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Pharmaceutical innovation collaboration, evaluation, and matching[☆]

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ABSTRACT

This paper theoretically studies pharmaceutical innovation collaborations, where heterogeneous firms compete for heterogeneous academics. At an interim stage, the firm evaluates the project, which allows it to monitor academics and decide whether to terminate the project to avoid the loss from a future failure. This paper explores the contract, project termination strategy, and collaboration matching. The firm's innovation strategy (exploitations or explorations) determines the evaluation structure, which may affect the market equilibrium. By considering different innovation strategies, this paper shows that in each case, the equilibrium matching is unique (either positive or negative assortative). Consequently, the chosen innovation strategy plays a pivotal role in shaping equilibrium matching outcomes. These findings provide theoretical insights into pharma-academic alliances, shed light on the observed positive or negative assortative properties in the market, and advocate for the consideration of innovation strategies and evaluation structures in future research endeavors. Moreover, this paper also provides several empirical and policy implications.

1. Introduction

In numerous industries, companies often partake in research collaborations with universities, a practice particularly prevalent in the pharmaceutical sector. Within the pharmaceutical industry, such collaborations are deemed vital for the development of novel therapeutic drugs. Many of these drugs stem from discovery-based R&D initiatives, which academic partners are often better equipped to conduct efficiently.[1] Numerous collaborative agreements serve as examples of fruitful partnerships in the pharmaceutical industry. For instance, the pharmaceutical group Chiesi has joined forces with the University of Alberta to explore the behavior of aerosol particles within a humid lung environment. Similarly, in Japan, the Takeda Pharmaceutical Company collaborates with Kyoto University, while Chugai Pharmaceutical collaborates with Osaka University on various drug development projects.[2]

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[1] Mansfield (1998) illustrates the importance of the products that have been developed with the substantial aid of academic research. Rafols et al. (2014) and Palmer and Chaguturu (2017) stress that many pharmaceutical companies outsource their own research departments to academics because this is more efficient and less costly.

[2] Hughes (2008) and Tralau-Stewart et al. (2009) show that the collaborations between pharmaceutical companies and academic institutions increase the efficiency of developing new drugs.

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In these collaborative agreements, academic partners strive to generate innovative and groundbreaking ideas that could serve as the foundation for firms to develop marketable products. However, these partnerships encounter two significant challenges. Firstly, the contribution of academics is subject to moral hazard due to the difficulty in monitoring their efforts. Academic researchers may prioritize enhancing their scientific reputation over aligning their agenda with the pharmaceutical partner's goals. This could lead to a focus on personal research projects, adhering strictly to schedules, or prioritizing the research value of the collaborative project over its commercial viability.[3]

Secondly, the development of a meaningful therapeutic product entails a highly intricate process. Bringing a new product to market profitably involves navigating through a lengthy, uncertain journey that demands considerable effort and resources, often incurring substantial costs for industry partners.[4] The potential commercial outcome is not immediately available, and academics cannot afford to wait for funding or payment in the distant future. To mitigate the risk of investing in a potentially unsuccessful marketable product and to compensate academics for their work, pharmaceutical partners typically adopt strategies such as monitoring progress and providing payment after the academic achieves proof-of-concept results. Additionally, at this interim stage, they evaluate the project's prospects, assess the significance of the gains and losses at stake, and consider possible adjustments to the plan. The project may be terminated if there is a high expectation of negative side effects, significant risks associated with marketing the new drug, or anticipation of substantial reputational damage. The company's evaluation provides further insights into the project, enabling it to decide whether to abandon the project if, at this interim stage, the evidence suggests that the likelihood of developing a marketable product is not sufficiently high.[5]

This paper undertakes a theoretical analysis of research collaborations between pharmaceutical companies and academia, framing such collaborations as an agency problem wherein each firm collaborates with one academic through an incentive contract. The academic's effort significantly impacts the likelihood of yielding meaningful results but is susceptible to moral hazard.

More importantly, the novelty and the significant distinct of the framework in this paper from the traditional principal–agent models are that, at an interim stage, the pharmaceutical company evaluates preliminary projects, leveraging this evaluation as a signal to decide whether to continue or terminate the project before the outcome is fully realized to avoid the potential damage from a possible failed marketable outcome in the future. Moreover, unlike the traditional setup in which the principal's ability usually decides the value of the final outcome, in this paper, the principal's ability determines the evaluation accuracy. Thus, in a collaborative project, the termination decision, incentive, and payment scheme all depend on the signal from the evaluation rather than the final outcome.

While collaborations between pharmaceutical companies and academics for drug development are widespread, the literature exploring the rationale behind these collaborations and factors influencing their formation remains limited. To delve deeper into these partnerships and underscore the significance of project evaluation, this paper models collaborative agreements within a market context wherein heterogeneous pharmaceutical companies with varying evaluation capabilities vie for academics differing in their productivity. In reality, the evaluation abilities of pharmaceutical firms can be quantified by factors such as team members' expertise, patenting prowess, and prior experience with academic researchers. Conversely, academics' effort productivity can be gauged by factors including academic tenure, title, publication record, and patent portfolio.

In this market, competition implies that the characteristics of collaboration matching patterns, the contracts signed in each collaborative agreement, and the utility levels obtained by participants are all endogenous. This paper investigates which pharmaceutical company collaborates with which academic in the market (i.e., a one-to-one matching), the contract they sign, and when the company chooses to abandon the project.

The role of the interim evaluation and the termination decision during the innovation process of developing therapeutic products is of paramount importance. Therefore, this paper adopts both agency theory and matching theory to conduct a novel framework to explore how evaluation can influence the incentive contract and shape collaboration. Moreover, it also provides a theoretical explanation of the matching pattern between firms and academics. To address this, I consider the following three general scenarios to elucidate how evaluation technology can affect the equilibrium outcome.

In real-world scenarios, some projects are exploitative, where some potential side effects are well-documented through previous research articles, patents, and reports. However, firms might not have full knowledge of the effectiveness of drugs developed through such collaborations. Hence, in the first case, I consider that all pharmaceutical companies have an equal probability of receiving a negative signal when project prospects are unfavorable. Conversely, they receive positive signals with varying probabilities if the future outcome is promising. Essentially, companies exhibit heterogeneity in making type I errors while having homogeneous likelihoods of making type II errors. I term this scenario the *sensitivity* case.

On the other hand, the pharmaceutical industry also sees a high number of explorative projects. Here, limited findings or studies may identify the side effects of products developed from collaborations, leading to uncertainty. Consequently, in the second scenario, I posit that companies vary in making type II errors. To streamline the analysis of explorative projects, this paper simplifies the assumption by considering firms as homogeneous in making type I errors. This scenario is referred to as the *specificity* case.[6] Lastly,

[3] Kloyer (2011), Kloyer and Scholderer (2012), and Hillerbrand and Werker (2019) show that academics and companies may have conflicts of interest, for instance, because the industry is profit-oriented while the academic's goal is the production of knowledge.

[4] DiMasi et al. (2003) state that the cost of developing a new drug is, on average, 1.3 billion dollars and takes 14 years. Pammolli et al. (2011) show that R&D investments tend to focus on new therapeutic targets characterized by high uncertainty and difficulty.

[5] Henstock (2019) emphasizes that the application of artificial intelligence helps the pharma company to predict the toxicology and even the clinical outcomes according to the cumulative data in the pharma company.

[6] Sensitivity and specificity are original statistical terms describing the accuracy of a diagnostic test. Sensitivity measures how well a test can identify true positives, and specificity measures how well a test can identify true negatives.

this paper delves into a complementary scenario where, regardless of the future outcome, pharmaceutical companies receive a noisy signal — either false positive or false negative — with equal probability. In this case, companies exhibit heterogeneous probabilities of making both type I and type II errors, termed the *symmetry* case.

In an equilibrium outcome, no company and no academic can form a new partnership that is more beneficial (or weakly beneficial) for both parties than their initial pair (i.e., there is no blocking pair). I analyze the characteristics of the equilibrium contract and matching. This paper demonstrates that in the equilibrium of all scenarios, pharmaceutical companies consistently terminate projects upon receiving a negative signal, indicating a high likelihood of failure. Furthermore, there exists a unique matching between pharmaceutical companies and academics in each case. In the sensitivity case, pharmaceutical companies with superior evaluation technologies, resulting in fewer type I errors, collaborate with more productive academics (positive assortative matching, or PAM). The rationale lies in the heterogeneity of firms in making type I errors: companies vary in their ability to detect success. Consequently, the increased probability of success brought by better academics becomes more significant. Superior firms can utilize more of this increase in probability; therefore, they are willing to pay more in the competition for better academics, outbidding weaker firms.

In the specificity and symmetry scenarios, superior pharmaceutical companies collaborate with less productive academics (negative assortative matching, or NAM). This phenomenon arises because, in both scenarios, firms exhibit heterogeneity in making type II errors, indicating varying accuracies in identifying potential failures. The pivotal factor is the extent to which more productive academics can reduce the probability of failure. Firms with less accurate evaluations are prone to making more type II errors, increasing the likelihood of experiencing a failure despite receiving a positive signal. Consequently, these inferior firms benefit more from the reduction in the probability of failure offered by better academics. To mitigate the risk of substantial losses from potential failures, these weaker firms are willing to invest more in securing collaborations with better academics, outbidding superior counterparts in the competition.

Additionally, I discuss the relationship between an academic's ability and their incentive payment. Unlike in an isolated pair without matching, where the relationship is typically positive, in this two-sided matching market considered in this paper, higher productivity can increase, decrease, or have a non-monotonic effect on incentive payments. This relationship tends to be positive when academics are more heterogeneous than firms. Conversely, when firms are more polarized than academics, the relationship tends to be negative.

Furthermore, this paper also analyzes the welfare effect of increasing participants' abilities. Intuitively, one might think that higher abilities would increase total welfare. However, a numerical example in the paper demonstrates that raising the lower bound of academic abilities may actually harm total welfare. Therefore, the more effective and safer approach to enhancing welfare is encouraging the best academics and most capable firms to join the collaboration market while not restricting the participation of academics with relatively lower abilities. Finally, this paper provides several extensions that partially remove the initial assumptions and show that the results can remain.

This paper makes significant contributions to the study of project evaluation within industry-academic alliances and their collaboration-matching patterns. The analysis highlights the substantial influence of innovation strategy and evaluation technology on the matching between firms and academics. Consequently, future investigations into these partnerships may benefit from distinguishing between different project types and taking into account the nuances of project evaluation. Moreover, this paper also provides several empirical implications that can be testable and policy implications that can potentially improve the total welfare. The results in this paper can offer valuable insights into optimizing collaboration outcomes and fostering mutually beneficial partnerships in the pharmaceutical and academic realms.

1.1. Brief literature review

This paper is related to different strands of the literature. Firstly, it is associated with the principal–agent theory, where the outcome of a relationship is unobservable. Chade and Kovrijnykh (2016) explore a similar inquiry, investigating a repeated moral hazard problem between a principal and an agent, wherein the principal's project outcome remains unobservable. In contrast, this paper examines a scenario where success probability depends on the agent's effort, with the firm overseeing information acquisition. Additionally, my paper studies the endogenous formation of partnerships within the matching market, providing insights into collaboration and their implications for innovation strategy and project evaluation within industry-academic alliances.

Secondly, this paper is linked to the literature on assortative matching with non-fully transferable utility. Legros and Newman (2007) provide the generalized differences condition as a sufficient criterion for assortative matching. Chade et al. (2017) introduce an alternative version. I employ their approach to identify the equilibrium matching.

This paper also contributes to a burgeoning literature exploring two-sided matching markets under moral hazard with non-fully transferable utility. For instance, Alonso-Paulí and Pérez-Castrillo (2012) examine a market where shareholders may offer incentive contracts or contracts incorporating a Code of Best Practice. Their findings indicate that matching is positively assortative if all participants sign the same type of contract. Similarly, Antón and Dam (2020) analyze incentive contracts under double-sided moral hazard, investigating market equilibrium between investors and entrepreneurs, revealing a negative assortative matching. More papers delve into the agency and matching theory with non-fully transferable utility, including Dam and Pérez-Castrillo (2006), Hong et al. (2020), Macho-Stadler et al. (2021), and Altınok (2023).[7] In addition, this paper also contributes to the literature on

[7] Other papers investigate assortative matching with fully transferable, for instance, Serfes (2005), Ghatak and Karaivanov (2014), and Macho-Stadler and Pérez-Castrillo (2020).

alliance formation; for instance, Cabral and Pacheco-de Almeida (2019) adopts the frictionless matching model to study assortative matching and firm value.

Several papers empirically explore the characteristics of industry-university alliances. For example, Mindruta (2013) studies collaborations between biotech companies and academics, employing an empirical matching approach from Fox (2018) and finds several firm's and academic's abilities are substitutable, e.g., patenting ability. Similarly, Banal-Estañol et al. (2018) investigate industry-university partnerships in the UK, revealing that firms' and academics' research orientations (applied or basic research) are complementary. However, there is a scarcity of theoretical papers investigating why and how companies and academics engage in partnerships, particularly emphasizing the importance of innovation strategy and evaluation structure. This paper fills this gap by offering a theoretical approach to address these two-sided matching issues.

To the best of my knowledge, this paper represents the inaugural endeavor to explore the impact of performance evaluation on incentive contracts and endogenous matching. It elucidates the effect of evaluation structure on equilibrium matching, offering novel insights into the collaboration matching market. Furthermore, this contribution extends beyond the realm of collaboration matching, offering implications for other matching markets where outcomes are unobservable, and principals have the capacity to evaluate agents' performance.

The rest of the paper is organized as follows. The model is introduced in Section 2. Section 3 states the equilibrium contract. Section 4 identifies the equilibrium matching. Section 5 studies the relationship between incentive payment and academic's ability. A welfare analysis is conducted in Section 6. Section 7 extends the results. Section 8 concludes and discusses the paper. All proofs are in Appendix.

2. Model

This section serves as an introduction to the model, providing a comprehensive overview of the matching market and important definitions.

2.1. Model setup and timing of the game

I consider a continuum of risk-neutral pharma companies P and a continuum of risk-neutral academics A. A pharma is denoted by p, p_i, $p_{i'}$, etc., and an academic is denoted by a, a_j, $a_{j'}$, etc. A collaboration involves one firm and one academic denoted by a pair (p_i, a_j).

