Buchdahl Limits Beyond General Relativity

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Abstract: Buchdahl's theorem in four dimensions establishes a universal bound on the degree of compactness of a static star of radius R. The bound is saturated by constant-density stars with infinite central pressure and in terms of the stars' Schwarzschild radius reads $R \geq \frac{9}{8} r_{\rm S}$. We show that the theorem holds for D-dimensional Einstein gravity and that the limit becomes $R \geq \left[\frac{(D-1)^2}{4(D-2)}\right]^{1/(D-3)} r_{\rm S}$. We show that the bound is modified when Einstein gravity is corrected by a quadratic higher-curvature term of the Gauss-Bonnet type. In that case, the degree of compactness can be significantly reduced with respect to the Einsteinian result.
Keywords: Buchdahl Theorem, General Relativity, Einstein-Gauss-Bonnet Gravity
SDGs: This work is related to SDG number 4: Quality Education

I. INTRODUCTION

In Einstein's theory of General Relativity (GR), gravity is described as a manifestation of the curvature of space-time produced by the presence of mass and energy. GR predicts a remarkable amount of new phenomena with respect to Newton's theory. Notably striking are gravitational collapse and black hole formation [1].

Black holes are regions of space-time where gravity is so intense that nothing can escape from them, not even light. They usually form when a large amount of mass is confined in a sufficiently small region, causing it to gravitationally collapse [2]. This typically occurs at the end of the life of very massive stars. Black holes are characterized, according to GR, by a central singularity and an event horizon. The singularity is a region where the curvature of space-time diverges, and the theory ceases to be valid. On the other hand, the event horizon is a boundary beyond which nothing can escape once crossed.

Gravitational collapse is the process by which some body, such as a star, loses the balance between the outward pressure (caused by nuclear fusion or quantum degeneracy pressure in the core) and the gravitational force pulling inwards. When this balance breaks down dominated by gravity, the body collapses. This gravitational collapse can lead to the formation of a white dwarf, a neutron star, or a black hole, depending on the star mass.

This naturally leads to a different question: how compact can a star of mass M become before the gravitational pull becomes unsustainable, forcing it to undergo collapse? This question is (at least partially) addressed by Buchdahl's limit, which establishes the maximum possible degree of compactness for a static and spherically symmetric perfect-fluid neutral and isotropic star compatible with hydrostatic equilibrium [3]. In four-dimensional GR, Buchdahl's theorem shows that for a star of mass M, its radius R must satisfy

$$R \geq \frac{9}{8} r_{\rm S} \,, \tag{1}$$

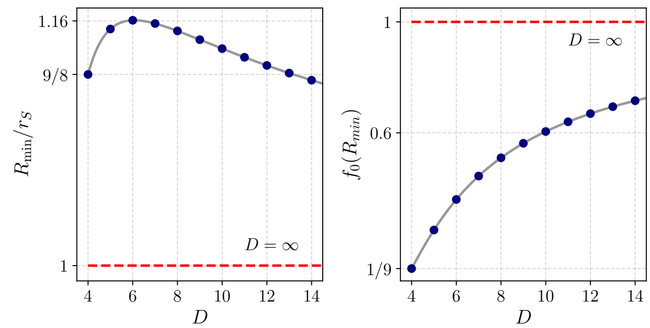

FIG. 1: (Left) Buchdahl's minimum radius as a function of the spacetime dimension. A maximum is achieved in $D = 6$. (Right) Metric function at the Buchdahl-type star's surface $f_0(R_{\rm min})$ as a function of the number of spacetime dimensions. This function is monotonically increasing. Both functions asymptotically approach 1 as $D \to \infty$.

where $r_{\rm S} = 2GM/c^2$ is the Schwarzschild radius. The result can be shown to hold generally provided the above assumptions are met, as long as the density of the star decreases monotonically as we move outwards from the star center. The "Buchdahl limit", in which the inequality is saturated, can be shown to correspond to constant-density stars for which the central pressure diverges.

