Phantom matter and vacuum energy: cosmological implications

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Abstract: While the vacuum energy and the cosmological constant are thought to be related, the exact connection is not well understood. In this work, we will consider the equation of state of the vacuum and other cosmic fluids potentially relevant for the description of the dark energy in the universe, in particular that of 'phantom matter', which is different from both dark matter and phantom dark energy. It is related to a form of false vacuum energy producing positive pressure, as ordinary matter (hence its name), but in contrast negative energy density. Some theoretical scenarios will be considered and also possible phenomenological implications.
Keywords: Cosmology, Cosmological Constant, Dark Energy, Inflation, Vacuum Energy.
SDGs: 4, 9.

I. INTRODUCTION

Since Albert Einstein introduced in 1917 the Cosmological Constant (CC) term Λ in the field equations of his General Relativity (GR) in [1], several interpretations of this term have been made related to a vacuum energy linked to some kind of Dark Energy (DE). While at the beginning it was conceived by Einstein as a way for the field equations to lead to a static universe solution, further observations published by Edwin Hubble in 1929 in [2] on the distances between galaxies related to cosmological redshifts, showing that the universe is expanding rather than static, prepared the ground to later reinterpret this constant as a term that triggers not only an expansion, but an accelerated expansion of the universe, constituting the prime form of DE of the universe.

The discovery of the Cosmic Microwave Background (CMB) in 1964, the focus on pure-baryonic models in the 70s trying to explain the formation of galaxies and the research focused on Cold Dark Matter (CDM) during the 80s led to the current cosmological model ΛCDM (see [3]) to describe the universe and to explain the observed data related to the CMB, Baryon Acoustic Oscillations (BAO) and Type Ia Supernovae (SNIa), among others such as the Hubble parameter $H(z)$ with respect to the redshift z and the Large Scale Structures (LSS) formation. This model assumes a constant value of the Λ term related to a Vacuum Energy Density (VED) of the form $\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$, being G Newton's gravitational constant.

However, the model has had difficulties facing problems such as the Cosmological Constant Problem (CCP) (see [4]) or the tensions related to the Hubble parameter and the growth of LSS. This is why in the last two decades diverse alternative models to the ΛCDM have entered the scene. Among them, the so-called Running Vacuum Model (RVM) and the composite DE models (see [4, 5]).

In this work we will briefly introduce those models and study some of the cosmological implications of a dynamical VED, such as the different kinds of Equations of State (EoS) involved, the profiles of the Hubble parameter and the vacuum and radiation energy densities in the early epochs, the new terms in Einstein's equations due to the Chern-Simons coupling of the Kalb-Ramond axion field to the gravitational anomalies and the 'phantom matter' (PM) and its paper explaining the LSS formation.

In what follows, natural units will be used ($\hbar = c = 1$).

II. MAIN COMPOSITE DE MODELS

We will begin by introducing three main models of composite DE as explained in [5]:

1. ΛXCDM: introduced in [6], it considers that DE consists of two dynamical components: a variable Λ, associated with a running VED, ρ_{vac}, that evolves with the expansion rate H and can behave as either quintessence (decreasing with the expansion) or phantom DE (increasing with the expansion) with an EoS departing from -1; and an extra component X. Thus the model has two EoS parameters, one for the VED and another for X, w_X.

2. wXCDM: used in [5], it is a simpler version of the previous model in which the running Λ is replaced with a dynamical component Y whose EoS may be quintessence-like, while X behaves phantom-like. This means $w_Y \gtrsim -1$ but $w_X \lesssim -1$ (see Fig. 2). However, X actually acts as PM, with energy density $\rho_X < 0$ and pressure $p_X > 0$ (in contrast to the usual DE), while $\rho_Y > 0$ and $p_Y < 0$. Component X acts above a transition redshift $z_t > 1$, whereas Y acts below z_t until the current time.

3. Λ_sCDM: analyzed in [7], it involves a Λ that transitions with a sudden change of sign from anti-de Sitter with $-\Lambda < 0$ to de Sitter with $+\Lambda > 0$ at a transition redshift z_t, so it is the composition of two phases of Λ. It is the particular case $w_X = w_Y = -1$ of the former.

Here we will focus on a general structure of the RVM through the expression of its VED.

