Microlensing Near Macro-Caustics

Check for updates

Luke Weisenbach¹ · Timo Anguita^{2,3} · Jordi Miralda-Escudé^{4,5,6} · Masamune Oguri^{7,8} · Prasenjit Saha⁹ · Paul L. Schechter¹⁰

Received: 12 October 2023 / Accepted: 1 July 2024 / Published online: 30 July 2024 © The Author(s) 2024

Abstract

Microlensing near macro-caustics is a complex phenomenon in which swarms of microimages produced by micro-caustics form on both sides of a macro-critical curve. Recent discoveries of highly magnified images of individual stars in massive galaxy cluster lenses, predicted to be formed by these micro-image swarms, have stimulated studies on this topic. In this article, we explore microlensing near macro-caustics using both simulations and analytic calculations. We show that the mean total magnification of the micro-image swarms follows that of an extended source in the absence of microlensing. Micro-caustics join into a connected network in a region around the macro-critical line of a width proportional to the surface density of microlenses; within this region, the increase of the mean magnification toward the macro-caustic is driven by the increase of the number of micro-images rather than individual magnifications of micro-images. The maximum achievable magnification in micro-caustic crossings decreases with the mass fraction in microlenses. We conclude with a review of applications of this microlensing phenomenon, including limits to the fraction of dark matter in compact objects, and searches of Population III stars and dark matter subhalos. We argue that the discovered highly magnified stars at cosmological distances already imply that less than $\sim 10\%$ of the dark matter may be in the form of compact objects with mass above $\sim 10^{-6} M_{\odot}$.

J. Miralda-Escudé miralda@icc.ub.edu

- ³ Millennium Institute of Astrophysics, Santiago, Chile
- ⁴ Institute of Cosmos Sciences, University of Barcelona, Barcelona 08028, Spain
- ⁵ Institució Catalana de Recerca i Estudis Avançats, Barcelona 08010, Spain
- ⁶ Institut d'Estudis Espacials de Catalunya, Castelldefels 08860, Spain
- ⁷ Center for Frontier Science, Chiba University, 1-33 Yayoi-cho, Inage-ku, Chiba 263-8522, Japan
- ⁸ Department of Physics, Graduate School of Science, Chiba University, 1-33 Yayoi-Cho, Inage-Ku, Chiba 263-8522, Japan
- ⁹ Physik-Institut, University of Zurich, Winterthurerstr. 190, 8057 Zurich, Switzerland
- ¹⁰ MIT Kavli Institute 37-635, Cambridge MA 02139, USA

¹ Institute of Cosmology and Gravitation, University of Portsmouth, Portsmouth PO1 3FX, UK

² Instituto de Astrofisica, Departamento de Ciencias Fisicas, Universidad Andrés Bello, Santiago, Chile

Keywords Gravitational lensing · Strong gravitational lensing · Gravitational microlensing · Caustic crossing

1 Introduction

Paczynski (1986) showed how a screen of point mass micro-lenses (stars) of sufficiently high surface density produces an extended swarm of micro-images, the sum of whose fluxes is, on average, equal to the flux expected for a point source lensed by a smooth mass distribution with the same surface density (see also Katz et al. 1986; Granot et al. 2003; Saha and Williams 2011).

A caustic is a closed one-dimensional locus in the source plane of a lens system that maps onto a one-dimensional locus in the image plane – a critical curve – along which images of point sources undergo infinite magnification in the geometric optics approximation. A physical source has a finite angular size that implies a maximum magnification, which is reached when the source straddles a caustic and gives rise to two merging mirror images straddling the corresponding critical curve. When a source crosses a caustic, a pair of mirror images either annihilates at or emerges from the critical curve (see e.g., Schneider et al. 1992).

These two distinct phenomena combine when one has micro-lensing near a caustic, with the emergence or annihilation of a pair of elongated swarms of micro-images at a critical curve. The combination of these two phenomena was considered by Wambsganss (1990) as a possible explanation for BL Lac systems. The discovery of a bright transient, thought to be a highly magnified star at redshift 1.5, near the critical curve of the lensing cluster MACS J1149.5+2223 (Kelly et al. 2018) led to renewed interest, triggering theoretical studies of the "trains" of micro-images, or elongated swarms, formed on the two sides of a macro-critical curve (Venumadhav et al. 2017; Diego et al. 2018; Oguri et al. 2018; Dai and Pascale 2021) as well as observational searches of more such events (Rodney et al. 2018; Chen et al. 2019; Kaurov et al. 2019; Dai et al. 2020).

In what follows, after reviewing the macro-lens model near macro-caustics (Sect. 2), we briefly review the behavior of extended sources in the vicinity of a caustic (Sect. 3). This is then combined with the behavior of micro-image swarms in regions of high optical depth discussed in the article by Vernardos et al. (2024, this collection) to show how these two phenomena modify the predictions for each phenomenon considered separately and how these can be used to interpret transients near macro-caustics (Sect. 4). Finally, we review recent observations of micro-caustic crossing near macro-caustics and discuss some applications of these microlensing phenomena, including the possibility of searching for more exotic substructure (Sect. 5).

2 The Macro Model

To study microlensing near macro-caustics, we define a macro mass model (i.e., the lens model without microlenses) that contains a caustic and a critical curve, and set the coordinate system origin of the image and source planes at a point on the critical curve and the caustic, respectively. The Fermat time-delay surface that satisfies this condition is written as

$$\tau = \frac{1}{2} (\boldsymbol{\beta} - \boldsymbol{\theta})^2 - \psi(\boldsymbol{\theta}) , \qquad (1)$$

where β and θ are two-dimensional vectors of the angular positions in the source plane and the image plane, respectively, with coordinates (β_1, β_2) and (θ_1, θ_2) . The Taylor-expanded deflection potential, up to third order at the origin, is

$$\psi(\boldsymbol{\theta}) = \frac{1}{2} (\psi_{11}\theta_1^2 + 2\psi_{12}\theta_1\theta_2 + \psi_{22}\theta_2^2) + \frac{1}{6} (\psi_{111}\theta_1^3 + 3\psi_{112}\theta_1^2\theta_2 + 3\psi_{122}\theta_1\theta_2^2 + \psi_{222}\theta_2^3) .$$
(2)

Note that the first order terms of the expansion disappear because of the requirement that the origin of the image plane maps to the origin of the source plane. Denoting convergence at the origin as κ_0 , i.e.,

$$\psi_{11} + \psi_{22} = 2\kappa_0 , \qquad (3)$$

and using the condition that the critical curve and the caustic pass through the origin, the second derivatives of the deflection potential can be described as

$$\psi_{11} = \kappa_0 + (1 - \kappa_0) \cos \omega ,$$

$$\psi_{22} = \kappa_0 - (1 - \kappa_0) \cos \omega ,$$

$$\psi_{12} = -(1 - \kappa_0) \sin \omega ,$$
(4)

where ω is an arbitrary constant parameter.

