Critical consensus formation in a symmetric honeybee nest-site selection process

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Abstract: This paper explores collective decision-making in honeybee swarms, focusing on their nest-site selection process. Using the agent-based model proposed by List, Estholtz, and Seeley, we investigate how individual exploration and social interactions influence the bees' ability to make group decisions. The model incorporates parameters like site quality, self-discovery probability, and interdependence to asses these interactions. We particularly analyze the dynamics of the system under equal-quality sites and fully interdependent conditions. A critical point is identified where the system undergoes a second order phase transition in out of equilibrium conditions. The transition resembles the one observed in a contact process, with finite size effects observed in numerical simulations. Finite-size effects allow the observation of fluctuations that, like in several real animal swarms, impact decisively the group decision-making process. Additionally, we briefly show how multiplicative noise in the system, modeled by Langevin equations, is responsible for the symmetry breaking between the equal quality nest-site options. The presence of a non-equilibrium critical point in this context provides new insight into the role of critical behavior in biological systems.
Keywords: Decision-making, agent model, criticality, finite size effects
SDGs: Quality education, climate action, terrestrial life

I. INTRODUCTION

Physics and complexity science have been found useful in solving problems where a high number of constituents is involved. Although originally centered in solving specific problems in their own disciplines, they are found helpful when confronting similar questions in other fields. This is the case of collective decision-making, a process in which individuals interact and exchange information with one another, ultimately leading to a consensus within the group. Such behaviors can be observed in both natural and artificial contexts, from human elections or insect colonies to robotic swarms. The complexity inherent in these processes requires interdisciplinary collaboration for effective solutions. Apart from aforementioned physics, fields such as biology, sociology, and computer science all contribute to a deeper understanding of how collective decision-making operates across different systems. There is abundance of this behavior in the natural realm, where the biology of the individuals involved may vary in a notable way. Among this diversity, one can emphasize the behavior of social insects, as these systems can be conceived as conformed by simple agents that interact to exhibit collective emergent decisions. More specifically, we will drive our attention to a honeybee nest-seeking process, which has led to a variety of collective decision models and swarming robot experiments to have a better understanding of the problem.

Bees may behave in multiple ways when participating in nest-site selection processes. Both independent and interdependent behavior can be appreciated. On one hand, each bee can individually scan its surroundings to try to find a proper nest site. On the other, a group of bees can interact with each other to decide which of the possible places fits better for them. These insects can advertise their options through intriguing signaling mechanisms such as the waggle-dance and, as a consequence, they can also change their opinion and become uncommitted. Indeed, bees can individually stop promoting a particular site and use cross-inhibition so that their peers change their opinion. Besides, bees can appreciate different qualities in the places they visit, and may advertise more insistingly with their signaling mechanisms to other bees the higher the quality of their option. This helps the group to make a decision that rejects the low quality sites in front of the high quality ones.

The theoretical insights of this paper are based on the agent-based model presented by List, Estholtz and Seeley, that we will call *the LES model*, which includes most of the features describing the process by which honeybee swarms take a collective decision. We may additionally remark that this model was later further developed analytically by T. Galla [5]. Within this modeling framework, locations have assigned qualities, which are perceived by individual exploration. This, together with social interaction or imitation, helps us to model honeybee collective decision-making processes. Throughout this paper, we will discuss the LES model into further detail. We will specifically focus on the role of social interactions in a symmetric scenario where nest-choices are equivalent. In this model, this is done by one parameter called the "interdependence" parameter. It shall be underlined that individual exploration has two sides. Bees need to explore its surroundings to decide from their available options, but an excessive exploration may lead to noise that makes it more difficult to reach consensus. Thus, social interaction is crucial to mitigate this effect.

Additionally, we will focus on a limit case of the model, which can be understood as a contact process, i.e. an statistical model presenting a non-equilibrium phase transition to an absorbing state [6]. Results show that agreement has its peak when critical behavior begins. This matches the hypothesis that maintains that numerous biological systems work near a critical point. The structure of the paper is the following: In Section II, we introduce the LES model. In section III, we present our main results including a subsection where we briefly explore how noise can break the symmetry of our decision-making scenario. Finally, in Sec. IV we summarize our main conclusions.

II. *LES* MODEL

The agent-based LES model reproduces the process by which honeybee swarms make a decision. Within this framework, locations have assigned qualities, which are perceived by individual exploration. This, besides social interaction, helps us to model decision-making processes.