III. THE DYNAMICAL VED

A. General expression of the dynamical VED in terms of the Hubble parameter

Starting our study from [8], we have a general Renormalization-Group-like form of differential equation for the dynamical VED associated with the RVM:

$$\frac{d\rho_\Lambda(\mu)}{d\ln(\mu^2)} = \frac{1}{(4\pi)^2}\sum_i \left[A_i M_i^2\mu^2 + B_i\mu^4 + C_i\mu^6 + \cdots\right] . \tag{1}$$

Solving it and setting $\mu^2 = aH^2 + b\dot{H}$, it leads to an expression of ρ_Λ in terms of powers of H and its derivatives:

$$\rho_\Lambda(H,\dot{H}) = a_0 + a_1\dot{H} + a_2H^2 + a_3\dot{H}^2 + a_4H^4 + \ldots . \tag{2}$$

Actual calculations within the context of Quantum Field Theory (QFT) in curved spacetime give the renormalized VED at low energy for the present universe as follows (see [4, 8]): $\rho_{vac}(H) \simeq \rho_{vac,0} + \frac{3\nu}{8\pi}(H^2 - H_0^2)m_{Pl}^2$, with $m_{Pl} = 1.22\times10^{19}$ GeV the Planck mass and $|\nu| \ll 1$ a calculable parameter (β-function of the VED running). When one considers the effect of higher powers of H, the QFT calculation yields the more general formula

$$\rho^\Lambda_{RVM}(H) = \frac{\Lambda(H)}{\kappa^2} = \frac{3}{\kappa^2}\left(c_0 + \nu H^2 + \alpha\frac{H^4}{H_I^2} + \cdots\right) , \tag{3}$$

where $\kappa^2 = 8\pi G = \frac{1}{M_{Pl}^2}$, $M_{Pl} = \frac{m_{Pl}}{\sqrt{8\pi}} \simeq 2.43\times10^{18}$ GeV, $0 < c_0$ and $0 < \alpha \lesssim 0.1$. In what follows, we will use this general expression and show how the parameter H_I and the H^4 term are connected with a new inflationary mechanism in the early universe.

B. Friedmann equations and solution for $H(a)$

Using $H = \frac{\dot{a}}{a}$, $\dot{H} = \frac{\ddot{a}a - \dot{a}^2}{a^2}$ and a generic RVM, which includes matter/radiation excitations with $p_m = w_m\rho_m$, where $w_m = \frac{1}{3}$ for radiation and $w_m = 0$ for matter, a running VED with EoS parameter $w_{RVM} = -1$ and the total pressure and energy density of matter/radiation and vacuum $p_{tot} = p_m + p_\Lambda = w_m\rho_m - \rho_\Lambda$, $\rho_{tot} = \rho_m + \rho_\Lambda$, into Friedmann equations without Gaussian curvature

$$\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho_m + 3p_m) + \frac{\Lambda}{3} , \\ \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho_m + \frac{\Lambda}{3} , \end{aligned} \tag{4}$$

we get the cosmological equations for a dynamical $\Lambda(H)$:

$$\begin{aligned} 3H^2 &= \kappa^2\rho_m + \Lambda(H) = \kappa^2\rho_{tot} , \\ -2\dot{H} - 3H^2 &= \kappa^2 p_m - \Lambda(H) = \kappa^2 p_{tot} . \end{aligned} \tag{5}$$

Adding both equations we get $-2\dot{H} = \kappa^2\rho_m(1 + w_m)$, which, substituted into the second Friedmann equation from (5), gives us $\frac{\rho_\Lambda}{\rho_m} = w_m - (1 + w_m)\left(\frac{3}{2}\frac{H^2}{\dot{H}} + 1\right)$, and both of this equations substituted in (3) lead to

$$\dot{H} + \frac{3}{2}(1 + w_m)H^2\left(1 - \nu - \frac{c_0}{H^2} - \alpha\frac{H^2}{H_I^2}\right) = 0 . \tag{6}$$

Using $\dot{H} = \frac{dH}{dt} = \frac{dH}{da}Ha$ and $\frac{H_I^2}{H^2} > \frac{\alpha}{1-\nu} \sim \alpha$ and ignoring the c_0 term against 1 and the powers of H for the early epochs, (6) can be analytically solved as

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{\frac{1}{2}}\frac{H_I}{\sqrt{1 + Da^{3(1+w_m)(1-\nu)}}} , \tag{7}$$

with $D = \left(\frac{H_I^2}{H_0^2}\frac{1-\nu}{\alpha} - 1\right)a_0^{-3(1+w_m)(1-\nu)} > 0$.

