Observer: An Information-Theoretic Perspective

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Abstract

The boundary between quantum and classical domains remains one of the most profound puzzles in physics, intimately tied to the nature of observation itself. This thesis advances a principled framework wherein observers are recast as *System Identification Algorithms* (SIAs), finite informational agents[1] whose capacity to observe and track external systems is governed by their Kolmogorov complexity. Grinbaum's hypothesis formalizes *observerness*[2] as an algorithmic resource and gives a *relational*[3] criterion for quantum-classical transitions: a system appears quantum to an observer only when its Kolmogorov complexity lies below that of the observer. Within this framework, classicality emerges as a thermodynamic necessity once memory saturation of the observer forces irreversible erasure, as dictated by Landauer's principle. We further integrate this perspective into the Local Friendliness experiment, revealing that violations of Local Friendliness inequalities are computationally constrained: they persist only within regimes where complexity gaps between agents remain open. The undecidability of Kolmogorov complexity implies that the precise location of the quantum-classical cut is itself algorithmically inaccessible. We finally interpret the notion of an epistemic horizon discussed in Claim 1 of Restriction A [JM25] through complexity constraints.

Keywords: Complexity, Observer, Undecidability, Local Friendliness

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[1] We will often use 'agent' and 'observer' interchangeably.

[2] The term "observerness" is adopted from [ZLR25].

[3] The term "relational" follows Rovelli's usage in [Rov96].

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I thank my supervisors.
Family and friends.
Muḥyiddīn, the Andalusian.
And Kashmir.

Contents

1 Introduction

The quantum–mechanical measurement problem exposes a tension between two apparently incompatible facts: (i) Schrödinger dynamics preserves coherent superpositions, and (ii) every real experiment yields a single, definite outcome [TMB25]. Traditional resolutions invoke an observer–often an implicitly classical agent–to "collapse" the wave-function, but seldom specify what physical resources make such an agent possible. Central to this discussion is the notion of the Heisenberg cut: the conceptual boundary separating the quantum system from the classical observer or measuring apparatus. Crucially, it remains unclear where the quantum–classical boundary lies and why it should be located there.

This thesis addresses that gap in two stages. First, it surveys the diverse quantitative notions of observerness that have appeared in the literature. Second, building on Grinbaum's SIA paradigm [Gri13], the observer is reframed as a finite algorithm responsible for identifying and tracking the relevant degrees of freedom of a physical system. Classicality emerges whenever the observer's limited descriptive resources are exhausted or when the observed system's complexity exceeds those resources. The SIA framework is therefore employed to clarify the observer's role in a range of quantum-mechanical experiments.

Motivation

Algorithmic Information Theory provides the formal tools necessary to quantify descriptive resources. For instance, Kolmogorov complexity–defined as the length of the shortest program that reproduces a given object–is, in general, undecidable; that is, exact complexities are, in principle, uncomputable. Such quantification imposes stringent epistemic limits, indicating that no observer can fully model a system whose descriptive complexity exceeds its own. By linking observerness to such an algorithmic quantity, one obtains a principled and relational criterion for the quantum–classical cut–one that can be tested experimentally, for instance through calorimetric tests of memory saturation [Gri13]. The local friendliness experiment is a foundational test in quantum mechanics designed to probe the limits of observer-dependent facts and the notion of objective reality. We use this algorithmic framework to reinterpret the local friendliness experiment, providing a rigorous quantitative basis for understanding its implications in terms of complexity and observer limitations.

Structure and thematic progression

- **Section 2** reviews Kolmogorov complexity, its key properties, and the undecidability that constrains exact knowledge of complexity. This foundation motivates later use of complexity bounds to demarcate quantum and classical regimes.
- **Section 3** surveys quantitative and operational markers of observerness–information capacity, branch factor, consciousness, mass, objectivity, irreversibility, and thoughtfulness–highlighting their differing motivations and the need for a unifying framework.
- **Section 4** introduces Grinbaum's Hypothesis: an observer X is an SIA whose finite Kolmogorov complexity $K(X)$ bounds the set of systems it can treat as quantum. Formally, a system S is quantum for an observer X iff $K(S) < K(X)$, whereas S appears classical if $K(S) \geq K(X)$ [Gri13]. Memory saturation forces information erasure, invoking Landauer's principle and an attendant thermodynamic cost [Lan61]. A proposed fullerene-calorimetry experiment operationalises this prediction.
- **Section 5** applies the SIA framework to Local Friendliness (LF) scenarios. Finite observer complexity constrains when LF inequalities can be violated; as the friend's

memory approaches saturation, quantum correlations must decay into classical statistics.

- **Section 6** summarises the work and gives the conclusion.

Relevance to current debates
The SIA perspective complements ongoing discussions in quantum foundations, information theory, and the philosophy of physics. It maintains the empirical success of quantum mechanics while attributing classical definiteness to finite computational resources (or limited memory size, as discussed in[Gri15]). By doing so, it offers a principled response to explanations that rely solely on non-information-theoretical parameters like consciousness or macroscopic mass: neither suffices without adequate algorithmic capacity. In synthesizing these themes, the thesis positions computational complexity as the fundamental driver behind both observation and the emergence of classical reality.

2 Kolmogorov Complexity and Undecidability

2.1 Kolmogorov Complexity

Kolmogorov complexity provides a rigorous measure of the information content of objects by considering the length of their shortest descriptions on a *universal Turing machine*[4]. Formally, for a fixed universal Turing machine U, the Kolmogorov complexity $K_U(x)$ of a finite binary string x is defined as the length (in bits) of the shortest program p such that $U(p) = x$ [Kol68, Cha66, GV08]. Intuitively, $K_U(x)$ represents the most efficient compressive description of x–the smallest number of bits needed to reproduce x. Highly regular or compressible strings possess low K, whereas random or structureless strings exhibit high K. For example, the string "01010101..." can be generated by a very short program (e.g., "print '01' n times"), while a truly random string requires a program essentially as long as the string itself. A fundamental invariant of this definition of $K_U(x)$ is that while it depends on the choice of reference machine U, different universal machines yield complexity values differing at most by a constant independent of x (the invariance theorem) [LV08]. Thus, one can speak of "the" Kolmogorov complexity $K(x)$ up to an additive constant, making the notion well-defined [LV08, GV08].

2.2 Relevant Properties of Kolmogorov Complexity

[GV08, HRSV00] explored the analogies between Kolmogorov complexity, which measures the algorithmic randomness of individual strings, and Shannon entropy, which quantifies the average uncertainty in a probabilistic source. Their work led to some key properties and relationships reviewed below that are relevant to this thesis.

The conditional Kolmogorov complexity $K(b \mid a)$ is defined [She15, Vit13, GV08] as the length of the shortest program that, given a as input, produces b as output. As shown in [HRSV00, ZL70, GV08], up to an additive term of order $O(\log m)$

$$K(b \mid a) \approx K(\langle a, b \rangle) - K(a) \tag{1}$$

where m represents the length of a and $\langle a, b \rangle$ denotes a computable encoding of the pair (a, b) into a single binary string.

[4]Introduction to Turing Machines can be found in Appendix A

Now, the algorithmic mutual information between a and b is defined as:

$$I(a:b) = K(b) - K(b \mid a), \tag{2}$$

which, using the approximation above, is symmetric up to $O(\log m)$:

$$I(a:b) \approx K(a) + K(b) - K(\langle a,b\rangle). \tag{3}$$

This quantity is non-negative up to additive logarithmic terms, mirroring mutual information in Shannon theory.
The conditional mutual information is similarly defined up to $O(\log m)$ as

$$I(a:b \mid c) \approx K(a \mid c) + K(b \mid c) - K(\langle a,b\rangle \mid c). \tag{4}$$

As proved in [HRSV00], using these definitions, the complexity of a triple $\langle a,b,c\rangle$ can be expressed as:

$$2K(\langle a,b,c\rangle) \approx K(\langle a,b\rangle) + K(\langle a,c\rangle) + K(\langle b,c\rangle) - I(a:b \mid c) - I(\langle a,b\rangle : c). \tag{5}$$

In particular, when a, b, and c are mutually algorithmically independent, this simplifies to:

$$K(\langle a,b,c\rangle) \approx K(a) + K(b) + K(c). \tag{6}$$

Another important property of conditional Kolmogorov complexity, as discussed in [LV08], is that for any constant q, and for all strings a and b, the following inequality holds:

$$K(b \mid a) \leq K(b) + q\,, \tag{7}$$

where q is independent of a and b, and reflects the overhead of providing a as auxiliary input to the universal Turing machine.