Equations (2) and (4) form a general expression of the lens model near the macro-caustic. We have the additional freedom to rotate the coordinate system to simplify it further. For instance, choosing the coordinate system with $\omega = 0$ realizes the locally orthogonal coordinate system around a fold caustic, which is our main interest here. We could also choose to instead rotate the coordinate system to eliminate one of the third derivatives of the deflection potential, although such a transformation makes discussion regarding the direction and extent of the image trains less intuitive.

It is also helpful to consider more restrictive cases for the microlensing analysis. One possible choice is to make convergence constant across the field, for which we can easily attain microlensing simulations with a constant stellar mass fraction to the total matter density. Since convergence κ is computed from Eq. (2) as

$$\kappa = \kappa_0 + \frac{1}{2}(\psi_{111} + \psi_{122})\theta_1 + \frac{1}{2}(\psi_{112} + \psi_{222})\theta_2 , \qquad (5)$$

we need $\psi_{111} = -\psi_{122}$ and $\psi_{112} = -\psi_{222}$ to realize constant convergence. Setting $\omega = 0$ and denoting $\psi_{111} = -1/d_{\parallel}$ and $\psi_{112} = -1/d_{\perp}$, the lens equation becomes

$$\beta_1 = \frac{\theta_1^2}{2d_{\parallel}} + \frac{\theta_1\theta_2}{d_{\perp}} - \frac{\theta_2^2}{2d_{\parallel}} ,$$

$$\beta_2 = 2(1 - \kappa_0)\theta_2 + \frac{\theta_1^2}{2d_{\perp}} - \frac{\theta_1\theta_2}{d_{\parallel}} - \frac{\theta_2^2}{2d_{\perp}} ,$$
(6)

which, apart from notation, is the same as the macro lens model adopted in e.g., Dai and Pascale (2021).

The parameters d_{\parallel} and d_{\perp} have dimensions of the angular variable θ , and are on the order of the Einstein radius of the macro lens model, which is $\mathcal{O}(1'')$ for galaxy-scale strong

lensing and $\mathcal{O}(10'')$ for cluster-scale strong lensing. In this paper we will use a dimensionless variable defined as the angular coordinates divided by the Einstein radius of each microlens, θ_{\star} , which is $\theta_{\star} = \mathcal{O}(10^{-6''})$ for typical stars acting as microlenses at cosmological distances. Thus d_{\parallel} and d_{\perp} are on the order of $\sim 10^6 - 10^7$ in this dimensionless variable.

Alternatively, we consider a restricted case where the critical curve and a train of microimages (which follows the direction of the principal axis that has zero eigenvalue at the critical curve) are perpendicular to each other. This is realized by setting $\psi_{112} = \psi_{122} = \psi_{222} = 0$ along with $\omega = 0$. Denoting $\psi_{111} = -1/E$, the lens equation becomes

$$\beta_1 = \frac{\theta_1^2}{2E} ,$$

$$\beta_2 = 2(1 - \kappa_0)\theta_2 .$$
(7)

The variable E again has dimensions of the angle θ , and is on the order of $\sim 10^6 - 10^7$ in the unit defined by θ_{\star} .

While this macro lens model is a restricted case, it makes the analysis much easier and hence is instructive; we will use it in Sect. 3. From our experience in modeling such systems, introducing other degrees of freedom related to further derivatives of the potential introduces modest quantitative changes but not major qualitative differences. In this model, convergence changes near the critical line as

$$\kappa = \kappa_0 - \frac{\theta_1}{2E} \ . \tag{8}$$

3 Extended Sources in the Vicinity of a Caustic

In this Section, we review the behavior of magnification of extended sources in the vicinity of a macro caustic in the absence of microlensing. We use the simple macro model described by Eq. (7). At an image plane position $\boldsymbol{\theta} = (\theta_1, \theta_2)$, magnification factors along the θ_1 and θ_2 directions, which we denote μ_1 and μ_2 respectively (equal to the inverse eigenvalues of the magnification matrix), are easily computed as

$$\mu_1 = \frac{E}{\theta_1} ,$$

$$\mu_2 = \frac{1}{2(1 - \kappa_0)} .$$
(9)

The magnification of a point image at that point is $\mu = \mu_1 \mu_2$. A point source at $\beta = (\beta_1, \beta_2)$ has two images on both sides of the critical curve at $\theta_1 = \pm \sqrt{2E\beta_1}$ if and only if $\beta_1 > 0$. Thus the magnification of these two images of a point source at β is

$$\mu(\boldsymbol{\beta}) = \frac{1}{(1-\kappa_0)} \sqrt{\frac{E}{2\beta_1}} , \qquad (10)$$

for $\beta_1 > 0$.

For an extended source with a Gaussian surface brightness profile with standard deviation σ , the magnification factor of the source at β is given by

$$\mu(\boldsymbol{\beta}) = \frac{\sqrt{E}}{2\sqrt{\pi}\sigma(1-\kappa_0)} \int_0^\infty \frac{d\beta_1'}{\sqrt{\beta_1'}} \exp\left[-\frac{(\beta_1'-\beta_1)^2}{2\sigma^2}\right].$$
 (11)

🖄 Springer



As an example, Fig. 1 shows magnification factors as a function of the source position β_1 , both for the point source and the extended source, for the case $E = 10^7$, $\kappa_0 = 0.5$ and $\sigma = 0.14$. The magnification of the point source diverges as the source approaches the caustic at $\beta_1 = 0$, while the magnification of the extended source saturates at a maximum value that can be estimated as the magnification of the point source (Eq. (10)) at $\beta_1 = \sigma$ (e.g., Miralda-Escudé 1991), i.e.,

$$\mu_{\max} \approx \frac{1}{(1-\kappa_0)} \sqrt{\frac{E}{2\sigma}} \,. \tag{12}$$

For the example in Fig. 1, this estimate yields $\mu_{\text{max}} \approx 12,000$, close to the maximum magnification directly derived from Eq. (11), $\mu_{\text{max}} \approx 12,200$.

4 Microlensing Near Macro-Caustics

We now describe the impact of microlensing on the magnification of sources.