In the LES model, there are N bees and k potential nest-sites. Each site has an intrinsic quality $q_\alpha \geq 0$ (with $\alpha = 1, ..., k$) and a fixed probability $\pi_\alpha \geq 0$ of being discovered independently by each bee. These parameters do not change over time. In the current project, we will focus our attention on the case in which all site qualities are the same.

The decision process evolves in discrete time steps. At a certain time t, each bee can be uncommitted or support a certain option. We will denote the first case as $s_i(t) = 0$ $(i = 1, ..., N)$ and the second one as $s_i(t) = \alpha$. Bees can change their opinion from uncommitted to any option and vice-versa, but cannot change directly from supporting a specific site to another one. This changes are regulated by two probabilities, $p_{\alpha,t+1}$ and $r_{\alpha,t+1}$. The first one is the probability of moving from an uncommitted state to a particular α option. The second one is the probability of becoming uncommitted at time $t+1$ when the bee was supporting a certain α option at time t.

The commitment probabilities are expressed by the following equation:

$$p_{\alpha,t+1} = (1-\lambda)\pi_\alpha + \lambda f_{\alpha,t}. \tag{1}$$

The first term of this equation indicates how likely it is that an exploring bee independently finds a site α. The second one expresses the probability of a bee changing its opinion because of the influence of the others. In this term, $f_{\alpha,t}$ represents the fraction of bees that support option α at time t. A crucial part of this equation is λ, the interdependence parameter. It varies between 0 and 1, and shows how much importance do bees give to individual exploration or if they act based on their peers opinions. If $\lambda = 0$, bees will commit to a site only taking into account their own exploration, whereas if $\lambda = 1$, they will support a certain option just based on what the other bees advertise.

The uncommitment probability $r_{\alpha,t}$ depends on the quality of the explored site. In the original LES model these transitions occurred deterministically after a certain period of time had passed. However, T. Galla substituted this deterministic process by a stochastic one in which this probability is expressed by:

$$r_\alpha = q_0 \left[\frac{\mu}{K} - \frac{1-\mu}{q_\alpha}\right], \alpha = 1, ..., k. \tag{2}$$

Here, we set $q_0 = 1$, and focus on the case where $\mu = 0$ (further detail is shown in Ref. [5]). The parameter q_0 ensures that $0 < r_\alpha \leq 1$ and represents the characteristic time scale of the problem. According to these assumptions, the average length of the advertisement for site α is $1/r_\alpha$, which is proportional to q_α.

A master equation can be used to derive a set of nonlinear differential equations that express the evolution of the average fraction values $\langle f_{\alpha,t}\rangle$. These will be referred as average dancing frequencies from now on. Assuming a fully connected, mean-field like system and taking into account mathematical details in [5] one can observe that:

$$\langle \dot{f}_{\alpha,t}\rangle = \langle f_{0,t}\rangle[(1-\lambda)\pi_\alpha + \lambda\langle f_{\alpha,t}\rangle] - r_\alpha\langle f_{\alpha,t}\rangle, \alpha = 1, ..., k \tag{3}$$

where $\langle f_{0,t}\rangle = 1 - \sum_{\alpha=1}^{k}\langle f_{\alpha,t}\rangle$. This equation can be easily integrated numerically, for example, using an Euler method. Stationary points, referred as f_α^*, can also be deterministically found by imposing $\langle \dot{f}_{\alpha,t}\rangle = 0$.

III. SYMMETRIC SELECTION SCENARIO

Simplified and limiting cases of this general model will be further discussed henceforth. In particular, we are interested in the case in which all site qualities are considered to be equal, that is $r_\alpha = r \ \forall\alpha$. In this situation the symmetry between the populations committed to the different options can only be broken by the discovery probabilities π_α. Moreover, if we consider this probabilities to be the same for all sites, the system will reach an impasse between all options, where $f_1^\star = ... = f_k^\star$. This means that no consensus will be reached.