2.3 Undecidability of Kolmogorov Complexity

A crucial fact about Kolmogorov complexity is that it is undecidable or uncomputable in the precise sense of computability theory. This means there does not exist any algorithm or Turing machine which, given an arbitrary string x as input, can effectively output the exact value of $K(x)$ [Cha66, ZL70, GV08]. Equivalently, no mechanical procedure can decide for an arbitrary x and integer n whether $K(x) \leq n$ [LV08]. Kolmogorov complexity thus defines a concrete example of a *total function*[5] from $\{0,1\}^*$[6] to $\mathbb{N}$ that is well-defined mathematically but provably non-computable. Informally, Kolmogorov complexity is an undecidable property of strings and it formalizes the intuitive notion of algorithmic (in)compressibility or algorithmic randomness [FZG08, SUV17], yet no general algorithm can determine this property for every instance.

The incomputability of $K(x)$ can be demonstrated through a paradoxical argument rooted in Berry's paradox (see [Ten25]) or the halting problem[7] [Cha75, LV08]. The halting problem asks whether an arbitrary computer program, when run on a particular input, will eventually stop or continue to run forever. The proof works by reducing the halting problem to our question: for any pair (x,n), deciding whether $K(x) < n$ is exactly as

[5] Function that is defined for every input in the domain

[6] $\{0,1\}^*$ represents the set of all finite binary strings, including the empty string

[7] see Appendix A

hard as solving the halting problem, and thus it is undecidable [ZL70]. In summary, there is no effective procedure to compute, or even reliably approximate from below, the value of $K(x)$; only upper bounds can be *semi-computed*[8] by searching for increasingly shorter descriptions [LV08, Vit20].

This undecidability of Kolmogorov complexity has significant implications in theoretical computer science, information theory, and philosophy. In data compression, it implies that no universal algorithm can compress all strings to their minimal possible representations. Any specific compressor (e.g., `zip`, `gzip`) is a computable function that exploits certain regularities but cannot achieve the absolute lower bound given by $K(x)$. If such an optimal compressor existed, it would effectively solve the Kolmogorov complexity problem. Hence, all practical compressors necessarily fall short of the theoretical optimum on some inputs [LV08].

2.4 Chaitin's Incompleteness Theorem

Gregory Chaitin [Cha74] showed that for any *consistent*[9], *recursively enumerable*[10] *formal axiomatic system*[11] that is *sufficiently expressive*[12] to formalise *elementary arithmetic*[13], there exists a natural number $N_{\mathcal{S}} \in \mathbb{N}$ such that the theory can never prove any explicit sentence of the form

$$K(x) \; > \; N_{\mathcal{S}}, \tag{8}$$

where $K(x)$ is the *prefix-free*[14] Kolmogorov complexity of the finite binary string x.

Chaitin's theorem thus provides a quantitative analogue to Gödel's incompleteness results [Poo14, Zis23, PECG+24]. Whereas Gödel's theorem established the existence of true but unprovable propositions within formal systems, Chaitin identified a specific class of such propositions: statements asserting that a string's Kolmogorov complexity exceeds a given bound. These statements are mathematically true but provably undecidable within any fixed formal system [Cha74, Cha75].

Importantly, these undecidable statements are not isolated or exceptional. Chaitin's result entails that every sufficiently powerful axiomatic system is systematically incomplete with respect to statements about algorithmic complexity. There exists an infinite set of true yet unprovable propositions regarding the incompressibility of individual strings, reflecting a pervasive epistemic boundary in the formal analysis of randomness [SUV17, LV08, GV08].

The implications of this result are both foundational and epistemological. It reveals that randomness, when rigorously defined in terms of algorithmic incompressibility, cannot be fully captured by any fixed formal system. This limitation arises not merely from logical constraints, but from the finite information content encoded in the axioms themselves. As a consequence, Kolmogorov complexity imposes an explicit upper bound on what a formal

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[8] Semicomputable here means an algorithm can list ever-smaller upper bounds for $K(x)$ but can never certify it has reached the exact value, so the search may run indefinitely.

[9] A theory is consistent if it never proves both a statement and its negation.

[10] It means there is an algorithm that can list all theorems of the system.

[11] It consists of a finite or recursively enumerable set of axioms together with explicit mechanical inference rules, so every proof can be checked by a computer.

[12] It indicates that the language and axioms can represent and reason about natural-number arithmetic, which lets the theory encode programs and their behaviour.

[13] First-order arithmetic over natural numbers with $+$, $\times$, equality and quantifiers.

[14] No valid program is a proper prefix of another.

system can prove: beyond a certain complexity level, true statements necessarily escape formal derivation [Cha75, LV08].

In summary, Chaitin's incompleteness theorem introduces a precise, quantifiable boundary to formal mathematical knowledge. It shows that certain truths–specifically those concerning high algorithmic complexity–are inherently unprovable within any given axiomatic framework. This insight deepens our understanding of the intrinsic limitations of formal reasoning, and underscores a profound connection between complexity theory, logic, and the philosophy of mathematics [Cha74, Cha75, LV08].

2.5 Summary of Section 2

Kolmogorov complexity provides a rigorous and objective measure of the informational content of finite objects by associating each with the length of its shortest possible description–or generative program–on a universal Turing machine. This notion extends naturally to conditional Kolmogorov complexity and algorithmic mutual information, through which classical entropy-like relations reappear, albeit up to additive logarithmic terms. Importantly, complexity exhibits additivity when the constituent components are algorithmically independent, offering a natural analog to statistical independence in classical information theory.

However, despite its conceptual appeal, Kolmogorov complexity is fundamentally uncomputable. No algorithm can, in general, decide whether a given string admits a shorter description than a specified bound. As a consequence, no universal data compression algorithm can achieve optimal performance across all possible inputs. This intrinsic undecidability has far-reaching implications: it constrains the practical efficacy of compression schemes and restricts algorithmic analysis to upper bounds or comparative assessments rather than exact values. Most profoundly, Chaitin's information-theoretic analogue of Gödel's incompleteness theorem demonstrates that for any formal axiomatic system, there exists a threshold beyond which statements of the form $K(x) > N$ are true but unprovable. This result establishes a precise epistemic boundary within which formal reasoning must operate and beyond which it necessarily fails.

Together, these findings describe the limits of what can be formally proven and what can be algorithmically compressed. Having explored this theoretical boundary in the context of symbolic strings, the following section shifts focus to the notion of the observer in quantum mechanics. By employing concepts such as algorithmic complexity, branching structure, consciousness, and environmental redundancy, we aim to formalize and quantify the "observerness," ranging from simple measurement devices to *anthropocentric*[15] observers.

3 Quantifying Observerness

Understanding what constitutes an observer in quantum mechanics, and what qualifies a system to be called an observer, is of fundamental importance. Multiple interpretations have been proposed to address this issue, highlighting the complexity and foundational significance of the status of observer within quantum theory, as well as the lack of consensus on the necessary physical or informational criteria. This section (following a scheme similar to [ZLR25]) systematically examines some candidate parameters proposed to understand observerness.

[15] Viewing humanity as the most significant entity, and interpreting the world chiefly in terms of human values and experience.

Specifically, we review information-theoretic models employing algorithmic complexity (Kolmogorov complexity) as a metric to evaluate an observer's capacity to record and process information. Additionally, we discuss computational complexity approaches, introducing the concept of a "branch factor" derived from quantum circuit complexity, which quantifies how classically definitive a system's measurement outcomes become–effectively measuring their resistance to quantum interference and reversal. Consciousness-based frameworks, which propose the necessity of a conscious mind for genuine observation, are also considered.

Alongside these models, this section also evaluates various operational criteria proposed as hallmarks of an observer. These criteria include the system's macroscopic scale or mass–under the assumption that larger systems induce more decoherence; the emergence of objective classical records through environmental redundancy (as seen in quantum Darwinism [Zur09]); the irreversibility of measurement interactions accompanied by entropy increase; and intrinsic observer attributes like *thoughtfulness*[16].