4.1 Average Magnification Profile Interpreted as an Effectively Extended Source

As shown in Neindorf (2003) and Venumadhav et al. (2017), and discussed in the article by Vernardos et al. (2024, this collection) for quasars, the total magnification of a source in the presence of microlensing, averaged over all random point mass realizations of the microlenses, is the magnification under the macro-lens model of the source profile convolved with the probability distribution function (PDF) of the microlens deflection angle:

$$\langle \mu(\boldsymbol{\beta}) \rangle = \int I_{\star}(\boldsymbol{\beta}') \mu_{\rm B}(\boldsymbol{\beta} + \boldsymbol{\beta}') \,\mathrm{d}^2 \boldsymbol{\beta}' \,, \tag{13}$$

where $\boldsymbol{\beta}$ is the position of the source, $\boldsymbol{\beta}'$ is a dummy variable of integration over the source plane, $I_{\star}(\boldsymbol{\beta}')$ is the source profile $I(\boldsymbol{\beta}')$ convolved with the PDF of the microlens deflection angle $p(\boldsymbol{\alpha}_{\star})$, and $\mu_{\rm B}(\boldsymbol{\beta})$ denotes the magnification factor of the background macro-model (i.e., the mass model after the point mass lenses are smoothed) for a source at $\boldsymbol{\beta}$. The mean effect of microlenses is encapsulated in the convolved source profile $I_{\star}(\boldsymbol{\beta}')$. The PDF of the deflection angle has a bivariate Gaussian core with a width of

$$\sigma_{\star} = \theta_{\star} \kappa_{\star}^{1/2} \left[\ln(2e^{1-\gamma_E} N_{\star}^{1/2}) \right]^{1/2}, \qquad (14)$$

Dispringer

Fig. 2 Magnification of a point source as a function of source position β_1 near the macro-caustic in the presence of microlenses (*blue solid line*), using the macro model of Eq. (7) with $\kappa_0 = 0.5$ and $E = 10^7$, and convergence of point masses of $\kappa_{\star} = 0.0025$. For comparison, magnification of an extended source with a Gaussian surface brightness distribution with $\sigma = \sigma_{\alpha} = 0.14$ (see also Eq. (14)) in the absence of microlenses is plotted by a dashed line



where κ_{\star} is the convergence of point mass lenses (stars), θ_{\star} is the Einstein radius of each point mass, $\gamma_E \approx 0.577$ is the Euler-Mascheroni constant, and N_{\star} is the number of point masses.

Although the deflection angle distribution $p(\alpha_{\star})$ is not precisely a Gaussian, it can be approximated this way whenever the deflection is contributed by many microlenses, according to the central limit theorem (this fails, of course, for the tails of this distribution caused by large deflections near a single microlens). The behavior of the mean magnification of point sources near a macro-caustic should then follow that of an extended Gaussian source in the absence of microlensing as studied in Sect. 3. To explicitly check this point, we make numerical simulations of the magnification of a point source in the simple macro model of Eq. (7), with $\kappa_0 = 0.5$ and $E = 10^7 \theta_{\star}$, adding randomly distributed microlenses of a single mass corresponding to an Einstein radius θ_{\star} , within a circle of radius $\theta_{\text{max}} = 15,000 \theta_{\star}$. A surface density contributing a convergence $\kappa_{\star} = 0.0025$ in this macro model is replaced with microlenses. This stellar mass fraction is similar to the expected value for the first discovery of a micro-caustic crossing event due to stars making up the intracluster light (see Sect. 5.1). With the choice of these parameters, from Eq. (14) we obtain the effective width of the Gaussian as $\sigma_{\star} \approx 0.14 \theta_{\star}$. We solve the lens equation using an adaptive mesh method implemented in the glafic software (Oguri 2010) together with a hierarchical tree algorithm to speed up calculations of deflection angles from an ensemble of point mass lenses (see e.g., Wambsganss 1999).

Figure 2 shows the result of moving the source along the β_1 axis, fixing $\beta_2 = 0$ and computing the total magnification as a function of the source position, or lightcurve. While many sharp peaks corresponding to micro-caustic crossings are present, the mean magnification closely follows the prediction of the total magnification for an extended source without microlenses as studied in Sect. 3, shown by a dashed line in Fig. 2. Note that the size of the extended source is not fitted to the lightcurve, but determined by Eq. (14). This confirms the average effect of microlensing in macro-caustic crossings can indeed be interpreted as an effectively extended source, as discussed in the literature (e.g., Dai and Pascale 2021).

The lightcurve in Fig. 2 exhibits large fluctuations originating from a random realization of positions of point masses. A better comparison with the model of an effectively extended source is obtained by averaging the calculation of the lightcurve over independent realizations of the point mass positions. The mean of 10 realizations is shown in Fig. 3, indicating a converging match of the lightcurve of an effectively extended source with the numerical result.



Fig. 4 An example of the micro-image swarms (or "trains"). Upper and lower panels show the source and image planes, respectively, and the middle panel shows enlarged view of micro-images in the plane around the swarms. The macro model caustic and critical curve (i.e., those in absence of microlenses) are vertical lines at $\beta_1 = \theta_1 = 0$. Micro-caustics and critical curves are shown by gray thin lines. The source position is $(\beta_1, \beta_2) = (1, 0)$, and corresponding multiple images in the image plane are shown by filled circles in the middle and lower panels. The size of the circles scales with the logarithm of the magnification factor of each image, and red and blue circles indicate micro-minimum and micro-saddle point images. In total there are 12 micro-minimum images on the negative parity side ($\theta_1 < 0$), and 15 micro-minimum images on the positive parity side ($\theta_1 > 0$). The set-up of the calculation is same as the one in Fig. 2

4.2 Effective Sizes

Accounting for the high-angle tail of the PDF of the microlens deflection angle, about 99% of the flux of a point source should be found within a region of size $10 \cdot \theta_* \kappa_*^{1/2}$ in the source plane, when considering the smooth mapping to the source place caused only by the smoothed lens and including the average effect of microlenses as an effective source, as explained above. Transforming to the image plane, one gets an ellipse of semi-major axes $r_1 \approx 10 \cdot \theta_* \kappa_*^{1/2} \mu_1$ and $r_2 \approx 10 \cdot \theta_* \kappa_*^{1/2} \mu_2$. This effective size of the micro image swarm is discussed as well in Vernardos et al. (2024) for the case of quasars.

Figure 4 shows an example of the micro image swarm (or "train") from calculations given in Sect. 4.1. The Figure indicates that there are two trains of the micro images on both sides of the macro critical curve at $\theta_1 = 0$ such that they are highly spread along horizontal di-

3

 β_1

 $\theta_1 > 0$

 $\theta_1 < 0$



rections. Since in our macro mass model $\mu_2 = 1$ and $\mu_1(\theta_1 = 4000) = 2500$, the discussion above suggests that the size of the swarm are $r_1 \approx 1250$ and $r_2 \approx 0.5$ in the dimensionless unit. The distribution of micro images shown in Fig. 4 appears to be consistent with this estimate.

