decays i.e. $\mathcal{O}(500)$ TeV [46].

7. Conclusions

This letter highlights that hadronic tau lepton decays remain to be not only a privileged tool for the investigation of the hadronization of QCD currents but also offer an interesting scenario as New Physics probes.

In this work, we have performed a global analysis of strangeness-conserving ($\Delta S = 0$) and strangeness-changing ($|\Delta S| = 1$) exclusive hadronic τ decays into one and two mesons. From the current experimental measurements of the corresponding decay spectra and/or branching ratios, we have set bounds on the NP effective couplings of the low-energy (dimension six) Standard Model Effective Field Theory Lagrangian. This has been possible due to a controlled theoretical determination of the necessary Standard Model hadronic input i.e. the form factors. For the description of the corresponding vector and scalar form factors, we have employed previous results, based on constraints from Chiral Perturbation Theory supplemented by dispersion relations, that show a nice agreement with the rich experimental data provided by the experiments. On the other hand, as there is no experimental data that can help us constructing the corresponding tensor form factors, they have been built under theoretical arguments only.

In general, our bounds on the NP couplings, Eqs. (13), (18) and (21), are competitive. This is specially the case for the combination of couplings $\epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e$, which is found to be in accord with the constraints placed from a combination of inclusive and exclusive (strangeness-conserving) tau decays [36], and for ϵ_T^τ, that can even compete with the constraints set by the theoretically cleaner $K_{\ell 3}$ decays (for the comparison, lepton flavor universality is assumed as mentioned throughout the main text). Our separate fits to both $\Delta S = 0$ and $|\Delta S| = 1$ decays reflect that we are not sensitive to the coefficients ϵ_P^τ and ϵ_R^τ individually but rather to a combination of them. It is still possible to fit them separately performing a global fit to both $\Delta S = 0$ and $|\Delta S| = 1$ sectors simultaneously. However, they carry a large error bar whose origin stems from the very strong correlation between them. As for ϵ_S^τ, it is impossible to compete with the limits coming from $K_{\ell 3}$ decays. Our limit, however, is found to be much weaker than previous constraints from tau decays. This is due to the fact that, for lack of experimental data, the decay $\tau^- \to \pi^-\eta\nu_\tau$ has not been taken into account in our analysis. These different bounds on ϵ_S^τ obtained with and without the $\pi\eta$ mode thus increase the interest of its measurement and demands refined theoretical descriptions accordingly.

Our study is presently limited by the fact that the Standard Model form factors, the input parameters of which have been fitted to data previously, may have absorbed some NP information, if this is in the data. We have tried to address this drawback through fits where not only the NP effective couplings are treated as free parameters to fit but also the Standard Model input parameters entering the corresponding form factors. In doing so, we have too many free parameters to fit and found no sensitivity to the NP couplings. This is indeed interesting to prove in the future, with a higher-quality data, but at present is not feasible. We thus hope our work can serve to encourage the experimental tau physics groups at Belle-II to measure these decays with higher accuracy.

Declaration of competing interest

We declare that we have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The work of S.GS has been supported in part by the National Science Foundation (PHY-1714253) and by the U.S. Department of Energy under Grants No. DE-FG02-87ER40365. The work of A. Miranda and J. Rendón has been granted by their Conacyt scholarships. P. R. thanks Conacyt funding through projects 250628 (Ciencia Básica) and Fondo SEP-Cinvestav 2018 (No. 142).