In this TFG we address the question of how Buchdhal's limit changes beyond $D = 4$ GR. Firstly, we show that the above results can be generalized to D-dimensional GR provided (1) is modified to

$$R \geq \left[\frac{(D-1)^2}{4(D-2)}\right]^{\frac{1}{(D-3)}} r_{\rm S} \,, \tag{2}$$

where $r_{\rm S}$ is the D-dimensional Schwarzschild radius— see Fig. 1. We argue that the inequality is generically satisfied for all D-dimensional stars under the aforementioned conditions and that saturation takes place for constant-density stars with infinite central pressure. We also compare the pressure profiles of more general constant-density D-dimensional stars, finding qualitative agreement with the $D = 4$ case.

We also study how the Buchdahl limit is modified for constant-density stars in the case of Einstein-Gauss-Bonnet gravity. This is a theory which corrects GR by introducing a higher-curvature correction to the Einstein-Hilbert action weighted by a new coupling. We find that varying this parameter it is possible to achieve stars whose radius, relative to the corresponding gravitational radius of the theory, can be made smaller than the GR minimum radius, thus allowing for relatively more compact stars.

Notation. Throughout the TFG we use an abbreviated notation for several physically relevant quantities associated to the D-dimensional stars. Let M, ρ and p be the ADM mass, the density and the pressure of the star. We define the spacetime mass contained within an area of coordinate radius r as

$$m(r) = \frac{1}{\Omega_{D-2}} \int_0^r \mathrm{d}x\, x^{D-2} \rho(x)\,, \tag{3}$$

and the average density of the star as

$$\bar{\rho}(r) = \frac{(D-1)\, m(r)}{\Omega_{D-2}\, r^{D-1}}\,. \tag{4}$$

We introduce the following reduced quantities

$$\{\mathsf{m}, \mathsf{M}\} \equiv \frac{8\pi G}{(D-2)\Omega_{D-2}} \{m, M\}\,, \tag{5}$$

$$\{\varrho, \bar{\varrho}, \mathsf{p}\} \equiv \frac{8\pi G}{(D-2)(D-1)} \{\rho, \bar{\rho}, p\}\,, \tag{6}$$

which we will often use.

II. EINSTEIN EQUATIONS IN D DIMENSIONS

Let us consider first the case of D-dimensional Einstein gravity. The Einstein field equations read

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}\,, \tag{7}$$

where G is the Newton constant, c the speed of light, $g_{\mu\nu}$ is the spacetime metric, $R_{\mu\nu}$ the Ricci tensor associated with that metric, and R its Ricci scalar. The matter stress tensor satisfies the conservation condition $\nabla_\mu T^{\mu\nu} = 0$.

In order to make progress, let us consider a general static and spherically symmetric ansatz

$$\mathrm{d}s^2 = -N(r)^2 f(r)\, \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\, \mathrm{d}\Omega^2_{D-2}\,, \tag{8}$$

where $\mathrm{d}\Omega^2_{D-2}$ represents the line element of a unit $(D-2)$-sphere. This metric consists of three distinct parts: the temporal component g_{tt}, the radial component g_{rr} and the angular sector associated with the unit sphere of dimensions $(D-2)$. $f(r)$ and $N(r)$ are functions of the radial coordinate and are to be determined by solving Einstein equations, together with the conservation law.

According to Birkhoff's theorem, any spherically symmetric solution to the Einstein equations in vacuum must be static. Therefore, even if the metric functions were considered to also depend on t, that is, $f(r,t)$ and $N(r,t)$, the symmetry and field equations would imply that the dependence on time would disappear for vacuum solutions. This is not the case in the presence of matter, and here we consider a static spacetime from the onset. The independent non-vanishing components of (7) read

$$tt: \quad \frac{\mathrm{d}}{\mathrm{d}r}\left(r^{D-3}(1-f)\right) = \frac{16\pi G}{D-2} \frac{r^{D-2}}{N^2 f} T^{tt}\,,\,, \tag{9}$$

$$rr: \quad \frac{1}{rN}\frac{\mathrm{d}N}{\mathrm{d}r} = \frac{1}{2}\frac{16\pi G}{D-2}\left(\frac{1}{N^2 f^2} T^{tt} + T^{rr}\right)\,. \tag{10}$$