4.3 Parity Dependence

As seen in Fig. 4, unless β_1 is small enough that the two swarms of images are merged, there are two image swarms appearing on each side of the macro-critical line. If the two swarms are well separated, they can be observationally resolved and their total magnification can be separately measured. To check the different behavior of each swarm, Fig. 5 shows the total magnifications of the two swarms located on the positive and negative parity sides as a function of the source position. A noticeable difference is that the total magnification of the negative parity side reaches significantly smaller values. As noted in Diego et al. (2018) and Oguri et al. (2018), this is because in the negative parity region, a source can be more strongly de-magnified by the microlenses de-magnified compared with the original macro model magnification (see also, e.g., Schechter and Wambsganss 2002). This effect becomes less pronounced as β_1 decreases because of averaging over many micro-images (see also Sect. 4.5).

4.4 Thickness of the Corrugated Macro-Caustic

The micro-critical lines are corrugated, that is to say, typically joined together in a largescale network, within a width θ_w around the original macro-critical curve, where the characteristic size of the micro-critical curve of a single microlens is equal to the mean separation between microlenses (Venumadhav et al. 2017). For microlenses with Einstein radius θ_{\star} and convergence contribution κ_{\star} , their mean separation is given by $\theta_{\star}\kappa_{\star}^{-1/2}$. At a separation θ from the macro-critical curve, the magnification eigenvalue of the smooth lens is θ/E , and the micro-critical curve is boosted to a size $\theta_{\star}(\theta/E)^{-1/2}$. Equating these two quantities we derive the thickness of the corrugated micro-critical curves, θ_w , as

$$\theta_{\rm w} = E\kappa_\star \ . \tag{15}$$

For our specific example shown in this paper, we find $\theta_w = 25,000$, which indicates that results shown in previous subsections are those for micro-images well within the corrugated



band. Within this thickness θ_w , the magnification of the individual micro-images is roughly constant, and given by roughly the magnification at the separation θ_w i.e.,

$$\mu_i \sim \frac{1}{2(1-\kappa_0)\kappa_\star} , \qquad (16)$$

which reduces to $\mu_i \sim 400$ in our example. Note that this typical magnification of an individual micro-image is not the same as the total magnification, which increases as we approach the macro-critical line owing to the increasing number of micro-images in the two swarms. As we will see below, the maximum magnification achieved in microcaustic crossings is also constant within the region of width r_w , in the same way as the typical magnification of a single micro-image.

4.5 Mean Number of Extra Micro-Image Pairs

Petters et al. (2009) provide a generic formula for the expected number of positive parity images in lensing systems (see their Eq. 11). In general, the formula relies on knowing the expected value of the magnification as a function of image plane position, conditional upon the fact that image plane positions are mapped to a particular source-plane position. In the case of constant surface mass density and shear, the formula simplifies greatly due to the fact that the expected value of the magnification is independent of image plane position. This is not the case near a macro-caustic, and the matter of the distribution of the magnification is currently an open question. We cannot therefore provide analytic estimates of the mean number of micro-minima, nor the mean magnification per micro-minima, as we could with constant surface mass density and shear. However, we can provide some commentary based on arguments from the simpler case.

For constant convergence and shear that produces an image of formally infinite magnification, the equations in Vernardos et al. (2024) are found to produce an infinite number of expected micro-minima. However, the random shear due to the micro-lenses combines in such a way that the mean magnification per minimum is finite. We know that near a macrocaustic, the magnification will not in fact diverge – as the discussion of the previous sections show, the point mass behaves as a an extended source. The number of micro-minima will not diverge then either.

Figure 6 shows the magnification divided by the number of micro-minima from calculations given in Sect. 4.1. The result indicates that the increase of the mean magnification as approaching the macro-caustic is achieved mainly by the increased number of micro-images, rather than the increase of magnifications of individual micro-images. It is found that the average magnification of micro-minima is $\sim 200 - 300$, which we find is reasonably close to a rough estimation given in equation (16).

The maximum number of images is reached when the two swarms merge, over a region in the source plane with the width of the effective source size induced by microlensing, $\theta_{\star}\kappa_{\star}^{1/2} \sim 10^{-7}$ arcsec for typical values. The time it would take for a source to cross this region is of the order of a year, so if a supermagnified source is close to the position of the smooth model caustic, it should be possible to measure the variation in the frequency of caustic crossings and average magnification that occurs at this position over this timescale.

4.6 Maximum Achievable Magnification

The maximum magnification in a microimage is reached when the source crosses a microcaustic, which occurs over a timescale determined by the source angular radius and the transverse velocities of source and lens relative to us (Miralda-Escudé 1991). The maximum magnification is determined by the eigenvalue gradient E at the critical line, and its characteristic value is limited by the presence of compact microlenses within the large-scale gravitational lens. The practical case this has been observed in is when the large-scale lens is a cluster of galaxies or a galaxy, and the compact microlenses are individual stars in cluster galaxies or the intracluster population.

As shown in Sect. 4.4, within the corrugated band, the average magnification of any random point (i.e., microimage) in the image plane is $\bar{\mu} \sim [2\kappa_{\star}(1-\kappa_0)]^{-1}$. This can also be interpreted as the eigenvalue with smallest absolute value fluctuating with a typical value of κ_{\star} with the other eigenvalue being fixed to $2(1-\kappa_0)$. The scale of variation of the small eigenvalue in the image plane is the mean distance between microlenses, $\theta_{\star}\kappa_{\star}^{-1/2}$, which in the source plane results in a typical separation between micro-caustics of $\theta_{\star}\kappa_{\star}^{1/2}$, implying that the maximum magnification that is typically reached for individual images of a source of angular radius θ_s in micro-critical lines is

$$\mu_{\text{peak}} \sim \frac{(\theta_{\star} \kappa_{\star}^{1/2} / \theta_{\text{s}})^{1/2}}{2\kappa_{\star} (1 - \kappa_0)} = \frac{\theta_{\star}^{1/2} \kappa_{\star}^{-3/4}}{2\theta_{\text{s}}^{1/2} (1 - \kappa_0)} \,. \tag{17}$$

An interesting observation from this result is that the maximum achievable magnification decreases with increasing stellar mass density κ_{\star} .

5 Applications of Observations

5.1 Summary of Observations

The first discovery of a micro-caustic crossing near a macro-caustic was reported by Kelly et al. (2018), as the rapid transient MACS J1149 Lensed Star 1 (also known as 'Icarus') near the macro critical curve of the massive cluster MACS J1149.5+2223 at z = 0.54, interpreted as a highly magnified individual star at z = 1.49. The timescale of the lightcurve near the peak (≤ 10 days) constrains the size of the background source to be $\leq 200 R_{\odot}$ for typical transverse velocities expected from large-scale structure (Oguri et al. 2018). Together with the spectral energy distribution and peak magnitude, this led Kelly et al. (2018) to conclude that the transient is an image of a blue supergiant at z = 1.49 magnified by more than a

factor of 2000. Stars making up the intracluster light can fully account for the observed microlensing event rate. Kelly et al. (2018) also reported a separate short transient event detected 0.26" from Lensed Star 1, which can be a counterimage of Lensed Star 1.