A pair (p_i, a_j) is formed via a contract W_{ij}, and in the collaboration, academic a_j supplies a non-contractual and non-observable effort e_j to develop the project for company p_i. This effort is associated with a cost $c(e_j) = \frac{1}{2}e_j^2$. The final product delivers either a success, which yields a good outcome $G > 0$, or a failure, which yields a loss $B < 0$. The effort e_j determines the project's prior probability of success, formally: $\pi(G) = \gamma_j e_j$, $\pi(B) = (1 - \gamma_j e_j)$, where γ_j denotes a_j's productivity, which is public information.[8] Academics are heterogeneous in γ_j, which is distributed on the interval $[\underline{\gamma}, \bar{\gamma}]$, with $\underline{\gamma} \geq 0$.[9]

I assume that if the project fails in the end, the loss is larger than the benefit when it succeeds (i.e., $(G + B) \leq 0$). This is because companies that experience a failed drug often face a plummeting stock price, need to reduce their workforce, close research sites, consolidate business units, potentially sell off various therapeutic areas to preserve the core business, or even worse, file for bankruptcy because the drug may cause fatal side effects for patients.[10]

However, the eventual result of the project (e.g., a marketable drug) typically occurs in the distant future. For instance, the development of a new drug often takes an average of ten years. Consequently, the outcome of the project is non-contractual due to the extended timeframe involved. Hence, at an interim stage, pharma p_i evaluates the progress of academic a_j's project. This evaluation serves multiple purposes: it enables the company to decide whether to terminate the project to avoid potential substantial losses ($B < 0$), to monitor the academic's progress, and to facilitate the corresponding payment to the academic. If the project is abandoned, the outcome is neither G nor B but zero. Hence, the project's ultimate outcome hinges on several factors: the academic's effort, more importantly, the company's evaluation, and the decision regarding project continuation.

After firms evaluate the project, the result takes the form of a signal, $s \in \{g, b\}$, i.e., either good (g) or bad (b). The signal is correlated with the final outcome as follows:

$$\pi(g|G) = \alpha_i; \quad \pi(b|G) = 1 - \alpha_i;$$

$$\pi(b|B) = \beta_i; \quad \pi(g|B) = 1 - \beta_i,$$

where α_i (β_i, respectively) denotes the accuracy of the evaluation if the future outcome is a success (a failure, resp.). For example, α_i represents the likelihood of receiving a good signal g if the future product is successful, and $(1 - \alpha_i)$ represents the probability

[8] An academic's ability (productivity) can be assessed by her number of patents, academic reputation, or publication record. This information is publicly available.

[9] Using the productivity γ_j is equivalent to assuming a heterogeneous cost parameter for the effort in the cost function.

[10] To complete the discussion, in Section 7.3, I explore how the results extend for the cases where $(G + B) > 0$ and show that the results remain under certain circumstances.

of making type I errors. Companies are heterogeneous in their evaluation ability (α_i, β_i), with $\alpha_i \in [\underline{\alpha}, \bar{\alpha}]$, $\beta_i \in [\underline{\beta}, \bar{\beta}]$, $0 \leq \underline{\alpha} < \bar{\alpha} \leq 1$, $0 \leq \underline{\beta} < \bar{\beta} \leq 1$, and $(\alpha_i + \beta_i) \geq 1$.

Although the evaluation technology is characterized by two parameters, for model simplicity, attention is restricted to one-dimensional heterogeneity. Thus, firms' evaluation abilities, denoted as t_i, can take only one of three possible forms: $t_i \in \{(\alpha_i, \beta), (\alpha, \beta_i), (\alpha_i, \beta_i = \alpha_i)\}$.[11] The first case refers to the *sensitivity* setup, where pharmaceutical companies differ in making type I errors but are homogeneous in making type II errors. The second case is the *specificity* setup, where companies are homogeneous in making type I errors but heterogeneous in making type II errors. Lastly, in the *symmetry* scenario, a firm has the same probability of making both types of errors, and firms are heterogeneous in this probability. The pharmaceutical company's evaluation ability is public information.

Given that the only verifiable element is the signal s, the contract only depends on the realization of this signal. For a pair (p_i, a_j), the compensation scheme is $W_{ij} = (w_{ij}^g, w_{ij}^b)$, where w_{ij}^g is the payment if the signal is g, and w_{ij}^b is the wage when the signal is b.

The timing of the game is the following: (1) Pharma companies compete for academics via the incentive contract W_{ij}; (2) When a partnership is formed, the academic exerts a non-observable effort e_j to conduct a project; (3) The pharma company evaluates the project and receives a signal $s \in \{g, b\}$; (4) According to the signal realization, the pharma company decides whether to terminate the project and delivers the payment to the academic.

2.2. Participants' utilities and decisions in a partnership

The academic's utility depends on the contract W_{ij} and the probabilities of the different signal realizations. Clearly, let $\pi(g) = \pi(G)\pi(g|G) + \pi(B)\pi(g|B)$ and $\pi(b) = \pi(G)\pi(b|G) + \pi(B)\pi(b|B)$ be the probability of receiving a good signal and a bad signal, respectively.

Academic a_j's expected utility is denoted by $Eu_j(W_{ij}, e_j; p_i) = \pi(g)w_{ij}^g + \pi(b)w_{ij}^b - \frac{1}{2}e_j^2$ for an effort e_j when she has been matched to pharma p_i under the contract W_{ij}. One can easily obtain the incentive compatibility constraint (*ICC*): $e_j(W_{ij}; p_i) = \arg\max_{e_j} Eu_j(W_{ij}, e_j; p_i)$.

Before introducing the company's expected profit, it is necessary to discuss the continuation decision under different signal realizations. This decision depends on the posteriors $\pi(G|g)$, $\pi(G|b)$, $\pi(B|b)$, and $\pi(B|g)$ by Bayes' rule. Note that these posteriors are determined by the characteristics of the participants in a pair and the academic's effort incentivized by the contract.

Under the assumption $(\alpha_i + \beta_i) \geq 1$, it is straightforward to demonstrate that signal s provides more information regarding the corresponding outcome, i.e., $\pi(G|g) \geq \pi(G|b)$ and $\pi(B|b) \geq \pi(B|g)$. In this scenario, suppose that p_i decides to abandon the project upon the revelation of signal g due to the high probability of failure $\pi(B|g)$. It follows logically that p_i should also abandon the project when the signal is b, as $\pi(B|b) \geq \pi(B|g)$. Consequently, the relationship is not profitable and cannot be formed at stage (1) because, regardless of the signal, the firm will abandon the project.

Therefore, a collaboration may only be formed when the firm expects to continue the project unconditionally upon the revelation of a good signal. In this scenario, the partnership is deemed profitable. Thus, in potential collaborations, firms only make the continuation decision when the evaluation reveals a bad signal b. The company's expected profit $Ev_i(W_{ij}; a_j)$ under different continuation decisions when signal b reveals are:

(1) If pharma p_i terminates the project, it will only receive the final outcome when a good signal is revealed and zero, otherwise. The expected profit depends on the probability of receiving each signal (i.e., $\pi(g)$ and $\pi(b)$), the posteriors corresponding to the good signal (i.e., $\pi(G|g)$ and $\pi(B|g)$), and the payment to the matched academic. Thus, its ex-ante expected profit $E\hat{v}_i$ is defined by:

$$E\hat{v}_i(W_{ij}; a_j) = \pi(g)[\pi(G|g)G + \pi(B|g)B - w_{ij}^g] + \pi(b)[0 - w_{ij}^b]. \tag{1}$$

(2) If pharma p_i continues the project, it will always receive the final outcome, whatever the signal is. The expected profit, in this case, also depends on the posteriors corresponding to the bad signal (i.e., $\pi(G|b)$ and $\pi(B|b)$). Its expected profit is $E\tilde{v}_i$ defined by:

$$E\tilde{v}_i(W_{ij}; a_j) = \pi(g)[\pi(G|g)G + \pi(B|g)B - w_{ij}^g] + \pi(b)[\pi(G|b)G + \pi(B|b)B - w_{ij}^b]. \tag{2}$$

The pharma designs the contract and chooses the continuation decision that yields a higher expected profit, denoted by $Ev_i(W_{ij}; a_j) = \max\{E\hat{v}_i(W_{ij}; a_j),\ E\tilde{v}_i(W_{ij}; a_j)\}$.

The contract signed by (p_i, a_j) must be acceptable to both of them. I assume the academic's outside option is of zero utility when she accepts no contract. Then, a contract $W_{ij} = (w_{ij}^g, w_{ij}^b)$ is acceptable to both a_j and p_i if the following constraints hold (denoted by ACC_a and ACC_p, respectively): $Eu_j(W_{ij}; p_i) \geq 0$ and $Ev_i(W_{ij}; a_j) \geq 0$, respectively. Besides, the academic is protected by limited liability (denoted by *LLC*): $w_{ij}^g, w_{ij}^b \geq 0$.

Definition 1. A contract W_{ij} for a pair (p_i, a_j) is feasible if it satisfies: (1) the wages are non-negative *(LLC)*; (2) the contract is acceptable to both p_i and a_j, i.e., (ACC_a) and (ACC_p) hold.

[11] I extend the model to a setup of multi-dimensional heterogeneity in Section 7.2.

2.3. Competition and stability in the market

This paper considers one-to-one matching, which is usually the case of collaborations in the pharma industry.[12] In this matching, first, each pharma p_i is assigned to an academic or stays unmatched; second, each academic a_j is assigned to a firm or stays unmatched; third, a pharma p_i is matched with an academic a_j if and only if a_j is matched with p_i. The formal definition of a matching in this economy is as follows.

Definition 2. A one-to-one matching is a mapping function $\mu : P \cup A \to P \cup A$, such that (1) $\mu(p_i) \in A \cup \{p_i\}$, $\forall p_i \in P$; (2) $\mu(a_j) \in P \cup \{a_j\}$, $\forall a_j \in A$; (3) for any pair $(p_i, a_j) \in P \times A$, $\mu(p_i) = a_j$ if and only if $\mu(a_j) = p_i$.[13]

The outcome of the game is a market allocation $(\mu, \mathcal{W})$. It consists of a matching μ and a menu of contracts $\mathcal{W}$, which contains the feasible contract W_{ij} between each matched pair $(p_i, a_j) \in P \times A$. In equilibrium, the solution concept must satisfy stability, which means that there is no pharma and academic can form a partnership by signing a feasible contract such that both are better off under the new deal compared to the initial equilibrium situation.

Definition 3. The market allocation $(\mu, \mathcal{W})$ is stable if there is no blocking pair (p_i, a_j) and a feasible contract W_{ij} such that $Ev_i(W_{ij}; a_j) \geq Ev_i(W_{i\mu(i)}; \mu(p_i))$ and $Eu_j(W_{ij}; p_i) \geq Eu_j(W_{\mu(j)j}; \mu(a_j))$, with at least one strict inequality.

The matching is endogenous; thus, when pharma p_i competes for an academic a_j, to design the optimal contract for a_j, it has to consider the expected utility a_j can obtain from p_i's potential competitors. Let $\underline{U}_j$ represent this reservation utility of academic a_j, which is endogenous, the maximum among all outside options of all possible alternative matches, and the reservation utility in equilibrium. In the relationship (p_i, a_j), a feasible contract with endogenous reservation utility is denoted by $W_{ij}(\underline{U}_j)$. Thus, a_j's participation constraint (PC) is: $Eu_j(W_{ij}(\underline{U}_j); p_i) \geq \underline{U}_j$.[14] The matching μ of interest in this paper is positive (i.e., better firms are matched with better academics) and negative assortative matching (i.e., better firms are matched with worse academics).[15]

Definition 4. For any two firms p_i and $p_{i'}$ with $t_i > t_{i'}$ in two matched pairs, a matching μ is positive assortative (PAM) if $\mu(t_i) > \mu(t_{i'})$, and it is negative assortative (NAM) if $\mu(t_i) < \mu(t_{i'})$.

3. Equilibrium contract in a stable market allocation

This section solves the equilibrium contract for a pair (p_i, a_j) in a stable market allocation $(\mu, \mathcal{W})$. The equilibrium contracts exhibit identical properties across the three scenarios, with the only distinction being the heterogeneity of the firm's parameters. The following assumption guarantees that the probability of success does not exceed one, i.e., $\pi(G) = \gamma_j e_j \leq 1$.[16]

Assumption 1. $[G\bar{\alpha} - (1 - \underline{\beta})B]\bar{\gamma}^2 \leq 1$.

For any pair (p_i, a_j), when designing the payment scheme $W_{ij}(\underline{U}_j) = (w^g_{ij}, w^b_{ij})$, pharma company p_i maximizes its expected profit subject to the constraints *(PC)*, *(ICC)*, and *(LLC)*. Upon receiving a signal b from the evaluation, the pharmaceutical company faces a decision: whether to terminate the project. This decision directly impacts the company's expected profit and, consequently, the optimal contract. While solving for the equilibrium, I consider both possibilities. However, for the sake of discussing crucial properties of the equilibrium contract, let us momentarily assume that the company terminates the project under a bad signal. This assumption will be further validated as we delve into the equilibrium analysis.

In accordance with *(ICC)*, payment w^g_{ij} surpasses w^b_{ij}. Furthermore, the company's inclination is to elevate w^g_{ij} while diminishing w^b_{ij} to zero (owing to the limited liability constraint), thereby crafting a contract with greater incentives and enhanced profitability. As is customary in moral hazard problems featuring a risk-neutral agent shielded by limited liability, the equilibrium contract is influenced by the reservation utility. Depending on $\underline{U}_j$, three scenarios emerge. Firstly, if $\underline{U}_j$ is sufficiently low, the company is compelled to offer a bonus w^g_{ij} to induce the academic to exert adequate effort. In this instance, the equilibrium contract remains unaffected by $\underline{U}_j$. The company achieves maximum profit, and the academic secures an expected utility surpassing $\underline{U}_j$, rendering *(PC)* non-binding.

Secondly, for a relatively higher level of $\underline{U}_j$, the company must elevate w^g_{ij} to the extent that the equilibrium contract guarantees the academic precisely the reservation utility, i.e., *(PC)* becomes binding, $Eu_j = \underline{U}_j$. The threshold of $\underline{U}_j$ that delineates these two

[12] It is uncommon for several pharmaceutical companies to collaborate with a single academic partner for the same project (many-to-one matching), or vice versa, due to considerations of project confidentiality and privacy. Similarly, many-to-many matching is also atypical in this context.

[13] In the following sections, depending on which expression or notation is more convenient, when (p_i, a_j) are matched under μ, then I will use interchangeably following expressions by the participant's notation, its ability, or its subscript: $\mu(p_i) = a_j$, $\mu(\alpha_i) = \gamma_j$, $\mu(\beta_i) = \gamma_j$, and $\mu(i) = j$. If the individual stays unmatched, then $\mu(p_i) = p_i$ or $\mu(a_j) = a_j$.

[14] Note that *(PC)* dominates (ACC_a). In addition, the contract for any matched pair (p_i, a_j) in the stable market allocation should be Pareto optimal. That is, in equilibrium, there is no matched (p_i, a_j) such that both participants can sign an alternative feasible contract to be better off.

[15] PAM and NAM will be shown to be the equilibrium matching later in the paper.