Besides, differentiating $\rho_\Lambda = -\rho_m - \rho_m(1+w_m)\frac{3}{2}\frac{H^2}{\dot{H}}$ and using $\ddot{H} = -\frac{\kappa^2}{2}\dot{\rho}_m(1 + w_m)$ from (5), we can get

$$-\dot{\rho}_\Lambda = \dot{\rho}_m + 3(1 + w_m)H\rho_m . \tag{8}$$

C. Behavior in the very early epochs: radiation and vacuum energy densities

From (7) we can deduce the behavior of H in the early epochs of the universe by setting $a \ll 1$. This gives $Da^{3(1+w_m)(1-\nu)} \sim Da^{3(1+w_m)} \ll 1$, which turns (7) into an approximately constant $H \simeq \frac{1}{\sqrt{\alpha}}H_I$ for a de Sitter phase. Moreover, in a radiation-dominated epoch with $w_m = \frac{1}{3}$, we get $Da^{3(1+w_m)(1-\nu)} = Da^{4(1-\nu)} \sim Da^4$, so

$$H(a) \simeq \frac{1}{\sqrt{\alpha}}\frac{H_I}{\sqrt{1 + Da^4}} . \tag{9}$$

This provides us a smooth transition from the early de Sitter era and a general description of $H(a)$ for $a \simeq 0$.

In order to connect this early universe epoch with the current era $a \simeq 1$, we use the equality point a_{eq} given by $\rho_\Lambda(a_{eq}) = \rho_r(a_{eq})$, i.e., the point at which inflation stops, and a rescaled form of a, $\hat{a} = \frac{a}{a_*}$, where a_* is defined as $D = \frac{1}{1-2\nu}a_{eq}^{-4(1-\nu)} = a_*^{-4(1-\nu)}$. Now, if we use this and set $\tilde{H}_I = \sqrt{\frac{1-\nu}{\alpha}}H_I$, (9) including the ν term turns into

$$H(\hat{a}) = \frac{\tilde{H}_I}{\sqrt{1 + \hat{a}^{4(1-\nu)}}} . \tag{10}$$

Setting $w_m = \frac{1}{3}$ for radiation in (5) gives us:

$$\begin{aligned} \rho_r &= -\frac{3}{2}\frac{\dot{H}}{\kappa^2} , \\ \rho_\Lambda &= \frac{3H^2}{\kappa^2} + \frac{3}{2}\frac{\dot{H}}{\kappa^2} . \end{aligned} \tag{11}$$

Defining $\rho_I = \frac{3H_I^2}{\kappa^2}$ and $\tilde{\rho}_I = \frac{3\tilde{H}_I^2}{\kappa^2}$ and differentiating (10), so $\dot{H} = -2\frac{(1-\nu)^2}{\alpha}H_I^2(\hat{a}^{4(1-\nu)} + 1)^{-2}\hat{a}^{4(1-\nu)}$, this leads to:

$$\begin{aligned} \rho_r(\hat{a}) &= \tilde{\rho}_I(1-\nu)\frac{\hat{a}^{4(1-\nu)}}{[1 + \hat{a}^{4(1-\nu)}]^2} , \\ \rho_\Lambda(\hat{a}) &= \tilde{\rho}_I\frac{1 + \nu\hat{a}^{4(1-\nu)}}{[1 + \hat{a}^{4(1-\nu)}]^2} . \end{aligned} \tag{12}$$

One can see that, for $a = 0$ (i.e., for $\hat{a} = 0$), $H(0) = \tilde{H}_I$ and $\rho_\Lambda(0) = \tilde{\rho}_I$, so $\tilde{H}_I$ and $\tilde{\rho}_I$ are the Hubble parameter and the VED at the start of the inflationary era.
The normalized energy densities from (12) with $\nu = 10^{-3}$ are shown in Fig. 1. It is clear that around $\hat{a} \simeq 1$ (i.e., at $a = a_{eq} \simeq a_*$), both densities are equal after a period of energy transfer from vacuum into radiation and matter preceded by a constant, maximal vacuum energy epoch.

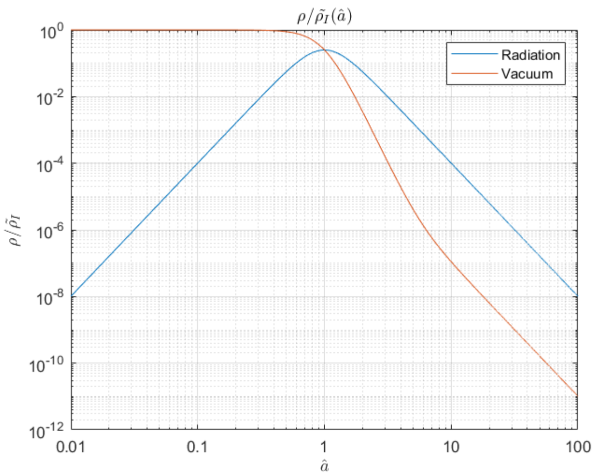

FIG. 1. Normalized densities ρ_r, ρ_Λ from (12), with $\nu = 10^{-3}$.