A. Exterior solution

First, we can solve the case of the metric in the exterior of the star, where $T^{\mu\nu} = 0$. We find that $N(r)$ is constant, so $N(r) = 1$ to guarantee asymptotic flatness. For $f(r)$ we have:

$$f(r) = 1 - \frac{2\mathsf{M}}{r^{D-3}}\,. \tag{11}$$

where M is an integration constant proportional to the mass of the solution. The D-dimensional Schwarzschild radius is given in terms of this as $r_{\mathrm{S}} = (2\mathsf{M})^{1/(D-3)}$.

B. Interior solution

In order to obtain the interior solution, the star must be modeled as a continuous distribution of matter. In this context, the energy-momentum tensor plays a fundamental role. A common and physically relevant idealization is to describe the matter content as a perfect fluid, an idealized fluid with no viscosity and no heat conduction. The corresponding energy-momentum tensor is given by

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu} \tag{12}$$

where $\rho = \rho(r)$ is the energy density, $p = p(r)$ the pressure, and u^μ the four-velocity of the fluid, which satisfies $u^\mu u_\mu = -1$. Assuming a static fluid configuration and using the metric (8) we can find the only non-zero component of the four velocity, $u^\mu = \frac{1}{N(r)\sqrt{f(r)}}\delta^\mu_t$. The non-zero components of the energy-momentum tensor $T^{\mu\nu}$ are:

$$T^{tt} = \frac{\rho N^2}{f}, \quad T^{rr} = \frac{p}{f}, \quad T^{\theta_i\theta_i} = p\, r^2 \prod_{j=1}^{i-1} \sin^2(\theta_j)\,. \tag{13}$$

Given this, we can tackle the problem of solving Einstein equations in the interior of the star under certain assumptions.

First, we can use the components $T^{\mu\nu}$ and plug them into the equations (9) and (10). Doing this, we find the equations for $f(r)$ and $N(r)$:

$$f(r) = 1 - \frac{2\mathsf{m}(r)}{r^{D-3}} = 1 - 2\bar{\varrho}r^2\,, \tag{14}$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\log N(r) = (D-1)\frac{\varrho + \mathsf{p}}{f}r\,. \tag{15}$$

On the other hand, using the conservation equation $\nabla_\mu T^{\mu\nu} = 0$, together with the components in (13), we derive the hydrostatic equilibrium (structure) equation:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -(\rho + p)\left(\frac{\mathrm{d}\log N}{\mathrm{d}r} + \frac{1}{2}\frac{\mathrm{d}\log f}{\mathrm{d}r}\right)\,. \tag{16}$$

Finally, the system is closed by specifying an equation of state of the form

$$p = p(\rho)\,. \tag{17}$$

1. TOV equation

We are now in a position to derive the generalization of the Tolman-Oppenheimer-Volkoff (TOV) equation [4] to a D-dimensional spacetime. Substituting equations (14) and (15) into equation (16), we find that it takes a form which we can later compare with its Newtonian counterpart.

$$\frac{\mathrm{d}\mathsf{p}}{\mathrm{d}r} = -(\varrho + \mathsf{p})\,r\,\frac{(D-3)\bar{\varrho} + (D-1)\mathsf{p}}{1 - 2\bar{\varrho}r^2} \tag{18}$$

$$\frac{\mathrm{d}\mathsf{p}}{\mathrm{d}r} = -\frac{(D-3)\varrho(r)\mathsf{M}}{r^{D-2}} \qquad \text{(Newton)} \tag{19}$$

$$\frac{\mathrm{d}\mathsf{p}}{\mathrm{d}r} = -\frac{(D-3)\varrho(r)\mathsf{m}(r)}{r^{D-2}}\left[1 + \frac{\mathsf{p}(r)}{\varrho(r)}\right]\left[\frac{1}{1 - \frac{2\mathsf{m}(r)}{r^{D-3}}}\right]\left[1 + \frac{(D-1)}{(D-3)}\frac{\mathsf{p}(r)}{\bar{\varrho}(r)}\right] \qquad \text{(Einstein)} \tag{20}$$

