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Prospective Teachers' Reflections on the Inclusion of Mathematical Modelling During the Transition Period Between the Face-to-Face and Virtual Teaching Contexts

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Abstract

Research in mathematics education highlights the importance of including modelling for the teaching of this subject. In 2020, this trend coexisted with a grave contingency situation caused by the COVID-19 pandemic but which, despite its negative aspects, provided a realistic and authentic context for modelling. Given this situation, it is relevant to study which aspects of the teaching and learning process prospective teachers related to mathematical modelling in their reflections on its inclusion during the transition period between the face-to-face and virtual teaching contexts. To this end, we used the didactic suitability criteria construct, proposed by the onto-semiotic approach, as a theoretical reference. This is qualitative research of a naturalistic type, since we did not interfere in the Master's Programme in Mathematics Teacher Education studied. We conducted a content analysis on 122 master's degree final projects written during the 2019–2020 course and implemented during the COVID-19 pandemic. We highlight the following results: (a) about 40% of the prospective teachers stated that they implemented modelling in their didactic units and reflected on its inclusion; (b) in their reflections, the prospective teachers positively assessed the inclusion of modelling mainly based on the epistemic, affective and ecological suitability criteria; and (c) 60% of the prospective teachers did not implement modelling, and we rule out that it was due to a lack of knowledge about this process or of a favourable context for modelling, but because they prioritised other aspects of the mathematical teaching and learning process, given the grave contingency situation.

Keywords Content analysis · Didactic suitability criteria · Master's degree final project · Mathematical modelling · Teacher reflection

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Introduction

There is a worldwide consensus on the development of competencies that involve the use of mathematics to solve real-world problems, among which the mathematical modelling competency stands out (Kaiser, 2020; Niss & Højgaard, 2019). On one hand, modelling is considered as a central aspect of the PISA international assessment for problem solving (Organisation for Economic Co-operation and Development, 2019); and, on the other hand, it is agreed that working with modelling has a series of benefits for learning mathematics (Blum, 2011), in addition to being indispensable for educating individuals to be capable of linking their mathematical knowledge to contemporary needs and demands (Maass et al., 2022). Therefore, in order to teach modelling skills to students, teachers need to be prepared and educated in teaching strategies associated with modelling so they can implement it in the classroom (Blum & Borromeo Ferri, 2009).

Various studies on the role of modelling in mathematics teacher education have been reported in the literature of the last decades by addressing the teaching and learning of this process (see further details in the "Mathematical Modelling in Teacher Education" subsection). Although such studies are in line with Maaß's (2007) idea that it is not enough to educate teachers in modelling, but they also need to experience it, the study reported in this article focuses on the prospective teachers' reflections on the inclusion of this process in their master's degree final projects (MFPs). In the Spanish context, prospective teachers must obtain a master's degree to teach mathematics at secondary and baccalaureate education levels (students aged 12–18). To do this, they must prepare an MFP, an original, autonomous and individual work, which allows the prospective teachers to show the formative content they have received and the general competencies acquired during the master's programme in an integrated way. In the MFP, they also must reflect and go deeper into analysing their own practice, as well as to propose elements for its improvement. The prospective teachers prepare their MFPs after an internship period in educational centres, where they must design and implement a didactic unit that, depending on certain factors (see further details in the "Research Context" subsection), can include working with modelling.

Therefore, due to the importance of modelling in mathematics teacher education, we consider it relevant to deepen the reflections that prospective teachers made on the inclusion of this process during their educational internship experiences in a particular context of implementation where a grave contingency occurred. During the year 2020, we lived through complex moments worldwide due to the COVID-19 pandemic which affected, among many other aspects, education at all levels (see a broader discussion in Engelbrecht et al., 2023). Given this situation, prospective teachers also saw their educational processes affected, as in the case of their internship experiences, many of which were carried out during the transition period between the face-to-face and virtual teaching contexts due to lockdowns. In this context, our study aims to show the importance (or not) of modelling for prospective teachers in a grave contingency situation, where some aspects of the mathematical teaching and learning process considered as relevant were prioritised and other aspects were relegated to a background (or suppressed). In this way, the gravity of the contingency made the relevance of modelling for prospective teachers evident.

This study raises the question: What aspects of the teaching and learning process did prospective secondary and baccalaureate education teachers relate to mathematical modelling in their reflections on its inclusion during the transition period between the face-to-face and virtual teaching contexts? To answer this, we analysed the prospective teachers' reflections in their MFPs on the design of their didactic units, which were implemented in their educational internship experiences during the transition period between the face-to-face and virtual teaching contexts due to the COVID-19 pandemic. We analysed these reflections using the didactic suitability criteria, which is one of the tools proposed by the onto-semiotic approach (Godino, Batanero & Font, 2007), and which was the same tool used by the prospective teachers to guide their reflections on their own educational practice. Specifically, our focus was on analysing the MFPs whose didactic units included working with modelling.

Theoretical Framework

In this section, we describe the theoretical references considered for this study.

Mathematical Modelling

In general terms, the modelling process is understood as a transition between the real world and mathematics for solving a problem-situation taken from reality. This process should not be understood in linear terms, since both the context of the problem and the mathematical aspects involved in the situation affect the mathematical model that is generated (Blomhøj, 2004; Borromeo Ferri, 2007). At the theoretical level, different cycles have been designed to explain this process (Borromeo Ferri, 2006), and different perspectives on implementing modelling in the classroom have emerged (Abassian et al., 2020). Although these differences are mainly due to the diversity of positions on modelling (Borromeo Ferri, 2013), the proposed cycles all tend to have certain phases in common (see Geiger et al. 2018). For this study, we consider the modelling cycle proposed by Blum and Leiß (2007) (see Fig. 1), since this is the cycle taught to prospective teachers in the master's programme in which we conducted this research. Along with this, we consider some consensual attributes that characterise working with modelling in the classroom.

An illustrative example of the use of this cycle can be found in Blum and Leiß (2007, pp. 225–227). The transit between the phases of the cycle is carried out through transitions or, in terms of Maaß (2006), modelling sub-competencies (numbered to the right of Fig. 1). The work with modelling in the classroom is usually carried out in small groups of students. They are generally presented with a real-world problem-situation that they must mathematise (Doerr & English, 2003; Shahbari & Tabach, 2019). Modelling tasks involve a cyclical process, with different

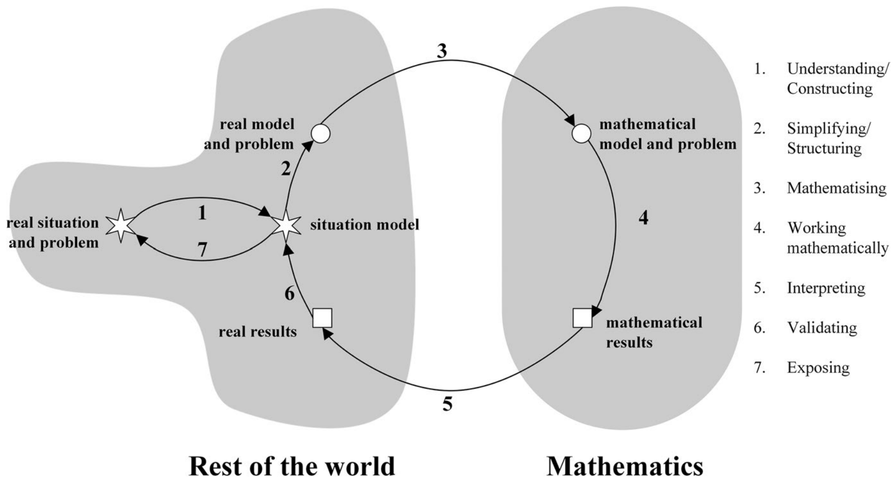


Fig. 1 Mathematical modelling cycle. Adapted from Blum and Leiß (2007, p. 225)

ways for obtaining a plausible solution that is consistent with the context of the situation posed (English, 2003; Lesh & Doerr, 2003). This situation, known as *modelling problem*, must meet certain consensual characteristics (Borromeo Ferri, 2018): it must be *open* and *complex*, in which its solving is not limited to a specific answer or procedure and where students must search for relevant data; it must be *realistic* and *authentic*, adding elements from the real world and showing a situation consistent with an event that has occurred or may occur in reality (Palm, 2007); and finally, it must be a *problem* (Schoenfeld, 1994) that can be *solved through a modelling cycle*, which implies the use of all the phases that make up this cycle.