A. Deterministic analysis

In this particular symmetric situation, the solution for $f_0^\star$ comes from a simple second degree polynomial, which has a physical and stable solution $f_0^\star = (B - \sqrt{B^2 - 4\lambda r})/2\lambda$, being $B = k\pi(1-\lambda) + \lambda + r$. If in this situation we take the fully interdependent limit ($\pi = 0$), we find two solutions $f_0^\star = 1$ and $f_0^\star = r/\lambda$, which are delimited by the stability threshold value $\lambda = r$. In the former case, all bees remain uncommitted, with no bees dancing for any possible site. On the latter case, the exact values of $f_\alpha^\star$ are not determined, as we are dividing by 0 in this process (for further details see Ref. [1]). Using linear stability analysis we know that any solution that satisfies $\sum_{\alpha=1}^{k} f_{\alpha,t} = 1 - r/\lambda$ is correct, and that if an initial state fulfills it, the frequency will remain unchanged, whereas other initial conditions will lead to stationary points where one option dominates and the others disappear. This helps us understand that the stationary state reached is highly sensible to initial conditions. The exact option that has an advantage in the beginning is the one which will remain. We will later remark that this is not necessarily the case when considering finite size fluctuations, for instance, when performing stochastic simulations of the LES model in finite system sizes.

B. Stochastic simulations of the agent-based *LES* model

Stochastic simulations of the LES model have been implemented using an agent-based approach. Particularly, we have implemented the following method to simulate bees' behavior in simulations. There are N bees and each bee can be in one of the three states determining the fractions f_0, f_1 or f_2. Although this analysis can be done with an arbitrary number of options, for the sake of simplicity, we will concentrate in an scenario with two possible nest-site options. At each time step, we update the transition probabilities in the LES model according to:

$$p_{0\to 1,2}(t+1) = \tilde{\lambda} f_{1,2}(t) + \tilde{\pi}, \tag{4}$$

with $\tilde{\pi} = (1-\lambda)\pi$, for bees passing from the uncommitted 0 state to a 1 or 2 state, and

$$p_{1,2\to 0}(t+1) = r, \tag{5}$$

for bees transferring from a 1 or 2 committed state to the uncommitted one. In equation (4), $f_{1,2}(t)$ is the amount of agents in state 1 (or 2) divided the total number of bees N at a certain time t. The process is repeated until we reach a steady state. These stochastic simulations allow us to distinguish relevant finite site fluctuations.

Under the assumption of equal qualities and in the fully interdependent limit ($\tilde{\pi} = 0$), as discussed in section IIA, a continuous phase transition between a fully uncommitted absorbing state and an active state with a finite fraction of bees promoting the available sites occurs at $\lambda = r$. This will be discussed in further detailed in the next subsection. However, we may succinctly highlight here the influence of finite size fluctuations. The system may behave as expected by the deterministic equations for certain time, but this tendency is eventually broken by finite size effects. Due to finite size fluctuations, the system eventually attains a situation where the winner takes it all, and thus ends up with no population in either state 1 or 2, which may differ in each particular realization (see figure 1). Notice that, in agreement with the deterministic results, not all agents end up in state 1 or 2, since some of them are in the sate 0 due to the finite value of the abandonment rate r. There may also be an initially favored option which ends up vanishing as a consequence of these fluctuations.

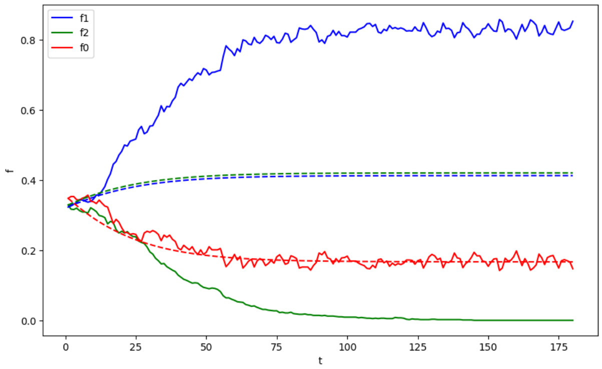

FIG. 1: Temporal evolution of a single experimental realization in the fully interdependent limit with equal qualities for $N = 1000$. Other parameters are $r = 0.1$ and $\tilde{\lambda} = 0.1$. Initially $f_\alpha \approx (1/3, 1/3, 1/3)$

In real swarms, these fluctuations are expected to be pertinent, as systems will consist of a finite, and relatively small, set of scout bees participating in the decision process. In ecological systems, when options have very similar qualities it may be preferable to choose more rapidly one of them instead of prolonging the discussion for a longer time, as this can mean an important waste of resources. Fluctuations facilitate this selection in a symmetric nest-site scenario, ensuring strong consensus for one of the available options. On the other hand, a deterministic treatment of the model will simply exhibit a decision dead-lock.