We observe that the pressure is a monotonically decreasing function of the radial coordinate as one moves away from the center of the star toward the surface. Just like in the $D = 4$ case, all relativistic corrections to the Newtonian expression are larger than 1, which means that hydrostatic equilibrium is harder to achieve in D-dimensional Einstein gravity than in its Newtonian counterpart. The first and third corrections are trivially greater than 1, but let us have a closer look to the second. Since $r = 0$ is a regular point of the manifold, spacetime must be locally flat there. This implies that the quotient between the length of a circle near that point and its proper radius must approach 2π as $r \to 0$. As a consequence, the mass function must go to zero at the star center in a way such that $\mathsf{m}(r)/r^{D-3} \overset{(r\to 0)}{\longrightarrow} 0$. By analyzing the pressure profile inside the star we can also argue that the radius must follow the relation $r^{D-3} > 2\mathsf{m}(r) \quad \forall r < R$. We begin by assuming that there exists some radius $r_1 < R$ such that $r_1^{D-3} = 2\mathsf{m}(r_1)$. Near this point $\frac{\mathrm{d}\mathsf{p}}{\mathrm{d}r} \to -\infty$ because of the TOV equation, so the pressure would become arbitrarily negative as $r \to r_1$ from below. However, since the pressure must vanish on the surface of the star, $p(R) = 0$, and must remain positive inside, this situation is unphysical. Therefore, such a radius r_1 cannot exist in order to maintain the decreasing pressure profile.

III. BUCHDAHL LIMITS IN D DIMENSIONS

A. Constant density stars

We will start by studying the interior solution of a star with constant density. We will assume that the energy density is uniform throughout the stellar interior, $\rho = \rho_0$. In this case, we see that $\bar{\varrho} = \varrho_0 = \frac{\mathsf{M}}{R_{D-1}}$ so $\mathsf{m}(r) = \frac{\mathsf{M}}{R_{D-1}}r^2$. Hence, we can write one of the functions as

$$f_0(r) = 1 - \frac{2\mathsf{M}}{R^{D-1}}r^2 = 1 - 2\varrho_0 r^2\,. \tag{21}$$

Using this relation and the condition $\mathsf{m}(0) = 0$, we can solve the TOV differential equation and find an expression for the pressure. It reads

$$\frac{\mathsf{p}(r)}{\varrho_0} = \frac{1 - \left(\frac{f_0(R)}{f_0(r)}\right)^{1/2}}{\frac{(D-1)}{(D-3)}\left(\frac{f_0(R)}{f_0(r)}\right)^{1/2} - 1}\,, \tag{22}$$

where we set the boundary condition $\mathsf{p}(R) = 0$. Using equation (22) we can represent the pressure profiles for different values of the mass. We show this in Fig. 2.

The central pressure $\mathsf{p}_c = \mathsf{p}(0)$ can be easily found from the above expression and reads

$$\frac{\mathsf{p_c}}{\varrho_0} = \frac{1 - f_0(R)^{1/2}}{\frac{(D-1)}{(D-3)}f_0(R)^{1/2} - 1}\,. \tag{23}$$