VED decay is then accompanied by a radiation epoch with a radiation energy density that, for $\hat{a} \gg 1$ (i.e., $a \gg a_{eq}$), behaves as $\rho_r(\hat{a}) \simeq \tilde{\rho}_I(1-\nu)\hat{a}^{-4(1-\nu)}$, so we (approximately) recover the $\rho_r \propto a^{-4}$ standard behavior.

D. Connection with the current era and brief comparison with Starobinsky inflation

We can see from (7) that in the RVM universe there is no initial singularity for $a = 0$ if $\alpha > 0$, but we cannot do $\alpha \to 0$ to connect the early universe solution to the current era. Instead, we set $\alpha = 0$ in (6) so the RVM physics from the H^4 term for the early universe erases, and assume $c_0 \ll H^2$ for a matter/radiation dominance era. For such case, the only non-trivial solution is

$$H(a)^{\text{matter/radiation dominance}}_{\alpha=0} = H_0 \left(\frac{a}{a_0}\right)^{-\frac{3}{2}(1+w_m)(1-\nu)}, \tag{13}$$

which, for $\nu = 0$, gives the standard ΛCDM description with a initial singularity at $a = 0$.
This shows that, under the standard RVM, the connection between a non singular solution in the early epochs driven by the H^4 term and a singular solution for the current era dominated by the H^2 term is made through a dynamical evolution of the dominance of this two terms from an H^4 dominance to an H^2 dominance and a current CC corrected by a νH^2 term still evolving nowadays.

In the stringy formulation, contributions to the current era CC arise from the condensation of weak gravitational waves (GW) that leads to a phase transition that affects the smoothness of the evolution of the universe. This condensation and gravitational anomalies give rise to the H^4 term of the VED and will be discussed later.
We end this section with a short comparison between RVM inflation and Starobinsky inflation as discused in [8] and [9]. Starobinsky model is based on the Einstein-Hilbert (EH) action with signature $(+,-,-,-)$

$$S = \int \sqrt{-g}\left(-\frac{R}{16\pi G} + \tilde{b}R^2\right) d^4x + S_{matter}\,, \tag{14}$$

with $\tilde{b} = \frac{m_{Pl}^2}{6M_{SC}^2}$ and M_{SC} a mass-dimension parameter.
The variation of the action in (14) with respect to the metric gives the field equations

$$G_{\mu\nu} - 32\pi G\tilde{b}\left(\nabla_\mu\nabla_\nu R + g_{\mu\nu}\Box R + RR_{\mu\nu} - \frac{g_{\mu\nu}}{4}R^2\right) = 8\pi G T_{\mu\nu}\,, \tag{15}$$

where $T_{\mu\nu} = -pg_{\mu\nu} + (\rho + p)U_\mu U_\nu$ is the energy-momentum tensor for a single matter ideal fluid-like component. Assuming a relativistic early component with $p_r = \frac{1}{3}\rho_r$ and writing the $(\mu,\nu) = (0,0), (i,j)$ equations of (15), one obtains, in the spatially flat FLRW metric,

$$2H^2 + \dot{H} + 48\pi G\tilde{b}(2\dddot{H} + 14\ddot{H}H + 24H^2\dot{H} + 8\dot{H}^2) = 0\,. \tag{16}$$

If $\tilde{b} = 0$, then (15) becomes the usual Einstein's field equations and (16) turns into $2H^2 + \dot{H} = 0$, so we get $a(t) = a_0\sqrt{1 + 2H_0(t - t_0)} \sim t^{1/2}$ as for a pure radiation era. If $\tilde{b} \neq 0$, (16) cannot be solved analytically, neither by a $H = $ const. solution. However, we can solve it for a initial phase of constant $\dot{H}$, which is essentially its behavior until near the end of the inflationary era (as can be seen from the numerical solution of (16), see [8, 9]). Thus, neglecting $\frac{\dot{H}}{H^2} \ll 1$ and higher derivatives terms, we get $576\pi G\tilde{b}\dot{H} = -1$, whose solution is $H(t) = H_0 - \frac{m_{Pl}^2}{576\pi\tilde{b}}(t - t_0) = H_I - \frac{m_{Pl}^2}{576\pi\tilde{b}}t$, leading to $a(t) = a_0 e^{H_I(t-t_0)} e^{-\frac{M_{SC}^2}{192\pi}(t^2 - t_0^2)}$. $\tilde{b} > 0$ is needed to have a stable inflationary solution whose phase is extinguished at $t_f = 192\pi\frac{H_I}{M_{SC}^2} \propto \tilde{b}$.
Unlike the RVM, this solution does not connect the inflationary era with the radiation era analytically, but the model provides a transition to it through a reheating stage due to a final period of oscillating H. Notice that while the H^4 terms are generated in the stringy RVM and induce inflation from a period of $H = $ const., in the Starobinsky case these terms are missing and inflation must be produced by a period of $\dot{H} = $ const.. Finally, in the Starobinsky model no dynamical term of the inflationary phase is left to influence the late universe. In contrast, after RVM inflation occurs, there are still H^2 terms in (3) that make the current DE dynamical. Clearly, they are very different inflationary mechanisms.