C. Absorbing phase transition

Although the analysis can be done with an arbitrary number of frequencies, here we simply consider, as previously, a simple two-site decision process with fractions (f_0, f_1, f_2). Moreover, a symmetrical model scenario assumes that $r_\alpha = r\ \forall\alpha$. It is also supposed that we work in the fully interdependent limit, where $\pi_\alpha = 0$ or $\lambda = 1$. Acknowledging the previous assumptions, the following equations can be found in this limit of the LES model:

$$\langle \dot{f}_{1,t} \rangle = \langle f_{0,t} \rangle \lambda \langle f_{1,t} \rangle - r \langle f_{1,t} \rangle, \tag{6}$$

$$\langle \dot{f}_{2,t} \rangle = \langle f_{0,t} \rangle \lambda \langle f_{2,t} \rangle - r \langle f_{2,t} \rangle. \tag{7}$$

From our previous explanations, we also know that $\langle f_{0,t} \rangle = 1 - (\langle f_{1,t} \rangle + \langle f_{2,t} \rangle)$. Now we can derive $\langle f_{0,t} \rangle$:

$$\langle \dot{f}_{0,t} \rangle = -\langle f_{0,t} \rangle \lambda (1 - \langle f_{0,t} \rangle) + r(1 - \langle f_{0,t} \rangle). \tag{8}$$

Let us introduce the density ρ of active bees, i.e. of bees dancing for any of the two sites, as $\rho = 1 - \langle f_{0,t} \rangle$. We can thus rewrite the previous equation as

$$\frac{d\rho}{dt} = \lambda(1-\rho)\rho - r\rho \tag{9}$$

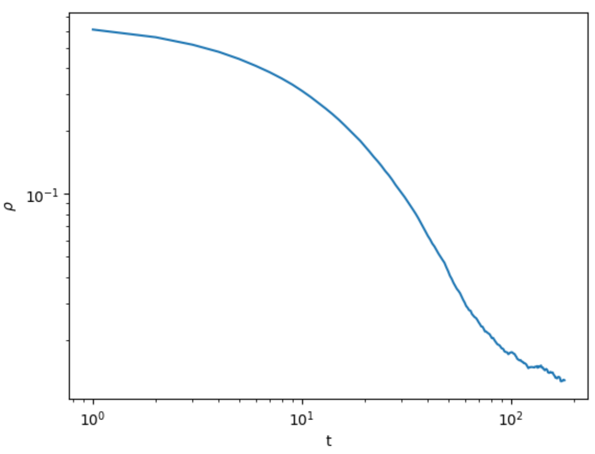

FIG. 2: Double logarithmic representation of the slow temporal evolution of ρ near the critical point $\tilde{\lambda} = 1$.

This equation shows us that our problem can be mapped to the renowned contact process [6], with which will share the same non-equilibrium critical behavior. The parameter ρ can be understood within this context as the fraction of agents promoting available sites, and it works as the order parameter of the transition. If we rescale time by r in the last equation, one can write:

$$\frac{d\rho}{d\tau} = (\widetilde{\lambda} - 1)\rho - \widetilde{\lambda}\rho^2 \qquad (10)$$

where $\widetilde{\lambda} = \lambda/r$ and $\tau = t/r$. This equation has two stationary points: $\rho^* = 0$ and $\rho^* = 1 - \widetilde{\lambda}^{-1}$, thus having the mean-field critical point at $\widetilde{\lambda}_c = 1$. As per the deterministic analysis in section IIA, the absorbing state is stable if $\widetilde{\lambda} < 1$ and the active state is stable if $\widetilde{\lambda} > 1$. Fig. 2 illustrates the slow temporal evolution of the system near the critical point. In figure 3, we can observe the stationary value of ρ versus $\widetilde{\lambda}$ for simulations together with the analytical result of the deterministic model. We can appreciate that the transition occurring exactly at $\widetilde{\lambda} = 1$ in the deterministic model is not occurring at the same location in stochastic simulations. Finite size effects give rise to slight deviations from this deterministic threshold.