Mathematical Modelling in Teacher Education

As mentioned above, literature of the last decades in mathematics education has broadly addressed the teaching and learning of modelling in teacher education.

In the Austrian context, Kuntze et al. (2013) study the teachers' self-perceptions about their *pedagogical content knowledge* (PCK) related to modelling, considering both the PCK necessary to help their students during the modelling process in the classroom and what they think of their own professional development at university level. The results made a need for professional development that not only address the PCK on modelling evident, but also the teaching of teacher strategies for pedagogical self-efficiency when implementing this process, for example, by using technological tools. In this same research line, a more recent study is that reported by Greefrath et al. (2022) in the German context, who propose the creation of their own problems as a strategy to develop of the modelling competency in prospective teachers. In the American context, Manouchehri (2017) reports the efforts to assist a group of practising mathematics teachers to develop knowledge about modelling and its implementation in school curriculum. This study reports the results of 25 of the 85 teachers involved in a course for professional development, making an increase of their knowledge on modelling evident through mathematical (construction and work with the mathematical model), pedagogical (strategies to develop this process in the classroom) and epistemological (obstacles during the modelling process) challenges that they had to face.

More recent studies have broadened the perspective on the role of modelling in teacher education, adding complementary mathematical tools and processes. For example, Alwast and Vorhölter (2022) focus on the noticing competencies, by examining the indicators for different kinds of validity (content, elemental and construct validity) of a video-based instrument to measure the prospective teachers' noticing competencies framed in modelling. Among the contributions of this instrument, it is worth noting that it makes it possible to measure the effects of interventions, to investigate learning trajectories of noticing, and evaluating the efficiency of teacher training courses related to noticing competencies for the teaching of modelling. In the Turkish context, Tekin (2019) focuses on the arguments constructed by prospective primary education teachers during the solving of a modelling task, using the Toulmin's (1954/2003) argumentation schema. Findings revealed the relationships between the components of the arguments constructed by the prospective teachers and the transitions of the modelling cycle.

Although these studies provide guidelines for the inclusion of modelling in mathematics teacher education, our article focuses on the reflection made by prospective teachers in their didactic proposals during their educational internship experiences, by using a tool that provides criteria to reflect on the improvement of teaching and which, as far as we know, has not been applied to the modelling process before.

Didactic Suitability Criteria

In mathematics education, different researchers have made attempts to compile criteria to guide the mathematics teacher's practice so that it is of quality (see Hill et al., 2008; Praetorius & Charalambous, 2018; Prediger et al., 2022; among others). The onto-semiotic approach (OSA) is one of the theoretical frameworks that has developed this research line, defining the notion of didactic suitability (Godino, 2013). The didactic suitability of a teaching and learning process is understood as the degree to which it (or a part of it) meets certain characteristics that allow it to be qualified as suitable (optimal or adequate) in order to achieve an adaptation between the *personal meanings* achieved by students (learning) and the intended or implemented *institutional meanings* (teaching), taking into account the circumstances and available resources (environment).

This multidimensional construct consists of six suitability criteria, each one focused on a specific aspect of the teaching and learning process. Each of the didactic suitability criteria (DSC) has its respective components, and their use requires defining a set of observable indicators for assessing the degree of suitability of each of the facets of the teaching and learning process. Table 1 presents the components of each DSC with the codes used in this research to label them, based on the guideline by Breda, Pino-Fan and Font (2017).

Table 1 Didactic suitability criteria and their components. Adapted from Breda et al. (2017)

Criteria	Description	Components
Epistemic	For assessing whether the mathematics that is taught is "good mathematics"	Errors (ES1); ambiguities (ES2); richness of processes (ES3); representativeness of the complexity of the mathematical object (ES4)
Cognitive	For assessing, before starting the teaching and learning process, whether what is intended to be taught is at a reasonable distance from what students know; and after, whether students learnt what was intended to be taught	Prior knowledge (CS1); curricular adaptation (CS2); learning (CS3); high cognitive demand (CS4)
Interactional	For assessing whether the interaction solves the students' doubts and difficulties	Teacher–student interaction (IS1); student interaction (IS2); autonomy (IS3); formative assessment (IS4)
Mediational	For assessing the adequacy of material resources and time used in the teaching and learning process	Material resources (MS1); number of students, class schedule and conditions (MS2); time (MS3)
Affective	For assessing the students' involvement (interest, motivation) in the teaching and learning process	Interests and needs (AS1); attitudes (AS2); emotions (AS3)
Ecological	For assessing the adaptation of the teaching and learning process to the school's educational project, the curricular guidelines, the conditions of the social and professional environment etc.	Adaptation to curriculum (EcS1); intra- and interdisciplinary connections (EcS2); social and labour usefulness (EcS3); didactic innovation (EcS4)

The DSC represent a rubric (with *criteria*, *components* and *indicators*) to help mathematics teachers assess their practice and then guide a redesign to improve it. However, the DSC are very different from the teaching guides whose purpose is to help teachers shape the teaching and learning processes, guiding their actions and decision-making (Remillard, 2018), as those included in textbooks. As an example, when this tool is taught to the prospective teachers in the master's programme where our study is contextualised, the importance of developing a mathematical activity rich in mathematical processes (such as problem solving, modelling, argumentation, connections) is stressed, so it is expected that they be able to include most or, at least, some of these processes in their didactic units. In the same way, it is explained that this mathematical activity requires that the proposed tasks/problems have a high cognitive demand, considering the work of many researchers in mathematics education that highlight this aspect as a theoretical support (e.g. Stein & Smith, 1998). Therefore, it is expected that the prospective teachers include, among others, the modelling process in their didactic units with tasks/problems that promote a high cognitive demand and, also, because of their reflection, that they give a special weight in their redesign proposal to those processes less developed. In addition, from the DSC perspective, the "richness of processes" (from the *epistemic criterion*) and "high cognitive demand" (from the *cognitive criterion*) components, two of the aspects that the prospective teachers must assess of their implemented didactic unit, reaffirm the importance of including relevant processes of mathematical activity.

The theoretical framework OSA (from which the DSC emerge) provides tools for analysing both the mathematical activity underlying the modelling process (see Ledezma, Font & Sala, 2022) and the mathematics teacher's knowledge and competencies to develop mathematical teaching and learning processes (see Pino-Fan, Castro & Font, 2023). Finally, the OSA considers that enhancing modelling is an aspect that improves the didactic suitability of mathematical teaching and learning processes (Ledezma, Sol, Sala-Sebastià & Font, 2022).

Methodological Aspects

We followed a qualitative research methodology of a naturalistic type (since we did not interfere in the research context) from an interpretative paradigm (Cohen et al., 2018), which consists of a content analysis (Schreier, 2012) conducted on a sample of 122 MFPs prepared by the prospective teachers of the 2019–2020 academic year at the end of their education in a master's programme, who had to develop their educational internship experiences, either partially or totally, in a virtual teaching context due to the COVID-19 pandemic. Based on the indirectly provided information in the MFPs, these prospective teachers did not have special problems of digital competency to face virtual teaching.

Research Context

This research was conducted in the context of the master's programme in teacher education for secondary and baccalaureate education (specialised in mathematics), taught by the public universities of Catalonia (Spain), during the 2019–2020 academic year.