By analyzing these diverse perspectives and their associated quantifiable measures, this section clarifies the essential characteristics distinguishing observers from non-observers within quantum mechanical frameworks. Ultimately, the discussion illuminates the observer's role in the measurement process, the emergence of classical reality, and the spectrum spanning simple physical detectors to fully-fledged anthropocentric objects. This section ends with a tabular summary given in 1.

3.1 Information

Grinbaum [Gri13, Gri15] and Müller (2020) [Mül20] connect observerness to information theory. Grinbaum proposes modeling an observer as SIA and emphasizes its Kolmogorov complexity. He explores how a system's description complexity determines whether it can play the role of an observer [Gri15]. Higher complexity in the observer (or its model) is associated with enhanced observerness in these works. More about this would be discussed in 4.

3.2 Higher branch factor

Taylor *et al.* (2025) [TM25] introduce the so-called branch factor (or branching), which they argue is a natural information-theoretic metric for quantifying observerness. In Extended Wigner Friend experiments, a high branch factor implies that operationally determining whether the friend is in a coherent superposition of the pointer states $|F_0\rangle$ and $|F_1\rangle$, or in a classical mixture of them, requires high complexity. This difficulty in distinguishing the superposition from a classical mixture is sufficient to classify the system as effectively classical. Building on this insight, Zeng *et al.* (2025) [ZLR25] employ the branch factor as a quantitative measure of the degree to which a quantum system exhibits "observer-like" behavior, focusing on two operational features of the friend pointer states $|F_0\rangle$ and $|F_1\rangle$: how easily they can be distinguished, and how difficult they are to interfere.

These two tasks are quantified using circuit complexity C, defined as the minimal number of one and two-qubit unitaries required to implement a given unitary transformation. Specifically, C_D denotes the minimal number of quantum gates needed to distinguish the states, while C_I quantifies the complexity required to interfere them. The branch factor is then defined as $BF = C_I - C_D$, with larger values indicating systems that are easy to measure but difficult to coherently recombine.

[16]Term used by Wiseman *et al.* in [WCR23]

Aaronson *et al.* (2020) [AAS20] formally define these two complexity proxies and branch factor for two orthogonal states $|\psi_0\rangle$ and $|\psi_1\rangle$, and a parameter $0 \leq \delta \leq 1$. According to their definitionn, the interference complexity proxy $C_I(|\psi_0\rangle, |\psi_1\rangle, \delta)$ is the minimum $C(U)$ such that $|\langle\psi_1|U|\psi_0\rangle + \langle\psi_0|U|\psi_1\rangle|/2 \geq \delta$, while the distinguishability complexity proxy $C_D(|\psi_0\rangle, |\psi_1\rangle, \delta)$ is the minimum $C(U)$ such that $|\langle\psi_0|U|\psi_0\rangle - \langle\psi_1|U|\psi_1\rangle| \geq \delta$. Each proxy approximates the corresponding true complexity up to an $O(1)$ factor. Thus, they define branch factor as $BF(|\psi_0\rangle, |\psi_1\rangle, \delta) = C_I(|\psi_0\rangle, |\psi_1\rangle, \delta) - C_D(|\psi_0\rangle, |\psi_1\rangle, \delta)$.

Zeng *et al.* also provide examples, such as GHZ and Dicke states, in which the branch factor increases with system size, thereby capturing the emergence of observer-like behavior in composite quantum systems.

3.3 Consciousness

Consciousness remains a highly controversial subject, with no universally accepted definition. Existing definitions are largely speculative and often fail to persuade those holding opposing views. Stapp (1999) [Sta99] argues that consciousness is essential for quantum dynamics–not as a passive observer, but as an active participant. Hameroff & Penrose (2014) [HP14] propose that consciousness arises from quantum processes within brain microtubules, which undergo orchestrated objective reductions (Orch OR) based on the idea that gravity plays a role in quantum state reduction. These reductions are not random or epiphenomenal but are tied to fundamental space-time geometry at the Planck scale, so that each orchestrated reduction selects a particular configuration of that geometry and thereby realizes a discrete "moment" of conscious experience reflective of objective features of reality. The observer, in this view, is not a separate metaphysical entity but is inherently linked to these moments, meaning consciousness and observation are intertwined through fundamental physical processes. Neven *et al.* (2024) [NZR+24] go further, proposing that any quantum superposition yields consciousness, and ultimately suggest a quantum biology experiment to validate their proposal. Bayne *et al.* (2024) [BSM+24] survey and classify consciousness tests (C-tests) designed to empirically assess consciousness in humans and other systems.

Taken together, these works imply that observerness is linked to consciousness: systems that are conscious or have internal awareness are considered observers. In these accounts, increasing a system's consciousness (having thoughts, awareness, or subjective experience) is treated as making it more observer-like in quantum measurements.

3.4 Mass

Fein *et al.* (2019) [FGZ+19] and Delić *et al.* (2020) [DRD+20] showed that increasing mass alone does not force a system to behave classically. Fein *et al.* prepared interference with molecules of approximately 25 kDa (4.15×10^{-23} kg) and reported "excellent agreement with quantum theory" that "cannot be explained classically." Delić *et al.* cooled an optically levitated nanoparticle (about 10^8 atoms with ~ 150 nm diameter) to its quantum ground state of motion from ambient temperature and noted this enables creating "superposition states involving large masses." These results imply that even very massive objects can remain quantum. In terms of observerness, the referenced works suggest that simply adding mass does not make a system an observer; heavy systems like those above would still require a measurement interaction to acquire a definite outcome.

3.5 Objectivity

Zurek (2009) [Zur09] and Chisholm *et al.* (2021) [CGPR+21] interpret objectivity as agreement among multiple observers. They emphasize that a quantum outcome becomes objective only when multiple independent observers can access the same information. Zurek interprets objectivity as the ability of many observers to independently and reliably determine the state of a system by accessing different parts of its environment without disturbing it. According to his view, which is termed in Quantum Darwinism, the environment acts as a witness by storing multiple copies of information about the system. When information about a system is redundantly recorded across many parts of the environment, it becomes accessible to different observers, leading to the emergence of classical, objective reality from the underlying quantum world. Thus, "more objectivity" arises when information is widely proliferated: many fragments (effective observers) see the same outcome. In these references, observerness is tied to a system becoming observable by many parties with consistent results–an objective state is one that can be stably witnessed by multiple observers.

3.6 Irreversibility

Manikandan *et al.* (2019) [MEJ19] and Jayaseelan *et al.* (2021) [JMJB21] link irreversibility with the measurement process. Manikandan *et al.* derive fluctuation theorems for continuous quantum measurements and find that "measurement-induced wave-function collapse exhibits absolute irreversibility"–the process is inherently one-way when information is gained. Jayaseelan *et al.* experimentally quantify this by studying *weak quantum measurements*[17] on ultracold ^{87}Rb atomic spins, demonstrating that the process is absolutely irreversible: although rare *arrow of time reversal*[18] events occur, the average arrow of time remains strictly positive and increases with measurement strength. Extending their study to an entangled many-body spin–orbit coupled Bose–Einstein condensate, they show that absolute irreversibility persists even in quantum many-body systems, highlighting a deep connection between wavefunction collapse and thermodynamic irreversibility. Thus, these works seem to emphasize that irreversibility stems from information gain during measurement; thus, greater irreversibility yields more classical, observer-like definiteness.

Based on the observation that spontaneous and irreversible processes in nature are typically associated with equilibration, Schwarzhans *et al.* (2023) [SBHL23] hypothesize that any measurement corresponds to an entropy-increasing transition toward equilibrium. They further explain that an ideal projective measurement is strictly impossible within their framework, but it can be approximated exponentially well as the measuring device becomes a large macroscopic ensemble; the many interacting subsystems then absorb the entropy produced and render the outcome effectively irreversible.

3.7 Thoughtfulness

Wiseman *et al.* (2023) [WCR23] explicitly associate thoughtfulness with observer status. In their "thoughtful" Local-Friendliness no-go theorem, they assume that if a system has "thoughts," it should count as an observer. The authors treat a "thought" as any piece

[17] They only partially collapse the wavefunction, extracting limited information about the quantum state without fully projecting it onto an eigenstate.

[18] Rare statistical fluctuations where the measured quantum trajectory locally appears to reverse the thermodynamic arrow of time

of information that a system could, in principle, report–*physically realised*[19] inside that system at a particular place and time–for a human, perhaps as a burst of neural activity; for an AI, as a pattern of bits on a chip. Thus, in their framework, adding the capacity for thought or internal reflection to a system makes it an observer. In short, having thoughts is taken to be a measure of observerness.