It should be noted that the independent discovery parameter π would play the role of an external field in the LES model. If $\pi \neq 0$, activity is readily introduced in the system by this field. Thus a non-zero value of π disrupts the absorbed phase and thus the corresponding phase transition, moving the system away from criticality, akin to the contact process. In our case, under stationary conditions, the order parameter at $\widetilde{\lambda} = 1$ should scale as $\rho \sim \pi^{1/2}$ (see Fig.4). As the contact process, the LES model in a symmetric decision scenario belongs to the directed percolation university class [6].

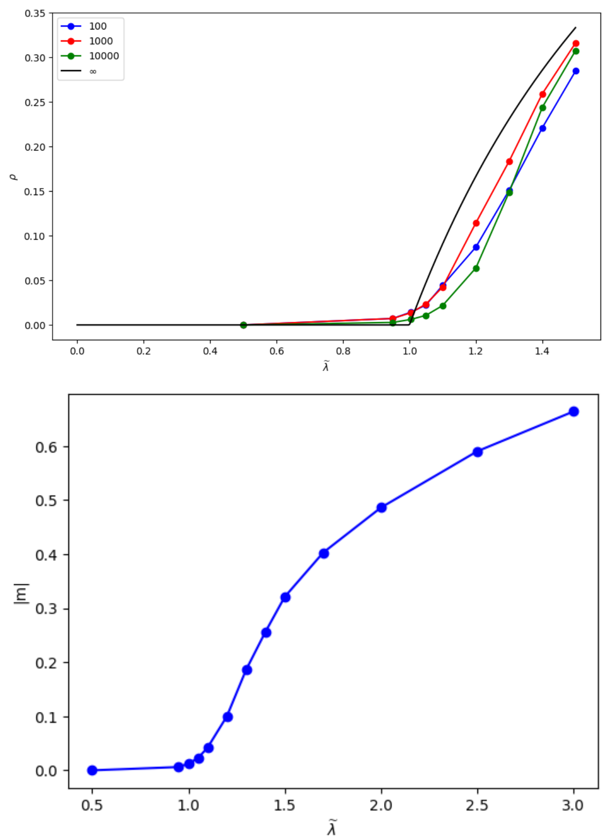

FIG. 3: Absorbing phase transition in the symmetric case ($r = 0.1$) and fully interdependent limit ($\pi = 0$). Top: Order parameter ρ versus $\tilde{\lambda}$. Bottom: Absolute value of the consensus parameter m versus $\tilde{\lambda}$.

D. Symmetry breaking induced by noise

In addition to the density of active particles, ρ, we can also define the *simple consensus* parameter $m = \langle f_1 \rangle - \langle f_2 \rangle$ comparing the fraction of bees dancing for each available site. From equations (6) and (7), we can obtain an equation for this *consensus* parameter as

$$\frac{dm}{dt} = m\left[\widetilde{\lambda}(1-\rho) - 1\right], \qquad (11)$$

which, exhibits a stable stationary solutions $m^* = 0$ when $\rho^* = 0$, and a marginally stable undetermined solution for m^* when $\rho^* = 1 - \widetilde{\lambda}^{-1}$. In the latter case, a noise-induced mechanism drives the system to a state in which one of the options disappears and the other takes it all such that $m^* = \rho^*$. This solution is currently being studied by Prof. Miguel's research group. At the lowest order, corrections due to finite size fluctuations can be described by Langevin stochastic differential equations with a multiplicative noise term of the form:

$$\frac{df_\alpha}{dt} = f_\alpha(\lambda f_0 - 1) + \frac{1}{\sqrt{N}} f_\alpha \left[\lambda f_0 + 1\right] \eta(t) \qquad (12)$$

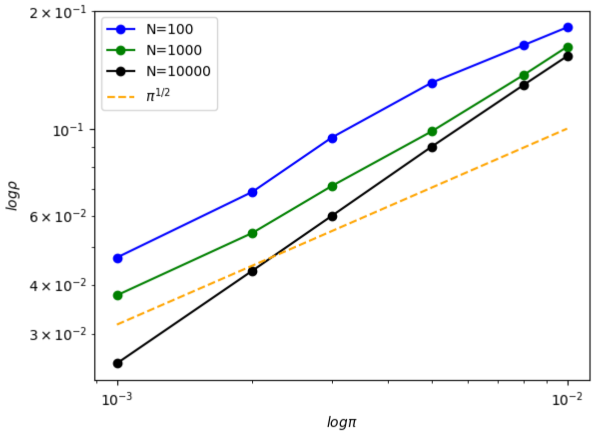

FIG. 4: Dancing activity ρ versus π. We can observe that for small values of π simulations yield a scaling consistent with the expected result $\pi^{1/2}$.