[16] In fact, under this assumption, the highest probability $\pi(G) \leq 1$ for the first-best effort. Thus, $\pi(G) = \gamma_j e_j \leq 1$ always holds for any effort level of the academic in a matched pair because the effort will not be higher than the first-best level.

scenarios hinges on the characteristics of the participants in a pair (α_i, β_i, and γ_j), denoted by $\hat{U}(\alpha_i, \beta_i, \gamma_j)$. Thirdly, if $\underline{U}_j$ is extremely high, there is no profitable contract between p_i and a_j, thus, (ACC_p) is violated. Let $\tilde{U}(\alpha_i, \beta_i, \gamma_j)$ denote the threshold above which pharma p_i prefers not to form a partnership with a_j. Therefore, for any partnership (p_i, a_j), the utility to academic a_j must be lower than $\tilde{U}(\alpha_i, \beta_i, \gamma_j)$.

Proposition 1 formally presents the optimal contract. Notation $X_{ij} = [G\alpha_i - B(1-\beta_i)](\alpha_i + \beta_i - 1)\gamma_j{}^2$ and $Y_{ij} = \sqrt{(1-\beta_i)^2 + 2(\alpha_i+\beta_i-1)^2\gamma_j{}^2 Eu_j}$ are used to simplify and shorten expressions, where Eu_j is the academic's expected utility.

Proposition 1. *In a stable market allocation $(\mu, \mathcal{W})$, for any matched pair (p_i, a_j) and any equilibrium reservation utility $\underline{U}_j \le \tilde{U}(\alpha_i, \beta_i, \gamma_j)$, the equilibrium contract $W_{ij}(\underline{U}_j)$ satisfies:*

(1) the company abandons the project when receiving a bad signal b;
(2) the academic's expected utility is $Eu_j = max\{\hat{U}(\alpha_i, \beta_i, \gamma_j), \underline{U}_j\}$;
(3) the payment scheme is $W_{ij}(\underline{U}_j) = (w^g_{ij} = \frac{Y_{ij}-(1-\beta_i)}{(\alpha_i+\beta_i-1)^2\gamma_j{}^2},\ w^b_{ij} = 0)$;
(4) the academic's optimal effort is $e_j(W_{ij}(\underline{U}_j); p_i) = \frac{Y_{ij}-(1-\beta_i)}{(\alpha_i+\beta_i-1)\gamma_j}$;
(5) the company's expected profit is $Ev_i(W_{ij}(\underline{U}_j); a_j) = \frac{(X_{ij}-Y_{ij})[Y_{ij}-(1-\beta_i)]+B(1-\beta_i)(\alpha_i+\beta_i-1)^2\gamma_j{}^2}{(\alpha_i+\beta_i-1)^2\gamma_j{}^2}$

In equilibrium, firms terminate projects when signal b is revealed since they anticipate that a bad signal b indicates a failure $B < 0$ with a high likelihood, and they design contracts according to the objective function (1).[17]

4. Equilibrium matching in a stable market allocation

This section analyzes the matching in the stable market allocation. The primary objective is to ascertain who can be matched in the market. To ensure that certain participants can be matched, thereby averting an empty stable market allocation, additional restrictions on parameters are necessary. Furthermore, the equilibrium matching will be delineated in each evaluation scenario (sensitivity, specificity, and symmetry).

In the stable market allocation, certain low-ability companies and academics may fail to find a match with a participant on the opposite side of the market due to the absence of a feasible contract. I denote the threshold of the company's ability as $(\hat{\alpha}, \hat{\beta})$, below which firms remain consistently unmatched. Similarly, there exists a threshold for the academic's productivity denoted by $\hat{\gamma}$, such that all academics with $\gamma_j \le \hat{\gamma}$ remain unmatched. I summarize this result formally in Lemma 1.

Lemma 1. *In a stable market allocation, there exist thresholds $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$, if $\hat{\alpha} \in [\underline{\alpha}, \bar{\alpha}]$, $\hat{\beta} \in [\underline{\beta}, \bar{\beta}]$, and $\hat{\gamma} \in [\underline{\gamma}, \bar{\gamma}]$, then firms with $\alpha_i < \hat{\alpha}$ and $\beta_i < \hat{\beta}$, and academics with $\gamma_j < \hat{\gamma}$ will be always unmatched in the market.*

To guarantee that the stable market allocation is not empty, the abilities of firms and academics need to satisfy the following constraints.[18]

Assumption 2. $\underline{\alpha} > \hat{\alpha}$, $\underline{\beta} > \hat{\beta}$ and $\underline{\gamma} > \hat{\gamma}$.

4.1. Sensitivity scenario: the case of exploitative projects

In the sensitivity case, pharma companies are heterogeneous in making type I errors but homogeneous in making type II errors. This situation may arise in projects characterized as exploitative, where findings concerning some side effects of exploitative compounds are publicly available from previous research and patents. Consequently, all firms possess equal chances of making type II errors if the project fails in the future. However, firms may not have comprehensive knowledge regarding the effectiveness of the drug. Therefore, the company's evaluation structure takes the following form: $\pi(g|G) = \alpha_i, \pi(b|G) = (1-\alpha_i)$; $\pi(b|B) = \beta, \pi(g|B) = (1-\beta)$.[19]

In this scenario, I set a homogeneous $\beta > \hat{\beta}$ for all firms in the equilibrium contract outlined in Section 3. Firstly, I will elucidate the method for identifying the equilibrium matching and the underlying intuition. Subsequently, I will solve the equilibrium matching and provide an explanation for why firms and academics are matched in such a way.

[17] Note that this paper investigates three different evaluation setups, and the properties of the equilibrium contract are the same. Therefore, in each scenario, we only need to change the heterogeneity of (α_i, β_i) in the contract.

[18] Note that this assumption only guarantees a non-empty stable market allocation. Since the distributions of firms and academics can be different, and the number of firms and academics in the market also varies, this assumption does not rule out the case in which some participants are unmatched in the stable market allocation.

[19] Note that in this scenario, β has no subscript because it is homogeneous across firms. Moreover, in the following specificity case, α has no subscript because it will be homogeneous.

Note that the utility is not fully transferable in this model because of the limited liability and moral hazard. To identify the equilibrium matching, I follow Legros and Newman (2007) and Chade et al. (2017), and the following marginal rate of substitution is key:

$$MRS_i = -\frac{\partial E v_i(W_{ij}(\underline{U}_j); a_j)/\partial \gamma_j}{\partial E v_i(W_{ij}(\underline{U}_j); a_j)/\partial \underline{U}_j}.$$

This MRS_i represents, for a given level of the expected profit, how much utility a pharma p_i is willing to provide to the academic for an increase in the academic's productivity. Clearly, the sign of MRS_i is positive because a firm has to ensure a more productive academic with a higher utility along the iso-profit curve. If the MRS_i increases in the pharma's ability α_i in this scenario (i.e., $\frac{\partial MRS_i}{\partial \alpha_i} > 0$), the equilibrium matching is positive assortative (PAM) because better firms can utilize more from matching with better academics and outbid worse firms in the market. In contrast, if the reverse case holds (i.e., $\frac{\partial MRS_i}{\partial \alpha_i} < 0$), then it is negative assortative (NAM).[20]

In this paper, I assume that firms have all the bargaining power, i.e., the equilibrium contract maximizes the firm's expected profit. If academics had all the bargaining power, they would have the highest utility from the partnership, leaving zero expected profit for companies. In this case, $\underline{U}_j$ reaches the maximum. However, the bargaining power will not influence results because the framework considers all feasible levels of $\underline{U}_j$. The stable matching in equilibrium is stated in Proposition 2.

Proposition 2. *In the sensitivity scenario, for the stable market allocation* $(\mu, \mathcal{W})$*, the equilibrium matching* μ *is positive assortative; that is,* $\mu(p_i) = a_j$ *and* $\mu(p_{i'}) = a_{j'}$*, where* $\alpha_i > \alpha_{i'}$ *and* $\gamma_j > \gamma_{j'}$*.*

The equilibrium matching is positive assortative (PAM) because superior firms derive greater benefits from collaborating with more productive academics. In other words, better firms exhibit higher MRS_i. Consequently, in the competitive environment, superior firms outbid those with lower abilities. The intuition is as follows: Firms base their decision to continue the project solely on receiving a positive signal (g). Therefore, their focus lies on the future outcomes when this positive signal is revealed. Despite this selective continuation criterion, there remains a possibility of encountering a failed product in the future ($\pi(B|g) \geq 0$) due to evaluation imperfections. In this context, collaborating with superior academics yields dual benefits for firms. Firstly, it enhances the likelihood of project success ($\pi(G)$), thereby increasing the chances of favorable outcomes upon receiving signal g. Secondly, superior academics reduce the likelihood of project failure ($\pi(B)$), consequently diminishing the probability of negative outcomes in the future.

In this sensitivity scenario, firms exhibit heterogeneity in their ability to detect success ($\pi(g|G)$), reflecting varying capacities to discern project viability. Additionally, for an academic a_j with a given reservation utility $\underline{U}_j$, a firm equipped with a more accurate evaluation mechanism can incentivize a_j with lower payment to elicit higher effort. This translates to an augmented probability of success ($\pi(G)$). Both $\pi(g|G)$ and $\pi(G)$ profoundly impact the firm's expected profit. High-ability firms can leverage these enhancements more effectively, given a cheaper incentive and a higher probability of success.

Despite firms' homogeneous ability to detect failure ($\pi(g|B)$), collaborating with superior academics still diminishes the likelihood of failure upon receiving a positive signal, thereby curbing expected losses in profit. Consequently, better firms can capitalize more on high-ability academics, leveraging both success probability increments and failure probability reductions more efficiently than their counterparts. This differential advantage is reflected in a higher Marginal Rate of Substitution (MRS_i) for better firms.

Consequently, in the competition for superior academics, firms endowed with higher α_i are willing to offer greater incentives, outbidding inferior firms. The resulting equilibrium matching exhibits positive assortative matching (PAM). Hence, in the context of pharma companies engaging in exploitative projects with academics, superior firms should be paired with more capable academics.

4.2. Specificity scenario: the case of explorative projects

This section considers a reversed scenario, where pharma companies are heterogeneous in making type II errors and homogeneous in making type I errors. The company's evaluation has the following form: $\pi(g|G) = \alpha, \pi(b|G) = (1-\alpha);\ \pi(b|B) = \beta_i, \pi(g|B) = (1-\beta_i)$.

In this scenario, characterized by exploratory projects, firms face uncertainty due to limited prior findings or studies regarding the potential side effects of drugs or treatments. Consequently, firms exhibit heterogeneity in making type II errors, with more experienced companies boasting a reduced likelihood of errors. To streamline the analysis of explorative projects, this paper simplifies the assumption by considering firms as homogeneous in making type I errors. We maintain a uniform $\alpha > \hat{\alpha}$ across all firms in the equilibrium contract outlined in Proposition 1. This allows us to compute MRS_i and determine the stable matching, as presented in Proposition 3.

Proposition 3. *In the specificity scenario, for a stable market allocation* $(\mu, \mathcal{W})$*, the equilibrium matching* μ *is negative assortative; that is,* $\mu(p_i) = a_{j'}$ *and* $\mu(p_{i'}) = a_j$*, where* $\beta_i > \beta_{i'}$ *and* $\gamma_j > \gamma_{j'}$*.*

[20] See, generalized increasing (decreasing) differences condition in Legros and Newman (2007), equivalent single-crossing condition in Chade et al. (2017).

In this market, the equilibrium matching is NAM because worse pharma companies have higher MRS_i (i.e., $\frac{\partial MRS_i}{\partial \beta_i} < 0$). That is, firms with less accurate evaluations benefit more from hiring better academics. The intuition is as follows. Remember that firms only care about what outcome they will receive in the future when signal g is revealed. In this specificity scenario, firms are heterogeneous in making type II errors ($\pi(b|B) = \beta_i$). A more productive academic decreases the probability of failure $\pi(B)$. Because the loss of a failed outcome is very high, both $\pi(b|B)$ and $\pi(B)$ influence the firm's expected profit significantly. Firms with lower $\pi(b|B) = \beta_i$ can utilize more the decrease in $\pi(B)$ brought by high-ability academics because $\pi(g|B)$ is higher. Although better firms can motivate academics to provide higher effort, hence higher $\pi(G)$ and lower $\pi(B)$, the decline in $\pi(B)$ from better academics is more critical for worse firms. Therefore, these firms with less accurate evaluations can benefit more from higher-ability academics and are willing to pay more to them to reduce the chance of failure. Consequently, worse firms with lower β_i outbid better firms in the competition. In equilibrium, the stable matching is negative assortative (NAM).

As a result, when pharma companies undertake explorative projects with academics, inferior firms will be matched with more capable academics.

4.3. Symmetry scenario: a complementary case

As a complementary case, I investigate the following symmetry scenario, in which a firm makes both type I and type II errors with the same probability. Formally: $\pi(g|G) = \alpha_i, \pi(b|G) = (1-\alpha_i)$; $\pi(b|B) = \alpha_i, \pi(g|B) = (1-\alpha_i)$. In this scenario, I assume $\beta_i = \alpha_i$ in the equilibrium contract introduced in Proposition 1. Proposition 4 states the equilibrium matching for the symmetry scenario.

Proposition 4. *In the symmetry scenario, for the stable market allocation $(\mu, \mathcal{W})$, the equilibrium matching μ is negative assortative; that is, $\mu(p_i) = a_{j'}$ and $\mu(p_{i'}) = a_j$, where $\alpha_i > \alpha_{i'}$ and $\gamma_j > \gamma_{j'}$.*

Let us leverage the insights garnered from the sensitivity and specificity cases to expound upon the rationale behind the matching outcome within this symmetry scenario. In this case, firms are characterized by an equal probability of making both type I and type II errors. It is important to note that firms opt to terminate the project upon receipt of signal b, thereby focusing solely on the outcome subsequent to signal g. Firms endowed with more accurate evaluations can effectively capitalize on the augmented likelihood of success stemming from collaborations with superior academics (i.e., $G\pi(g|G)\pi(G)$ in the profit function). Conversely, weaker firms stand to maximize the reduction in the probability of failure through engagement with superior academics (i.e., $B\pi(g|B)\pi(B)$ in the profit function (1)). These dual utilization effects delineate the differential benefits that firms derive from collaborating with superior academics, as reflected in their respective MRS_i. Given the heightened significance of loss B (i.e., $(G+B) \le 0$), the latter effect supersedes the former, implying that weaker firms stand to gain more from superior academic collaborations. Consequently, weaker firms prevail over their more proficient counterparts, resulting in a negative assortative matching (NAM).

5. Relationship between productivity and incentive

This section explores the relationship between the academic's productivity γ_j and equilibrium incentive w^g_{ij}, which can be testable if the funding or payment is available.[21] Considering an isolated principal–agent model, there is no competition, and the academic will obtain exactly $\hat{U}(\alpha_i, \beta_i, \gamma_j)$ since the outside option is zero. One can easily solve that in all cases, an academic's bonus payment is increasing in productivity, i.e., $\frac{\partial w^g_{ij}}{\partial \gamma_j} > 0$. However, in a two-sided matching market, this relationship will be affected by the competition and the equilibrium matching. To simplify the discussion, let us assume that the distributions of pharma companies and academics are both uniform. The main discussion focuses on the sensitivity scenario, but the results are consistent in all three cases.