3.8 Summary

Table 1 summarizes the parameters of observerness discussed above.

Table 1: Quantitative and operational markers of observerness.

Parameter / Criterion	**Central Insight**	**Representative Evidence / Examples**	**Possible Consequence for Observerness**
Information-theoretic complexity	An observer can be modelled as a finite information string; its Kolmogorov complexity limits what it can witness.	Grinbaum's SIA [Gri13]; Müller's induction from an observer's bit-string history [Mül20].	Higher description complexity correlates with richer observational capacity.
Branch factor (circuit complexity)	Classicality of a measurement record is quantified by the gap between interference and discrimination of pointer states [ZLR25].	Branch factor = Difference between gate complexity of interfering and distinguishing.	Large branch factor signals effectively irreversible detectors.
Consciousness	Gravity plays a role in quantum state reduction.	Stapp's participatory mind [Sta99]; Hameroff–Penrose Orch OR [HP14]; Neven's pan-consciousness [NZR+24]; Bayne's C-tests [BSM+24].	Systems that are conscious are considered observers.
Mass / Scale	Large size alone may not enforce classical behaviour; massive objects may remain quantum if isolated.	Interference at 25 kDa [FGZ+19]; ground-state levitated bulky nanoparticle [DRD+20].	Mass is insufficient; a measurement interaction is still required for definite outcomes.

Continued

[19]Instantiated as a definite physical process

Table 1 (continued)

Parameter / Criterion	**Central Insight**	**Representative Evidence / Examples**	**Possible Consequence for Observerness**
Objectivity via redundancy	Classical reality emerges when many fragments of the environment encode the same outcome.	Quantum Darwinism [Zur09]; multi-observer consistency tests [CGPR+21].	Environmental redundancy yields objective, observer-like states.
Irreversibility / Entropy gain	Wave-function collapse becomes absolutely irreversible once information is acquired.	Quantum fluctuation theorems [MEJ19]; arrow-of-time experiments [JMJB21]; entropy-observer ensembles [SBHL23].	Greater irreversibility aligns with more classical, definite observation.
Thoughtfulness	Having internal "thoughts" is proposed as a direct marker of observer status.	Thoughtful Local-Friendliness theorem [WCR23].	Cognitive-style internal dynamics elevate a system to full observerhood.

Despite their valuable insights, the proposed parameters surveyed above are still partly *ad hoc* and are not necessarily compatible with each other. Molecules of arbitrarily large mass can retain quantum interference; consciousness rests on highly debatable and controversial metaphysical assumptions; environmental redundancy and irreversibility lacks a clean quantum-classical demarcation; and "thoughts" are not directly quantifiable. In what follows, we therefore distill the concrete, testable implications for observerness that can nonetheless be extracted from these diverse proposals.

Only two effectively measurable proposals remain: the branch factor and Grinbaum's SIA. Even so, the branch factor must be estimated via proxy circuit complexities and becomes intractable beyond toy models. By contrast, Grinbaum's SIA hypothesis treats any observer as an algorithm whose size is given by its Kolmogorov complexity. While this complexity is not exactly algorithmically computable in general, the idea is clear and allows us to make useful estimates and comparisons even in larger objects.

Because algorithmic complexity (i) is substrate-independent, (ii) ties directly to information erasure costs through Landauer's principle, and (iii) meshes naturally with the language of quantum states, it furnishes a principled, minimal framework into which all observer attributes can be recast. The next section therefore pivots to Grinbaum's algorithmic perspective as a coherent foundation for quantifying observerness in quantum mechanics.

4 System Identification Algorithm (SIA)

As discussed in 3.1, Grinbaum models observers as SIAs [Gri13], each object X characterized by its Kolmogorov complexity $K(X)$. This formalism replaces the classical/quantum divide with a precise, information-theoretic criterion.

Observation leads to memory accumulation, and once this memory saturates, Landauer's principle implies a thermodynamic cost for erasure [Lan61]. Grinbaum proposes a concrete experimental setup involving fullerenes and photon absorption, which could serve to validate this framework. Thus, this hypothesis forges a novel and testable link among quantum mechanics, information theory, and thermodynamics.

4.1 Observer as SIA with Bounded Complexity

The SIA hypothesis proposed by Grinbaum treats an observer in strictly physical and computational terms [Gri13, Gri15]. An observer is any entity–human, machine-like, biological, silicon-based, or otherwise–that possesses information about a physical system and carries out system identification: the continual recognition, measurement, and maintenance of that system's identity. In quantum mechanics this identity is preserved across state changes (an electron remains the same electron after a spin measurement) [Gri17].

Mathematically, an observer is the collection of all finite strings whose Kolmogorov complexity remains below a fixed bound, i.e., every bit-string that can be compressed to within that preset limit [Gri15].

Formally, the observer's task is executed by an algorithm on a universal Turing machine that receives, on its tape, the full list of degrees of freedom present in the environment. The algorithm marks those degrees of freedom that constitute the target system S and continues to do so as S evolves in time. Hence the observer is specified not by its material substrate but by the computational function it realizes. This perspective is sharpened by noting that every SIA can, in principle, be reconstructed from a single minimal binary program; the Kolmogorov complexity of this program is an invariant distinguishing one observer from another and supersedes anthropocentric notions of observerness.

Operationally, the SIA maintains a memory register into which each new measurement outcome is appended. Because the register has finite capacity, denoted $K(X)$, it eventually saturates; writing fresh data afterward requires erasing existing bits and therefore dissipates heat in accordance with Landauer's principle. The observer's informational limits are thus inseparable from thermodynamic cost.

The same capacity $K(X)$ also bounds the complexity of any binary string that can occur inside the observer: only strings with Kolmogorov complexity $\leq K(X)$ are admissible descriptions, whereas more complex strings cannot be tracked by the observer. Since Kolmogorov complexity is defined relative to a universal Turing machine, this cut-off is substrate-independent; biological brains or silicon processors with identical $K(X)$ are informationally equivalent.

4.2 Quantum vs. Classical Systems

An observer X is capable of fully tracking all the degrees of freedom of the system S only if $K(S) < K(X)$, where $K(X)$ gives a measure of the informational capacity or descriptive power of the observer. This leads to the following information-theoretic criterion: a system S is said to be quantum with respect to an observer X if $K(S) < K(X)$; otherwise, it is classical with respect to X. This formulation replaces traditional, often *ad hoc* distinctions between quantum and classical regimes with a precise, observer-dependent condition grounded in computational and information-theoretic principles.

For example, consider a single qubit–such as the spin state of an electron–which has $d = 1$, yielding $K(S) \approx \mathcal{O}(1)$. A human observer, possessing a significantly greater descriptive capacity ($K(X) \gg \mathcal{O}(1)$), will thus perceive the qubit as quantum. In contrast, a macroscopic system like a measurement apparatus may exhibit $K(S) \approx K(X)$, making it effectively classical from the standpoint of X.

Crucially, this classification is relational: it depends on the observer's informational resources. Suppose another observer Y satisfies $K(Y) > K(X)$ and $K(Y) > K(S)$. In this case, both X and S are quantum from Y's perspective, and their interactions are modeled as those between quantum systems. Conversely, if $K(X) \approx K(Y) \gg K(S)$, then Y may treat S as quantum, while simultaneously regarding X as classical.

4.3 Zurek's Redundancy and Proliferation

Zurek's key insight is that a complete physical measure of entropy must incorporate both the inherent randomness of outcomes and the complexity of the process generating this randomness [Zur89]. In his formulation, the physical entropy $\mathcal{S}$ of a system is given by the sum of two distinct components:

$$\mathcal{S} = H + K \tag{9}$$

Here, H denotes the conventional ensemble entropy, defined as $H(\rho) = -\mathrm{Tr}(\rho \log_2 \rho)$, which quantifies thermodynamic randomness. The term K represents the algorithmic (or Kolmogorov) entropy associated with the system's description, capturing its algorithmic randomness. When the state of the system is well known, K becomes the dominant contribution. This formulation thus yields a more comprehensive measure of entropy by accounting for both the stochastic nature of outcomes (via H) and the descriptive complexity of the system's state (via K). As a result, $\mathcal{S}$ provides a unified framework that connects thermodynamic entropy with algorithmic information (for a better understanding, see [GV08]).