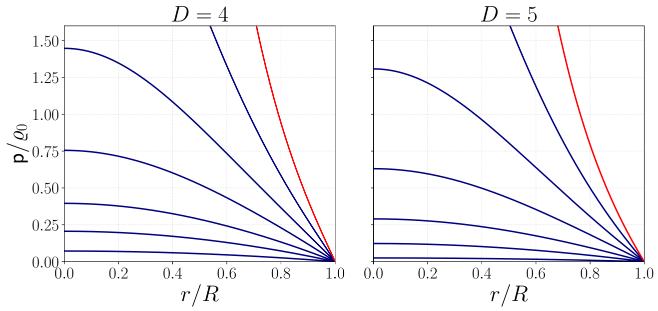

FIG. 2: Radial pressure profiles $p(r)$ for spacetime dimensions $D = 4, 5$. Each plot shows a family of curves corresponding to compactness parameters $r_S/R = (1/4,\ 1/2,\ 2/3,\ 4/5,\ 8/9,\ 21/22) \cdot r_S/R_{\min}$ and includes in red the limiting case for the minimum stable radius $R = R_{\min}$.

It is interesting to see that for a fixed value of the star radius R, the value for the central pressure decreases as D grows. This means that for higher dimensional spacetimes the star will be more stable and the collapse will be more difficult to occur.

To prevent the central pressure from becoming infinite, the denominator of the expression for p_c must remain non-zero. Setting it to zero identifies the critical compactness limit. The limiting condition reads

$$f_0(R_{\min}) = \left(\frac{D-3}{D-1}\right)^2 . \tag{24}$$

This gives the minimum radius allowed as a multiple of the Schwarzschild radius. This is the generalized Buchdahl limit in D dimensions [5]. It establishes a fundamental constraint: no static, spherically symmetric star of constant density can exist with a radius below this limit: more compact stars would simply be gravitationally unsustainable. Explicitly, it reads

$$\frac{R_{\min}}{r_S} = \left[\frac{(D-1)^2}{4(D-2)}\right]^{\frac{1}{(D-3)}} , \tag{25}$$

as anticipated in (2). In the $D = 4$ case, the result reduces to the well-known Buchdahl bound $R_{\min} = \frac{9}{8} r_S$. In Fig. 1, we have plotted the minimum radius relative to the Schwarzschild as a function of D. It reaches a maximum at $D = 6$ which therefore corresponds to the case with the smallest possible degree of compactness compatible with a finite central pressure.

B. General equations of state

We now extend the above result to a more general equation of state under the hypothesis that its average density decreases as one moves outwards from the center of the star. Starting from equations (16) and (15), and applying the variable changes $x \equiv r^2$, $\zeta \equiv N f^{1/2}$ and $\mathrm{d}\xi \equiv \frac{\mathrm{d}x}{\sqrt{f}}$, it is possible to find a differential equation for the combination of metric functions ζ which does not involve pressure, namely,

$$\zeta_{,\xi\xi} = g(\xi)\zeta \,, \quad \text{where} \quad g(\xi) \equiv \frac{D-3}{2}\, \bar{\varrho}_{,x} \,, \tag{26}$$

and where we momentarily use the notation $f_{,x} \equiv \mathrm{d}f/\mathrm{d}x$ Under our hypothesis $\bar{\varrho}_{,x} < 0$, it is clear that the value of $\zeta_{,\xi}$ at the center of the star ("c") is greater than at the boundary ("b"),

$$(\zeta_{,\xi})_{\rm c} \geq \zeta_{,\xi} \geq (\zeta_{,\xi})_{\rm b} \tag{27}$$

At the star boundary $r = R$ we have

$$(\zeta_{,\xi})_{\rm b} = \left.\frac{\mathrm{d}\sqrt{f}}{\mathrm{d}x/\sqrt{f}}\right|_{r=R} = \frac{(D-3)}{2}\, \bar{\varrho}_{\rm b} \,. \tag{28}$$

And from the second inequality one finds

$$\zeta_{,\xi} \mathrm{d}\xi \geq \frac{(D-3)}{2}\, \bar{\varrho}_{\rm b} \mathrm{d}\xi \,. \tag{29}$$