IV. COSMOLOGICAL IMPLICATIONS

A. Kalb-Ramond field for stringy axions and gravitational Chern-Simons term

Following with the discussion in [8], and later with that of [10], some string-inspired RVM features must be introduced in order to deduce cosmological implications. The first one is the existence of stiff matter, composed of a Kalb-Ramond (KR) axion field, in order to embed the RVM formalism into the string theory, and other stringy axions. Such axions lead to a stiff matter dominated pre-inflationary era that remains undiluted during the inflationary era, and they couple to the gravitational anomalies present in the early phases of the universe through a CP-violating coupling to the gravitational Chern-Simons terms.

The KR axion field is represented by an antisymmetric tensor field $B_{\mu\nu}$ and a pseudoscalar massless excitation field $b(x)$ in a 4-dimensional spacetime. The coupling of this axion field to the early universe gravitational anomalies is described through the effective action

$$
\begin{aligned}
S_B^{eff} &= \int \sqrt{-g}\left[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b \partial^\mu b\right] d^4x \\
&+ \int \sqrt{-g}\left[\sqrt{\frac{2}{3}}\frac{\alpha'}{96\kappa} b(x) R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} + \cdots\right] d^4x\,,
\end{aligned}
\tag{17}
$$

where $\alpha' = \frac{1}{M_S^2}$, with M_S being the string mass scale, $\tilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\pi}R^{\lambda\pi}{}_{\rho\sigma}$ the dual of the Riemann tensor, $\varepsilon_{\mu\nu\rho\sigma} = \sqrt{-g}\epsilon_{\mu\nu\rho\sigma}$ the 4-dimensional contravariant Levi-Civita tensor density and the last term in the integral is the gravitational Chern-Simons term, which accounts for the CP-violating coupling to the gravitational anomalies.

We can see from (17) that, when the interaction of the b field with the gravitational anomalies in the early universe vanishes for a FLRW spacetime, we have the action $S^b = \int \sqrt{-g}\frac{1}{2}\partial_\mu b \partial^\mu b d^4x$, so if $T^b_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S^b(b, g_{\alpha,\beta})}{\delta g^{\mu\nu}}$,

$$
T^b_{\mu\nu} = \partial_\mu b \partial_\nu b - \frac{1}{2}g_{\mu\nu}\left(\partial_\alpha b \partial^\alpha b\right)\,, \tag{18}
$$

which is the stress tensor for the massless KR axions that, according to [11], have a stiff matter EoS $p = \rho$.

B. The Cotton tensor in Einstein's equations and primordial gravitational waves and anomalies

If we take into account the interaction of the b field with the gravitational anomalies, then the variation of the action in (17) yields a new conserved stress tensor:

$$
\kappa^2\tilde{T}^{\mu\nu}_{b+gCS} = \sqrt{\frac{2}{3}}\frac{\alpha'\kappa}{12}C^{\mu\nu} + \kappa^2 T^{\mu\nu}_b \implies \partial_\mu \tilde{T}^{\mu\nu}_{b+\Lambda+gCS} = 0\,, \tag{19}
$$

where the first term is obtained from the variation of the Chern-Simons term and the Cotton tensor is

$$
\begin{aligned}
C^{\mu\nu} = &-\frac{1}{2}v_\sigma\left(\varepsilon^{\sigma\mu\alpha\beta}R^\nu{}_{\beta;\alpha} + \varepsilon^{\sigma\nu\alpha\beta}R^\mu{}_{\beta;\alpha}\right) \\
&-\frac{1}{2}v_{\sigma\tau}(\tilde{R}^{\tau\mu\sigma\nu} + \tilde{R}^{\tau\nu\sigma\mu})\,,
\end{aligned}
\tag{20}
$$

with $v_\sigma = \partial_\sigma b = b_{;\sigma}$ and $v_{\sigma\tau} = v_{\tau;\sigma} = b_{;\tau;\sigma}$. $g_{\mu\nu}C^{\mu\nu} = 0$, so it is gravitationally traceless, and $C^{\mu\nu}{}_{;\mu} \neq 0$, which implies that the stress tensor in (18) is not conserved due to the exchange of energy during the interaction. For a flat FLRW spacetime, $C^{\mu\nu} = 0$, so the tensor $\tilde{T}_{b+gCS}$ in (19) becomes the tensor T_b in (18).