In the sensitivity setup, the equilibrium matching is PAM, and the mass of the matched companies must be equal to that of the matched academics,[22] which yields the following matching function:

$$\int_{\alpha_i}^{\bar{\alpha}} \frac{1}{\bar{\alpha}-\underline{\alpha}} d\alpha = \int_{\gamma_j}^{\bar{\gamma}} \frac{1}{\bar{\gamma}-\underline{\gamma}} d\gamma \Rightarrow \alpha_i = \mu(\gamma_j) = \frac{\bar{\alpha}-\underline{\alpha}}{\bar{\gamma}-\underline{\gamma}}\gamma_j + \frac{\underline{\alpha}\bar{\gamma}-\bar{\alpha}\underline{\gamma}}{\bar{\gamma}-\underline{\gamma}}. \tag{MF_{sen}}$$

Clearly, this matching function (MF_{sen}) increases in γ_j because of PAM. Then, the slope of the equilibrium bonus given an academic's productivity is:

$$\frac{dw^g_{ij}}{d\gamma_j} = \underbrace{\frac{\partial w^g_{ij}}{\partial \gamma_j}}_{\text{direct effect}<0} + \underbrace{\frac{\partial w^g_{ij}}{\partial \alpha_i}\frac{d\mu(\gamma_j)}{d\gamma_j}}_{\text{matching effect}<0} + \underbrace{\frac{\partial w^g_{ij}}{\partial \underline{U}_j}\frac{d\underline{U}_j}{d\gamma_j}}_{\text{competition effect}>0}. \tag{DF_{sen}}$$

The academic's productivity γ_j influences the payoff w^g_{ij} through the combination of three effects in this case: (i) a direct effect on the incentive. In (DF_{sen}), this direct effect is negative because a more productive academic needs lower incentives to

[21] The payment schemes in all setups have the form $(w^g_{ij}, 0)$. Moreover, both the effort and success rate depend on the bonus payment; hence, I only focus on the bonus payment.

[22] See, for instance, Legros and Newman (2002) and Serfes (2005).

provide sufficient effort. (ii) An indirect matching effect, which is also negative. Because a higher-ability academic is matched with a company of more accurate evaluation (i.e., $\frac{d\mu(\gamma_j)}{d\gamma_j} > 0$). On the other hand, with the sensitivity evaluation, better firms can motivate an academic to exert effort with cheaper incentives and, thus, pay her less (i.e., $\frac{\partial w^g_{ij}}{\partial \alpha_i} < 0$). (iii) An indirect competition effect associated with the variation of the endogenous equilibrium reservation utility $\underline{U}_j$. Competition effect is the only positive part because $\frac{\partial w^g_{ij}}{\partial \underline{U}_j} > 0$ and $\frac{d\underline{U}_j}{d\gamma_j} = -\frac{\partial Ev_i/\partial \gamma_j}{\partial Ev_i/\partial \underline{U}_j} > 0$. That is, to partner with a more productive academic with higher reservation utility, the company has to pay her more.

For the specificity scenario, we can obtain the matching function $\beta_i = \mu(\gamma_j)$ and $\frac{dw^g_{ij}}{d\gamma_j}$ following the same steps. The equilibrium matching is NAM, which yields $\frac{\partial \mu(\gamma_j)}{\partial \gamma_j} < 0$. All the effects have the same sign as those in (DF_{sen}). However, the intuition for the indirect matching effect is different because $\frac{\partial w^g_{ij}}{\partial \beta_i} > 0$ and $\frac{d\mu(\gamma_j)}{d\gamma_j} < 0$, given that the matching is NAM. In this case, better firms are willing to pay more for higher-ability academics, but the matching is NAM. A better academic is matched with a worse firm that pays her less.

In consequence, depending on the combination of these three effects, the relationship between the academic's productivity and the incentive can always be positive, negative, or non-monotonic. Proposition 5 summarizes the results. Let Δt denote either range $(\bar{\alpha} - \underline{\alpha})$ or $(\bar{\beta} - \underline{\beta})$ decided by the heterogeneity in each scenario, and $\Delta\gamma$ denotes $(\bar{\gamma} - \underline{\gamma})$.

Proposition 5. *There exists a threshold $\Omega(\gamma_j, \underline{U}_j)$ for each matched γ_j through the matching function $t_i = \mu(\gamma_j)$ and the equilibrium reservation utility $\underline{U}_j$, such that $\forall \gamma_j \in [\hat{\gamma}, \bar{\gamma}]$:*

(i) when $\frac{\Delta\gamma}{\Delta t} > \Omega(\gamma_j, \underline{U}_j)$, $\frac{dw^g_{ij}}{d\gamma_j} > 0$.

(ii) when $\frac{\Delta\gamma}{\Delta t} < \Omega(\gamma_j, \underline{U}_j)$, $\frac{dw^g_{ij}}{d\gamma_j} < 0$.

(iii) otherwise, in general, the equilibrium payment is non-monotonic with respect to productivity.

In case (i), we consider that the distribution of academics is polarized compared to the firm's distribution, or one can consider firms to be almost homogeneous as an extreme case. Since all firms have close evaluation abilities, academics with different productivity are matched with similar firms, almost eliminating the indirect matching effect. On the other hand, academics' productivity is more polarized; thus, higher-ability academics are extremely desirable in the market, which yields a high competition effect. This positive competition effect dominates the negative direct and matching effects. Therefore, the relationship between productivity and incentive is always positive in case (i).

In contrast, in case (ii), the academic's productivity is not as heterogeneous as the firm's evaluation ability, or one can consider academics to be almost homogeneous as an extreme case. A slight increase in the academic's productivity will result in the academic being matched with a firm of significantly different ability, which yields a pronounced negative indirect matching effect. Furthermore, because academics have similar abilities, better academics are not that desirable in the market, which implies a weak competition effect. Hence, in this case, the former dominates the latter, which yields a negative relationship between the incentive and academic productivity. Case (iii) is more complicated, where the relationship is, in general, non-monotonic because the sign of $\frac{dw^g_{ij}}{d\gamma_j}$ changes in the interval $[\underline{\gamma}, \bar{\gamma}]$.

Note that even though I assume the participants are uniformly distributed, it is still very difficult to provide a detailed sufficient condition for the slope of the equilibrium bonus payment because it depends on the participants' intervals and the equilibrium reservation utility $\underline{U}_j$, which does not have an analytical solution. If the participants are not uniformly distributed, the analysis is more complicated, and in general, the relationship between payment and productivity is non-monotonic. Because, for one of the matched pairs, in a small neighborhood of this pair, the density function of academics may be sufficiently higher than that of firms, which yields a similar situation as case (i), $\frac{dw^g_{ij}}{d\gamma_j} > 0$. In contrast, if the density function of firms is sufficiently higher than that of academics, then $\frac{dw^g_{ij}}{d\gamma_j} < 0$, which is a similar scenario to case (ii). Since the distributions are not uniform, both cases mentioned before may occur several times in the interval $[\underline{\gamma}, \bar{\gamma}]$, which implies that the sign of $\frac{dw^g_{ij}}{d\gamma_j}$ also changes several times; hence, the equilibrium payment is, in general, non-monotonic with respect to the academic's productivity.

6. Welfare analysis: a numerical example

It is natural to question how participants' abilities influence overall welfare. However, due to the complexity of the utility function within a competitive matching market, deriving a formal welfare function and analytically discussing total welfare is not feasible for continuous participants or even for a simple two-firm and two-academic example. This complexity arises because the utilities for both firms and academics depend on the equilibrium reservation utility, $\underline{U}_j$, which is determined by market competition. To gain further insights into welfare, I conduct a two-by-two numerical analysis. The primary objective of this analysis is to identify more effective ways to improve welfare. Interestingly, increasing the ability of weaker academics may actually harm the overall welfare, which provides insights and precautions to policymakers.

In this numerical analysis, I adopt the sensitivity scenario, in which the stable matching is PAM, and the results are consistent for all scenarios. Consider two firms p_1 and p_2 with the evaluation ability $(\alpha_1 = 0.9, \beta_1 = 1)$ and $(\alpha_2 = 0.8, \beta_2 = 1)$, respectively. For

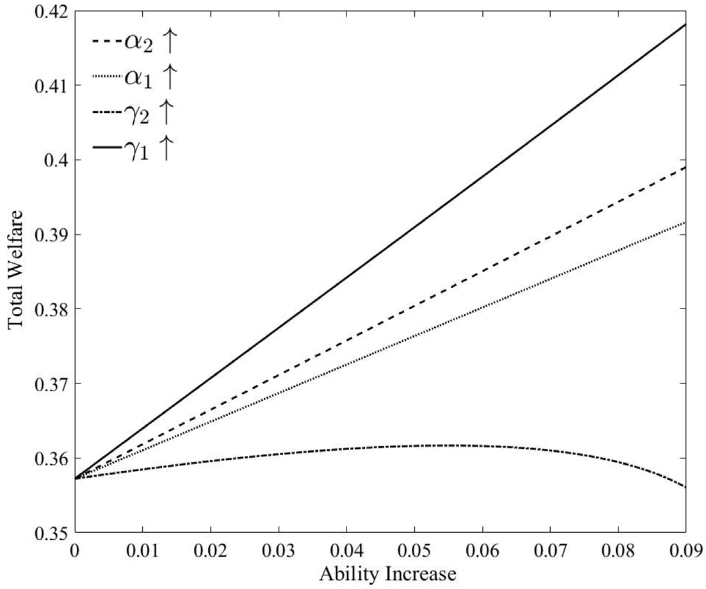

Fig. 1. The effect of each participant's ability on total welfare.

two academics a_1 and a_2, they have productivity $\gamma_1 = 0.9$ and $\gamma_2 = 0.8$, respectively. The final outcome is $(G = 0.9, B = -1)$. For an increase of each participant's ability (i.e., α_1, α_2, γ_1, and γ_2) in the interval $[0, 0.09]$, I solve the total welfare in equilibrium.[23] Fig. 1 illustrates how increasing each participant's ability affects the total welfare.

The dashed curve and dotted curve in this figure show that for the same level of ability increase in the firm's evaluation ability, the increase in the worse firm's ability benefits the total welfare more than the increase in the better firm's ability in this numerical example. The reason is the following. A higher α_2 directly raises worse firm p_2's profit and academic a_2's utility. Besides, it also increases academic a_1's utility because firm p_2 has a better evaluation and is willing to pay more for a_1, which yields a stronger competition for a_1. For this reason, p_1 earns less since it has to pay more for academic a_1 in equilibrium. The former three positive effects dominate the latter negative effect. In contrast, an increase in α_1 has a weaker effect on total welfare because it only increases firm p_1's and academic a_1's utilities.

The other solid and dash-dotted curves in Fig. 1 show that for the same level increase in the academic's productivity, the increase in γ_1 raises the total welfare, but an increase in worse academic's γ_2 may harm the total welfare when γ_2 is already high enough (i.e., the decreasing part of the dash-dotted curve). The former is clear since higher γ_1 increases both this academic's and the matched firm's utility; thus, I focus on the latter. A higher γ_2 increases utilities of p_2 and a_2; hence p_2 is less willing to offer a high outside option for better academic a_1, which yields a much lower utility for a_1. Although a better a_2 benefits all three participants except a_1, this cannot compensate the utility loss for a_1 when γ_2 is high enough and becomes closer to γ_1. Note that this result is also aligned with case (ii) in Proposition 5, when two academic's productivity becomes more homogeneous, higher productivity yields a lower payment, hence a lower utility to better academic a_1 and lower total welfare.

To enhance total welfare, the most effective strategy is to boost the productivity of top academics, as shown in this example. In general, policymakers should encourage the best researchers and more firms with higher ability to participate in the collaboration market, thereby raising the upper bound of productivity and increasing the overall firm's ability, enhancing the total welfare. Conversely, policymakers should be careful to restrict the involvement of researchers with relatively lower abilities in these alliances. In certain cases, if all academics possess similar high abilities, overall welfare may tend to be lower.

7. Extensions

7.1. Heterogeneous evaluation structure

This subsection extends the model by introducing variability in the evaluation structures of the firms. Specifically, we examine a scenario where two firms, denoted as Firm "α" and Firm "β", possess distinct evaluation structures. Firm α is characterized by a "perfect" sensitivity evaluation structure, parameterized by $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, with $\beta = 1$. Conversely, Firm β operates under a "perfect" specificity evaluation structure, where $\alpha = 1$ and $\beta \in [\underline{\beta}, \bar{\beta}]$.

[23] I solve each participant's utility and conduct the numerical analysis in MATLAB, the codes of which are available upon request.

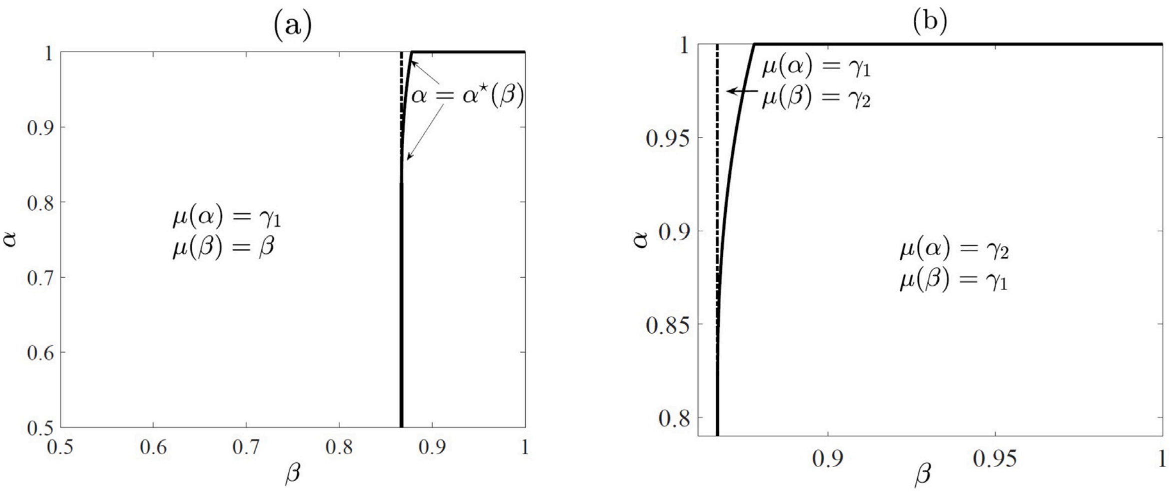

Fig. 2. Illustration of the equilibrium matching under heterogeneous evaluation structures.

In this setting, we consider two academics, denoted by a_1 and a_2, with $\gamma_1 > \gamma_2$. Notably, for any $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, it is invariably profitable for firm α to engage in collaboration with an academic, as $\pi(G|g) = 1$ by Bayes' rule. However, should β fall below a certain threshold (e.g., lower than $\hat{\beta}$ in Lemma 1), firm β will find collaboration with any academic unprofitable due to a high $\pi(B|g)$.