The master's study programme includes, within the "complements of disciplinary formation" module, a submodule on modelling. This submodule consists of four sessions (one per week), and its (mainly expository) methodology is as follows: in the first session, the prospective teachers are introduced to what is meant by modelling, and the cycle proposed by Blum and Leiß (2007) is presented to them; during the second and third sessions, a series of examples of modelling problems are presented, and the prospective teachers have to solve some of them in class; and in the fourth session, the prospective teachers must expose the final task of the submodule in front of the class. This task consists of presenting a modelling problem that includes the wording and solving of the problem and the curricular location of the mathematical contents necessary for its solving.

The master's study programme also involves, within the "internships" module, carrying out educational internship experiences in collaboration with various institutions established through university agreements and which are recognised as internship centres. The internship period consists of two phases: an observation (during 2 weeks of November) and an intervention (during 6 weeks from February) phase, both developed under the supervision of a mentor teacher from the internship centre. In the intervention phase, the prospective teachers must implement a didactic unit previously designed by them, which is determined by the internship centre, the student educational level and the time of the school year in which they carry out their intervention. Given this situation, even though the prospective teachers are expected to include modelling, among other mathematical processes, in the implementation of their didactic units, the available margin to do it is subject to the factors mentioned above, but this is not the case of the redesigns proposed in their MFPs.

Structure of a Master's Degree Final Project

To obtain the master's degree in mathematics teacher education for secondary and baccalaureate education, prospective teachers must prepare an MFP, which must be an original, autonomous and individual work. This master's programme offers the prospective teachers two modalities to prepare an MFP. The first modality consists of a written reflection on educational practice during the internship experiences, with a professionalising orientation and where the level of research skills required is lower than for a research-oriented master's thesis. The second modality is oriented towards educational research (master's thesis). At the beginning of the master's programme, the prospective teachers must decide which of the two modalities they want to develop during the course, in agreement with their tutor professor. For this research, we considered the MFPs of the first modality mentioned above.

The DSC and the modified version of the guidelines to these criteria, components and descriptors (see Breda et al., 2017) are presented to the prospective teachers so they can apply them in the elaboration of this type of MFPs. It is suggested that the prospective teachers use these tools to assess the didactic unit that they implemented

Table 2 Chapters that structure an MFP

Chapters	Description
Introduction	In which the context of the educational centre where the internship experience was carried out and the curricular aspects of the implemented didactic unit are presented
Implementation analysis	In which the didactic suitability of the implemented teaching and learning process is assessed using the DSC as a tool. This chapter ends with the overall assessment of didactic suitability, using a hexagonal radial graph based on the assessments assigned to each DSC (see an example in Ledezma, Sala, Breda & Sánchez, 2021)
Redesign proposal	In which a reformulation of the implemented didactic unit is proposed to improve its didactic suitability, based on the reflections made in the previous chapter
Competency self-assessment	In which prospective teachers must compare their level for each competency (based on the proposal by Font, Giménez, Zorrilla, Larios, Dehesa, Aubanell & Benseny 2012), considering the level they had when they started the master's programme and the level that they achieved at the end of their educational process
Annexes	In which evidence of the implementation, the planning of the implemented didactic unit, the cited references, among others, can be included

in their MFPs with the aim that they can propose changes that can help improve the suitability of the teaching and learning process. In Table 2, we describe the five chapters that structure an MFP.

Although prospective teachers are encouraged to justify the improvements of their redesigned didactic units with the results of research in didactics of mathematics on the topic of their educational internship experiences, in general, few references are cited in the MFPs of the modality considered for this study.

Content Analysis

For this study, we considered 122 MFPs, corresponding to the 2019–2020 academic year. For their qualitative analysis, we followed steps like those used by Sánchez (2021), which are described below.

In a *first step*, according to specialised literature and our knowledge on the topic, we drew up a list of keywords related to modelling (*context*, *model*, *problem*, *real*) to search for in the MFPs. These terms allowed us to identify the references to modelling in the evaluative comments made by the prospective teachers in their MFPs.

In a *second step*, we recorded the data (author, title, educational level, mathematical content) of each MFP. We organised the mathematical contents based on the curricular guidelines for secondary (Departament d'Educació, 2019) and baccalaureate education (Departament d'Ensenyament, 2008) in Catalonia, which were grouped into seven thematic areas: algebra, functions, geometry, numbers, probability, statistics and trigonometry. This *second step* allowed us to obtain an organised database that we could consult for each MFP, and therefore, we could keep an initial record of which MFPs included the keywords from the *first step*.

When reviewing the database obtained in the *second step*, we were able to observe a regularity in the distribution of the keywords within the MFPs. That is, we found MFPs that did not include the keywords; MFPs that included the keywords mainly in the *Implementation analysis* and/or *Redesign proposal* chapters; and MFPs that included the keywords throughout the entire document. Given this situation, we decided, in a *third step*, to classify the MFPs according to four levels of reference to modelling that we could identify in these documents, as described in Table 3.

During this *third step*, once we established the four levels of reference to modelling in Table 3, we conducted a triangulation in the following way: firstly, each author classified the MFPs according to these levels; secondly, we compared our classifications, achieving an agreement percentage of 96% among the three of us; and finally, we discussed our differences of classification and achieved a consensus, due to our experience in this type of analysis.

In a *fourth step*, we used the DSC to categorise the comments that referred to modelling. Various studies have addressed the topic of teacher reflection in mathematics teacher education processes (see Breda, 2020, for didactic analysis; Sánchez, Font & Sala, 2022, for the development of creativity; Hidalgo-Moncada et al., 2023, for self-regulation practices; among others), using a content analysis methodology to make the use of the DSC components evident. In this research, we considered these components as a priori categories (Schreier, 2012), in order to identify the aspects of the teaching and learning process that the prospective teachers related to modelling. For the purposes of the content analysis of the MFPs, in this *fourth step*, we considered the evaluative comments in the *Implementation analysis* chapter from the documents classified at levels L_2 and L_3, since they contain the prospective teachers' reflections on modelling in their implementation. Due to the agreement that we

Table 3 Levels of reference to mathematical modelling

Levels	Description
Level 0 (L_0)	Corresponds to the MFPs that did not refer to the terms related to modelling; that is, that did not consider working with this process in the implemented didactic units, or MFPs that included some of the established keywords, but without these being directly related to modelling
Level 1 (L_1)	Corresponds to the MFPs that, although they did not consider working with modelling in the implemented didactic units, they proposed including modelling in the redesign proposal. More specifically, at this level, we considered the MFPs that only included comments about modelling in the *Redesign proposal* chapter
Level 2 (L_2)	Corresponds to the MFPs that included modelling problems in the implemented didactic units, reflecting on their implementation; however, they did not propose improvements to enhance this process. More specifically, at this level, we considered the MFPs that only included evaluative comments about modelling (using the DSC) in the *Implementation analysis* chapter, but that did not propose concrete changes for improving this process in the *Redesign proposal* chapter
Level 3 (L_3)	Corresponds to the MFPs similar to those classified as L_2, but which did propose improvements in their redesign to enhance modelling. More specifically, at this level, we considered the MFPs that included evaluative comments about modelling (using the DSC) in the *Implementation analysis* chapter and that also proposed concrete changes (in addition to comments) for improving this process in the *Redesign proposal* chapter

achieved during the *third step*, we conducted this *fourth step* with no discrepancies, since the assessment of a certain component of the DSC in each MFP containing (or not) an evaluative comment on modelling is an objective fact.

Table 4 exemplifies how we applied the four content analysis *steps* to the MFPs #005, #042, #060 and #076. The choice of these documents is justified by the fact that we classified each of them at a different level of reference to modelling.