References

[1] M. Tanabashi, et al., Particle Data Group, Phys. Rev. D 98 (2018) 030001.
[2] A. Pich, Prog. Part. Nucl. Phys. 75 (2014) 41, arXiv:1310.7922 [hep-ph].
[3] D. Boito, M. Golterman, K. Maltman, J. Osborne, S. Peris, Phys. Rev. D 91 (3) (2015) 034003, arXiv:1410.3528 [hep-ph].
[4] A. Pich, A. Rodríguez-Sánchez, Phys. Rev. D 94 (3) (2016) 034027, arXiv:1605.06830 [hep-ph].
[5] K. Maltman, C.E. Wolfe, S. Banerjee, J.M. Roney, I. Nugent, Int. J. Mod. Phys. A 23 (2008) 3191, arXiv:0807.3195 [hep-ph].
[6] M. Antonelli, V. Cirigliano, A. Lusiani, E. Passemar, J. High Energy Phys. 1310 (2013) 070, arXiv:1304.8134 [hep-ph].
[7] R.J. Hudspith, R. Lewis, K. Maltman, J. Zanotti, Phys. Lett. B 781 (2018) 206, arXiv:1702.01767 [hep-ph].
[8] K.G. Chetyrkin, J.H. Kuhn, A.A. Pivovarov, Nucl. Phys. B 533 (1998) 473, arXiv:hep-ph/9805335.
[9] A. Pich, J. Prades, J. High Energy Phys. 9910 (1999) 004.
[10] J. Kambor, K. Maltman, Phys. Rev. D 62 (2000) 093023, arXiv:hep-ph/0005156.
[11] S. Chen, M. Davier, E. Gamiz, A. Hocker, A. Pich, J. Prades, Eur. Phys. J. C 22 (2001) 31, arXiv:hep-ph/0105253.
[12] E. Gámiz, M. Jamin, A. Pich, J. Prades, F. Schwab, J. High Energy Phys. 0301 (2003) 060, arXiv:hep-ph/0212230.
[13] E. Gámiz, M. Jamin, A. Pich, J. Prades, F. Schwab, Phys. Rev. Lett. 94 (2005) 011803, arXiv:hep-ph/0408044.
[14] P.A. Baikov, K.G. Chetyrkin, J.H. Kuhn, Phys. Rev. Lett. 95 (2005) 012003, arXiv:hep-ph/0412350.
[15] S. Aoki, et al., Flavour Lattice Averaging Group, arXiv:1902.08191 [hep-lat].
[16] F. Guerrero, A. Pich, Phys. Lett. B 412 (1997) 382, arXiv:hep-ph/9707347.
[17] A. Pich, J. Portolés, Phys. Rev. D 63 (2001) 093005, arXiv:hep-ph/0101194.
[18] D. Gómez Dumm, P. Roig, Eur. Phys. J. C 73 (8) (2013) 2528, arXiv:1301.6973 [hep-ph].
[19] S. Gonzàlez-Solís, P. Roig, Eur. Phys. J. C 79 (5) (2019) 436, arXiv:1902.02273 [hep-ph].
[20] M. Jamin, A. Pich, J. Portolés, Phys. Lett. B 640 (2006) 176, arXiv:hep-ph/0605096.
[21] M. Jamin, A. Pich, J. Portolés, Phys. Lett. B 664 (2008) 78, arXiv:0803.1786 [hep-ph].
[22] D.R. Boito, R. Escribano, M. Jamin, Eur. Phys. J. C 59 (2009) 821, arXiv:0807.4883 [hep-ph].
[23] D.R. Boito, R. Escribano, M. Jamin, J. High Energy Phys. 1009 (2010) 031, arXiv:1007.1858 [hep-ph].
[24] R. Escribano, S. Gonzàlez-Solís, M. Jamin, P. Roig, J. High Energy Phys. 1409 (2014) 042, arXiv:1407.6590 [hep-ph].
[25] G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B 321 (1989) 311.
[26] R. Escribano, S. Gonzàlez-Solís, P. Roig, J. High Energy Phys. 1310 (2013) 039, arXiv:1307.7908 [hep-ph].
[27] R. Escribano, S. Gonzalez-Solis, P. Roig, Phys. Rev. D 94 (3) (2016) 034008, arXiv:1601.03989 [hep-ph].
[28] S. Descotes-Genon, B. Moussallam, Eur. Phys. J. C 74 (2014) 2946, arXiv:1404.0251 [hep-ph].
[29] E. Kou, et al., Belle-II Collaboration, to be published on PTEP, arXiv:1808.10567 [hep-ex].
[30] J. Alcaraz, et al., ALEPH and DELPHI and L3 and OPAL Collaborations and LEP Electroweak Working Group, arXiv:hep-ex/0612034.
[31] A. Filipuzzi, J. Portoles, M. Gonzalez-Alonso, Phys. Rev. D 85 (2012) 116010, arXiv:1203.2092 [hep-ph].
[32] J.P. Lees, et al., BaBar Collaboration, Phys. Rev. D 85 (2012) 031102, Erratum: Phys. Rev. D 85 (2012) 099904, arXiv:1109.1527 [hep-ex].
[33] Y. Grossman, Y. Nir, J. High Energy Phys. 1204 (2012) 002, arXiv:1110.3790 [hep-ph].
[34] E.A. Garcés, M. Hernández Villanueva, G. López Castro, P. Roig, J. High Energy Phys. 1712 (2017) 027, arXiv:1708.07802 [hep-ph].
[35] J.A. Miranda, P. Roig, J. High Energy Phys. 1811 (2018) 038, arXiv:1806.09547 [hep-ph].
[36] V. Cirigliano, A. Falkowski, M. González-Alonso, A. Rodríguez-Sánchez, Phys. Rev. Lett. 122 (22) (2019) 221801, arXiv:1809.01161 [hep-ph].