In the presence of this gravitational anomalies in the early universe, the resulting Einstein's field equations are

$$
R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = \Lambda(H)g^{\mu\nu} + \sqrt{\frac{2}{3}}\frac{\alpha'\kappa}{12}C^{\mu\nu} + \kappa^2 T^{\mu\nu}_b\,, \tag{21}
$$

and, as a result of the CP-violating primordial GW-condensation perturbing the FLRW background, the gravitational anomaly term yields, as explained in [8, 11],

$$
\langle R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}\rangle = \frac{16}{a^4}\kappa^2\int\frac{1}{(2\pi)^3}\frac{H^2}{2k^3}k^4\Theta d^3k\,, \tag{22}
$$

with $\langle ... \rangle$ the condensation integrating over the momentum k with a cutoff at an UV momentum scale $\mu \lesssim k^{-1}$, $\mu \sim 10^3 M_S$ and $\Theta = \sqrt{\frac{2}{3}}\frac{\alpha'\kappa}{12}H\dot{b}$ assumed small.

This description allows us to write $b(t) = b(0) + Ct$, with $C = \sqrt{2\epsilon}M_{Pl}H$, and this GW-condensation in an effective action involving $\langle \bar{b}R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}\rangle$, assuming $|b(0)| \gtrsim 10M_{Pl}$ and $\epsilon \sim 10^{-2}$, leads to $\Lambda > 0$ in (21) approximated by a constant during the inflationary period, where $H \simeq H_I = \text{const.}$, which induces inflation.

Then, $\kappa^2\tilde{T}^{\mu\nu}_{b+\Lambda+gCS} = \kappa^2\tilde{T}^{\mu\nu}_{b+gCS} + \Lambda g^{\mu\nu}$ and, as shown in [10, 11], it ensures a positive-definite VED during the stringy inflationary universe, with contributions from the axions and the condensates, of the form

$$
\begin{aligned}
\rho_{total} &\simeq 3\kappa^{-4}\left[\nu(\kappa H)^2 + \alpha(\kappa H)^4\right] \\
&\simeq 3\kappa^{-4}\left[c_1 + c_2\right](15)^4(\kappa H_I)^2 > 0\,,
\end{aligned}
\tag{23}
$$

where $c_1 = -0.34\times 10^{-5}\epsilon$, $c_2 \simeq 2,8\times 10^{-5}|\bar{b}(0)|\kappa\sqrt{\epsilon}$.

C. Resulting EoS for phantom vacuum and phantom matter

We focus now on [10] to use the previous description to calculate the vacuum p_{total} and ρ_{total} with all its contributions $p_{total} = p^b + p^{gCS} + p^{condensate}$ and $\rho_{total} = \rho^b + \rho^{gCS} + \rho^{condensate}$. As we have seen, the ground state of the KR axion field b satisfies a stiff EoS $p^b = \rho^b$. The gravitational tracelessness of the Cotton tensor leads to a radiation-like relation of its contribution $p^{gCS} = \frac{1}{3}\rho^{gCS}$, where the first term is associated to C_{ii} and the second to C_{00}, and the condensate satisfies a de Sitter EoS $p^{condensate} = -\rho^{condensate}$. As shown in [11], $\rho^{gCS} < 0$, hence $p^{gCS} < 0$, and $\rho^b = -\frac{2}{3}\rho^{gCS}$, so:

$$\begin{aligned} \rho^b + \rho^{gCS} &= -\frac{2}{3}\rho^{gCS} + \rho^{gCS} = \frac{1}{3}\rho^{gCS} < 0\,, \\ p^b + p^{gCS} &= \rho^b + \frac{1}{3}\rho^{gCS} = -\frac{2}{3}\rho^{gCS} + \frac{1}{3}\rho^{gCS} = -\frac{1}{3}\rho^{gCS} \\ &= -(\rho^b + \rho^{gCS}) > 0\,, \end{aligned} \tag{24}$$

This state corresponds to the red line in Fig. 2, and it is transitory until the standard dynamical vacuum state of the RVM (green line) with $\rho_{total} > 0$ is reached.