Regarding the *fourth step* of content analysis, we consider it important to clarify that an MFP can include more than one phrase/sentence with references to the terms related to modelling in the assessment of a particular DSC component. For example, in the assessment of the "richness of processes" (ES3) component, we could find the definitions of the processes "modelling", "contextualisation" and "problem solving" distributed differently: either in three cells within a table, in three different sentences within a single paragraph, or in three disjointed paragraphs within the assessment of this component. Given this situation, we decided to consider the set of these phrases/sentences that included terms related to modelling as "one comment" in the assessment of each DSC.

Presentation and Analysis of Results

In this section, we present (subsections "Classification of the MFPs According to the Levels of Reference to Modelling" and "Classification of the Comments of the MFPs According to the DSC Components") and analyse (subsections "On the MFPs with References to Mathematical Modelling" and "On the Evaluative Comments About Mathematical Modelling") the main results from the content analysis conducted on the MFPs.

Classification of the MFPs According to the Levels of Reference to Modelling

Based on the search for keywords in the 122 MFPs (*first step* of content analysis), as a first result, we found terms related to modelling in 86 of these MFPs. After recording each MFP (*second step* of content analysis), we proceeded to classify them according to the levels of reference to modelling (*third step* of content analysis), and thus, we obtained the results presented in Table 5.

Table 5 presents a notion on the use of the terms related to modelling in the MFPs analysed and on the importance that the prospective teachers gave to this process within their didactic units. In this sense, 36 MFPs did not include references directly related to modelling (MFPs classified as L_0), and 41 MFPs, although they did not include this process in the implemented didactic units, they considered it for their redesign proposals (MFPs classified as L_1). We did not consider these 77 MFPs in the subsequent analyses since they were not in line with the objectives of our study.

The *first step* of content analysis also allowed us to identify that three MFPs were presented as a master's thesis (as described in subsection "Structure of a Master's Degree Final Project"). We classified these three MFPs at the L_0 level, and we did not consider them for the subsequent analyses of our study.

Table 4 Examples of the content analysis of MFPs #005, #042, #060 and #076

Content analysis	Analysed content
MFP #005	
First step	We identified the keywords "model" and "context"
Second step	It is a didactic proposal for teaching statistics in the third grade of secondary education (students aged 14–15)
Third step	The references to the identified keywords are related to the "educational model of the centre" and the "implementation context of the didactic unit". The redesign does not consider the inclusion of modelling. Therefore, this MFP was classified at L_0
Fourth step	–
MFP #042	
First step	We identified the keywords "modelling" and "problem"
Second step	It is a didactic proposal for teaching geometry in the first grade of secondary education (students aged 12–13)
Third step	We did not find evaluative comments about modelling (using the DSC) in the *Implementation analysis* chapter. We found the following comment in the *Redesign proposal* chapter: "this would be a good problem for initiating students in modelling and can be proposed as a challenge for those students who successfully solved the first problem" (p. 24). Therefore, this MFP was classified at L_1
Fourth step	–
MFP #060	
First step	We identified the keywords "model" and "contextualised"
Second step	It is a didactic proposal for teaching geometry in the third grade of secondary education (students aged 14–15)
Third step	We found evaluative comments about modelling in the assessment of the DSC in the *Implementation analysis* chapter. We found the following comment in the *Redesign proposal* chapter: "we work focusing on contextualised problems" (p. 25), but no improvements are proposed. Therefore, this MFP was classified at L_2
Fourth step	Among many others, we found the following comment in the assessment of the ES3 component: "in the first case, contextualised in the pandemic situation that we had at that time, the student will be asked to model a bit [sic] to be able to see the existence of two right triangles" (p. 11)
MFP #076	
First step	We identified the keywords "modelling" and "real"
Second step	It is a didactic proposal for teaching functions in the third grade of secondary education (students aged 14–15)
Third step	We found evaluative comments about modelling in the assessment of the DSC in the *Implementation analysis* chapter. We found the proposal of a dossier with new contextualised problems in the *Redesign proposal* chapter. Therefore, this MFP was classified at L_3
Fourth step	Among many others, we found the following comment within the assessment of the CS4 component: "tasks with a certain cognitive demand, which are those that have allowed modelling to be carried out" (p. 14). In addition, we found the following comment in the assessment of the AS1 component: "mathematics linked to the real world awakened the students' motivation" (p. 14)

Table 5 Number of MFPs according to the levels of reference to mathematical modelling

Levels of reference	Number of MFPs	Percentages*
L_0	36	29.5%
L_1	41	33.6%
L_2	21	17.2%
L_3	24	19.7%
Total	122	100%

*percentages rounded to the first decimal place

The *third step* of content analysis also allowed us to identify that three other MFPs did not make explicit references to modelling in the *Implementation analysis* chapter, but they did mention activities that, in their extended description within the *Redesign proposal* and *Annexes* chapters, referred to modelling and even proposed improvements to work on this process. We classified these three MFPs at L_3, although we did not consider them in the *fourth step* of content analysis because their reflections did not refer to any particular DSC.

Based on the considerations mentioned above, we obtained as a second result that in 45 of the 122 MFPs, the prospective teachers reflected on the implementation of modelling in their didactic units (corresponding to the levels of reference L_2 and L_3). The results that we present in the following subsection (*fourth step* of content analysis) include the analysis of 42 of these 45 MFPs (excluding the three MFPs mentioned in the previous paragraph).

Classification of the Comments of the MFPs According to the DSC Components

Based on the classification presented in Table 5 (*third step* of content analysis), we proceeded to categorise the evaluative comments related to modelling according to the DSC component on which the prospective teacher was reflecting when made the comment (*fourth step* of content analysis). We obtained the results presented in Table 6.

Based on the categorisation in Table 6 (*fourth step* of content analysis), we obtained two results. First, we identified 169 comments explicitly or implicitly referring to modelling in the 42 MFPs considered in this analysis. Regarding this aspect, we did not consider it relevant to attribute a fixed number of comments identified to each MFP since, for example, one document could include comments related to modelling in eight different DSC components and other could include comments in only three components. Since this type of data refinement did not add richness to our study, we have excluded it from the analyses performed. Second, we identified that the largest number of comments were made on the *epistemic criterion*, followed by the *affective* and *ecological criteria*. We analyse these results in the next two subsections.

Table 6 Number of comments according to the DSC component

Component	No. of comments	Component	No. of comments	Component	No. of comments	Component	No. of comments	Component	No. of comments	Component	No. of comments
ES1	1	CS1	5	IS1	3	MS1	4	AS1	32	EcS1	2
ES2	2	CS2	1	IS2	4	MS2	0	AS2	3	EcS2	14
ES3	40	CS3	1	IS3	1	MS3	4	AS3	5	EcS3	20
ES4	11	CS4	12	IS4	0					EcS4	4
Total	54	Total	19	Total	8	Total	8	Total	40	Total	40

The codes of the DSC components from Table 1 were used

Table 7 Mathematical contents and student educational level where modelling was (not) implemented

Mathematical contents	Secondary education				Baccalaureate ed.		*n*/impl.	Total
	1st	2nd	3rd	4th	1st	2nd		
Algebra		1	2	1			10	14
Functions		2	7	6	3	1	9	28
Geometry	3	4	1	1	1		31	41
Numbers	1			1			15	17
Probability				1			4	5
Statistics	2		1	1			3	7
Trigonometry				5			2	7
Total	6	7	11	16	4	1	74	119

N/impl. = no implementation of modelling

On the MFPs with References to Mathematical Modelling

From the database generated during the *second step*, and the classification of comments made during the *third step*, Table 7 shows (a) the mathematical contents addressed by the didactic units in the MFPs; (b) the number of MFPs which implemented modelling, according to the mathematical content and educational level in which the internship experiences were carried out; (c) the number of MFPs which did not implement this process; and (d) the total number of didactic units for each mathematical content (Table 7 does not consider the three MFPs presented as master's thesis).