Rodney et al. (2018) reported two peculiar fast transients (collectively nicknamed 'Spock') in a giant arc at z = 1.0054 lensed by the massive cluster MACS J0416.1–2403 at z = 0.397. While they interpret the transients as eruptions of a luminous blue variable star or a recurrent nova, they also noted these events might be micro-caustic crossings near the macro-critical curve. Improved mass modeling of this cluster to constrain the shape of the critical curves would be helpful to discriminate these possibilities.

The discovery of Icarus has triggered searches of micro-caustic crossings in archival *Hubble Space Telescope* images. Chen et al. (2019) and Kaurov et al. (2019) reported the discovery of a highly magnified star (blue supergiant) at redshift z = 0.94 in a giant arc behind the massive cluster MACS J0416.1–2403. The possible counterimage of the background star at an offset of ~ 0.1" is also reported. The discovery suggests that micro-caustic crossing events may be ubiquitously found in deep imaging of massive clusters of galaxies.

Dai et al. (2020) reported several asymmetric surface brightness features in a giant arc at z = 2.93 produced by the galaxy cluster SDSS J1226+2152 at z = 0.43. While such asymmetric features can be produced by micro-caustic crossings, Dai et al. (2020) concluded that they are more likely to be produced by subhalos with masses of $\sim 10^6 - 10^8 M_{\odot}$, based on the absence of notable time variation over a six-year baseline.

Several other observations of supermagnified stars in lensing clusters have been reported recently, some of which are being given special names, and are continuing to be discovered with the ongoing observations with JWST: an additional star called *Mothra* in MACSJ0416 (Diego et al. 2023b), the star *Earendel* in WHL0137-08 (Welch et al. 2022b,a), Godzilla in PSZ1 G311.65 (Diego et al. 2022), *Quyllur* in the El Gordo cluster (Diego et al. 2023a), and several other stars in Abell 2744 (Chen et al. 2022), MACSJ0647 (Meena et al. 2023b; Furtak et al. 2024), and Abell 370 (Kelly et al. 2022; Meena et al. 2023a). In this short paper we cannot do justice to all the wonderful science that is made possible by this blossoming of discoveries.

5.2 Some Applications

We now discuss some applications that the phenomenon of super-magnification in the microcaustics of lensing clusers of galaxies has already demonstrated or may be developed in the future. This list is by no means complete and is limited by the length of this paper; in general any source, such as supernovae or quasars, may be studied in special ways when highly magnified; in the case of gravitational waves, diffraction effects mean that the interest of lensing observations shifts to masses larger than stellar ones, corresponding to a Schwarzshild radius larger than the observed wavelengths.

5.2.1 Compact Objects in the Dark Matter

As discussed in Sect. 4, one of the most important model parameters that control the property of microlensing near macro-caustics is the surface density of microlenses, κ_{\star} . Normally only stars in lensing clusters or galaxies are considered as microlenses, whose abundance near macro-caustics can be inferred from observations of intracluster light. However, if a fraction of dark matter is composed of dark compact objects, such as primordial black holes (Carr and Hawking 1974), the surface density of microlenses may be much higher. This, in turn, means we can constrain the abundance of compact objects from observations of microcaustic crossings in a manner complementary to other constraints (see e.g., Sasaki et al. 2018, for a review of constraints on primordial black holes), including quasar microlensing (Mediavilla et al. 2017).

Constraints on compact objects from the Icarus highly magnified star have extensively been explored in Kelly et al. (2018) as well as in follow-up papers by Venumadhav et al. (2017), Diego et al. (2018), and Oguri et al. (2018). For instance, as explained in Sect. 4.6 the maximum achievable magnification is reduced as $\kappa_{\star}^{-3/4}$. Following this idea, Oguri et al. (2018) adopted a simple analytic model to derive constraints on the compact object abundance for a wide range of mass ranging from $10^{-5} M_{\odot}$ to $10^2 M_{\odot}$. In addition, a large κ_{\star} reduces the time variability of lensed sources due to averaging effects, which also yields constraints (Kelly et al. 2018; Diego et al. 2018).

Furthermore, as discussed in Sect. 4.4, the compact object abundance determines the width of the corrugated band around the macro-critical curve within which most highly magnified images appear, so an independent constraint is obtained from the spread of microcaustic crossings around macro critical curves (Venumadhav et al. 2017). A limit can then be derived on κ_{\star} in compact objects from the set of highly magnified stars at cosmological distances reported so far. The width of the corrugated band is $\theta_w = d_{\parallel}\kappa_{\star}$, where d_{\parallel} is the gradient of the small magnification eigenvalue. In the first event named Icarus, the macro model predicts $d_{\parallel} \simeq 4 \operatorname{arcmin}^{-1}$, and the separation of the image from the predicted critical curve is ~ 0.13 arcsec (Kelly et al. 2018; Venumadhav et al. 2017). In the MACS J0416.1-2403 event, the macro model predicts $d_{\parallel} = 7 \operatorname{arcmin}^{-1}$ and the separation of the image is 0.1 arcsec (Diego et al. 2018; Oguri et al. 2018). The ratios of the observed image separations compared to the maximum separation where images with the highest magnifications are observed is therefore $0.012\kappa_{\star}$ and $0.009\kappa_{\star}$, respectively. The random probability these two values are observed at least as small as they are is $\sim 10^{-4} \kappa_{\star}^2$. Requiring this probability to be at least larger than 1% we obtain the limit $\kappa_{\star} < 0.1$. This is valid for a broad range of microlens mass, roughly $10^{-6} M_{\odot} < M$, over which other effects like diffraction or source size are not important (see Fig. 9 in Venumadhav et al. 2017). Naturally, as the number of discovered events increases in cluster lenses that can be adequately modeled, this limit on any objects that are sufficiently compact to act as microlenses near macro-caustics will improve essentially to the value of the microlenses accounted by intracluster stars.

Recently, this argument has been developed in detail using most of the discovered supermagnified stars by Vall Müller and Miralda-Escudé (2024), and in the future this limit should rapidly improve as more examples of these stars with extreme magnifications are revealed by the potential of JWST and other telescopes.

5.2.2 Observing Population III Stars

Microlensing near the macro-caustics provides a means of observing distant individual stars that cannot be observed without the high magnification achieved in micro-caustic crossings. Windhorst et al. (2018) explored the possibility of observing individual Population III (Pop III) stars, which are metal-free stars formed from pristine gas, at $z \simeq 7 - 17$, and concluded that direct observations of highly magnified Pop III stars would be possible by monitoring a few to a few tens of mass clusters over a decade with the James Webb Space Telescope.