Integrating the LHS, we have

$$\int_{\rm c}^{\rm b} \zeta_{,\xi} \mathrm{d}\xi = \zeta_{\rm b} - \zeta_{\rm c} = \sqrt{f(R)} - N(0) \,, \tag{30}$$

where we used that $f(0) = 1$ and $N(R) = 1$. On the other hand, in the RHS we have

$$\int_{\rm c}^{\rm b} \bar{\varrho}_{\rm b} \mathrm{d}\xi = \int_{\rm c}^{\rm b} \frac{2\bar{\varrho}_{\rm b} r \mathrm{d}r}{\sqrt{f}} = \int_{\rm c}^{\rm b} \frac{2\bar{\varrho}_{\rm b} r \mathrm{d}r}{\sqrt{1 - 2\bar{\varrho}(r) r^2}} \,, \tag{31}$$

where we omitted the $(D-3)/2$ factor. Now, taking into account that $\bar{\varrho}_{,r} \leq 0$ by hypothesis, we have $(1 - 2\bar{\varrho}(r)r^2)^{-1/2} \geq (1 - 2\bar{\varrho}_{\rm b} r^2)^{-1/2}$ $\forall r \in [0, R]$. Hence,

$$\int_{\rm c}^{\rm b} \bar{\varrho}_{\rm b} \mathrm{d}\xi \geq \int_{\rm c}^{\rm b} \frac{2\bar{\varrho}_{\rm b} r \mathrm{d}r}{\sqrt{1 - 2\bar{\varrho}_{\rm b} r^2}} = 1 - \sqrt{f(R)} \,. \tag{32}$$

Combining (29) with (30) and (32), we finally have

$$\sqrt{f(R)} - N(0) \geq \frac{(D-3)}{2} \left(1 - \sqrt{f(R)}\right) . \tag{33}$$

And from this, we finally have

$$f(R) \geq \left(\frac{D - 3 + 2N(0)}{D-1}\right)^2 \geq \left(\frac{D-3}{D-1}\right)^2 , \tag{34}$$

where in the second inequality we used that $N(0) \geq 0$. The limiting value for $f(R)$ corresponds to stars with $N(0) = 0$, for which the metric would describe a zero-size horizon at the center. Among those, the smallest possible value of $f(R)$ coincides precisely with (24), which means that the compactness limit for any star is saturated by constant-density stars with infinite central pressure. This generalizes the $D = 4$ result to general dimensions.

IV. EINSTEIN-GAUSS-BONNET GRAVITY

Einstein gravity is expected to receive higher-curvature corrections at sufficiently high energies. Here we explore the effects of one of such terms on the Buchdahl limit of constant-density stars. Our action contains now a quadratic term of Lovelock type [6] known as "Gauss-Bonnet" density, $\mathcal{Z}_2 = R^2 + R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$. The gravitational Lagrangian now reads $\mathcal{L} = R + \alpha\mathcal{Z}_2$, where α is a new coupling constant with dimensions of length^2. This represents the simplest non-trivial extension of General Relativity in higher dimensions ($D \geq 5$).

In the case for a constant density star, the metric function takes now the form

$$f_0(r) = 1 - \psi_0 r^2 , \quad \psi_0 \equiv \frac{1}{2\alpha}\left(\sqrt{1+8\alpha\varrho_0} - 1\right) . \quad (35)$$

Similarly, the gravitational radius of the star is modified with respect to the Einstein gravity result, and now reads $r_h/r_S = (1+\alpha\psi_0)^{1/(3-D)}$. Following the same procedure as for Einstein gravity, we can solve the generalized TOV equation and find the modified pressure profile. The result for the central pressure reads now (compare with (23))

$$\frac{\mathsf{p}_c}{\varrho_0} = \frac{1 - f_0(R)^{1/2}}{\frac{(D-1)}{\Delta(D-3)}\, f_0(R)^{1/2} - 1} , \quad (36)$$

where we have defined

$$\Delta \equiv \frac{1}{D-3}\left(D - 1 - 2\,\frac{\psi_0 h'(\psi_0)}{h(\psi_0)}\right) , \quad (37)$$