As mentioned in subsection "Research Context", among the determining factors for the development of educational internship experiences are the student educational level and the time of the school year in which the prospective teachers carry out their intervention; that is, both factors determined the mathematical content of the didactic units, and its choice did not depend on the prospective teachers. Regarding the student educational level, Table 7 shows that, at all educational levels, to a greater or lesser extent, the prospective teachers implemented modelling in their didactic units, focusing on the 3rd (students aged 14–15) and 4th (students aged 15–16) grades of secondary education. Regarding the time of the school year, in the context of our study, the educational internship experiences were carried out during 6 weeks from February 2020 (period from February to April, approximately).

Table 7 also shows that the most used mathematical content to implement modelling was "functions", followed by "geometry". This tendency of the prospective teachers to use modelling for teaching functions is in line with the Michelsen's (2006) position, who highlights the role of this mathematical content as a tool for developing the modelling process in the classroom. Regarding the "geometry" content, we realised that, compared to the total number of didactic units that addressed it (41), only about a quarter of them (10) implemented modelling for its teaching. This result is partially consistent with the Girnat and Eichler's (2011) findings that teachers tend not to consider the teaching of geometry linked to modelling.

On the Evaluative Comments About Mathematical Modelling

The *fourth step* of content analysis revealed the DSC components in which the prospective teachers made their reflections on the implementation of modelling through evaluative comments. As mentioned in subsection "Structure of a Master's Degree Final Project", the prospective teachers assess the didactic suitability of the implemented teaching and learning process in the *Implementation analysis* chapter of the MFP, using the DSC for this. More specifically, the prospective teachers write evaluative comments in each DSC component, where they reflect, among other aspects, on the implementation of modelling in their didactic units. Therefore, we identified that the 45 MFPs (see L_3 and L_4 levels in Table 5) that included modelling in their didactic units concentrated the largest number of comments in the *epistemic*, *affective* and *ecological criteria*. In this subsection, we analyse these evaluative comments since they are evidence of the prospective teachers' reflection on the implementation of this process in their didactic units.

In the *epistemic criterion*, there is the "richness of processes" (ES3) component, which indicates that the sequence of tasks considers relevant processes in mathematical activity (modelling, argumentation, problem solving, connections, etc.) (Breda et al., 2017), and it was the component that had the largest number of comments about modelling. This is because the mathematical processes worked on during the implemented didactic unit are defined and exemplified in this component. In the assessment of this component, we found important differences between the definitions of the modelling process in the analysed MFPs, which are shown in Table 8 with three representative examples.

MFP #002 is representative of a common situation in some documents, in which the definitions of and comments on the "problem solving", "modelling" and "contextualisation" processes were slightly differentiated. More specifically, in some MFPs, it was stated that, if a statement was posed within the mathematical world, then "problem solving" was carried out; however, if it was posed in the real world, then "modelling" was carried out; or, if a problem developed "contextualisation",

Table 8 Definitions of mathematical modelling found in MFPs #002, #028 and #115

MFP	Definition or comment
#002	Contextualisation (modelling): use of real-world problems (p. 9)
#028	Modelling: includes structuring the situation to be modelled; translating "reality" into a mathematical structure; working with a mathematical model; validating the model; reflecting, analysing and criticising a model and its results; communicating effectively about the model and its results (including limitations); and monitoring and controlling the modelling process (pp. 9–10)
#115	In my didactic unit, modelling was one of the central processes. We worked on this process a lot, above all, by transforming a purely written statement into a mathematical situation that can be represented with a drawn model of a right triangle (or a set of them) (p. 11)

then it was a "modelling" problem. MFP #028 is also representative of another common situation, in which more detailed definitions of the modelling process were given (similar to that provided by Geiger et al., 2018), but without citing the source of the definition. This is because, since an MFP is not oriented towards research, but rather towards reflection on their own practice, the prospective teachers do not always respect the rules of citing or making a reference list in their works. Finally, MFP #115 is representative of another common situation, in which modelling was commented on as if this process consisted only in translating the wording of a problem from natural language to a mathematical representation, which suggests a double interpretation. On one hand, some prospective teachers tended to reduce the modelling process to the idea of *horizontal mathematisation* (in terms of Freudenthal, 1991), without considering the other phases of the modelling cycle. According to the classification proposed by Maaß (2010), this type of tasks can be considered as those focused only on the development of the mathematisation subcompetency, but not on the development of the modelling process as a whole. On the other hand, we interpret that some prospective teachers tended to overlap the modelling process with treating and converting registers of semiotic representation (in terms of Duval, 2017) of the mathematical objects involved in this type of problems.

In the *affective criterion*, there is the "interests and needs" (AS1) component, which indicates that the selection of tasks is interesting for students, and the situations presented enable them to evaluate the practicality of mathematics in everyday situations and professional life (Breda et al., 2017). This component was the second one with the largest number of comments about modelling, which pointed out that these types of problems, being "contextualised" and "realistic", caught (or were intended to catch) the students' attention, because many of them took advantage of the context of COVID-19 and lockdown as central topics in their wordings. For example, we found the following comment in one of the assessments of this component:

> Regarding the contexts in which the didactic unit was carried out, I consider that the fact that all the activities were contextualised around COVID-19 has motivated the students. (MFP #118, p. 21)

This use of problems with wordings related to COVID-19 to work on modelling occurred with different mathematical contents. In this sense, COVID-19 as a contingent issue encouraged modelling where it was not commonly done, for example, in the case of trigonometry (see Ledezma, Font, & Sala, 2021). In addition, in the assessment of this component, we highlight that the implementation of modelling problems was intended to arouse the students' interest about daily life decisions, as made evident by the following comments:

> The posed activities […] were contextualised in possible situations or close to the students' daily life. In this way, their interest was caught, and importance was given to the need to take and introduce these new mathematical concepts in real and professional life. (MFP #024, p. 16)

> An example was the decision-making activity in which it was necessary to choose the best electricity supply option based on real criteria, such as cost, payback period, sustainability, or reliability of supply. (MFP #035, p. 15)

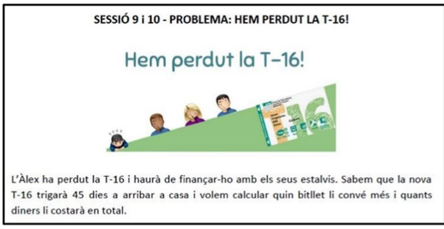
In the *ecological criterion*, there are the “intra and interdisciplinary connections” (EcS2) and “social and labour usefulness” (EcS3) components. The EcS2 component indicates that the taught contents are related to other mathematical topics (more advanced or of the same educational level), or to the contents from other disciplines (extra-mathematical contexts or from other subjects of the same educational level); and the EcS3 component indicates that the contents are useful for social and professional insertion (Breda et al., 2017). Both components included comments referring to modelling as a tool for relating mathematics both to the curricular contents from other subjects (especially physics and history) and to the students’ context (contextualised problems in their social and work environment). For example, we found the following comments in the assessments of these two components:

> The only interdisciplinary incursion has been with the profitability study to connect, in a certain way, with economy, making calculations to find out the profitability by identifying costs and incomes, and making comparisons with the minimum interprofessional salary, explained during the first day as an objective of this problem. (Assessment of the EcS2 component; MFP #052, p. 11)

> Certainly, contextualisation is the only way of social utility of this DU [Didactic Unit]. Since the [educational] centre is specialised in artistic education, we can say that contextualisation in this field can serve as a labour utility. (Assessment of the EcS3 component; MFP #108, p. 13)

We need to make a special mention to the *cognitive criterion*, where there is the “high cognitive demand” (CS4) component, which indicates that relevant cognitive processes are activated in mathematical activity (generalisation, intra-mathematical connections, changes of representation, conjectures, etc.) and metacognitive processes are promoted (Breda et al., 2017). The comments in this component highlighted that the implemented modelling problems made it possible to work on other relevant mathematical processes. For example, we found the following comment in one of the assessments of this component:

> From the beginning, we tried that throughout the entire DU [Didactic Unit] there were activities and sessions that came out of formal and traditional teaching and were more competent to work on cognitive and mathematical processes, such as communication of concepts, mathematical reasoning and argumentation, abstraction to imagine and reason fictious situations, modelling, and problem solving. (MFP #024, pp. 12–13)



We have lost the T-16!