5.2.3 Dark Matter Subhalos

Dai et al. (2018) proposed a novel method to constrain the abundance of dark matter subhalos with masses of $10^6 - 10^8 M_{\odot}$, whose existence is naturally predicted by the standard cold dark matter paradigm (e.g., Diemand et al. 2008), using observations of image pairs of sources near caustics. The method uses astrometry of these highly magnified image pairs of stars or other luminous objects (star clusters or star-forming regions) in caustic-straddling giant arcs and searches for perturbations of the shape of macro critical curves, which are expected to be smooth on small scales in the absence of subhalos. Subhalos with masses of $10^6 - 10^8 M_{\odot}$ produce irregularities on the critical curve shape (see also Abe et al. 2024), and the astrometric perturbations of image pairs, at the level of 20 - 80 mas, which can be detected by accurate centroid measurements of several magnified pairs by the James Webb Space Telescope. Known cluster galaxies produce irregularities on larger scales only, corresponding to their higher masses.

5.2.4 Axion Minihalos

Finally, axion minihalos may also be detectable using highly magnified stars. Axion minihalos are expected in many variations of the theory of axions for dark matter that solve the strong QCD problem, with masses of $\sim 10^{-12} M_{\odot}$. These are extremely difficult to detect, not only because of their tiny size but also because their surface density is highly subcritical to lensing. However, when a star crosses one of the micro-caustics affected by micro-lensing from intracluster stars, axion minihalos would create a secondary level of perturbations on the magnification at smaller scales that would cause perturbations of the lightcurve of micro-caustic crossing events on small time scales. This was predicted in Dai and Miralda-Escudé (2020), proposing a possible route to the detection of axion dark matter using only astrophysical methods.

6 Summary

In this artcile we have explored the expected microlensing magnification of sources close to macro-caustics. For this purpose, we have performed simulations and compared the results to the analytical expectations of the behavior of microimages, also described in Venumadhav et al. (2017), Dai and Pascale (2021), Vernardos et al. (2024, this collection).

We have shown that the mean magnification due to lens plane microlenses of point sources in the vicinity of the macro-caustic follows that of an extended source in the vicinity of a macro-caustic in the absence of microlensing. This is precisely correct when the source profile matches the probability distribution function of the microlens deflection angle.

Similarly, we have shown that the two swarms of microimages at each side of the macrocaustic are spread within an ellipse that results from mapping the microlens deflection angle PDF, or the corresponding "extended source", to the lens plane. We note that even though the sizes of the two image swarms are symmetric, the swarm of images at the negative parity side of the macro-caustic can show significantly lower magnification values. This is consistent with the findings of Schechter and Wambsganss (2002) who show that microlensed sources in macro-saddle points are more prone to be de-magnified than in macro-minima.

We have reviewed the characteristic number and average magnification of the microimages within the microlens-distorted (or corrugated) macro-caustic. We observe that the magnification per micro-image remains roughly constant within this corrugated caustic and with a value given by the separation of micro-image pairs from micro-caustics. The maximum individual micro-image magnification is limited by the angular size of the source, and decreases with increasing stellar mass density. The increase of the mean magnification when approaching the position of the macro-caustic is driven by the increase in the number of micro-images rather than their individual magnifications. Analogous to the fact that micro-images behave as a lensed extended source and thus the total magnification is not infinite when crossing the macro-caustic, the number of micro-images reaches a maximum when the two image swarms merge.

The microlensing phenomena occurring near macro-caustics that we have reviewed in this Chapter have not been observed until recently. Since the dawn of the era of time-domain astronomy, in the past few years several transients have been shown to be consistent with such microlensing in the vicinity of macro-caustics. These discoveries imply that microcaustic crossing events should be ubiquitous when monitoring observations of massive galaxy clusters, and will surely be increased by observations with JWST reaching for fainter magnitudes. With this in mind, we have non-exhaustively reviewed some future (and current) applications of the different observational signatures, ranging from identifying population III stars from individual caustic crossing events to constraining the abundance and nature of primordial black holes and dark matter subhalos from the statistical properties of observed caustic crossing events and probing the distorted macro-caustic from astrometric perturbations in magnified background stars.

Funding Open Access funding provided thanks to the CRUE-CSIC agreement with Springer Nature. This work was supported in part by JSPS KAKENHI Grant Number JP22H01260, JP20H05856, and by Spanish grant PID2019-108122GB-C32. TA acknowledges support the Millennium Science Initiative ICN12_009 and the ANID BASAL project FB210003.

Declarations

Competing Interests The authors declare no competing interests.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

- Abe KT, Kawai H, Oguri M (2024) Analytic approach to astrometric perturbations of critical curves by substructures. Phys Rev D 109:083517. https://doi.org/10.1103/PhysRevD.109.083517. arXiv:2311.18211 [astro-ph.CO]
- Carr BJ, Hawking SW (1974) Black holes in the early universe. Mon Not R Astron Soc 168:399–416. https:// doi.org/10.1093/mnras/168.2.399
- Chen W, Kelly PL, Diego JM, Oguri M, Williams LLR, Zitrin A, Treu TL, Smith N, Broadhurst TJ, Kaiser N, Foley RJ, Filippenko AV, Salo L, Hjorth J, Selsing J (2019) Searching for highly magnified stars at cosmological distances: discovery of a redshift 0.94 blue supergiant in archival images of the galaxy cluster MACS J0416.1-2403. Astrophys J 881(1):8. https://doi.org/10.3847/1538-4357/ab297d
- Chen W, Kelly PL, Treu T, Wang X, Roberts-Borsani G, Keen A, Windhorst RA, Zhou R, Bradac M, Brammer G, Strait V, Broadhurst TJ, Diego JM, Frye BL, Meena AK, Zitrin A, Pascale M, Castellano M, Marchesini D, Morishita T, Yang L (2022) Early results from GLASS-JWST. VIII. An extremely magnified blue supergiant star at redshift 2.65 in the A2744 cluster field. Astrophys J Lett 940(2):54. https://doi.org/10.3847/2041-8213/ac9585