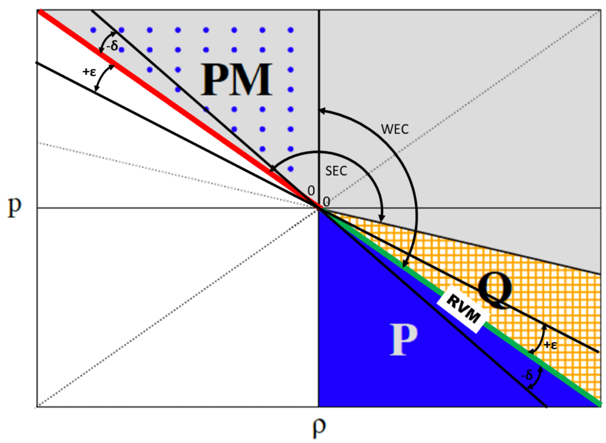

FIG. 2. Possible energy conditions for the cosmic fluids, with $w_X = -1 - \delta$ and $w_Y = -1 + \epsilon$. The standard WEC and SEC regions are shown, together with PM (phantom matter), P (phantom DE) and Q (quintessence). Adapted from [5].

The anomaly terms contribute negatively to the energy density over the free b terms. This reminds us of phantom matter (PM in Fig. 2), a substance X with $p > -\rho$ and $\rho < 0$ introduced in [6] that satisfies the strong energy conditions (SEC), $\rho + p \geq 0$, $\rho + 3p \geq 0$. As explained in detail in [5, 6], the existence of PM bubbles as a transitory phantom vacuum when approaching a de Sitter era could trigger a higher rate of LSS at relatively large redshifts, as shown in the data discussed in [5].

V. CONCLUSIONS

- In this work we have studied the description of the dynamical Vacuum Energy Density within the context of the Running Vacuum Model and explained its implications for the dynamics of the universe.
- We have discussed the dominance of the H^4 (very early universe) and H^2 (late universe) terms and compared the RVM with Starobinsky inflation.
- We have shown how the stringy version of the RVM is introduced through an axion field and its interaction with the primordial GW-condensate and gravitational anomalies, leading to a new conserved stress tensor, Einstein's field equations and a total energy density for the early era of the universe.
- The description of this gravitational framework has led us to an EoS that has motivated the notion of 'phantom matter' in the very early epochs and its possible implications on structure formation.

ACKNOWLEDGMENTS

I would like to thank my advisor, Dr. Joan Solà, for letting me know about this field of study, for his guidance during my learning on the subject and for making me enjoy cosmology even more. Thanks also to my parents for his support, and to Ana for this great years together.

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Matèria fantasma i energia del buit: implicacions cosmològiques

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Resum: Tot i que es creu que l'energia del buit i la constant cosmològica estan relacionades, la connexió exacta no s'entén del tot. En aquest treball considerarem l'equació d'estat del buit i d'altres fluids còsmics potencialment rellevants per a la descripció de l'energia fosca a l'univers, en particular la de la 'matèria fantasma', que és diferent tant de la matèria fosca com de l'energia fosca fantasma. Està relacionada amb una forma d'energia de buit fals que produeix pressió positiva, com la matèria ordinària (d'aquí el seu nom), però en canvi té densitat d'energia negativa. Es consideraran alguns escenaris teòrics i també possibles implicacions fenomenològiques.
Paraules clau: Cosmologia, Constant Cosmològica, Energia Fosca, Inflació, Energia del Buit.
ODSs: Aquest TFG està relacionat amb els Objectius de Desenvolupament Sostenible (SDGs) 4.3 i 9.5.

OBJECTIUS DE DESENVOLUPAMENT SOSTENIBLE (ODSS O SDGS)

1. Fi de la pobresa		10. Reducció de les desigualtats	
2. Fam zero		11. Ciutats i comunitats sostenibles	
3. Salut i benestar		12. Consum i producció responsables	
4. Educació de qualitat	X	13. Acció climàtica	
5. Igualtat de gènere		14. Vida submarina	
6. Aigua neta i sanejament		15. Vida terrestre	
7. Energia neta i sostenible		16. Pau, justícia i institucions sòlides	
8. Treball digne i creixement econòmic		17. Aliança pels objectius	
9. Indústria, innovació i infraestructures	X		

El contingut d'aquest TFG, part d'un grau universitari de Física, es relaciona amb l'ODS 4, i en particular amb la fita 4.3, ja que contribueix a l'educació a nivell universitari. També es pot relacionar amb l'ODS 9, fita 9.5, perquè promou la millora de la investigació científica i fomenta la innovació i l'augment de treballadors en recerca.

SUPPLEMENTARY MATERIAL

The image shown below has been extracted from [5], and it is a compendium of plots showing the constraints at 68% and 95% Confidence Intervals of pairs of characteristic parameters for various models of ΛCDM and composite DE, as well as their individual one-dimensional distributions, as obtained with the fitting numerical analyses explained in [5] with the data set CMB+CCH+SNIa+SH0ES+BAO+$f\sigma_{12}$. In Table 1 of [5], one can find the mean values, uncertainties at 68% Confidence Intervals and best-fit values for these parameters. Here we only show the full triangle contour plot as a visual representation of the fitting analyses in the corresponding parameter spaces.