This section seeks to ascertain which firm is predominantly matched with the more productive academic a_1 in equilibrium and elucidate the underlying intuition, which is summarized in Proposition 6.

Proposition 6. *When $\beta \geq \hat{\beta}$, there is a threshold $\alpha^{\star}(\beta)$ for α as a function of β such that the matching equilibrium is:*
(i) if $\alpha^{\star}(\beta) \geq 1$, then $\mu(\alpha) = \gamma_2$ and $\mu(\beta) = \gamma_1$;
(ii) if $\alpha^{\star}(\beta) < 1$ and $\alpha > \alpha^{\star}(\beta)$, then $\mu(\alpha) = \gamma_1$ and $\mu(\beta) = \gamma_2$;
(iii) if $\alpha^{\star}(\beta) < 1$ and $\alpha \leq \alpha^{\star}(\beta)$, then $\mu(\alpha) = \gamma_2$ and $\mu(\beta) = \gamma_1$.
When $\beta < \hat{\beta}$, the equilibrium matching is: $\mu(\alpha) = \gamma_1$ and $\mu(\beta) = \beta$, i.e., firm β is unmatched.

To provide further clarity on the results, I present a numerical example depicted in Fig. 2, where $\alpha \in [0.5, 1]$, $\beta \in [0.5, 1]$, $(G = 0.75, B = -1)$, $\gamma_1 = 1$, and $\gamma_2 = 0.8$, illustrating the (β, α) space when companies possess differing evaluation structures. The solid curve denotes $\alpha^{\star}(\beta)$, while the dash-dotted line represents $\hat{\beta}$. Areas to the left of the dash-dotted line ($\beta < \hat{\beta}$) indicate where firm β remains unmatched. Fig. 2(b) offers a detailed view of the upper segment of Fig. 2(a) and elucidates the threshold $\alpha^{\star}(\beta)$.

The underlying intuition behind Proposition 6 is as follows: firm β faces the risk of encountering a failed outcome B even when the signal is g (i.e., $\pi(B|g) > 0$). To mitigate this potential loss, firm β is inclined to offer higher bids to secure the collaboration of academic a_1. Consequently, if β surpasses a certain threshold, such that $\alpha^{\star}(\beta) \geq 1$, firm β consistently outbids firm α for academic a_1 (i.e., case (i)). Conversely, when β falls short of this threshold and α exceeds a certain level (i.e., $\alpha > \alpha^{\star}(\beta)$ in case (ii)), firm α prevails in the bidding war and secures the collaboration of a_1 in equilibrium. In case (iii), if firm α fails to exhibit sufficiently high ability, firm β can outbid it for academic a_1.

These results underscore the inclination of firms susceptible to type II errors to offer more enticing contracts to high-ability academics, thereby mitigating the risk of project failure.

7.2. Multi-dimensional firm heterogeneity

The symmetry scenario is a particular case of multi-dimensional firm heterogeneity, which already provides evidence of the determinant role of making type II errors that can lead to a stable matching of NAM. However, it can be more valuable to extend the results by using a more general setup to discuss the two-dimensional errors. Therefore, I consider a correlation between α_i and β_i; without loss of generality, let β_i to be a function of α_i denoted by $\beta_i = m(\alpha_i)$, which is twice-differentiable. Note that in the symmetry scenario, the function is linear $m(\alpha_i) = \alpha_i$. The equilibrium matching between the firm's ability α_i and the academic's productivity are summarized in Proposition 7.

Proposition 7. *There exists a threshold $\Lambda(\alpha_i, \gamma_j, \underline{U}_j) \in (0, 1)$, such that $\forall \alpha_i \in [\hat{\alpha}, \bar{\alpha}]$, $\forall \gamma_j \in [\hat{\gamma}, \bar{\gamma}]$, and any possible level of $\underline{U}_j$:*
(i) if $m'(\alpha_i) < \Lambda(\alpha_i, \gamma_j, \underline{U}_j)$ for all matched pair (p_i, a_j), the stable matching is PAM;
(ii) if $m'(\alpha_i) > \Lambda(\alpha_i, \gamma_j, \underline{U}_j)$ for all matched pair (p_i, a_j), the stable matching is NAM;
(iii) otherwise, the stable matching is, in general, non-assortative.

The intuition is clear by using the discussion of the main results in this paper. Remember that the heterogeneity of α_i affects the matching to be PAM, but the heterogeneity of β_i affects the matching to be NAM. In case (i), when $m'(\alpha_i) > 0$, this can be a case in the real world; that is, better firms make both fewer type I and II errors due to better firms' experience and size. Since $m'(\alpha_i) < \Lambda(\alpha_i, \gamma_j, \underline{U}_j) < 1$, firms are more heterogeneous in making type I errors but much less heterogeneous in making type II errors. Thus, the effect of the heterogeneity of α_i dominates the effect of β_i on the stable matching, and the stable matching is PAM. Note that $m'(\alpha_i) < 0$ may also be an example in the real world. For instance, firms may have limited budgets to reduce the errors; thus, risk-tolerant firms may use the budget to reduce more type I errors and have higher α_i but lower β_i. In contrast, risk-averse firms may reduce more type II errors and have lower α_i but higher β_i. In this case, the stable matching is PAM, and the intuition is very much aligned with that in the sensitivity and specificity scenarios because both higher α_i and lower β_i for risk-tolerant firms allow them to outbid others for more productive firms.

In case (ii), note that $\Lambda(\alpha_i, \gamma_j, \underline{U}_j) < 1$; therefore, even though the heterogeneity of making type II errors is lower than that of type I errors, the matching is NAM because to avoid failure, firms with lower β_i outbid better firms. Interestingly, this result is also aligned with the discussion of heterogeneous evaluation structure in Section 7.1, that is, in most cases, firms that make type II errors are matched with better academics. In other cases, the matching is, in general, non-assortative because depending on function $m(\alpha_i)$, the sign of $(m'(\alpha_i) - \Lambda(\alpha_i, \gamma_j, \underline{U}_j))$ may change several times.[24]

7.3. Alternative assumption on the final outcome

The initial model assumes that $(G + B) \leq 0$, which is usually the case for innovative product development in the pharmaceutical and high-tech industries. However, one may consider that in some particular cases (e.g., developing me-too drugs), the loss from a failure is lower than the gain from a success (i.e., $(G + B) > 0$). This section conducts a comprehensive analysis in two steps: (1) examining the continuation decision and equilibrium contract and (2) studying the equilibrium matching.

Under the premise of $(G+B) > 0$, two noteworthy facts need to be pointed out. Firstly, firms now may continue the project upon receiving signal b, motivated by the substantial benefits G, which may overcome the risk associated with a potential failed product. Secondly, this change of continuation decision may affect equilibrium matching.

This transition underscores the intricate interplay between risk perception, project viability, and partnership dynamics in scenarios where the potential benefits of success outweigh the detriments of failure. This analysis provides valuable insights into the strategic decisions and matching outcomes prevalent in industries characterized by innovative pharmaceutical product development.

Firstly, I compare again the two expected profit functions (1) and (2) stemming from different continuation decisions under $(G + B) > 0$. Proposition 8 focuses solely on delineating the firm's optimal continuation decision when receiving an unfavorable signal b. Secondly, I identify the equilibrium matching, which is summarized in Proposition 9.

Proposition 8. *There exists a threshold $\underline{U}^{\star}_{ij}(\alpha_i, \beta_i, \gamma_j)$ as a function of participants' abilities for a pair (p_i, a_j), such that when the signal is b:*

(i) if $\underline{U}_j \leq \underline{U}^{\star}_{ij}(\alpha_i, \beta_i, \gamma_j)$, firm p_i abandons the project as in the initial model;
(ii) if $\underline{U}_j > \underline{U}^{\star}_{ij}(\alpha_i, \beta_i, \gamma_j)$, firm p_i continues the project.

The intuition behind Proposition 8 is as follows. When academic a_j's endogenous reservation utility $\underline{U}_j$ is relatively low, which means a weak competition for academics, firm p_i can suffice by offering a modest incentive to secure the academic's acceptance of the contract. Consequently, the academic's effort level tends to be subdued, resulting in a diminished probability of success $\pi(G)$. Consequently, firm p_i foresees a high likelihood of project failure upon observing signal b (i.e., $\pi(B, b)$ is elevated), prompting termination of the project. In contrast, when $\underline{U}_j$ surpasses a certain threshold $\underline{U}^{\star}_{ij}(\alpha_i, \beta_i, \gamma_j)$, which means a relatively stronger competition for academics, firm p_i offers a more substantial bonus to incentivize the academic, thereby eliciting a heightened effort level and correspondingly increased $\pi(G, b) = \pi(G)\pi(b|G)$. Consequently, given the alternative assumption of $(G+B) > 0$, continuing the project even subsequent to receiving signal b may yield superior profitability owing to the high benefit from a possible success.

Proposition 9. *In each evaluation structure setup, if in a stable market allocation $(\mu, \mathcal{W})$, the contracts in $\mathcal{W}$ are such that all firms choose to continue the project under a signal b, then the equilibrium matching μ is always negative assortative (NAM).*

This result arises from the fact that firms' evaluations solely serve to monitor academics and facilitate payment based on evaluation outcomes. Firms endowed with more precise evaluations can effectively incentivize even lower-ability academics to exert sufficient effort while maintaining a high probability of success. Conversely, firms with less accurate evaluations must engage higher-ability academics to bolster the project's success. Consequently, superior firms exhibit a reduced willingness to either offer higher compensation to superior academics or outbid inferior firms, leading to a negative assortative matching.

In consequence, the positive assortative matching (PAM) observed in the sensitivity scenario within the initial model transitions to NAM under the assumption of $(G + B) > 0$, provided all firms elect to continue the project despite a negative signal. In the sensitivity case, if both continuation and termination decisions coexist, the matching pattern does not exhibit assortativity.

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[24] Previous literature usually considers firms to be heterogeneous in the value of the final outcome (i.e., benefit G from a success and damage B from a failure in this paper). To analyze this case of multidimensional heterogeneity of the outcome value and evaluation ability, we can also consider a correlation between them. I will not conduct this analysis in this paper since the analysis process will be the same.

7.4. Breakthrough and me-too drugs

This paper theoretically analyzes the innovative collaboration of explorative and exploitative drugs. A similar discussion can be applied to further examine the development of breakthrough and me-too drugs during the early stages of collaboration. Previous literature has focused on the incentives for developing breakthrough and me-too drugs and the impact of reference pricing on pharmaceutical innovation (see, for instance, Bardey et al., 2010; González et al., 2016).

Breakthrough (first-in-class) drugs typically explore new mechanisms, pathways, and targets, representing significant innovation associated with substantial cost, risk, and uncertainty. Therefore, collaborating on developing a breakthrough drug is usually a specificity or symmetry scenario. The equilibrium matching in the collaboration market of breakthrough drugs should be NAM, as weaker firms need to collaborate with stronger academics to reduce the risk of significant losses.

In contrast, developing me-too drugs involves lower costs, less risk, and greater promise, with firms facing lower potential losses if the project fails. In this case, the collaboration is more likely to be the case in the sensitivity scenario but offers benefits much greater than the losses (i.e., $G + B \gg 0$). As discussed in Section 7.3, firms may always continue the project after the early-stage evaluation, and the matching is still NAM. This is because in developing a me-too drug, a better firm can leverage its evaluation capabilities to monitor a weaker academic to ensure sufficient effort, eliminating the need to collaborate with a more productive academic.

In consequence, we may observe that firms make different termination decisions for collaboration projects to develop breakthrough and me-too drugs. However, we may find NAM between firms and academics in both cases.

8. Conclusion and discussion

This paper investigates the collaboration between pharmaceutical companies and academics within a two-sided matching market framework. At a crucial interim stage, companies meticulously scrutinize the academic's proof-of-concept findings, utilizing this evaluation to gauge project feasibility and make pivotal decisions regarding project termination. In this market, heterogeneous firms vie for collaboration with academics of diverse abilities, implying a competitive equilibrium characterized by a menu of equilibrium contracts and a stable matching. The study delves into three distinctive scenarios delineated by the firm's evaluation structure, unraveling the evaluative impact on equilibrium matching.

Operating under the premise of substantial losses accompanying project failure, a salient finding emerges: firms uniformly opt to terminate projects upon receiving adverse evaluation signals. More importantly, the analysis unveils the pivotal role of innovation strategy and evaluation technology structure in shaping equilibrium matching. In the sensitivity scenario, where the projects are more exploitative, positive assortative matching prevails. In the remaining scenarios, where the projects are more explorative, negative assortative matching dominates. These findings offer valuable insights into how evaluation mechanisms influence matching patterns. Moreover, they underscore the importance of considering innovation strategies and evaluation structures in understanding pharma-academic alliances, thereby enriching theoretical discourse and offering avenues for future research exploration.

The theoretical results in this paper have several empirical implications, which can be testable using available data. Regarding the testable matching of PAM and NAM in exploitative and explorative drug development projects, we can identify the type of collaborative project from the database on the drug development pipeline and adopt an empirical matching approach (see, for instance, Fox (2018)) to learn the stable matching in each case.[25] If the funding and payment information is available, we can further study the academic benefit of the collaboration. We expect to observe that better academics benefit more if they are relatively more heterogeneous than firms, and they obtain less if more polarized firms are involved in the market. Moreover, this paper can also provide explanations for the current empirical results about the industry-academic alliances that the existing literature cannot explain (see, for instance, Banal-Estañol et al., 2018; Mindruta, 2013). Further empirical study in these aspects may be interesting.

In addition, this paper provides important policy recommendations. Based on the numerical welfare analysis, policymakers should encourage the best researchers to join the collaboration market to raise the upper bound of academic ability, thereby increasing total welfare. Moreover, it is beneficial to involve pharmaceutical companies with higher abilities in these alliances, as this raises the overall ability of firms and enhances total welfare. However, policymakers should avoid restricting the participation of researchers with relatively lower abilities. Encouraging only researchers of similar high abilities can lead to homogeneity among academics, which may harm better academics and reduce total welfare.

CRediT authorship contribution statement

Qianshuo Liu: Writing – review & editing, Writing – original draft, Methodology, Investigation, Funding acquisition, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

[25] For the database, see, for instance, Citeline Phamaprojects and Clarivate Cortellis.

Appendix

Proof of Proposition 1. I divide this proof into three parts. For a pair (p_i, a_j), first, I solve the optimal contract if p_i discontinues the project under a bad signal. Second, I solve the optimal contract if p_i continues the project when a bad signal is revealed. Third, I compare the expected profits in these two cases, and the equilibrium contract is the one that yields a higher expected profit.