[37] J. Rendón, P. Roig, G. Toledo Sánchez, Phys. Rev. D 99 (9) (2019) 093005, arXiv:1902.08143 [hep-ph].
[38] S. Gonzàlez-Solís, A. Miranda, J. Rendón, P. Roig, arXiv:d [hep-ph].
[39] I. Baum, V. Lubicz, G. Martinelli, L. Orifici, S. Simula, Phys. Rev. D 84 (2011) 074503, arXiv:1108.1021 [hep-lat].
[40] V. Cirigliano, J. Jenkins, M. Gonzalez-Alonso, Nucl. Phys. B 830 (2010) 95, arXiv:0908.1754 [hep-ph].
[41] T. Bhattacharya, V. Cirigliano, S.D. Cohen, A. Filipuzzi, M. Gonzalez-Alonso, M.L. Graesser, R. Gupta, H.W. Lin, Phys. Rev. D 85 (2012) 054512, arXiv:1110.6448 [hep-ph].
[42] V. Cirigliano, M. Gonzalez-Alonso, M.L. Graesser, J. High Energy Phys. 1302 (2013) 046, arXiv:1210.4553 [hep-ph].
[43] V. Cirigliano, S. Gardner, B. Holstein, Prog. Part. Nucl. Phys. 71 (2013) 93, arXiv:1303.6953 [hep-ph].
[44] H.M. Chang, M. González-Alonso, J. Martin Camalich, Phys. Rev. Lett. 114 (16) (2015) 161802, arXiv:1412.8484 [hep-ph].
[45] A. Courtoy, S. Baeßler, M. González-Alonso, S. Liuti, Phys. Rev. Lett. 115 (2015) 162001, arXiv:1503.06814 [hep-ph].
[46] M. González-Alonso, J. Martin Camalich, J. High Energy Phys. 1612 (2016) 052, arXiv:1605.07114 [hep-ph].
[47] M. González-Alonso, J. Martin Camalich, arXiv:1606.06037 [hep-ph].
[48] S. Alioli, V. Cirigliano, W. Dekens, J. de Vries, E. Mereghetti, J. High Energy Phys. 1705 (2017) 086, arXiv:1703.04751 [hep-ph].
[49] M. González-Alonso, J. Martin Camalich, K. Mimouni, Phys. Lett. B 772 (2017) 777, arXiv:1706.00410 [hep-ph].
[50] M. Gonzalez-Alonso, O. Naviliat-Cuncic, N. Severijns, Prog. Part. Nucl. Phys. 104 (2019) 165, arXiv:1803.08732 [hep-ph].
[51] W. Buchmuller, D. Wyler, Nucl. Phys. B 268 (1986) 621.
[52] B. Grzadkowski, M. Iskrzynski, M. Misiak, J. Rosiek, J. High Energy Phys. 1010 (2010) 085, arXiv:1008.4884 [hep-ph].
[53] J. Erler, Rev. Mex. Fis. 50 (2004) 200, arXiv:hep-ph/0211345.
[54] R. García-Martín, R. Kaminski, J.R. Peláez, J. Ruiz de Elvira, F.J. Ynduráin, Phys. Rev. D 83 (2011) 074004, arXiv:1102.2183 [hep-ph].
[55] K.M. Watson, Phys. Rev. 95 (1954) 228.
[56] G.P. Lepage, S.J. Brodsky, Phys. Lett. B 87 (1979) 359, Phys. Rev. D 22 (1980) 2157.
[57] M. Fujikawa, et al., Belle Collaboration, Phys. Rev. D 78 (2008) 072006, arXiv:0805.3773 [hep-ex].
[58] D. Epifanov, et al., Belle Collaboration, Phys. Lett. B 654 (2007) 65, arXiv:0706.2231 [hep-ex].
[59] K. Inami, et al., Belle Collaboration, Phys. Lett. B 672 (2009) 209, arXiv:0811.0088 [hep-ex].
[60] Z.H. Guo, J.A. Oller, Phys. Rev. D 84 (2011) 034005, arXiv:1104.2849 [hep-ph].
[61] Z.H. Guo, J.A. Oller, J. Ruiz de Elvira, Phys. Rev. D 86 (2012) 054006, arXiv:1206.4163 [hep-ph].
[62] Z.H. Guo, L. Liu, U.G. Meißner, J.A. Oller, A. Rusetsky, Phys. Rev. D 95 (5) (2017) 054004, arXiv:1609.08096 [hep-ph].
[63] M. Jamin, J.A. Oller, A. Pich, Nucl. Phys. B 622 (2002) 279, arXiv:hep-ph/0110193.
[64] V. Cirigliano, A. Crivellin, M. Hoferichter, Phys. Rev. Lett. 120 (14) (2018) 141803, arXiv:1712.06595 [hep-ph].
[65] M. Hoferichter, B. Kubis, J. Ruiz de Elvira, P. Stoffer, Phys. Rev. Lett. 122 (12) (2019) 122001, arXiv:1811.11181 [hep-ph].

[66] R. Decker, M. Finkemeier, Nucl. Phys. B 438 (1995) 17, arXiv:hep-ph/9403385.
[67] V. Cirigliano, I. Rosell, J. High Energy Phys. 0710 (2007) 005, arXiv:0707.4464 [hep-ph].
[68] J.L. Rosner, S. Stone, R.S. Van de Water, arXiv:1509.02220 [hep-ph].
[69] J.P. Lees, et al., BaBar Collaboration, Phys. Rev. D 86 (2012) 092010, arXiv:1209.2734 [hep-ex].
[70] S. Descotes-Genon, A. Falkowski, M. Fedele, M. González-Alonso, J. Virto, J. High Energy Phys. (1905) 172, arXiv:1812.08163 [hep-ph], 2019.