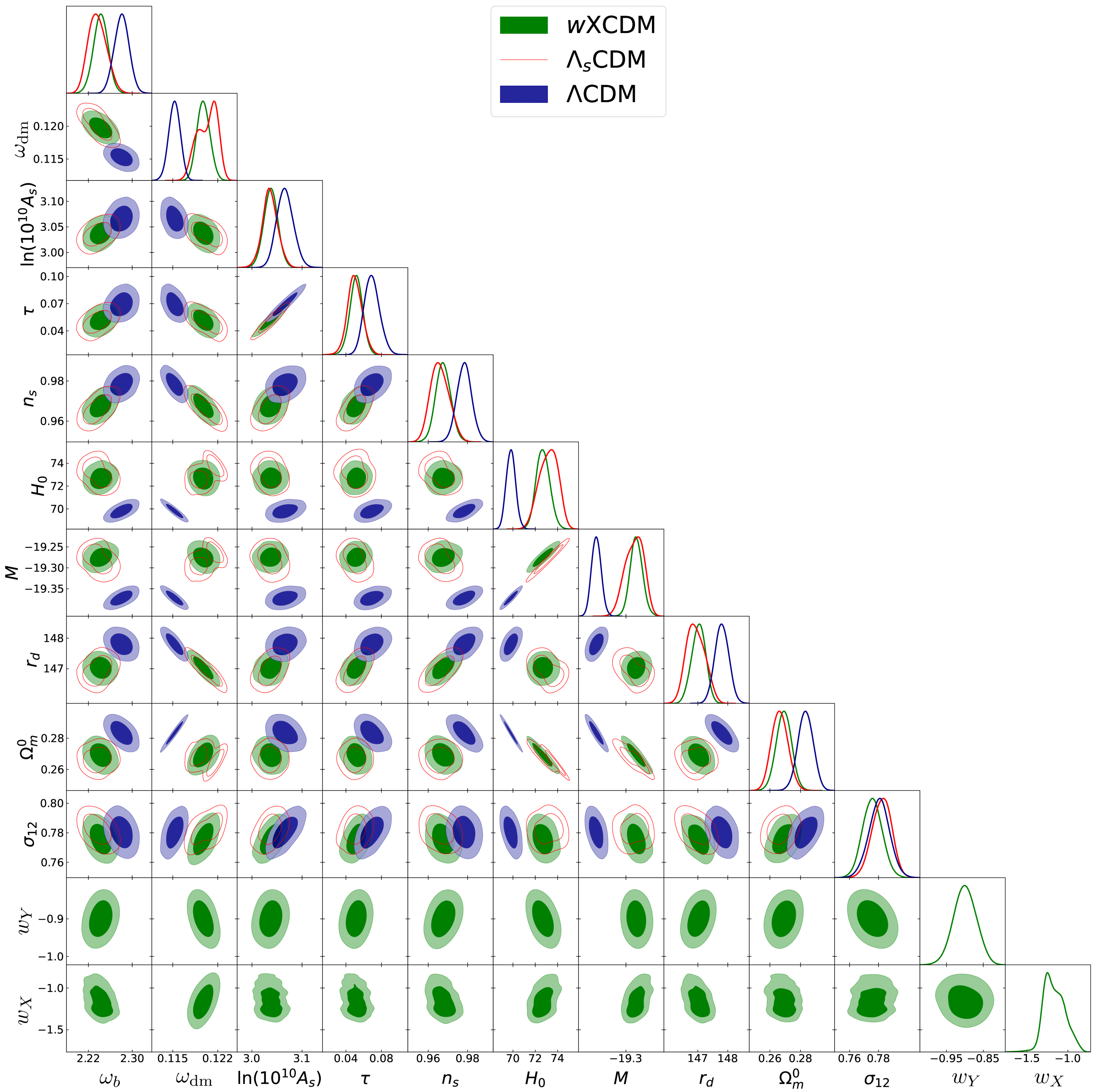

FIG. 3. Triangle of plots showing individual one-dimensional distributions and the 68% and 95% Confidence Intervals for pairs of characteristic parameters of the ΛCDM, Λ_sCDM and wXCDM models, such as the current Hubble parameter H_0 (given in $km/s/Mpc$), the σ_{12} tension (related to the measurement of the growth of LSS) or the EoS parameters w for the substances X and Y, baryons and dark matter.