- Dai L, Miralda-Escudé J (2020) Gravitational lensing signatures of axion dark matter minihalos in highly magnified stars. Astron J 159(2):49. https://doi.org/10.3847/1538-3881/ab5e83
- Dai L, Pascale M (2021) New Approximation of Magnification Statistics for Random Microlensing of Magnified Sources. arXiv e-prints arXiv:2104.12009 [astro-ph.GA]
- Dai L, Venumadhav T, Kaurov AA, Miralda-Escudé J (2018) Probing dark matter subhalos in galaxy clusters using highly magnified stars. Astrophys J 867(1):24. https://doi.org/10.3847/1538-4357/aae478
- Dai L, Kaurov AA, Sharon K, Florian M, Miralda-Escudé J, Venumadhav T, Frye B, Rigby JR, Bayliss M (2020) Asymmetric surface brightness structure of caustic crossing arc in SDSS J1226+2152: a case for dark matter substructure. Mon Not R Astron Soc 495(3):3192–3208. https://doi.org/10.1093/mnras/ staa1355
- Diego JM, Kaiser N, Broadhurst T, Kelly PL, Rodney S, Morishita T, Oguri M, Ross TW, Zitrin A, Jauzac M, Richard J, Williams L, Vega-Ferrero J, Frye B, Filippenko AV (2018) Dark matter under the microscope: constraining compact dark matter with caustic crossing events. Astrophys J 857(1):25. https://doi.org/ 10.3847/1538-4357/aab617
- Diego JM, Pascale M, Kavanagh BJ, Kelly P, Dai L, Frye B, Broadhurst T (2022) Godzilla, a monster lurks in the sunburst galaxy. Astron Astrophys 665:134. https://doi.org/10.1051/0004-6361/202243605
- Diego JM, Meena AK, Adams NJ, Broadhurst T, Dai L, Coe D, Frye B, Kelly P, Koekemoer AM, Pascale M, Willner SP, Zackrisson E, Zitrin A, Windhorst RA, Cohen SH, Jansen RA, Summers J, Tompkins S, Conselice CJ, Driver SP, Yan H, Grogin N, Marshall MA, Pirzkal N, Robotham A, Ryan RE, Willmer CNA, Bradley LD, Caminha G, Caputi K, Carleton T, Kamieneski P (2023a) JWST's PEARLS: a new lens model for ACT-CL J0102–4915, "El Gordo," and the first red supergiant star at cosmological distances discovered by JWST. Astron Astrophys 672:3. https://doi.org/10.1051/0004-6361/202245238
- Diego JM, Sun B, Yan H, Furtak LJ, Zackrisson E, Dai L, Kelly P, Nonino M, Adams N, Meena AK, Willner SP, Zitrin A, Cohen SH, Silva JCJD, Jansen RA, Summers J, Windhorst RA, Coe D, Conselice CJ, Driver SP, Frye B, Grogin NA, Koekemoer AM, Marshall MA, Pirzkal N, Robotham A, Rutkowski MJ, Ryan Russell E J, Tompkins S, Willmer CNA, Bhatawdekar R (2023b) JWST's PEARLS: Mothra, a new kaiju star at z = 2.091 extremely magnified by MACS0416, and implications for dark matter models. Astron Astrophys 679:A31. https://doi.org/10.1051/0004-6361/202347556. arXiv:2307.10363 [astro-ph.CO]
- Diemand J, Kuhlen M, Madau P, Zemp M, Moore B, Potter D, Stadel J (2008) Clumps and streams in the local dark matter distribution. Nature 454(7205):735–738. https://doi.org/10.1038/nature07153
- Furtak LJ, Meena AK, Zackrisson E, Zitrin A, Brammer GB, Coe D, Diego JM, Eldridge JJ, Jiménez-Teja Y, Kokorev V, Ricotti M, Welch B, Windhorst Abdurro'uf RA, Andrade-Santos F, Bhatawdekar R, Bradley LD, Broadhurst T, Chen W, Conselice CJ, Dayal P, Frye BL, Fujimoto S, Hsiao TY-Y, Kelly PL, Mahler G, Mandelker N, Norman C, Oguri M, Pirzkal N, Postman M, Ravindranath S, Vanzella E, Wilkins SM (2024) Reaching for the stars JWST/NIRSpec spectroscopy of a lensed star candidate at z = 4.76. Mon Not R Astron Soc 527(1):7–13. https://doi.org/10.1093/mnrasl/slad135
- Granot J, Schechter PL, Wambsganss J (2003) The mean number of extra microimage pairs for macrolensed quasars. Astrophys J 583(2):575–583. https://doi.org/10.1086/345447
- Katz N, Balbus S, Paczynski B (1986) Random scattering approach to gravitational microlensing. Astrophys J 306:2. https://doi.org/10.1086/164313
- Kaurov AA, Dai L, Venumadhav T, Miralda-Escudé J, Frye B (2019) Highly magnified stars in lensing clusters: new evidence in a galaxy lensed by MACS J0416.1-2403. Astrophys J 880(1):58. https://doi. org/10.3847/1538-4357/ab2888
- Kelly PL, Diego JM, Rodney S, Kaiser N, Broadhurst T, Zitrin A, Treu T, Pérez-González PG, Morishita T, Jauzac M, Selsing J, Oguri M, Pueyo L, Ross TW, Filippenko AV, Smith N, Hjorth J, Cenko SB, Wang X, Howell DA, Richard J, Frye BL, Jha SW, Foley RJ, Norman C, Bradac M, Zheng W, Brammer G, Benito AM, Cava A, Christensen L, de Mink SE, Graur O, Grillo C, Kawamata R, Kneib J-P, Matheson T, McCully C, Nonino M, Pérez-Fournon I, Riess AG, Rosati P, Schmidt KB, Sharon K, Weiner BJ (2018) Extreme magnification of an individual star at redshift 1.5 by a galaxy-cluster lens. Nat Astron 2:334–342. https://doi.org/10.1038/s41550-018-0430-3
- Kelly PL, Chen W, Alfred A, Broadhurst TJ, Diego JM, Emami N, Filippenko AV, Keen A, Li Kei S, Lim J, Meena AK, Oguri M, Scarlata C, Treu T, Williams H, Williams LLR, Zhou R, Zitrin A, Foley RJ, Jha SW, Kaiser N, Mehta V, Rieck S, Salo L, Smith N, Weisz DR (2022) Flashlights: More than a Dozen High-Significance Microlensing Events of Extremely Magnified Stars in Galaxies at Redshifts z = 0.7-1.5. arXiv e-prints arXiv:2211.02670 [astro-ph.CO]. https://doi.org/10.48550/arXiv.2211.02670
- Mediavilla E, Jiménez-Vicente J, Muñoz JA, Vives-Arias H, Calderón-Infante J (2017) Limits on the mass and abundance of primordial black holes from quasar gravitational microlensing. Astrophys J Lett 836(2):18. https://doi.org/10.3847/2041-8213/aa5dab
- Meena AK, Chen W, Zitrin A, Kelly PL, Golubchik M, Zhou R, Alfred A, Broadhurst T, Diego JM, Filippenko AV, Li SK, Oguri M, Smith N, Williams LLR (2023a) Flashlights: an off-caustic lensed star

at redshift z = 1.26 in Abell 370. Mon Not R Astron Soc 521(4):5224–5231. https://doi.org/10.1093/mnras/stad869