Case Discontinuation: Assuming that company p_i discontinues the project when receiving b, the optimal contract is the solution to the following maximization problem:

$$\max_{e,w_{ij}^g,w_{ij}^b} \quad G\underbrace{\alpha_i\gamma_j e}_{\pi(G,g)} + B\underbrace{(1-\beta_i)(1-\gamma_j e)}_{\pi(B,g)} - w_{ij}^g\underbrace{[\alpha_i\gamma_j e + (1-\beta_i)(1-\gamma_j e)]}_{\pi(g)} - w_{ij}^b\underbrace{[1-\alpha_i\gamma_j e - (1-\beta_i)(1-\gamma_j e)]}_{\pi(b)} \tag{A.1}$$

$$\text{s.t.} \quad w_{ij}^g[\alpha_i\gamma_j e + (1-\beta_i)(1-\gamma_j e)] + w_{ij}^b[1-\alpha_i\gamma_j e-(1-\beta_i)(1-\gamma_j e)] - \frac{1}{2}e^2 \geq \underline{U} \tag{PC}$$

$$e = (\alpha_i+\beta_i-1)\gamma_j(w_{ij}^g - w_{ij}^b) \tag{ICC}$$

$$w_{ij}^b \geq 0 \tag{LLC}$$

We substitute (ICC) into (A.1) and (PC) and solve the maximization problem. Let X_{ij} denote $[G\alpha_i - B(1-\beta_i)](\alpha_i+\beta_i-1)\gamma_j{}^2$ to reduce the length of expressions, by Kuhn–Tucker conditions, there are only four possible cases:

Case 1: Both (PC) and (LLC) are not binding. One can easily solve this case and get $-1 = 0$ from the first-order condition with respect to w_{ij}^b in the Kuhn–Tucker conditions, which is not possible.

Case 2: (PC) is binding but (LLC) is not. One can easily solve the expected profit $Ev = B(1-\beta_i) - (1-\beta_i)\frac{G\alpha_i - B(1-\beta_i)}{\alpha_i+\beta_i-1} - w_{ij}^b < 0$, which is not feasible.

Case 3: (LLC) is binding but (PC) is not, the payment schemes, effort, and a_j's and p_i's expected utilities are: $w_{ij}^g = \frac{X_{ij}-(1-\beta_i)}{2(\alpha_i+\beta_i-1)^2\gamma_j{}^2}$, $w_{ij}^b = 0$, $e_j(W_{ij}(\underline{U}_j); a_j) = \frac{X_{ij}-(1-\beta_i)}{2(\alpha_i+\beta_i-1)\gamma_j}$, $Eu_j = \frac{[X_{ij}+3(1-\beta_i)][X_{ij}-(1-\beta_i)]}{8(\alpha_i+\beta_i-1)^2\gamma_j{}^2} = \hat{U}(\alpha_i,\beta_i,\gamma_j) > \underline{U}_j$, $E\hat{v}_i = \frac{[X_{ij}-(1-\beta_i)]^2}{4(\alpha_i+\beta_i-1)^2\gamma_j{}^2} + B(1-\beta_i)$. This case is only possible when $\underline{U}_j \leq \hat{U}(\alpha_i,\beta_i,\gamma_j)$. Besides, $w_{ij}^g \geq 0$ implies (A.2):

$$X_{ij} - (1-\beta_i) \geq 0. \tag{A.2}$$

Case 4: Both (LLC) and (PC) are binding, the academic's expected utility is $Eu_j = \underline{U}_j$, and $w_{ij}^b = 0$. From (PC), I solve the wage $w_{ij}^g = \frac{\sqrt{(1-\beta_i)^2+2(\alpha_i+\beta_i-1)^2\gamma_j{}^2\underline{U}_j}-(1-\beta_i)}{(\alpha_i+\beta_i-1)^2\gamma_j{}^2}$. Let Y_{ij} denote $\sqrt{(1-\beta_i)^2+2(\alpha_i+\beta_i-1)^2\gamma_j{}^2 Eu_j}$, then the effort and expected profit are: $e_j(W_{ij}(\underline{U}_j); a_j) = \frac{Y_{ij}-(1-\beta_i)}{(\alpha_i+\beta_i-1)\gamma_j}$, $E\hat{v}_i = \frac{(X_{ij}-Y_{ij})[Y_{ij}-(1-\beta_i)]}{(\alpha_i+\beta_i-1)^2\gamma_j{}^2} + B(1-\beta_i)$. This case is possible when $\underline{U}_j \geq \hat{U}(\alpha_i,\beta_i,\gamma_j)$, which implies that

$$Y_{ij} \geq \frac{X_{ij}+(1-\beta_i)}{2}. \tag{A.3}$$

Moreover, the expected profit must satisfy (ACC_p), i.e., $E\hat{v}_i \geq 0$. Solving this inequality as a function of Y_{ij}:

$$\frac{[X_{ij}+(1-\beta_i)] - \sqrt{[X_{ij}-(1-\beta_i)]^2 + 4(\alpha_i+\beta_i-1)^2\gamma_j{}^2(1-\beta_i)B}}{2} \leq Y_{ij}; \tag{A.4}$$

$$Y_{ij} \leq \frac{[X_{ij}+(1-\beta_i)] + \sqrt{[X_{ij}-(1-\beta_i)]^2 + 4(\alpha_i+\beta_i-1)^2\gamma_j{}^2(1-\beta_i)B}}{2}. \tag{A.5}$$

Therefore, Y_{ij} has to satisfy (A.3), (A.4) and (A.5) simultaneously, which implies that:

$$\underline{U}_j \geq \hat{U}_j(\alpha_i,\beta_i,\gamma_j) = \frac{[X_{ij}+3(1-\beta_i)][X_{ij}-(1-\beta_i)]}{8(\alpha_i+\beta_i-1)^2\gamma_j{}^2} \tag{A.6}$$

$$\underline{U}_j \leq \tilde{U}(\alpha_i,\beta_i,\gamma_j) = \frac{[X_{ij}+(1-\beta_i)+\sqrt{(X_{ij}-(1-\beta_i))^2+4(\alpha_i+\beta_i-1)^2\gamma_j{}^2(1-\beta_i)B}]^2 - 4(1-\beta_i)^2}{8(\alpha_i+\beta_i-1)^2\gamma_j{}^2} \tag{A.7}$$

Thus, academic a_j's reservation utility should satisfy both (A.6) and (A.7). Let me summarize the contract in the following way: when $\underline{U}_j \leq \tilde{U}(\alpha_i,\beta_i,\gamma_j)$, $Eu_j = \max\{\hat{U}_j(\alpha_i,\beta_i,\gamma_j), \underline{U}_j\}$; the payment scheme, effort and expected profit are:

$$w_{ij}^g = \frac{Y_{ij}-(1-\beta_i)}{(\alpha_i+\beta_i-1)^2\gamma_j{}^2}, \quad w_{ij}^b = 0; \tag{A.8}$$

$$e_{ij}(W_{ij}(\underline{U}_j);p_i) = \frac{Y_{ij} - (1-\beta_i)}{(\alpha_i+\beta_i-1)\gamma_j}; \tag{A.9}$$

$$E\hat{v}_i(W_{ij}(\underline{U}_j);p_i) = \frac{(X_{ij}-Y_{ij})[Y_{ij}-(1-\beta_i)]}{(\alpha_i+\beta_i-1)^2{\gamma_j}^2} + B(1-\beta_i). \tag{A.10}$$

Case Continuation: Assuming that the company continues the project under signal b. I will not investigate the details of this contract but only discuss the expected profit $E\tilde{v}_i$. The maximization problem is:

$$\max_{e,w_{ij}^g,w_{ij}^b} \quad G\underbrace{\gamma_j e}_{\pi(G)} + B\underbrace{(1-\gamma_j e)}_{\pi(B)} - w_{ij}^g\underbrace{[\alpha_i\gamma_j e + (1-\beta_i)(1-\gamma_j e)]}_{\pi(g)} - w_{ij}^b\underbrace{[1-\alpha_i\gamma_j e-(1-\beta_i)(1-\gamma_j e)]}_{\pi(b)}$$

$$s.t. \quad (PC),\ (ICC),\ (LLC)$$

One can easily solve the problem and obtain that (LLC) is always binding. Thus, the possible cases are the following two.

Case 1: (LLC) is binding but (PC) is not, let $\tilde{X}_{ij}$ denote $(G-B)(\alpha_i+\beta_i-1){\gamma_j}^2$, the expected profit is: $E\tilde{v}_i = \frac{[\tilde{X}_{ij}-(1-\beta_i)]^2}{4(\alpha_i+\beta_i-1)^2\gamma_j^2} + B$.

Case 2: Both (LLC) and (PC) are binding, $Eu_j = \underline{U}_j$. Solving the expected profit, which is: $E\tilde{v}_i = \frac{(\tilde{X}_{ij}-Y_{ij})[Y_{ij}-(1-\beta_i)]}{(\alpha_i+\beta_i-1)^2\gamma_j^2} + B$.

Note that when $\underline{U}_j = \frac{[\tilde{X}_{ij}+3(1-\beta_i)][\tilde{X}_{ij}-(1-\beta_i)]}{8(\alpha_i+\beta_i-1)^2\gamma_j^2}$, **Case 1** and **Case 2** coincide. Thus, writing the expected profit as a function of Eu_j. The expected utility $Eu_j = \max\{\frac{[\tilde{X}_{ij}+3(1-\beta_i)][\tilde{X}_{ij}-(1-\beta_i)]}{8(\alpha_i+\beta_i-1)^2\gamma_j^2}, \underline{U}_j\}$, then expected profit is

$$E\tilde{v}_i = \frac{(\tilde{X}_{ij}-Y_{ij})[Y_{ij}-(1-\beta_i)]}{(\alpha_i+\beta_i-1)^2\gamma_j^2} + B. \tag{A.11}$$

We now prove that this profit (A.11) is always negative. Notice that the expected profit $E\tilde{v}_i = \frac{[\tilde{X}_{ij}-(1-\beta_i)]^2}{4(\alpha_i+\beta_i-1)^2\gamma_j^2} + B$ in **Case 1** is always higher than that in **Case 2** because the utility provided to academics is lower. Thus, we only need to prove $E\tilde{v}_i = \frac{[\tilde{X}_{ij}-(1-\beta_i)]^2}{4(\alpha_i+\beta_i-1)^2\gamma_j^2} + B < 0$ in **Case 1**.

To guarantee $\pi(G) \le 1$, we need a similar assumption for this case as Assumption 1. That is, for any pair (p_i, a_j), the probability $\pi(G) = \frac{\tilde{X}_{ij}-(1-\beta_i)}{2(\alpha_i+\beta_i-1)} < 1$, which implies $\frac{\tilde{X}_{ij}-(1-\beta_i)}{2} \le (\alpha_i+\beta_i-1) \le 1$. Therefore, $E\tilde{v}_i = \frac{[\tilde{X}_{ij}-(1-\beta_i)]^2}{4(\alpha_i+\beta_i-1)^2\gamma_j^2} + B \le \frac{[\tilde{X}_{ij}-(1-\beta_i)]}{2(\alpha_i+\beta_i-1)^2\gamma_j^2}(\alpha_i+\beta_i-1) + B$, and simplifying the right-hand side of the inequality, and by assumption $(G+B) \le 0$, we have: $\frac{(G+B)(\alpha_i+\beta_i-1)^2{\gamma_j}^2-(\alpha_i+\beta_i-1)(1-\beta_i)}{2(\alpha_i+\beta_i-1)^2\gamma_j^2} < 0$. Thus, $E\tilde{v}_i = \frac{[\tilde{X}_{ij}-(1-\beta_i)]^2}{4(\alpha_i+\beta_i-1)^2\gamma_j^2} + B$ is always negative.

To conclude, the expected profit (A.11) is always negative; hence, the equilibrium contract is the one in **Case Discontinuation**, where firms always abandon the project when receiving a bad signal b from the evaluation. The equilibrium contract, effort, and profit are: (A.8), (A.9), and (A.10), respectively. □

Proof of Lemma 1. Note that only when inequality (A.5) exists and is satisfied the expected profit is positive. Thus, a condition such that (A.5) holds is that the radicand term is not negative:

$$[X_{ij}-(1-\beta_i)]^2 + 4(\alpha_i+\beta_i-1)^2{\gamma_j}^2(1-\beta_i)B \ge 0 \Rightarrow \tag{A.12}$$

$${\gamma_j}^2 \ge \frac{(1-\beta_i)[G\alpha_i-(2\alpha_i+\beta_i-1)B+2\sqrt{\alpha_i(\alpha_i+\beta_i-1)(G-B)(-B)}]}{[G\alpha_i-B(1-\beta_i)]^2(\alpha_i+\beta_i-1)} = \hat{\gamma}(\alpha_i,\beta_i)^2 \tag{A.13}$$

In (A.13), $\hat{\gamma}(\alpha_i,\beta_i)$ is the productivity of the academic such that $E\hat{v}_i = 0$ and $\tilde{U}(\alpha_i,\beta_i,\hat{\gamma}(\alpha_i,\beta_i)) = \hat{U}(\alpha_i,\beta_i,\hat{\gamma}(\alpha_i,\beta_i))$. Thus, p_i will not form a partnership with any academic with $\gamma_j < \hat{\gamma}(\alpha_i,\beta_i)$. Let $\hat{\gamma}$ denote $\hat{\gamma}(\bar{\alpha},\bar{\beta})$ and the academic with $\gamma_j < \hat{\gamma}$ is always unmatched. Similarly, given a fixed β (resp., α) in the sensitivity (resp., specificity) scenario, we can find $\hat{\alpha} = \hat{\gamma}^{-1}(\bar{\gamma})$ (resp., $\hat{\beta} = \hat{\gamma}^{-1}(\bar{\gamma})$) such that firms with $\alpha_i < \hat{\alpha}$ (resp., $\beta_i < \hat{\beta}$) is always unmatched. □

Lemma 2. *For any matched pair, (p_i, a_j), their abilities must satisfy $(\alpha_i + 2\beta_i - 2) > 0$.*

Proof of Lemma 2. Because (A.12) is strictly increasing in α_i, β_i and γ_j, then $\frac{d\hat{\gamma}(\alpha_i,\beta_i)}{d\alpha_i} < 0$ and $\frac{d\hat{\gamma}(\alpha_i,\beta_i)}{d\beta_i} < 0$. By Assumption 1, where $\bar{\gamma}^2 \le \frac{1}{[G\bar{\alpha}-B(1-\underline{\beta})]}$, which implies $\hat{\gamma}(\alpha_i,\beta_i)^2 < \bar{\gamma}^2 \le \frac{1}{[G\bar{\alpha}-B(1-\underline{\beta})]} \le \frac{1}{[G\alpha_i-B(1-\beta_i)]}$; hence, $\hat{\gamma}(\alpha_i,\beta_i)^2 < \frac{1}{[G\alpha_i-B(1-\beta_i)]}$, formally:

$$2(1-\beta)\sqrt{\alpha_i(\alpha_i+\beta_i-1)(G-B)(-B)} < G\alpha_i(\alpha_i+2\beta_i-2) + \alpha_i(1-\beta_i)B \tag{A.14}$$

Inequality (A.14) implies that $(\alpha_i + 2\beta_i - 2) > 0$ must be satisfied; otherwise, the right-hand side of (A.14) is always negative. □

Proof of Proposition 2. We have to identify the sign of $\frac{\partial MRS_i}{\partial t_i}$ for any possible levels of $\underline{U}_j$.

$$\frac{\partial MRS_i}{\partial t_i} = \frac{\partial^2 Ev_i}{\partial t_i \partial \gamma_j} - \frac{\partial Ev_i/\partial \gamma_j}{\partial Ev_i/\partial \underline{U}_j}\frac{\partial^2 Ev_i}{\partial t_i \partial \underline{U}_j}, \tag{A.15}$$

where $t_i \in \{\alpha_i, \beta_i\}$ depending on the firm's heterogeneity in each scenario. In this sensitivity case, $t_i = \alpha_i$ because β is homogeneous across firms. When $\underline{U}_j \leq \hat{U}(\alpha_i, \beta, \gamma_j)$, $\frac{\partial E\hat{v}_i}{\partial \underline{U}_j} = 0$, thus, from (A.15), if the cross partial derivative $\frac{\partial^2 E\hat{v}_i}{\partial \alpha_i \partial \gamma_j}$ is positive, then the matching is PAM.

$$\frac{\partial^2 E\hat{v}_i}{\partial \alpha_i \partial \gamma_j} = \frac{G(\alpha_i + \beta - 1)^2 \gamma_j^2 X_{ij} + (1-\beta)}{(\alpha_i + \beta - 1)^3 \gamma_j^3} > 0.$$

Therefore, in the case where the reservation utility is low, the matching is PAM.