4.4 Experimental Proposal

Grinbaum proposes an experiment to verify his hypothesis, involving a single fullerene molecule C_{60} placed within a highly sensitive calorimeter. Photons are sent one at a time toward the molecule. Each photon, having few degrees of freedom, possesses low Kolmogorov complexity $K(S)$, while the fullerene has high complexity $K(C_{60}) \gg K(S)$. Thus, the molecule can function as a observer.

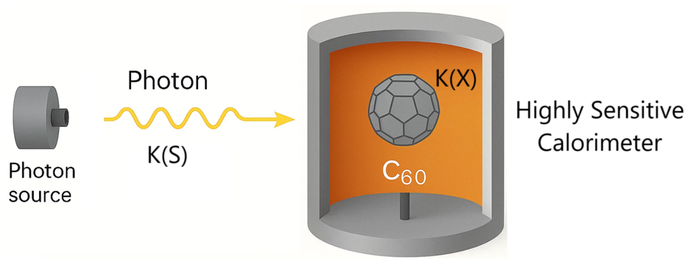

Figure 1: Single-photon calorimetry of an isolated. fullerene

Each absorbed photon establishes a correlation between its degrees of freedom and those of the fullerene. Informationally, this is equivalent to appending new bits to the fullerene's "memory". After n such absorptions, assuming the photons are uncorrelated, it follows from (6):

$$K(M_n) \approx n \cdot K(S). \tag{10}$$

Let N be the smallest n such that $K(M_n) \approx K(C_{60})$. Empirical studies indicate that fullerenes can absorb roughly 10 photons (with wavelength, $\lambda = 308$ nm) before dissociation due to heat, suggesting $N \approx 10$.

For $n > N$, memory saturation necessitates information erasure to accommodate new measurements. According to Landauer's principle, this results in heat dissipation. A calorimeter should detect a sudden heat spike at the $(N + 1)$th absorption, corresponding to this erasure cost.

Such a result would confirm that the molecule acts as an observer until its finite memory is exhausted, at which point thermodynamic costs emerge. Therefore, this experiment provides a physical test of Grinbaum's hypothesis.

4.5 Undecidability of Kolmogorov Complexity and SIA

Since Kolmogorov complexity is provably uncomputable [ZL70], there exists no general algorithm that, given an arbitrary description of a system or agent, outputs its exact Kolmogorov complexity. Concretely, even if we could formally define the binary string encoding for an agent's state (including memory, measurement records, and processing machinery), no Turing-machine procedure can determine, in general, the minimal description length of that string. Consequently, one cannot, in general, pinpoint a precise threshold at which $K(\textit{observer})$ becomes equal to or exceeds $K(\textit{observed})$. Any attempt to calculate or compare these complexities will necessarily involve only upper bounds or heuristic estimates [Vit20].

4.6 Summary

Collectively, the preceding arguments reconceptualize the observer as SIA, whose finite Kolmogorov complexity bound serves as a foundational parameter governing the quantum-classical boundary. Within this framework, Grinbaum proposes a novel, relational criterion to determine whether a physical system S is considered quantum relative to an observer X: namely, that the Kolmogorov complexity of the system's description must be strictly less than that of the observer's, $K(S) < K(X)$. This condition replaces the conventional, device-dependent quantum-classical boundary with a precise and information-theoretically motivated alternative.

Observers are thought to gather information over time, but once their memory is full, they must erase some of it to make room for more. According to Landauer's principle, this erasure comes with a non-zero thermodynamic cost. Grinbaum's experimental proposal puts this idea to the test by suggesting that this cost could appear as a measurable burst of heat: specifically, a sharp spike in temperature when the $(N + 1)^{\text{th}}$ photon is detected. At this point, the molecular observer (such as a fullerene) has reached its limit, able to store information about only N photons. Taking in any additional information forces it to erase old data, which releases energy as heat. If this temperature spike can be detected, it would provide experimental evidence supporting the SIA hypothesis.

With this theoretical apparatus established, we are now prepared to examine how computational constraints impact foundational experiments, particularly those probing the boundaries of locality and realism. In the following section, we extend the Kolmogorov complexity framework to so-called extended Wigner-type scenarios [Bru16, Bru18, BUAG$^+$20, FR18]. We demonstrate how the bounded algorithmic capacity of observers restricts–and in some cases outright prohibits–the realization of certain expected violations of joint probability distributions, thereby offering a computational perspective on recent "no-go" theorems in quantum foundations.

5 Local Friendliness with SIAs

We reinterpret the Local Friendliness (LF) experiment [BUAG+20, WCR23] through the lens of Kolmogorov complexity, incorporating the SIA hypothesis [Gri13] about objects as computationally bounded systems.

5.1 The Wigner–Friend Experiment

Let's start with the Wigner–Friend (WF) thought experiment. First proposed by Wigner in 1961 [Wig95], it extends Schrödinger's cat paradox to an observer who performs a measurement inside a sealed laboratory. Inside the lab, the *friend* measures a quantum system and, according to standard quantum mechanics, records a definite outcome. An external observer (Wigner), however, can consistently assign a pure entangled state to the entire laboratory, treating the friend, apparatus, and system as a superposition. This dual description exposes a tension between the unitary evolution prescribed by the Schrödinger equation and the definiteness of observed events.

Modern analyses refine the WF scenario to probe the limits of observer-independent facts. Brukner derived a no-go theorem for observer-independent facts based on WF assumptions [Bru18], and Proietti *et al.* reported an optical test that violates inequalities derived under those assumptions [PPG+19]. These developments motivate experimental scenarios that combine multiple WF laboratories with Bell-type spacelike separations–precursors to the Local Friendliness tests considered next.

5.2 Assumptions considered for the Local Friendlines Experiment

We commence by articulating three foundational assumptions mentioned in [BUAG+20, WCR23, JM25]:

1. **Absoluteness of Observed Events (AOE):** Each measurement outcome constitutes a single, real, observer–independent event.

2. **Locality:**

   Let A and B be spacelike-separated agents who freely choose the measurement settings x and y and record the corresponding outcomes a and b, respectively. Let λ denote the complete set of physical parameters contained within the intersection of their past light cones. Locality requires that, conditioned on λ, $P(a, b \mid x, y, \lambda) = P(a \mid x, \lambda)\, P(b \mid y, \lambda)$, so that $P(a \mid x, y, \lambda) = P(a \mid x, \lambda)$ and $P(b \mid x, y, \lambda) = P(b \mid y, \lambda)$. Hence, all correlations between a and b must originate within their shared past light cone; no influence propagates superluminally.

3. **No Superdeterminism (NSD):** Events situated on a given spacelike hypersurface are statistically uncorrelated with any freely chosen actions executed after that spacelike hypersurface.

It is worth noting that, in such a scenario, Assumptions 2 and 3 can equivalently be recast, as discussed in [BUAG+20, WCR23], into a single postulate, "local agency", first introduced in [WC16].

It is intuitively expected that with each recorded measurement outcome, the conditional Kolmogorov complexity of the observer decreases, as the observer's degrees of freedom are progressively consumed. This observation is consistent with the inequality in (7), which implies that conditioning on additional information—such as prior measurement records—can only reduce the complexity up to a constant.

Also, the Kolmogorov complexity of either agent remains unaffected by knowledge of the distant, spacelike-separated setting: the algorithmic information content of one party's outcome can neither be reduced nor enhanced by conditioning on the other party's freely chosen measurement setting.

5.3 Minimal Experimental Scenario

Consider the minimal experimental scenario illustrated in Figure 2, allowing for violation of LF inequalities [WCR23, JM25], involving three agents, Charlie, Alice, and Bob, whose Kolmogorov complexities are denoted as follows:

- Charlie, the "friend," has complexity $K(C)$,
- Alice (the "superobserver") has complexity $K(A)$,
- Bob has complexity $K(B)$ and is spacelike-separated from Alice.