with $\alpha = 1, 2$, and where $\eta(t)$ represents an uncorrelated Gaussian noise of zero mean and unit variance. Thus, we can see that when $N \to \infty$, the deterministic equations reappear as noise becomes negligible. It is also crucial to note that this multiplicative noise makes it impossible for the disappeared option to come back once $f_\alpha = 0$. This is what leads to $m^* = \rho^*$, as can bee observed in the bottom panel of Fig. 3.

IV. CONCLUSIONS

We have delved into modeling honeybees behavior when it comes to finding a new nest site. This has been done by exploring the LES model. This model with further corrections helps us understand this problem, by taking into account different parameters. Quality, self-discovery probability and interdependence parameters are underlined. From the stochastic model, one can develop a deeper theoretical analysis including a master equation, its deterministic limit and lowest order noise-induced effects, or perform agent-based numerical simulations. For the former, we find the fixed points, which helps us understand the system dynamics. We have particularly centered our attention in a two equal quality site scenario in the fully interdependent limit. Here, we have shown the appearance of consensus at the onset of a critical absorbing phase transition induced by finite size effects. The symmetry breaking between the equivalent options is due to finite size fluctuations and it allow bees to take a decision more rapidly, instead of wasting resources while facing a potential consensus deadlock. Moreover, the absorbing phase transition observed, with no dancing activity for $\lambda < \lambda_c$ and an active state for $\lambda > \lambda_c$, can be mapped to the contact process, which falls in the universality class of directed percolation. Finite size effects introduce a pseudo-critical point slightly away from the deterministic prediction valid for an infinite system. Besides, we have seen that if $\pi \neq 0$, this parameter act as an external field which drives the system away from criticality. Although for values of π small enough we can find results compatible with a mean-field scaling law. Finally, we have briefly discussed how a multiplicative noise, which appears as one introduces lowest order corrections to the deterministic treatment, can explain the strong consensus observed in numerical simulations where we observe $m^* = \rho^*$, i.e. the activity of the system is fully concentrated in one site. This may well represent the decision making process of real insect swarms where a relatively small number of bees are involved in the consensus formation.

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Formació de consens per abelles en cerca de niu

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Resum: Aquest treball explora el procés de presa de decisions col·lectives en les abelles, utilitzant un model basat en agents (anomenat LES) per simular el comportament de les abelles a l'hora de seleccionar un niu. El model es centra en com els individus interactuen socialment però també actuen individualment, i han de prendre decisions conjuntament amb informació d'aquestes dues fonts. S'analitzen en major profunditat els casos on tots els llocs tenen la mateixa qualitat i la probabilitat de trobar un niu independentment és zero. Observem com tractar aquest problema de manera determinista o mitjançant simulacions numèriques ens dona diferents resultats degut a l'efecte de grandària finita, que són crucials a l'hora de consensuar una decisió en un temps prudencial. Estudiem també un cas particular que és anàleg al procés de contacte, on veiem una transició de fase fora d'equilibri. Això concorda amb la qualitat d'altres sistemes biològics complexos per a optimitzar el consens prop d'un punt crític. Finalment, remarquem com el soroll multiplicatiu és el que trenca la simetria en els casos anteriors.
Paraules clau: Presa de decisions, model d'agents, criticalitat, efectes de mida finita
ODSs: Aquest TFG està relacionat amb els Objectius de Desenvolupament Sostenible (SDGs)

Objectius de Desenvolupament Sostenible (ODSs o SDGs)

1. Fi de la es desigualtats		10. Reducció de les desigualtats	
2. Fam zero		11. Ciutats i comunitats sostenibles	
3. Salut i benestar		12. Consum i producció responsables	
4. Educació de qualitat	X	13. Acció climàtica	X
5. Igualtat de gènere		14. Vida submarina	
6. Aigua neta i sanejament		15. Vida terrestre	X
7. Energia neta i sostenible		16. Pau, justícia i institucions sòlides	
8. Treball digne i creixement econòmic		17. Aliança pels objectius	
9. Indústria, innovació, infraestructures			

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SUPPLEMENTARY MATERIAL