Alex has lost a T-16 and he will have to pay it with his savings. We know that the new T-16 will take 45 days to arrive home and we want to calculate which ticket is best for him and how much money it will cost him.

Fig. 2 Modelling problem implemented in the didactic unit of MFP #024 (p. 48)

In the assessment of the CS4 component, the author of MFP #024 referred to a particular modelling problem (see Fig. 2) as an example of task with a high cognitive demand.

The statement in Fig. 2 is contextualised in the loss of the T-16 card (intended for children and adolescents to have free trips on the Barcelona Metropolitan Transport system), with the aim of calculating which other travel card would be the best option. This statement is characterised by being *open* and *complex*, since it requires the students to find out the prices of the other public transport cards and compare their costs; it is also *realistic* and *complex* because, in addition to being situated in a context close to the students (the city of Barcelona), it presents a situation that can happen in reality (in fact, the students use the T-16 card). Since this statement is in line with the characteristics mentioned in subsection "Mathematical Modelling", it can be considered as a *problem* that is *solvable through a modelling cycle*.

Finally, the *interactional* and *mediational criteria* included the fewest comments on modelling. Due to the virtual teaching context, the prospective teachers commented that they had a lot of difficulties getting their students to carry out collaborative work, as suggested for modelling activities (in terms of Doerr & English, 2003; Shahbari & Tabach, 2019), which clearly affected the *interactional criterion*. Similarly, we only found some references to the use of the GeoGebra software or manipulative resources for solving problems posed to students, but without specifically delving into modelling.

Discussion and Conclusions

The content analysis carried out on the 122 MFPs from the 2019 to 2020 academic year, in the context of a master's degree programme for mathematics teacher education, allowed us to make evident the decisions made by the prospective teachers, both during their educational internship period and in the improvement proposals derived from the reflection made in their MFPs on the inclusion of modelling in their didactic units.

The first aspect to highlight of these results is that about 60% of these prospective teachers did not include modelling as a relevant process in their didactic units (see L_0 and L_1 levels in Table 5). We discard the explanation that they did not have knowledge about modelling and its inclusion in the mathematical teaching and learning process, since the master's programme in which our study is contextualised dedicates a submodule specifically to teaching this process. We also discard the fact that the COVID-19 pandemic was a context that did not encouraged modelling as an explanation, since the media included information that made it possible to design modelling tasks/problems, as well as models to represent the evolution of the pandemic, which made modelling acquire relevant social value. However, a plausible explanation is that, in terms of the DSC, a teacher must try a priori that these *criteria* are met as much as possible; however, the implementation context forces him/her to make decisions about what aspects to prioritise, push to the background or simply skip. In the case of our study, although one of the *indicators* from the "richness of processes" component gives importance to the development of the modelling process in the classroom, the grave contingency that the transition period between the face-to-face and virtual teaching contexts implied made these prospective teachers prioritised other aspects of the mathematical teaching and learning process, such as the availability of time, the diversity of students, the work system of the internship centre and other relevant processes of mathematical activity. These results are consistent with other studies that have revealed the difficulties of including modelling in the classroom (e.g. Niss, 2001), given the complexity of the aspects that must be taken into consideration by a teacher when implementing a lesson.

The second aspect to highlight concerns the remaining almost 40% of these prospective teachers. Even though this master's programme includes a submodule on modelling in which the cycle proposed by Blum and Leiß (2007) is presented, we did not find references in the MFPs to the use of this cycle (or any other) to perform analyses on the implemented problems or those proposed in the redesign and that were considered as *modelling problems*. That is, the prospective teachers stated that they had implemented modelling in their didactic units, although in their reflections they did not rely on a modelling cycle to justify this affirmation. A plausible explanation is that, in addition to the grave contingency previously commented, since an MFP is an autonomous work, the reflection on modelling, whether from a theoretical perspective or not, is a decision made by the author of the MFP in agreement with his/her tutor professor and that, surely, takes into account, among other aspects, the page and time constrictions for its preparation. Furthermore, as we mentioned in subsection "Structure of a Master's Degree Final Project", the purpose of an MFP is for the prospective teacher to reflect on his/her own practice, which does not compulsorily imply a reflection from a purely theoretical perspective.

The third aspect to highlight is that, based on the evidence from the analysed MFPs, for most of these prospective teachers, given a mathematical or extra-mathematical situation, finding a mathematical object of which the situation is an instantiation was considered modelling. In this sense, this process was understood as the relationship between a general mathematical object and a particular case, which may or may not be an extra-mathematical situation. On the other hand, there were also cases in which *modelling* was considered as a *change of representation model*, in semiotic terms. A plausible explanation is that the master's programme, where our study is contextualised, did not make a general reflection on the notion of mathematical process, its different types and the relationships and differences between them, because this master's programme only prioritised a specific work with modelling and problem solving in the respective submodules. On one hand, this situation contradicts Rubio's (2012) position, who justifies the importance of carrying out general work with mathematical processes and not on specific processes. On the other hand, these results partially coincide with the study by Villa-Ochoa (2015), in the sense that teachers tend to make statements, considered as *modelling problems*, that only evaluate students' skills to register, through a symbolic expression, a mathematical relationship wrapped up in a relatively realistic word problem. This particular aspect reinforces the study by Kuntze et al. (2013), regarding the emphasis of teacher professional development on modelling to improve its implementation in the classroom.

The fourth aspect to highlight is that we made evident, in the process of reviewing the reflection carried out by the prospective teachers, that their tutor professors did not make them realise that, in order to state that the modelling process was implemented in a didactic unit, as a minimum, it is necessary that the problem posed to the students meets the characteristics described in subsection "Mathematical Modelling", as in the case of the problem in Fig. 2. This weakness in feedback was make evident, for example, in some MFPs that adopted a definition of modelling closer to that of contextualisation or word problem solving (as in the case of MFP #002 in Table 8), where the prospective teachers stated that they had implemented modelling by posing this type of problems (contextualisation or word problems) but not modelling problems as such. Therefore, the feedback process to prepare an MFP would be an aspect that the master's programme where we conducted our study could improve, considering the results of our research.

Finally, we found some MFPs in which the prospective teachers had to reduce the duration of the time allocated for their lessons and even eliminate some sessions. This was due to the abrupt interruption of face-to-face lessons and the difficulties of some educational centres to implement the virtual teaching system with their students. A paradigmatic example of this situation was MFP #052, where the prospective teacher planned to develop a modelling project during six sessions, but he had to reduce then to only two sessions, dispensing with the modelling process, and only focusing on solving specific tasks within the project (see a detailed analysis of this MFP in Ledezma et al., 2021b). In terms of the DSC, this fifth aspect is somewhat related to the first one that we highlighted, with the difference that here the prospective teachers did consider a priori the inclusion of modelling in their didactic units. However, since this process was finally supressed or reduced to its minimum expression, this importance of modelling may have been less that other aspects, such as the conceptual contents included in the curriculum.