In addition, Charlie and Bob share a bipartite physical system S_{BC} whose joint Kolmogorov complexity is $K(S_{BC})$. Based on the SIA hypothesis [Gri13], we have the following initial relationships:

$$\begin{aligned} &K(S_C) = K(\mathrm{Tr}_B\, S_{BC}) < K(C) < K(A), \\ &K(S_B) = K(\mathrm{Tr}_C\, S_{BC}) < K(B), \\ &K(C, S_C) < K(A) \end{aligned} \tag{11}$$

ensuring that Charlie's and Bob's memory can record their measurement outcomes and that Alice can track what Charlie is doing to S_C. Also, Alice being a super-observer, can, in principle, reverse the entire state of Charlie and his subsystem.

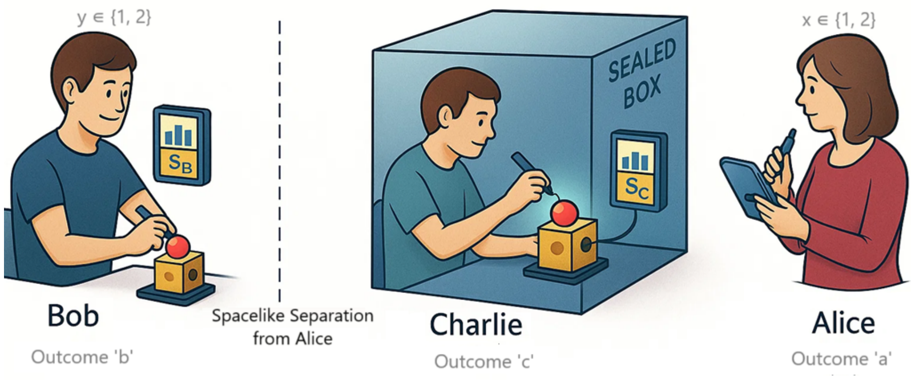

Figure 2: Local Friendlines setup.

Charlie is isolated in a shielded laboratory and performs a projective measurement on his subsystem and records the result $c \in \{0, 1\}$. The act of registration updates Charlie's memory, reducing its algorithmic complexity from $K(C)$ to $K(C \mid c)$.

Subsequently, Alice and Bob choose their measurement settings $x, y \in \{1, 2\}$ by independent random processes executed within space–time regions that lie outside the past light-cones of the events labelled c and b (for the choice of x) and of c and a (for the choice of y). These settings yield outcomes $a, b \in \{0, 1\}$, respectively.

The protocol for a single trial includes:

- **Case** $x = 1$. Alice queries Charlie for his recorded outcome and simply adopts it, setting $a = c$. The information she acquires reduces the description of her own state from $K(A)$ to $K(A \mid c)$.
- **Case** $x = 2$. Alice first applies the inverse unitary $U^\dagger$ that reverses Charlie's interaction, thereby restoring the subsystem S_C to its pre-measurement state. She then performs an independent measurement that produces a fresh outcome a. After registering a, her memory complexity contracts to $K(A \mid a)$. Because Charlie's laboratory has been unitarily reset, no record of c survives for Alice to access.

After each trial, we record the tuple (x, y, a, b, c). Analogous to Grinbaum's procedure (see 4.4), where a new photon is sent toward the fullerene molecule after the previous one has been measured, the local-friendliness experiment proceeds in the same manner: once the bipartite system is measured as described above, a fresh entangled pair is prepared and dispatched to Charlie and Bob in their respective laboratories, and the entire sequence is repeated.

5.4 Empirical Statistics

Now, the existence of $P(a, b \mid x, y)$ is implied by AOE. Also, No-Superdeterminism implies $P(c \mid x, y) = P(c)$, and Locality implies that $P(a \mid c, x, y) = P(a \mid c, x)$ and $P(b \mid c, x, y) = P(b \mid c, y)$.

Normally as seen in [BUAG+20, WCR23, JM25], repeated trials yield the *empirical distribution* $\varphi(ab|xy)$[20], constrained under the three assumptions as follows:

$$\varphi(ab|xy) = \begin{cases} \sum_c \delta_{a,c} P(b|cy)P(c), & \text{if } x = 1, \\ \sum_c P(ab|cxy)P(c), & \text{if } x \neq 1, \end{cases} \tag{12}$$

These distributions must therefore lie within the LF polytope defined by the assumptions. But, Bong *et al.* [BUAG+20] constructed explicit quantum states and measurement settings whose joint outcome statistics violate the LF inequalities. This result furnishes a no-go theorem, demonstrating that quantum mechanics cannot be reconciled with the three assumptions underlying the LF framework.

But what would happen to the statistics with repeated trials, especially when the SIA hypothesis is considered? As discussed in 5.2, with each measurement, the complexity gap between relative systems becomes narrower. In the following, let's see and discuss what happens when this gap begins to fade away.

5.5 Implications and Limits

The structure above parallels the derivation of LF inequalities [BUAG+20, WCR23], but reframed to account for computational limitations. We now consider the case where an observer complexity bound is nearly saturated by the system being measured. When for example the complexity of the system approaches the observer's capacity, e.g., when $K(CS_C) \approx K(A)$, further measurements become thermodynamically costly [Gri13, Zur89, Zur98].

According to Landauer's principle [Lan61], memory erasure beyond this point causes heat dissipation, disrupting reversibility [Zur89] for Alice. Similarly, if $K(S_B) \approx K(B)$,

[20] Related by marginalisation, i.e., $\varphi(ab|xy) = \sum_c \varphi(abc|xy)$.

Bob's effective capacity to treat his system as quantum (according to the criterion mentioned in 4.2) degrades, and the shared system decoheres into classical correlations, driving $\varphi(a, b \mid x, y)$ back into the classical polytope.

Thus, the Kolmogorov complexity of observer-system configurations naturally bounds the domain (which is, in general, non-computable) in which LF violation can be observed. This reframing highlights the possible role of algorithmic information as both a limiting and explanatory principle in the emergence of classicality from quantum theory.

5.6 Undecidability

4.5 discusses the uncomputability of Kolmogorov complexity and SIA. And hence, it is impossible to determine, in general, an exact point at which, say, $K(CS_C)$ matches or surpasses $K(A)$. Thus, this inherent undecidability of Kolmogorov complexity carries a direct implication for the quantum–classical transition in our Local Friendliness framework. Suppose an experimenter wishes to know exactly when Charlie's lab–viewed as a computational object with Kolmogorov complexity $K(CS_C)$–saturates Alice's processing capacity $K(A)$. Because we cannot compute, in general, $K(CS_C)$ or $K(A)$ exactly, we cannot assert with certainty that a given measurement by Charlie will be reversible or irreversible with respect to Alice. In other words, no finite procedure can declare, "At this moment t, the system has crossed from the quantum-coherent regime into the classical regime." At best, one can show that $K(CS_C)$ is bounded from above by some function of system size or physical parameters, but not that it equals Alice's complexity bound.

Thus, any physical claim about when classicality emerges–when reversibility breaks down or when the LF inequalities revert to classical bounds–must be formulated in terms of approximate complexity bounds or empirical thresholds rather than exact values. In practice, one could identify conditions under which it is extremely likely that $K(CS_C) \approx K(A)$ (for instance, by counting degrees of freedom using some suitable proxy or estimating memory requirements), but one cannot prove that the transition occurs at precisely that point in general. This limitation establishes an unavoidable epistemic boundary: although algorithmic information theory guides our understanding of when thermodynamic or memory-based costs force a system to decohere, it also predicts that the exact location of the "quantum–classical border" cannot be determined by any computation.

In summary, integrating the uncomputability of Kolmogorov complexity into the Local Friendliness argument shows that the division between quantum and classical descriptions is not merely practically difficult to locate–it is, in principle, undecidable. As a result, the emergence of classicality from quantum mechanics remains, fundamentally, an algorithmically undecidable phenomenon.

5.7 Implications on Claim 1 of Restriction A in [JM25]

In their work, Jones and Mueller (2025) [JM25] propose *Restriction A* as a foundational constraint on physical theories: for certain experiments, the theory cannot furnish a joint probabilistic description of all agents' observations. Formally, given a physical theory, potentially supplemented by additional reasonable assumptions such as locality or causality, there may exist scenarios where the theory does not admit a single probability space $(\Omega, \mathcal{F}, P)$ that encompasses the observed outcomes of all agents as random variables. Here, Ω denotes the sample space of all possible outcomes, $\mathcal{F}$ is a σ-algebra of measurable events (subsets of Ω), and P is a probability measure assigning probabilities to events in $\mathcal{F}$, satisfying $P(\Omega) = 1$ and countable additivity.