When $\hat{U}(\alpha_i, \beta, \gamma_j) \leq \underline{U}_j \leq \tilde{U}(\alpha_i, \beta, \gamma_j)$, we solve all partial derivatives in (A.15), and let notation Z denote $(1-\beta)$ to simplify the expressions:

$$\frac{\partial E\hat{v}_i}{\partial \gamma_j} = \frac{(Y_{ij} - Z)[X_{ij}(Y_{ij} + Z) - Y_{ij} Z + Z^2]}{(\alpha_i + \beta - 1)^2 \gamma_j^3 Y_{ij}}; \quad \frac{\partial E\hat{v}_i}{\partial \underline{U}_j} = -\frac{2Y_{ij} - X_{ij} - Z}{Y_{ij}};$$

$$\frac{\partial^2 E\hat{v}_i}{\partial \alpha_i \partial \gamma_j} = \frac{\begin{array}{c} G(\alpha_i + \beta - 1)^2 \gamma_j^2 (Y_{ij}^2 - Z^2) Y_{ij}^2 + \cdots \\ \ldots + Z(Y_{ij} - Z)[X_{ij} Y_{ij} Z + X_{ij} Z^2 + (Y_{ij} - Z)(Y_{ij}^2 - 2Y_{ij} Z - Z^2)] \end{array}}{(\alpha_i + \beta - 1)^3 \gamma_j^3 Y_{ij}^3};$$

$$\frac{\partial^2 E\hat{v}_i}{\partial \alpha_i \partial \underline{U}_j} = \frac{G(\alpha_i + \beta - 1)^2 \gamma_j^2 Y_{ij}^2 + Z(X_{ij} Z - Y_{ij}^2 + Z^2)}{(\alpha_i + \beta - 1) Y_{ij}^3}.$$

Substituting the derivatives into (A.15), of which the sign depends on the right-hand side of (A.16):

$$\text{sgn}\,\left(\frac{\partial MRS_i}{\partial \alpha_i}\right) = \text{sgn}\,\left(\underbrace{G(\alpha_i + \beta - 1)^2 \gamma_j^2 Y_{ij}^2}_{>0} + \underbrace{(Y_{ij} - 2Z)}_{>0}\underbrace{(Y_{ij} - X_{ij} - Z)}_{<0} Z\right) \tag{A.16}$$

First, let me explain the sign of each term. Clearly, $G(\alpha_i + \beta - 1)^2 \gamma_j^2 Y_{ij}^2 > 0$. From inequality (A.5), $(Y_{ij} - X_{ij} - Z) < 0$. Inequality $(Y_{ij} - 2Z) > 0$ can be implied by (A.3) and $X_{ij} \geq 3Z$. We can prove $X_{ij} \geq 3Z$ in the following:

$$X_{ij} - 3(1-\beta) > 0 \Rightarrow {\gamma_j}^2 > \frac{3(1-\beta)}{(\alpha_i + \beta - 1)[G\alpha_i - B(1-\beta)]} \tag{A.17}$$

From the inequality (A.13), $\gamma_j^2 \geq \hat{\gamma}(\alpha_i, \beta)^2 = \frac{(1-\beta)[G\alpha_i - (2\alpha_i + \beta - 1)B + 2\sqrt{\alpha_i(\alpha_i + \beta - 1)(G-B)(-B)}]}{[G\alpha_i - B(1-\beta)]^2(\alpha_i + \beta - 1)}$. I can prove that $\hat{\gamma}(\alpha_i, \beta)^2 > \frac{3(1-\beta)}{(\alpha_i + \beta - 1)[G\alpha_i - B(1-\beta)]}$, which yields (A.17).

$$\begin{aligned} &\text{sgn}\,\left([\hat{\gamma}(\alpha_i, \beta)^2 - \frac{3(1-\beta)}{(\alpha_i + \beta - 1)[G\alpha_i - B(1-\beta)]}]\right) = \\ &\text{sgn}\,\left(\underbrace{[(3\alpha_i + 4\beta - 4)(-B) - G\alpha_i]}_{\gtreqless 0}\underbrace{[(1-\beta)(-B) + G\alpha_i]}_{>0}\right) \end{aligned} \tag{A.18}$$

By Lemma 2 and assumption $(G + B) \leq 0$, we have (A.18) is always positive:

$$\underbrace{[(\alpha_i + 2(\alpha_i + 2\beta - 2))(-B) - G\alpha_i]}_{>0}\underbrace{[(1-\beta)(-B) + G\alpha_i]}_{>0} > 0$$

Thus, (A.17) is proved and $(Y_{ij} - 2Z) > 0$. Hence, the first term of (A.16) is positive, and the second term is negative. Therefore, the sign of (A.16) is determined by the size of each term. I will prove that the sign of (A.16) is always positive. First, because $Y_{ij} > (Y_{ij} - 2Z) > 0$, then if $[G(\alpha_i + \beta - 1)^2 \gamma_j^2 Y_{ij}^2 + Y_{ij}(Y_{ij} - X_{ij} - Z)Z] > 0$, then (A.16) is always positive. Second, $[G(\alpha_i + \beta - 1)^2 \gamma_j^2 Y_{ij} + (Y_{ij} - X_{ij} - Z)Z]$ is increasing in Y_{ij}, thus, if I can prove that $G(\alpha_i + \beta - 1)^2 \gamma_j^2 \geq Z$, then $[G(\alpha_i + \beta - 1)^2 \gamma_j^2 Y_{ij} + (Y_{ij} - X_{ij} - Z)Z] > [Z Y_{ij} + Z(Y_{ij} - X_{ij} - Z)] = Z(2Y_{ij} - X_{ij} - Z) > 0$, because of inequality (A.3). In consequence, I only need to prove that $G(\alpha_i + \beta - 1)^2 \gamma_j^2 \geq Z = (1-\beta)$ for any possible parameter combination $(G, B, \alpha_i, \beta, \gamma_j)$.

We have proven (A.17), thus, if $G(\alpha_i + \beta - 1)^2 \gamma_j^2 \geq \frac{1}{3} X_{ij}$, then $G(\alpha_i + \beta - 1)^2 \gamma_j^2 \geq Z = (1-\beta)$, and (A.16) is always positive. That is,

$$\begin{aligned} &G(\alpha_i + \beta - 1)^2 \gamma_j^2 > \frac{1}{3}[G\alpha_i - B(1-\beta)](\alpha_i + \beta - 1)\gamma_j^2 \iff G(\alpha_i + \beta - 1) \geq \frac{1}{3}[G\alpha_i - B(1-\beta)] \\ &\quad \iff G(\frac{2}{3}\alpha_i + \beta - 1) + B\frac{(1-\beta)}{3} \geq 0 \end{aligned}$$

From (A.14) in Lemma 2, we have: $G(\frac{1}{2}\alpha_i + \beta - 1) + B\frac{(1-\beta)}{2} > 0$, which implies that $G(\frac{2}{3}\alpha_i + \beta - 1) + B\frac{(1-\beta)}{3} > G(\frac{1}{2}\alpha_i + \beta - 1) + B\frac{(1-\beta)}{2} > 0$. Thus, inequality $G(\alpha_i + \beta - 1)^2 \gamma_j^2 \geq Z = (1-\beta)$ always holds. Therefore, (A.16) is always positive; hence, the equilibrium μ is always PAM in this sensitivity scenario. □

Remark 1. In the specificity market, the threshold $\hat{U}(\alpha, \beta_i, \gamma_j)$ decreases in β_i.

Proof of Remark 1. The first order derivative of $\hat{U}(\alpha, \beta_i, \gamma_j)$ w.r.t. β_i is:

$$\frac{\partial \hat{U}(\alpha, \beta_i, \gamma_j)}{\partial \beta_i} = \frac{\overbrace{B[X_{ij} + (1-\beta_i)](\alpha+\beta_i-1)^2\gamma_j^2}^{<0} - \overbrace{[X_{ij} - 3(1-\beta_i)]}^{>0}}{4(\alpha+\beta_i-1)^3\gamma_j^2}.$$

From Lemma 2 and assumption $(G+B) \le 0$, we can again obtain $[X_{ij} - 3(1-\beta_i)] > 0$ by following the same steps to get inequality (A.17). Therefore, $\frac{\partial \hat{U}(\alpha,\beta_i,\gamma_j)}{\partial \beta_i} < 0$. □

Proof of Proposition 3. There are two possible cases depending on $\underline{U}_j$: (1) $\underline{U}_j \le \hat{U}(\alpha, \beta_i, \gamma_j)$, the expected profit does not depend on $\underline{U}_j$; (2) $\underline{U}_j > \hat{U}(\alpha, \beta_i, \gamma_j)$ and $Eu_j = \underline{U}_j$, the expected profit depends on $\underline{U}_j$. Before further proving this proposition, let me show that case (1) never exists.

By Remark 1, $\hat{U}(\alpha, \beta_i, \gamma_j)$ decreases in β_i, in the competition, a better company with β_i has to ensure a better academic at least $\underline{U}_j > \hat{U}(\alpha, \beta_i, \gamma_j)$ because a worse firm has already offered a utility higher than $\hat{U}(\alpha, \beta_i, \gamma_j)$. In consequence, for each pair (p_i, a_j), $\underline{U}_j > \hat{U}(\alpha, \beta_i, \gamma_j)$. Case (1) never exist in this scenario. I only need to show that when $\underline{U}_j > \hat{U}(\alpha, \beta_i, \gamma_j)$, the $\frac{\partial MRS_i}{\partial \beta_i}$ is always negative, which implies NAM. Let notation Z_i denote $(1-\beta_i)$ to reduce the length of expressions. I first solve all derivatives needed in (A.15):

$$\frac{\partial E\hat{v}_i}{\partial \gamma_j} = \frac{(Y_{ij} - Z_i)[X_{ij}(Y_{ij} + Z_i) - Y_{ij}Z_i + Z_i^2]}{(\alpha+\beta_i-1)^2{\gamma_j}^3 Y_{ij}} > 0, \quad \frac{\partial E\hat{v}_i}{\partial \underline{U}_j} = -\frac{2Y_{ij} - X_{ij} - Z_i}{Y_{ij}} < 0,$$

$$\frac{\partial^2 E\hat{v}_i}{\partial \beta_i \partial \gamma_j} = \frac{\begin{array}{l} B(\alpha+\beta_i-1)^2{\gamma_j}^2(Y_{ij}^2 - Z_i^2)Y_{ij}^2 + \cdots \\ \ldots + \alpha(Y_{ij} - Z_i)[X_{ij}Z_i^2 + X_{ij}Y_{ij}Z_i + (Y_{ij} - Z_i)(Y_{ij}^2 - 2Y_{ij}Z_i - Z_i^2)] \end{array}}{(\alpha+\beta_i-1)^3{\gamma_j}^3 Y_{ij}^3},$$

$$\frac{\partial^2 E\hat{v}_i}{\partial \beta_i \partial \underline{U}_j} = \frac{B(\alpha+\beta_i-1)^2{\gamma_j}^2 Y_{ij}^2 + \alpha(X_{ij}Z_i - Y_{ij}^2 + Z_i^2)}{(\alpha+\beta_i-1)Y_{ij}^3}.$$

Substituting the derivatives into (A.15), by simplifying:

$$\text{sgn}\left(\frac{\partial MRS_i}{\partial \beta_i}\right) = \text{sgn}\left(\underbrace{B(\alpha+\beta_i-1)^2\gamma_j^2 Y_{ij}^2}_{<0} + \alpha\underbrace{(Y_{ij} - 2Z_i)}_{>0}\underbrace{(Y_{ij} - Z_i - X_{ij})}_{<0}\right) < 0$$

Inequality (A.5) and inequality $[X_{ij} - 3(1-\beta_i)] > 0$ imply that $(Y_{ij} - Z_i - X_{ij}) < 0$ and $(Y_{ij} - 2Z_i) > 0$, respectively. Then sgn $(\frac{\partial MRS_i}{\partial \beta_i})$ is always negative. The equilibrium matching is NAM. □

Remark 2. In the symmetry scenario, $\hat{U}(\alpha_i, \gamma_j)$ decreases in α_i.