Resuming our research question on what aspects of the teaching and learning process prospective secondary and baccalaureate education teachers related to mathematical modelling when they reflected on its inclusion during the transition period between the face-to-face and virtual teaching context, we found that the *epistemic*, *affective* and *ecological* (and, to a lesser extent, the *cognitive*) *criteria* were the aspects of the teaching and learning process that they mostly related to modelling in the context of our study. In general terms, the results of our study showed that the COVID-19 pandemic influenced these prospective teachers on mainly two aspects. On one hand, COVID-19 provided a realistic and authentic context to pose modelling tasks/problems close to the students although, on the other hand, represented a worldwide change in the way of developing teaching and learning processes (Engelbrecht et al., 2023) by affecting, for example, the student interaction. In other words, these results showed how the prospective teachers had to decide which *criteria* and *components* of the DSC to give more or less relevance to when implementing their lessons, forced by the serious contingency situation. Although the prospective teachers who included modelling in their didactic units commented on the, especially, pedagogical challenges to implement this process in the classroom (similar to those reported by Manouchehri, 2017), they did assess the inclusion of modelling, by using arguments similar to those given in the literature to justify its use in mathematical teaching and learning processes (see Blum, 2011; among others).

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References

Abassian, A., Safi, F., Bush, S., & Bostic, J. (2020). Five different perspectives on mathematical modeling in mathematics education. *Investigations in Mathematics Learning, 12*(1), 53–65. https://doi.org/10.1080/19477503.2019.1595360

Alwast, A., & Vorhölter, K. (2022). Measuring pre-service teachers' noticing competencies within a mathematical modeling context – An analysis of an instrument. *Educational Studies in Mathematics, 109*(2), 263–285. https://doi.org/10.1007/s10649-021-10102-8

Breda, A. (2020). Características del análisis didáctico realizado por profesores para justificar la mejora en la enseñanza de las matemáticas [Characteristics of the didactic analysis carried out by teachers to justify the improvement of mathematics teaching]. *BOLEMA: Boletim de Educação Matemática, 34*(66), 69–88. https://doi.org/10.1590/1980-4415v34n66a04

Breda, A., Pino-Fan, L., & Font, V. (2017). Meta didactic-mathematical knowledge of teachers: Criteria for the reflection and assessment on teaching practice. *Journal of Mathematics Science and Technology Education, 13*(16), 1893–1918. https://doi.org/10.12973/eurasia.2017.01207a

Blomhøj, M. (2004). Mathematical modelling: A theory for practice. In B. A. Clarke, D. M. Clarke, G. Emanuelsson, B. Johansson, D. V. Lambdin, F. Lester, A. Wallby, & K. Wallby (Eds.), *International Perspectives on Learning and Teaching Mathematics* (pp. 145–159). National Center for Mathematics Education

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in Teaching and Learning of Mathematical Modelling: ICTMA 14* (pp. 15–30). Springer. https://doi.org/10.1007/978-94-007-0910-2_3

Blum, W., & Leiß, D. (2007). How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics* (pp. 222–231). Woodhead Publishing Limited. https://doi.org/10.1533/9780857099419.5.221

Blum, W., & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45–58

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *Zentralblatt für Didaktik der Mathematik, 38*(2), 86–95. https://doi.org/10.1007/bf02655883

Borromeo Ferri, R. (2007). Personal experiences and extra-mathematical knowledge as an influence factor on modelling routes of pupils. In D. Pitta-Pantazi & C. Philippou (Eds.), *European Research in Mathematics Education V: Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 2080–2089). University of Cyprus

Borromeo Ferri, R. (2013). Mathematical modelling in European education. *Journal of Mathematics Education at Teachers College, 4*(2), 18–24

Borromeo Ferri, R. (2018). *Learning How to Teach Mathematical Modeling in School and Teacher Education*. Springer. https://doi.org/10.1007/978-3-319-68072-9

Cohen, L., Manion, L., & Morrison, K. (2018). *Research Methods in Education* (8th ed.). Routledge

Departament d'Educació. (2019). *Currículum Educació Secundària Obligatòria* [Compulsory Secondary Education Curriculum]. Generalitat de Catalunya

Departament d'Ensenyament. (2008). *Currículum Batxillerat* [Baccalaureate Education Curriculum]. Generalitat de Catalunya

Doerr, H. M., & English, L. D. (2003). A modeling perspective on students' mathematical reasoning about data. *Journal for Research in Mathematics Education, 34*(2), 110–136. https://doi.org/10.2307/30034902

Duval, R. (2017). *Understanding the Mathematical Way of Thinking – The Registers of Semiotic Representations*. Springer. https://doi.org/10.1007/978-3-319-56910-9

Engelbrecht, J., Borba, M. C., & Kaiser, G. (2023). Will we ever teach mathematics again in the way we used to before the pandemic? *ZDM – Mathematics Education*. Advanced online publication. https://doi.org/10.1007/s11858-022-01460-5

English, L. (2003). Mathematical modelling with young learners. In S. J. Lamon, W. A. Parker, & K. Houston (Eds.), *Mathematical Modelling: A Way of Life – ICTMA 11* (pp. 3–17). Horwood

Font, V., Giménez, J., Zorrilla, J. F., Larios, V., Dehesa, N., Aubanell, A., & Benseny, A. (2012). Competencias del profesor y competencias del profesor de matemáticas. Una propuesta [Teacher's competencies and mathematics teacher's competencies. A proposal]. In V. Font, J. Giménez, V. Larios, & J. F. Zorrilla (Eds.), *Competencias del Profesor de Matemáticas de Secundaria y Bachillerato* (pp. 59–68). Publicacions i Edicions de la Universitat de Barcelona.

Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Springer. https://doi.org/10.1007/0-306-47202-3

Geiger, V., Mulligan, J., Date-Huxtable, L., Ahlip, R., Jones, D. H., May, E. J., Rylands, L., & Wright, I. (2018). An interdisciplinary approach to designing online learning: Fostering pre-service mathematics teachers' capabilities in mathematical modelling. *ZDM – Mathematics Education, 50*(1–2), 217–232. https://doi.org/10.1007/s11858-018-0920-x

Girnat, B., & Eichler, A. (2011). Secondary teachers' beliefs on modelling in geometry and stochastics. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in Teaching and Learning of Mathematical Modelling: ICTMA 14* (pp. 75–84). Springer. https://doi.org/10.1007/978-94-007-0910-2_9

Godino, J. D. (2013). Indicadores de la idoneidad didáctica de procesos de enseñanza y aprendizaje de las matemáticas [Indicators of the didactic suitability of mathematics teaching and learning processes]. *Cuadernos de Investigación y Formación en Educación Matemática, 8*(11), 111–132

Godino, J. D., Batanero, C., & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM – Mathematics Education, 39*(1), 127–135. https://doi.org/10.1007/s11858-006-0004-1

Greefrath, G., Siller, H.-S., Klock, H., & Wess, R. (2022). Pre-service secondary teachers' pedagogical content knowledge for the teaching of mathematical modelling. *Educational Studies in Mathematics, 109*(2), 383–407. https://doi.org/10.1007/s10649-021-10038-z

Hidalgo-Moncada, D., Díez-Palomar, J., & Vanegas, Y. (2023). Prácticas de autorregulación en la propuesta didáctica de un futuro profesor de matemáticas: Un instrumento para la reflexión [Self-regulation practices in the teaching proposal of a future mathematics teacher: An instrument for reflection]. *PARADIGMA, XLIV*(2), 112–146. https://doi.org/10.37618/PARADIGMA.1011-2251.2023.p112-146.id1384

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400. https://doi.org/10.5951/jresematheduc.39.4.0372

Kaiser, G. (2020). Mathematical modelling and applications in education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (2nd ed., pp. 553–561). Springer. https://doi.org/10.1007/978-3-030-15789-0_101

Kuntze, S., Siller, H.-S., & Vogl, C. (2013). Teachers' self-perceptions of their pedagogical content knowledge related to modelling – An empirical study with Austrian teachers. In G. A. Stillman, G. Kaiser, W. Blum, & J. P. Brown (Eds.), *Teaching Mathematical Modelling: Connecting to Research and Practice* (pp. 317–326). Springer. https://doi.org/10.1007/978-94-007-6540-5_13

Ledezma, C., Font, V., & Sala, G. (2021a). Análisis de la reflexión realizada por un futuro profesor sobre el papel de la modelización matemática en la mejora de un proceso de instrucción para enseñar trigonometría [Analysis of a future teacher's reflection on the role of mathematical modelling for improving an instructional process for the teaching of trigonometry]. *PARADIGMA, (Extra 2)*(XLII), 290–312. https://doi.org/10.37618/PARADIGMA.1011-2251.2021.p290-312.id1043

Ledezma, C., Sala, G., Breda, A., & Sánchez, A. (2021b). Analysis of a preservice teacher's reflection on the role of mathematical modelling in his master's thesi. In M. Inprasitha, N. Changsri, & N. Boonsena (Eds.), *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 195–204). PME.