Claim 1 regarding Restriction A asserts that, within an empirically complete theory, Restriction A applies exclusively to scenarios in which the observations of all agents cannot be jointly assessed externally, a situation described as featuring an epistemic horizon [Sza18, FGDLC25]. Claim 1 thus becomes a direct application of SIA criterion for an observer, particularly within the context of Wigner's friend or Local Friendliness scenarios. This perspective characterizes observers as "quantum to each other," exhibiting quantum-correlated interactions solely when specific complexity inequalities hold; otherwise, their relationship defaults to classical interactions. Within a three-agent Local Friendliness scenario discussed above, these complexity conditions are formally expressed as $K(S_C) < K(C) < K(A)$, $K(S_B) < K(B)$, and $K(C, S_C) < K(A)$. When these complexity gaps exist, each observer has a sufficient "algorithmic buffer" to keep track of all the degrees of freedom of the other as a quantum system, so their interactions can display quantum correlations (e.g., violations of Local Friendliness inequalities).

Critically, when these complexity gaps narrow, the conditions articulated in Claim 1 begin to fail. Specifically, if the complexity of the observed system approaches or exceeds that of the observer, interactions can no longer sustain quantum characteristics, becoming effectively classical. This reflects an intrinsic epistemic constraint: an observer of limited complexity cannot encode all the information coherently of an equally complex system. Hence, as the observer's computational or memory capacity becomes nearly saturated–such as when $K(C, S_C) \approx K(A)$ in a Local Friendliness scenario–any further measurements entail trade-offs in information storage. Formally, the Kolmogorov complexity of the observer-system setup bounds the domain in which quantum-coherent (non-classical) behavior can occur. When this bound is reached, the system's degrees of freedom are no longer completely accessible to the observer, so the interaction defaults to classical, yielding definite outcomes and obeying local realistic descriptions. Thus, this complexity-induced quantum-to-classical transition embodies the emergence of the epistemic horizon described in Claim 1.

5.8 Summary

Taken together, the arguments developed in this section recast the observers in the Local Friendliness experiment within an explicitly algorithmic setting. By treating each agent and subsystem as a computationally bounded object–whose capacity is measured by Kolmogorov complexity–we showed that

- Violations of LF inequalities can persist only while distinct "complexity gaps" remain open: $K(S_C) < K(C) < K(A)$ for Alice-Charlie and $K(S_B) < K(B)$ for Bob, where S_C and S_B denote the respective subsystems of a bipartite quantum state, distributed to Charlie and Bob. As these gaps close, the probability distribution gets driven back into the classical polytope.
- Because exact Kolmogorov complexities are uncomputable, no algorithm can pinpoint the moment at which an observer's capacity is saturated; the quantum-to-classical crossover is therefore undecidable in principle and accessible only through approximate bounds.
- The term "Epistemic horizon" mentioned in Claim 1 of [JM25] is directly linked to inequalities involving the complexity gap between the agents

In sum, algorithmic information theory not only limits where LF violations can be observed but also explains why the quantum-classical divide remains intrinsically fuzzy in general.

6 Conclusion

In this thesis, we have attempted to address one of the most profound questions in quantum foundations: the precise characterization of an observer and the implications of this characterization for quantum experimental outcomes. Through a detailed survey of various proposed criteria for quantifying observerness—including metaphysical constructs, anthropocentric concepts, mass-dependence, and computational complexity—we identified information-theoretic approaches, particularly those leveraging Kolmogorov complexity, as possessing unique clarity, rigor, and experimental relevance.

Central to our analysis is Grinbaum's hypothesis, which posits observers fundamentally as system identification algorithms (SIAs) constrained by finite Kolmogorov complexity. This hypothesis provides a robust and computationally-grounded description of the quantum-classical boundary, transcending traditional ambiguities associated with the Heisenberg cut. By explicitly recognizing observers as finite computational entities, observerness becomes quantifiable, relational, and independent of substrate, thus aligning elegantly with the principles of relational quantum mechanics.

Within this computational paradigm, observers inevitably encounter informational saturation, an intrinsic threshold dictated by their Kolmogorov complexity. At this juncture, observers must erase information to accommodate new observations, incurring a definitive thermodynamic cost via Landauer's principle. Thus, quantum-to-classical transitions acquire a thermodynamic interpretation, linking foundational quantum mechanics to information theory and statistical mechanics. Crucially, the inherent undecidability of Kolmogorov complexity further reveals that exact demarcation of the quantum-classical boundary is intrinsically uncomputable. Consequently, the emergence of classicality from quantum mechanics is itself undecidable.

When applying this information-theoretic framework to the Local Friendliness experiment, we demonstrated explicitly that quantum violations are sustained only within well-defined complexity regimes. As observer complexity attains informational saturation, quantum correlations begin to diminish and inevitably revert to classical correlations, thus connstraining the polytope in which quantum violations can be sustained.

Moreover, we have shown how our results offer significant insights into contemporary foundational debates, notably Claim 1 of Restriction A in [JM25]. Here, the notion of an epistemic horizon is naturally interpreted through complexity constraints: observers become quantum entities to each other precisely when sufficient complexity "gaps" are maintained. As these gaps close, quantum coherence gives way to classical definiteness, embodying an epistemic boundary rooted in algorithmic incompleteness rather than empirical limitations.

In conclusion, by situating observerness within an algorithmic complexity framework, we establish a rigorous, principled, and experimentally accessible foundation for understanding the quantum-classical boundary, thereby advancing our understanding of the observer at its most fundamental level.

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A The Turing Machine

A.1 Historical Motivation

In 1936, Alan M. Turing introduced the *Turing Machine* (TM)[21] to address Hilbert's *Entscheidungsproblem*—the quest for a mechanical procedure that decides the truth of every first-order statement. Turing's negative answer, together with Church's independent work on λ–calculus, laid the foundations of modern computability theory [Tur37, unk25].

A.2 Formal Definition

A *deterministic* TM is specified by the seven–tuple

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}}), \tag{13}$$

where

- Q is a finite set of *states*[22];
- Σ is the finite *input alphabet*[23];
- $\Gamma \supseteq \Sigma \cup \{\sqcup\}$ is the *tape alphabet* [24];
- $\delta\colon (Q \setminus \{q_{\text{acc}}, q_{\text{rej}}\}) \times \Gamma \to Q \times \Gamma \times \{L, R\}$ is the *transition function*[25];
- $q_0 \in Q$ is the *start state*;
- q_{acc}[26], q_{rej}[27] $\in Q$ are distinct halting states.

The model consists of an infinite, one–dimensional *tape*[28] and a *read-write head*[29] controlled by the finite state machine (13). A configuration of M is a triple recording (i) the current tape contents, (ii) the head position, and (iii) the current state. Computation proceeds by iterating δ on successive configurations until a halting state is reached, or forever if none is.

[21] A Turing Machine is an abstract mathematical device, not a physical apparatus. It formalises the intuitive notion of an algorithm. See [Tur37] for the original paper.

[22] Think of Q as the "memory" of a finite control unit; it records only finitely many possibilities.

[23] The symbols originally present on the tape; $\sqcup \notin \Sigma$ is reserved for blank cells.

[24] The set of symbols that may appear on the tape at any time, including the blank symbol $\sqcup$.

[25] Given the current state and scanned symbol, δ supplies the next state, the symbol to write, and whether the head moves left (L) or right (R).

[26] Accepting halting state: once entered, M halts and accepts the input.

[27] Rejecting halting state: once entered, M halts and rejects the input.

[28] The tape supplies unbounded memory; only finitely many cells are non–blank at any finite time.

[29] The head scans a single cell each step, writes a symbol, and moves left or right.

A.3 Computability and Universality

Turing used this model to show that the *Halting Problem*[30] is undecidable, thereby proving no algorithm can solve every decision problem [Tur37, Dav13, unk25]. Moreover, there exists a *Universal Turing Machine* (UTM) U that, given the *Gödel encoding* $\langle M, w\rangle$, perfectly simulates TM M on the input w. Universality captures the essence of *programmability* and anticipates the stored-program architecture of real computers [unk25].

B Introduction to Quantum Turing Machines

The *Quantum Turing Machine* (QTM)[31] is a theoretical model of quantum computation that extends the classical Turing machine by incorporating key principles of quantum mechanics, namely superposition, entanglement, and unitary evolution. Independently developed by Paul Benioff (1980) and formally defined by Deutsch (1985) [Deu85], the QTM supplies a rigorous framework for analysing the ultimate limits of computation in a quantum-mechanical universe.