- Meena AK, Zitrin A, Jiménez-Teja Y, Zackrisson E, Chen W, Coe D, Diego JM, Dimauro P, Furtak LJ, Kelly PL, Oguri M, Welch Abdurro'uf B, Andrade-Santos F, Adamo A, Bhatawdekar R, Bradač M, Bradley LD, Broadhurst T, Conselice CJ, Dayal P, Donahue M, Frye BL, Fujimoto S, Hsiao TY-Y, Kokorev V, Mahler G, Vanzella E, Windhorst RA (2023b) Two lensed star candidates at z ~ 4.8 behind the galaxy cluster MACS J0647.7+7015. Astrophys J Lett 944(1):6. https://doi.org/10.3847/2041-8213/acb645
- Miralda-Escudé J (1991) The magnification of stars crossing a caustic. I. Lenses with smooth potentials. Astrophys J 379:94. https://doi.org/10.1086/170486
- Neindorf B (2003) A probability theoretical access to extragalactic microlensing. Astron Astrophys 404:83–92. https://doi.org/10.1051/0004-6361:20030098
- Oguri M (2010) The mass distribution of SDSS J1004+4112 revisited. Publ Astron Soc Jpn 62:1017. https:// doi.org/10.1093/pasj/62.4.1017
- Oguri M, Diego JM, Kaiser N, Kelly PL, Broadhurst T (2018) Understanding caustic crossings in giant arcs: characteristic scales, event rates, and constraints on compact dark matter. Phys Rev D 97(2):023518. https://doi.org/10.1103/PhysRevD.97.023518
- Paczynski B (1986) Gravitational microlensing at large optical depth. Astrophys J 301:503. https://doi.org/ 10.1086/163919
- Petters AO, Rider B, Teguia AM (2009) A mathematical theory of stochastic microlensing. II. Random images, shear, and the Kac-Rice formula. J Math Phys 50(12):122501. https://doi.org/10.1063/1.3267859
- Rodney SA, Balestra I, Bradac M, Brammer G, Broadhurst T, Caminha GB, Chirivi G, Diego JM, Filippenko AV, Foley RJ, Graur O, Grillo C, Hemmati S, Hjorth J, Hoag A, Jauzac M, Jha SW, Kawamata R, Kelly PL, McCully C, Mobasher B, Molino A, Oguri M, Richard J, Riess AG, Rosati P, Schmidt KB, Selsing J, Sharon K, Strolger L-G, Suyu SH, Treu T, Weiner BJ, Williams LLR, Zitrin A (2018) Two peculiar fast transients in a strongly lensed host galaxy. Nat Astron 2:324–333. https://doi.org/10.1038/s41550-018-0405-4
- Saha P, Williams LLR (2011) Understanding micro-image configurations in quasar microlensing. Mon Not R Astron Soc 411(3):1671–1677. https://doi.org/10.1111/j.1365-2966.2010.17797.x
- Sasaki M, Suyama T, Tanaka T, Yokoyama S (2018) Primordial black holes—perspectives in gravitational wave astronomy. Class Quantum Gravity 35(6):063001. https://doi.org/10.1088/1361-6382/aaa7b4
- Schechter PL, Wambsganss J (2002) Quasar microlensing at high magnification and the role of dark matter: enhanced fluctuations and suppressed saddle points. Astrophys J 580(2):685–695. https://doi.org/10. 1086/343856
- Schneider P, Ehlers J, Falco EE (1992) Gravitational Lenses. Astronomy and Astrophysics Library. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-662-03758-4
- Vall Müller C, Miralda-Escudé J (2024) Limits on Dark Matter Compact Objects implied by Supermagnified Stars in Lensing Clusters. arXiv e-prints arXiv:2403.16989 [astro-ph.CO]. https://doi.org/10.48550/ arXiv.2403.16989
- Venumadhav T, Dai L, Miralda-Escudé J (2017) Microlensing of extremely magnified stars near caustics of galaxy clusters. Astrophys J 850(1):49. https://doi.org/10.3847/1538-4357/aa9575
- Vernardos G, Sluse D, Pooley D, Schmidt RW, Millon M, Weisenbach L, Motta V, Anguita T, Saha P, O'Dowd M, Peel A, Schechter PL (2024) Microlensing of strongly lensed quasars. Space Sci Rev 220(1):14. https://doi.org/10.1007/s11214-024-01043-8
- Wambsganss J (1990) Gravitational microlensing. PhD thesis. Ludwig-Maximilians-Universität, Munich
- Wambsganss J (1999) Gravitational lensing: numerical simulations with a hierarchical tree code. J Comput Appl Math 109(1):353–372
- Welch B, Coe D, Diego JM, Zitrin A, Zackrisson E, Dimauro P, Jiménez-Teja Y, Kelly P, Mahler G, Oguri M, Timmes FX, Windhorst R, Florian M, de Mink SE, Avila RJ, Anderson J, Bradley L, Sharon K, Vikaeus A, McCandliss S, Bradač M, Rigby J, Frye B, Toft S, Strait V, Trenti M, Sharma S, Andrade-Santos F, Broadhurst T (2022a) A highly magnified star at redshift 6.2. Nature 603(7903):815–818. https://doi. org/10.1038/s41586-022-04449-y
- Welch B, Coe D, Zackrisson E, de Mink SE, Ravindranath S, Anderson J, Brammer G, Bradley L, Yoon J, Kelly P, Diego JM, Windhorst R, Zitrin A, Dimauro P, Jiménez-Teja Abdurro'uf Y, Nonino M, Acebron A, Andrade-Santos F, Avila RJ, Bayliss MB, Benítez A, Broadhurst T, Bhatawdekar R, Bradač M, Caminha GB, Chen W, Eldridge J, Farag E, Florian M, Frye B, Fujimoto S, Gomez S, Henry A, Hsiao TY-Y, Hutchison TA, James BL, Joyce M, Jung I, Khullar G, Larson RL, Mahler G, Mandelker N, McCandliss S, Morishita T, Newshore R, Norman C, O'Connor K, Oesch PA, Oguri M, Ouchi M, Postman M, Rigby JR, Ryan REJ, Sharma S, Sharon K, Strait V, Strolger L-G, Timmes FX, Toft S, Trenti M, Vanzella E, Vikaeus A (2022b) JWST imaging of earendel, the extremely magnified star at redshift z = 6.2. Astrophys J Lett 940(1):1. https://doi.org/10.3847/2041-8213/ac9d39

Windhorst RA, Timmes FX, Wyithe JSB, Alpaslan M, Andrews SK, Coe D, Diego JM, Dijkstra M, Driver SP, Kelly PL, Kim D (2018) On the observability of individual population III stars and their stellar-mass black hole accretion disks through cluster caustic transits. Astrophys J Suppl Ser 234(2):41. https://doi. org/10.3847/1538-4365/aaa760

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.