Ledezma, C., Font, V., & Sala, G. (2022). Analysing the mathematical activity in a modelling process from the cognitive and onto-semiotic perspectives. *Mathematics Education Research Journal*. Advanced online publication. https://doi.org/10.1007/s13394-022-00411-3

Ledezma, C., Sol, T., Sala-Sebastià, G., & Font, V. (2022). Knowledge and beliefs on mathematical modelling inferred in the argumentation of a prospective teacher when reflecting on the incorporation of this process in his lessons. *Mathematics*, *10*(18), Article 3339. https://doi.org/10.3390/math10183339

Lesh, R., & Doerr, H. M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching* (pp. 3–33). Lawrence Erlbaum

Maaß, K. (2006). What are modelling competencies? *Zentralblatt für Didaktik der Mathematik, 38*(2), 113–142. https://doi.org/10.1007/bf02655885

Maaß, K. (2007). Modelling in class: What do we want the students to learn? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics* (pp. 63–78). Woodhead Publishing Limited. https://doi.org/10.1533/9780857099419.2.63

Maaß, K. (2010). Classification scheme for modelling tasks. *Journal für Mathematik-Didaktik, 31*(2), 285–311. https://doi.org/10.1007/s13138-010-0010-2

Maass, K., Artigue, M., Burkhardt, H., Doorman, M., English, L. D., Geiger, V., Krainer, K., Potari, D., & Schoenfeld, A. (2022). Mathematical modelling – A key to citizenship education. In N. Buchholtz, B. Schwarz, & K. Vorhölter (Eds.), *Initiationen mathematikdidaktischer Forschung: Festschrift zum 70. Geburtstag von Gabriele Kaiser* (pp. 31–50). Springer. https://doi.org/10.1007/978-3-658-36766-4_2

Manouchehri, A. (2017). Implementing mathematical modelling: The challenge of teacher educating. In G. Stillman, W. Blum, & G. Kaiser (Eds.), *Mathematical Modelling and Applications: Crossing and Researching Boundaries in Mathematics Education* (pp. 421–432). Springer. https://doi.org/10.1007/978-3-319-62968-1_35

Michelsen, C. (2006). Functions: A modelling tool in mathematics and science. *Zentralblatt für Didaktik der Mathematik, 38*(3), 269–280. https://doi.org/10.1007/bf02652810

Niss, M. (2001). Issues and problems of research on the teaching and learning of applications and modelling. In J. F. Matos, W. Blum, K. Houston, & S. P. Carreira (Eds.), *Modelling and Mathematics Education: ICTMA 9 – Applications in Science and Technology* (pp. 72–88). Woodhead Publishing Limited. https://doi.org/10.1533/9780857099655.1.72

Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics, 102*(1), 9–28. https://doi.org/10.1007/s10649-019-09903-9

Organisation for Economic Co-operation and Development. (2019). *PISA 2018 Assessment and Analytical Framework*. OECD Publishing. https://doi.org/10.1787/b25efab8-en

Palm, T. (2007). Features and impact of the authenticity of applied mathematical school tasks. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (pp. 201–208). Springer. https://doi.org/10.1007/978-0-387-29822-1_20

Pino-Fan, L. R., Castro, W. F., & Font, V. (2023). A macro tool to characterize and develop key competencies for the mathematics teacher's practice. *International Journal of Science and Mathematics Education, 21*(5), 1407–1432. https://doi.org/10.1007/s10763-022-10301-6

Praetorius, A.-K., & Charalambous, C. Y. (2018). Classroom observation frameworks for studying instructional quality: Looking back and looking forward. *ZDM– Mathematics Education, 50*(3), 535–553. https://doi.org/10.1007/s11858-018-0946-0

Prediger, S., Götze, D., Holzäpfel, L., Rösken-Winter, B., & Selter, C. (2022). Five principles for high-quality mathematics teaching: Combining normative, epistemological, empirical, and pragmatic perspectives for specifying the content of professional development. *Frontiers in Education, 7*, Article 969212. https://doi.org/10.3389/feduc.2022.969212

Remillard, A. (2018). Examining teachers' interactions with curriculum resource to uncover pedagogical design capacity. In L. Fan, L. Trouche, C. Qi, S. Rezat, & J. Visnovska (Eds.), *Research on Mathematics Textbooks and Teachers' Resources: Advances and Issues* (pp. 69–88). Springer. https://doi.org/10.1007/978-3-319-73253-4_4

Rubio, N. (2012). *Competencia del Profesorado en el Análisis Didáctico de Prácticas, Objetos y Procesos Matemáticos* [Teachers' competency on the didactic analysis of mathematical practices, objects, and processes] [Doctoral dissertation, University of Barcelona]. Dipòsit Digital de la Universitat de Barcelona. https://hdl.handle.net/2445/65704

Sánchez, A. (2021). *Perspectivas de los Futuros Profesores de Matemáticas de Educación Secundaria sobre la Creatividad y su Desarrollo en las Clases* [Future secondary education mathematics teachers' perspectives about creativity and its development in the classroom] [Doctoral dissertation, University of Barcelona]. Dipòsit Digital de la Universitat de Barcelona. https://hdl.handle.net/2445/187046

Sánchez, A., Font, V., & Breda, A. (2022). Significance of creativity and its development in mathematics classes for preservice teachers who are not trained to develop students' creativity. *Mathematics Education Research Journal, 34*(4), 863–885. https://doi.org/10.1007/s13394-021-00367-w

Schoenfeld, A. H. (1994). Reflections on doing and teaching mathematics. In A. H. Schoenfeld (Ed.), *Mathematical Thinking and Problem Solving* (pp. 53–70). Erlbaum

Schreier, M. (2012). *Qualitative Content Analysis in Practice*. SAGE.

Shahbari, J. A., & Tabach, M. (2019). Adopting the modelling cycle for representing prospective and practising teachers' interpretations of students' modelling activities. In G. A. Stillman & J. P. Brown (Eds.), *Lines of Inquiry in Mathematical Modelling Research in Education* (pp. 179–196). Springer. https://doi.org/10.1007/978-3-030-14931-4_10

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268–275. https://doi.org/10.5951/MTMS.3.4.0268

Tekin, A. (2019). Arguments constructed within the mathematical modelling cycle. *International Journal of Mathematical Education in Science and Technology, 50*(2), 292–314. https://doi.org/10.1080/0020739X.2018.1501825

Toulmin, S. (2003). *The Uses of Argument* (2nd ed.). Cambridge University Press. (Original work published 1954)

Villa-Ochoa, J. A. (2015). Modelación matemática a partir de problemas de enunciados verbales: Un estudio de caso con profesores de matemáticas [Mathematical modelling based on verbal-stated problems: A case study with mathematics teachers]. *Magis: Revista Internacional de Investigación en Educación, 8*(16), 133–148. https://doi.org/10.11144/Javeriana.m8-16.mmpe