A QTM has a finite set of internal states, an infinite *quantum tape* (a superposition[32] of classical configurations), and a transition function given by a unitary operator. If $|\Psi_t\rangle$ is the joint state of head, tape, and internal register at discrete time t, the machine evolves according to

$$|\Psi_{t+1}\rangle \;=\; U\,|\Psi_t\rangle, \tag{14}$$

where the global operator U is unitary, hence reversible. Equation (14) is the discrete-time counterpart of the continuous Schrödinger equation[33].

This model not only formalizes the notion of quantum algorithmic processes, but also underpins the equivalence between quantum circuit and machine models [CCY93, BV97], thereby reinforcing the *quantum Church–Turing thesis*[34], which is fundamental for the realization of quantum computations.

C Quantum Kolmogorov Complexity

We start by reviewing several important formulations of quantum Kolmogorov complexity, a quantum–information–theoretic analogue of classical Kolmogorov complexity. Although the technical details differ, each definition measures the shortest description that enables a universal quantum computer to reproduce a given quantum state within a prescribed accuracy.

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[30] Given a TM M and input w, decide whether M ever halts on w.

[31] A QTM is the quantum analogue of a classical Turing machine: it manipulates qubits on an infinite tape and its global configuration evolves coherently under a unitary operator.

[32] Because the tape is quantum, its classical configurations occur simultaneously with complex amplitudes.

[33] The Schrödinger equation $i\hbar\partial_t|\psi(t)\rangle = H|\psi(t)\rangle$ governs the unitary time evolution generated by a Hamiltonian H.

[34] The quantum Church–Turing thesis posits that any physically realizable computation in a quantum world can be efficiently simulated by a quantum Turing machine.

C.1 Vitányi's definition

Vitányi's seminal approach extends classical algorithmic information to pure quantum state while insisting on *classical descriptions*[35][Vit00, Vit01, Mü07]. Let U be a universal quantum Turing machine (QTM) that receives a finite, prefix-free binary program p and produces the (generally pure) output state $U(p) = |\phi_p\rangle$. The prefix-free quantum Kolmogorov complexity of a target pure state $|\psi\rangle$ is

$$K_{Vit,U}(|\psi\rangle) \;:=\; \min_{p}\Big\{\,\ell(p) \;-\; \log_2(|\langle\psi \mid \phi_p\rangle|^2)\Big\}, \tag{15}$$

where $\ell(p)$ is the bit-length of p and $|\langle\psi \mid \phi_p\rangle|^2$ represents the fidelity between $U(p)$ and the target state. The second term in (15) acts as an approximation penalty: when $|\phi_p\rangle$ coincides with $|\psi\rangle$, the penalty vanishes, whereas larger infidelity is penalised logarithmically [Vit01, Miy11]. Because fidelity is continuous, neighbouring states possess similar complexities, and every pure state has finite $K_{V,U}$. Thus $K_{V,U}$ quantifies the minimum classical information required to prepare a quantum state to any prescribed accuracy on a universal QTM [Vit01, Gá01].

Similarly, if y is an auxiliary input, which may be classical or quantum, then [Vit01]

$$K_{Vit,U}(|\psi\rangle \mid y) \;:=\; \min_{p}\Big\{\,\ell(p) + \lceil -\log|\langle\psi \mid z\rangle|^2\rceil \;\Big|\; U(p,y) = |z\rangle\Big\}. \tag{16}$$

C.2 Berthiaume-van Dam-Laplante definition

Berthiaume *et al.* propose [BDL01, Mü07] that for a universal QTM U, a pure target state $|\psi\rangle$, and a convergence function $f : \mathbb{N} \to [0,1]$ with $f(k) \to 1$, the so-called f-approximation quantum Kolmogorov complexity is defined as

$$K^{f}_{Bert,U}(|\psi\rangle) := \min_{p}\Big\{\,\ell(p) \;\Big|\; \forall k \in \mathbb{N},\; |\langle\psi \mid z\rangle| \;\geq\; f(k)\Big\}, \tag{17}$$

where $|z\rangle = U(p, 1^k)$. Also, p represents a finite quantum program (qubit string) supplied to U. Thus, simply $K^{f}_{Bert,U}(|\psi\rangle)$ is the length of the shortest quantum input $|z\rangle$ that produces $|\psi\rangle$ with fidelity greater or equal to $f(k)$. A single program must meet the fidelity requirement for all k. Special cases include perfect-fidelity complexity ($f \equiv 1$) and fixed-error complexity ($f(k) = 1-\varepsilon$), but it is shown that only the vanishing-error version $f(k) \to 1$ enjoys invariance under changes of the universal QTM [BDL01]. Complexity is measured in qubits; hence $K^{f}_{Bert,U}$ quantifies the minimum amount of quantum information that must be supplied so that a universal QTM can generate $|\psi\rangle$ with arbitrarily high accuracy. When $|\psi\rangle$ encodes a classical string, this quantum complexity equals the classical Kolmogorov complexity up to an additive constant, demonstrating consistency with the classical theory [BDL01, Mü07].

C.3 Müller's definition

Müller refines the approximation criterion by employing the trace distance $D(\rho,\sigma) = \frac{1}{2}\|\rho-\sigma\|_1$, an metric that upper-bounds the statistical distinguishability of quantum states [Mü09, Mü07]. For $0 < \delta < 1$ and a universal QTM U, the fixed-error complexity of a density operator ρ is

$$K^{\delta}_{\text{Mül},U}(\rho) := \min_{\sigma}\Big\{\,\ell(\sigma) \;\Big|\; D(\rho,\, U(\sigma)) \leq \delta\Big\}, \tag{18}$$

[35] Finite binary (classical bit) program that, when run on a fixed universal quantum Turing machine, prepares (or approximates) a desired pure quantum state.

where σ is a quantum program state of length $\ell(\sigma)$. To obtain a robust measure, Müller further defines the approximation-scheme quantum Kolmogorov complexity

$$K_{M\ddot{\mathrm{u}}l,U}(\rho) := \min_{\sigma}\Big\{\, \ell(\sigma) \;\Big|\; \forall k \in \mathbb{N},\ D(\rho, U(\sigma, k)) \leq 1/k \Big\}, \tag{19}$$

requiring a single program that enables arbitrarily accurate approximations. Müller proves invariance of (19) with respect to strongly universal QTMs and shows that, for classical strings, the quantum and classical complexities coincide up to constants [Mü09]. Moreover, for any fixed $0 < \delta < 1/\sqrt{2}$, $K^{\delta}_{M\ddot{\mathrm{u}}l,U}(\rho)$ and $K_{M\ddot{\mathrm{u}}l,U}(\rho)$ differ by at most a constant factor for sufficiently large program lengths, indicating that permitting a small constant error does not drastically reduce description size.

C.4 Summary

Quantum Kolmogorov complexity is the quantum-information-theoretic counterpart of classical Kolmogorov complexity: it asks how much information, in the form of a finite program, must be supplied to a universal quantum computer so that the machine can generate a chosen quantum state as closely as we like. Vitányi's version insists that the program be an ordinary binary string. Its complexity is the number of bits in the shortest such string, plus a small surcharge that grows only when the state the computer actually prepares diverges from the target one; in other words, it quantifies the minimum classical information needed to recreate the state within any desired degree of fidelity. Berthiaume, van Dam, and Laplante instead let the program itself be a quantum state. Here, the complexity equals the fewest qubits that must be provided in a single, fixed program that will let the universal computer approximate the target state to whatever fidelity we later demand. Instead of fidelity, Müller judges accuracy with the trace distance (a yardstick for how distinguishable two quantum states are). He gives both a fixed-error version, where one allows a small, constant deviation, and a stronger version that requires the same program to support arbitrarily fine approximations; both turn out to be stable under changes of the underlying universal machine and reduce to ordinary Kolmogorov complexity when the states encode classical data.

All three approaches agree that for classical strings they reproduce the familiar classical measure up to an additive constant. Also, as in the classical case, none of the quantum extensions above is computable: there exists no algorithm that, given an arbitrary quantum state, outputs its exact complexity under any of these definitions [Vit00, Vit01].