Proof of Remark 2. In this scenario, $\beta_i = \alpha_i$.

$$\frac{\partial \hat{U}(\alpha_i, \gamma_j)}{\partial \alpha_i} = \frac{\overbrace{(G+B)(2\alpha_i-1)^2\gamma_j^2[X_{ij} + (1-\alpha_i)]}^{<0} - \overbrace{[X_{ij} - 3(1-\alpha_i)]}^{>0}}{4(2\alpha_i-1)^3\gamma_j^2} < 0.$$

From Lemma 2 and assumption $(G+B) \le 0$, we can again obtain $[X_{ij} - 3(1-\alpha_i)] > 0$ by following the same steps to get inequality (A.17). Therefore, $\frac{\partial \hat{U}(\alpha_i,\gamma_j)}{\partial \alpha_i} < 0$. □

Proof of Proposition 4. There are two possibilities depending on the level of $\underline{U}_j$. (1) $\underline{U}_j \le \hat{U}(\alpha_i, \gamma_j)$, the expected profit does not depend on $\underline{U}_j$; (2) $\underline{U}_j > \hat{U}(\alpha_i, \gamma_j)$ and $Eu_j = \underline{U}_j$, the expected profit depends on $\underline{U}_j$. Similarly, by 2, case (1) never exist. I only need to explore the sign of (A.15) when $\underline{U}_j > \hat{U}(\alpha_i, \gamma_j)$. The derivatives of the expected profit are the following (to reduce the expressions' length, I use notation $\hat{Z}_i = (1-\alpha_i)$, and $\beta_i = \alpha_i$):

$$\frac{\partial E\hat{v}_i}{\partial \gamma_j} = \frac{(Y_{ij}-\hat{Z}_i)[X_{ij}(Y_{ij}+\hat{Z}_i) - Y_{ij}\hat{Z}_i + \hat{Z}_i^2]}{(2\alpha_i-1)^2{\gamma_j}^3 Y_{ij}} > 0; \ \frac{\partial E\hat{v}_i}{\partial \underline{U}_j} = -\frac{2Y_{ij}-X_{ij}-\hat{Z}_i}{Y_{ij}} < 0;$$

$$\frac{\partial^2 E\hat{v}_i}{\partial \alpha_i \partial \gamma_j} = \frac{(G+B)(2\alpha_i-1)^2{\gamma_j}^2(Y_{ij}^2-\hat{Z}_i^2)Y_{ij}^2 + \cdots \quad \ldots + (Y_{ij}-\hat{Z}_i)[X_{ij}\hat{Z}_i^2 + X_{ij}Y_{ij}\hat{Z}_i + (Y_{ij}-\hat{Z}_i)(Y_{ij}^2 - 2Y_{ij}\hat{Z}_i - \hat{Z}_i^2)]}{(2\alpha_i-1)^3{\gamma_j}^3 Y_{ij}^3};$$

$$\frac{\partial^2 E\hat{v}_i}{\partial \alpha_i \partial \underline{U}_j} = \frac{(G+B)(2\alpha_i-1)^2{\gamma_j}^2 Y_{ij}^2 + X_{ij}\hat{Z}_i - (Y_{ij}^2 - \hat{Z}_i^2)}{(2\alpha_i-1)Y_{ij}^3}.$$

Substituting the derivatives into (A.15). By simplifying,

$$\text{sgn}\left(\frac{\partial MRS_i}{\partial \alpha_i}\right) = \text{sgn}\left(\underbrace{(G+B)(2\alpha_i-1)^2{\gamma_j}^2 Y_{ij}^2}_{\le 0} + \underbrace{(Y_{ij}-2\hat{Z}_i)}_{>0}\underbrace{(Y_{ij}-X_{ij}-\hat{Z}_i)}_{<0}\right) < 0.$$

Inequality (A.5) and $[X_{ij} - 3(1-\alpha_i)] > 0$ imply that $(Y_{ij}-X_{ij}-\hat{Z}_i) < 0$ and $(Y_{ij}-2\hat{Z}_i) > 0$, respectively. Therefore, $\frac{\partial MRS_i}{\partial \alpha_i} < 0$, the equilibrium matching is NAM. □

Proof of Proposition 5. I only provide the proof for the sensitivity scenario because the proofs for the other cases will be the same. The slope $\frac{dw_{ij}^g}{d\gamma_j}$ is given by:

$$\frac{dw_{ij}^g}{d\gamma_j} = \underbrace{-\frac{(Y_{ij}-Z_i)^2}{(\alpha_i+\beta-1)^2\gamma_j^3 Y_{ij}}}_{\frac{\partial w_{ij}^g}{\partial \gamma_j}} \underbrace{-\frac{(Y_{ij}-Z_i)^2}{(\alpha_i+\beta-1)^3\gamma_j^2 Y_{ij}}\frac{\bar{\alpha}-\underline{\alpha}}{\bar{\gamma}-\underline{\gamma}}}_{\frac{\partial w_{ij}^g}{\partial \alpha_i}\frac{d\alpha_i(\gamma_j)}{d\gamma_j}} + \underbrace{\frac{(Y_{ij}-Z_i)[X_{ij}(Y_{ij}-Z_i) - Y_{ij}Z_i + Z_i^2]}{(2Y_{ij}-X_{ij}-Z_i)(\alpha_i+\beta-1)^2\gamma_j^3 Y_{ij}}}_{\frac{\partial w_{ij}^g}{\partial \underline{U}_j}\frac{d\underline{U}_j}{d\gamma_j}}$$

We Simplify $\frac{dw_{ij}^g}{d\gamma_j}$ and denote $(\bar{\alpha}-\underline{\alpha})$ by $\Delta\alpha$, and $(\bar{\gamma}-\underline{\gamma})$ by $\Delta\gamma$:

$$\underbrace{\frac{(Y_{ij}-Z_i)^2}{(2Y_{ij}-X_{ij}-Z_i)(\alpha_i+\beta-1)^3\gamma_j^3 Y_{ij}\Delta\gamma}}_{>0}[2(X_{ij}-Y_{ij})(\alpha_i+\beta-1)\Delta\gamma - (2Y_{ij}-X_{ij}-Z_i)\gamma_j\Delta\alpha],$$

where $\alpha_i = \mu(\gamma_j)$. The sign of $\frac{dw_{ij}^g}{d\gamma_j}$ depends on the sign of the second term. By simplification, we have:

$$sgn\left(\frac{dw_{ij}^g}{d\gamma_j}\right) = sgn\left(\frac{\Delta\gamma}{\Delta\alpha} - \Omega(\gamma_j, \underline{U}_j)\right),$$

where $\Omega(\gamma_j, \underline{U}_j) = \frac{(2Y_{ij}-X_{ij}-Z_i)\gamma_j}{2(X_{ij}-Y_{ij})(\alpha_i+\beta-1)}$. Therefore, through the matching function $\alpha_i = \mu(\gamma_j)$, for $\forall \gamma_j \in [\hat{\gamma}, \bar{\gamma}]$ and equilibrium reservation utility $\underline{U}_j$, if $\frac{\Delta\gamma}{\Delta\alpha} > \Omega(\gamma_j, \underline{U}_j)$, $\frac{dw_{ij}^g}{d\gamma_j} > 0$; if $\frac{\Delta\gamma}{\Delta\alpha} < \Omega(\gamma_j, \underline{U}_j)$, $\frac{dw_{ij}^g}{d\gamma_j} < 0$; otherwise, the equilibrium incentive is not non-monotonic in productivity. □

Proof of Proposition 6. Let me use expression $Ev(t_i, \gamma_j, \underline{U}_j)$ to denote the company t_i's expected profit when it is matched with γ_j with reservation utility $\underline{U}_j$. Suppose that $Ev(\beta, \gamma_1, \hat{U}(\beta, \gamma_2)) > 0$, otherwise, company β is always unmatched. We solve the reservation utility for better academic a_1, $\underline{U}(t_i, \gamma_1)$, at which firms are indifferent between hiring a_1 and hiring a_2 at a_2's lowest utility level $\hat{U}(t_i, \gamma_2)$. If for firm β, hiring the worse academic is not profitable (i.e., $Ev(\beta, \gamma_2, \hat{U}(\beta, \gamma_2)) < 0$), then it is willing to pay the better academic the most utility it can, which is $\underline{U}(\beta, \gamma_1) = \tilde{U}(\beta, \gamma_1)$. Clearly, if $\underline{U}(\beta, \gamma_1) > \underline{U}(\alpha, \gamma_1)$, firm β is willing to pay more to academic a_1 and outbids firm α in the competition. By solving the following equation, we obtain $\underline{U}(\beta, \gamma_1)$:

$$Ev(\beta, \gamma_1, \underline{U}(\beta, \gamma_1)) = \max\{0, Ev(\beta, \gamma_2, \hat{U}(\beta, \gamma_2))\} \Rightarrow$$

$$\underline{U}(\beta, \gamma_1) = \frac{{Y^\star}^2 - (1-\beta)^2}{2\beta^2\gamma_1^2} \qquad \text{if } Ev(\beta, \gamma_2, \hat{U}(\beta, \gamma_2)) > 0,$$

$$\underline{U}(\beta, \gamma_1) = \tilde{U}(\beta, \gamma_1) \qquad \text{if } Ev(\beta, \gamma_2, \hat{U}(\beta, \gamma_2)) \le 0,$$

where $Y^\star = \frac{\hat{X}_1 + (1-\beta) + \sqrt{(\hat{X}_1+(1-\beta))^2 - (\hat{X}_2+(1-\beta))^2\gamma_1^2/\gamma_2^2}}{2}$, $\hat{X}_1 = [G - B(1-\beta)]\beta\gamma_1^2$, $\hat{X}_2 = [G - B(1-\beta)]\beta\gamma_2^2$.

We obtain $\underline{U}(\alpha,\gamma_1)$ from the following equation:

$$Ev(\alpha,\gamma_1,\underline{U}(\alpha,\gamma_1)) = Ev(\alpha,\gamma_2,\hat{U}(\alpha,\gamma_2)) \Rightarrow \underline{U}(\alpha,\gamma_1) = \frac{G^2\alpha^2(\gamma_1+\sqrt{\gamma_1^2-\gamma_2^2})^2}{8}.$$

Solving $\underline{U}(\beta,\gamma_1) > \underline{U}(\alpha,\gamma_1)$, we have:

$$\begin{aligned} &\alpha < \frac{2\sqrt{Y^{\star 2}-(1-\beta)^2}}{G\beta\gamma_1(\gamma_1+\sqrt{\gamma_1^2-\gamma_2^2})} = \alpha(\beta), && \text{if } Ev(\beta,\gamma_2,\hat{U}(\beta,\gamma_2)) > 0,\\ &\alpha < \frac{2\sqrt{2\tilde{U}(\beta,\gamma_1)}}{G\alpha(\gamma_1+\sqrt{\gamma_1^2-\gamma_2^2})} = \alpha(\beta), && \text{if } Ev(\beta,\gamma_2,\hat{U}(\beta,\gamma_2)) \le 0. \end{aligned}$$

Therefore, there exists $\alpha^\star(\beta) = \min\{\alpha(\beta),1\}$. When $\alpha < \alpha^\star(\beta)$, firm β is matched with better academic a_1. □

Proof of Proposition 7. $\beta_i = m(\alpha_i)$, which is twice-differentiable. We obtain all partial derivatives in (A.15). Note that $\frac{\partial E\hat{v}_i}{\partial \gamma_j}$ and $\frac{\partial E\hat{v}_i}{\partial \underline{U}_j}$ are the same as that in the proof of Propositions 2 and 3.

$$\begin{aligned} \frac{d^2E\hat{v}_i}{d\alpha_i d\gamma_j} &= \frac{\partial^2 E\hat{v}_i}{\partial\alpha_i\partial\gamma_j} + \frac{\partial^2 E\hat{v}_i}{\partial\beta_i\partial\gamma_j}\frac{\partial m(\alpha_i)}{\partial\alpha_i}\\ \frac{d^2E\hat{v}_i}{d\alpha_i d\underline{U}_j} &= \frac{\partial^2 E\hat{v}_i}{\partial\alpha_i\partial\underline{U}_j} + \frac{\partial^2 E\hat{v}_i}{\partial\beta_i\partial\underline{U}_j}\frac{\partial m(\alpha_i)}{\partial\alpha_i}. \end{aligned}$$

Substituting all the derivatives into (A.15), then we have:

$$\text{sgn}\,(\frac{\partial MRS_i}{\partial\alpha_i}) = \text{sgn}\,([G+Bm'(\alpha_i)](\alpha_i+\beta_i-1)^2\gamma_j{}^2Y_{ij}^2 + [m'(\alpha_i)\alpha_i+Z_i](Y_{ij}-2Z_i)(Y_{ij}-X_{ij}-Z_i)),$$

where $\beta_i = m(\alpha_i)$. By simplification, we obtain the equilibrium matching is determined by the following inequality:

$$m'(\alpha_i) < (>)\ \Lambda(\alpha_i,\gamma_j,\underline{U}_j) = \frac{\overbrace{G[\alpha_i+m(\alpha_i)-1]^2\gamma_j^2Y_{ij}^2}^{>0} + \overbrace{[1-m(\alpha_i)]}^{>0}\overbrace{(Y_{ij}-2Z_i)}^{>0}\overbrace{(Y_{ij}-X_{ij}-Z_i)}^{<0}}{\underbrace{-B[\alpha_i+m(\alpha_i)-1]^2\gamma_j^2Y_{ij}^2}_{<0} - \alpha_i\underbrace{(Y_{ij}-2Z_i)}_{>0}\underbrace{(Y_{ij}-Z_i-X_{ij})}_{<0}} \to \text{PAM (NAM)}.$$

From assumption and proofs of Propositions 2 and 3, that is, $|B| \ge G$ and $\alpha_i + m(\alpha_i) - 1 \ge 0$, we obtain the sign of the numerator is positive, and that of the denominator is positive; besides, the right hand side $\Lambda(\alpha_i,\gamma_j,\underline{U}_j) \in (0,1)$. From this inequality, when $m'(\alpha_i) < \Lambda(\alpha_i,\gamma_j,\underline{U}_j)$ ($m'(\alpha_i) > \Lambda(\alpha_i,\gamma_j,\underline{U}_j)$, resp.), the stable matching is PAM (NAM, resp.); otherwise, it is non-assortative. □

Proof of Proposition 8. With the new assumption $(G+B) > 0$, we re-consider the expected profits $E\hat{v}_i$ and $E\tilde{v}_i$ in the proof of Proposition 1. Because $(G+B) > 0$, $E\tilde{v}_i$ can be positive. We compare these two equilibrium candidates and solve the inequality $(E\hat{v}_i - E\tilde{v}_i) \ge 0$ as a function of $\underline{U}_j$, which implies:

$$\underline{U}_j \le \frac{[2(1-\alpha_i)(1-\beta_i)G-(\alpha_i-\beta_i+1)\beta_i B][-(\alpha_i+\beta_i-1)\beta_i B]}{2(\alpha_i+\beta_i-1)^2\gamma_j^2[(1-\alpha_i)G-\beta_i B]^2} = \underline{U}_{ij}^\star(\alpha_i,\beta_i,\gamma_j).$$

Thus, when $\underline{U}_j \le \underline{U}_{ij}^\star(\alpha_i,\beta_i,\gamma_j)$, firm p_i chooses a contract in which it abandons the project when the signal is b, and when $\underline{U}_j > \underline{U}_{ij}^\star(\alpha_i,\beta_i,\gamma_j)$, it chooses the one where it always continues the project. □

Proof of Proposition 9. In the case where firms always continue the project when the signal is b, one can solve $\frac{\partial MRS_i}{\partial t_i}$ in each scenario, and by simplification:

$$\text{sgn}\,(\frac{\partial MRS_i}{\partial t_i}) = \text{sgn}\,(\underbrace{(Y_{ij}-2Z_i)}_{>0}\underbrace{(Y_{ij}-\tilde{X}_{ij}-Z_i)}_{<0}Z_i) < 0.$$

One can easily show that $(Y_{ij}-2Z_i) > 0$ and $(Y_{ij}-\tilde{X}_{ij}-Z_i) < 0$ as in the proof of Proposition 2. We only need to change the heterogeneity of the parameters for each scenario, and we obtain the same results, where the equilibrium matching is NAM. □

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