$$h(\psi_0) \equiv \psi_0 + \alpha\psi_0^2 , \quad (38)$$

which reduce to $\Delta = 1$ and $h(\psi_0) = \psi_0$ for Einstein gravity. If we take the divergent value of p_c, we find the modified Buchdahl limit:

$$f_0(R_{\min}) = \left(\frac{D-3}{D-1}\right)^2 \Delta^2 , \quad (39)$$

which differs from the Einstein gravity result by the Δ^2 factor. From this, we have

$$\left(\frac{R_{\min}}{r_S}\right)^{D-3} = \frac{\psi_0}{h(\psi_0)} \frac{(D-1)^2}{(D-1)^2 - (D-3)^2\Delta^2} . \quad (40)$$

In Fig. 3 we study this modified Buchdahl limit in the $D = 5$ case for different values of α. We observe that as the parameter α increases, the maximum allowed compactness of the star also increases. This implies that more compact configurations are allowed without triggering gravitational collapse. In the plot, the white region corresponds to physically viable stars, whereas the orange region indicates stars which could not be sustained without collapsing. Finally, the gray region corresponds to black holes.

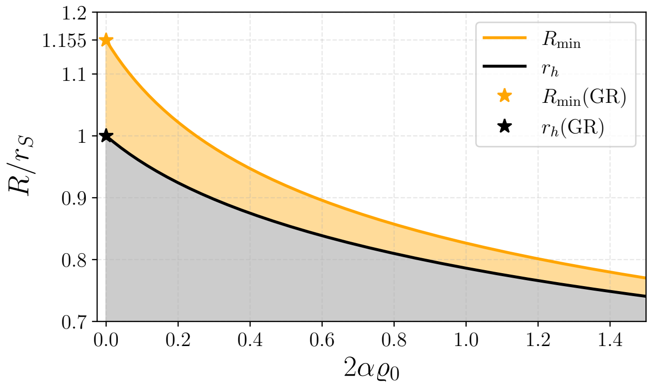

FIG. 3: Buchdahl radius $R_{\min}$ (in orange) and event horizon radius r_h (in black) as functions of the dimensionless parameter $2\alpha\varrho_0$ in $D = 5$. Both quantities approach the Einstein gravity result as $2\alpha\varrho_0 \to 0$.

V. CONCLUSIONS

We have examined Buchdahl's theorem and the limitations it imposes on relativistic stars with constant density in D-dimensional Einstein gravity. Furthermore, for general stars with monotonically decreasing density profiles, we have shown that the limit derived for constant-density configurations serves as an absolute lower bound. We have also analyzed the case of a constant-density star in Einstein–Gauss–Bonnet gravity. Our results indicate that the Buchdahl bound is modified, allowing for more compact star configurations. It would be of interest to explore more complex theories, such as including additional higher-curvature terms from the Lovelock series.

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Límits de Buchdahl més enllà de la Relativitat General

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Resum: El teorema de Buchdahl en quatre dimensions estableix un límit universal per al grau de compacitat d'una estrella estàtica de radi R. Aquest límit és saturat per estrelles de densitat constant amb pressió central infinita i, en termes del radi de Schwarzschild de l'estrella, s'expressa com $R \geq \frac{9}{8} r_{\mathrm{S}}$. Mostrem que el teorema també és vàlid per a la gravetat d'Einstein en D dimensions, i que el límit esdevé $R \geq \left[\frac{(D-1)^2}{4(D-2)}\right]^{1/(D-3)} r_{\mathrm{S}}$. Mostrem també que aquest límit es modifica quan la gravetat d'Einstein es corregeix amb un terme quadràtic de curvatura del tipus Gauss-Bonnet. En aquest cas, el grau de compacitat pot reduir-se significativament respecte al resultat Einsteinià.
Paraules clau: Teorema de Buchdahl , Relativitat General, Gravetat de Einstein-Gauss-Bonnet
ODSs: Aquest treball es relaciona amb la ODS 4: Educació de Qualitat

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