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Surveying the Types of Tables in Ancient Greek Texts

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Abstract: We may take tables for granted. However, due to a variety of factors, tables were a rarity in the history of ancient Greek culture, used only limitedly in very special contexts and generally in a non-systematic way, except in astronomy. In this paper I present the main types of tables that can be found in ancient Greek texts: non-ruled columnar lists (accounts and other types of informal tables), ruled columnar lists (mostly astronomical tables), and symmetric tables (mainly Pythagorean displays of numbers).

Keywords: Greek tables; ancient astronomy; Pythagorean mathematics; administrative tables; mathematical tables

From our perspective, we may assume that tables have always been there. They are ubiquitous in most non-literary types of texts, helping in the presentation of complex information, be it numerical or non-numerical. Given the great amount of information contained in some of the ancient Greek philosophical and scientific genres, we have the intuition that tables would have been extremely useful. However, only what I will define as unruled columnar lists (the simplest kind) were used with some frequency in ancient Greek texts, especially in documentary texts.

This paper will examine the types of tables used in ancient Greek culture, tracing a history of their development. It will not be an exhaustive survey, in the sense that I will not count all the tables in the ancient Greek documents. In my view, the evidence is more clearly described as classified into different types, periods, and genres than with bare counting, mostly because of the extreme discontinuities of document survival. Unruled columnar lists were relatively frequent throughout all periods in public and private accounting, but ruled tables were apparently exclusive of concrete textual genres such as astronomy or arithmology/arithmetic. Counting the absolute number of astronomical tables in Ptolemy (the only astronomical author whose corpus has reached us) would be

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little informative, but I discuss all the tables in the arithmetical treatises of Nicomachus and Theon of Smyrna (see below, Section 2.2) because they are not numerous and serve to illustrate the concerned type of table.[1] At least in terms of preserved treatises, I can claim that I have surveyed all the evidence: Nicomachus and Theon are the only other ancient mathematical treatises that contain tables. Nothing in Euclid and the Hellenistic geometers, nor in the mechanical authors Philo and Hero. I will also deal with a couple of tables in astrological works, and with the unruled columnar lists in Ptolemy and Aristotle.[2] I have not found tables elsewhere: they were probably regarded as generally unsuited for prose-writing.

Regarding the question of transmission, it is evident that we are on safer ground with tables stemming from ancient artifacts than when gathered from the manuscript tradition. Indeed, we will see three cases in which tables were substantially modified in the course of transmission (see Sections 1.2 and 1.4.1 for Ptolemy, Section 1.3 for Aristotle). However, we can detect such changes due to the fact that authors tend to give indications about the original appearance of their tables.[3] The original form can generally be traced to a certain point in the manuscript evidence,[4] the modifications themselves constituting new evidence of the use of tables in antiquity. In any case, tables seem to have been more prone to changes than the lettered diagrams of geometrical treatises,[5] and they should be treated with due care when found in the medieval manuscripts.

1 Jones 1999, 7 counts the datable astronomical papyri and concludes that the number of papyri in each century roughly corresponds to the total number of Greek papyri datable in that century. Having in mind that most astronomical papyri consist of tables, this can serve to estimate that the distribution of astronomical tables through the centuries (starting with the 1st c. BC, the date of the first astronomical papyri) is rather uniform. In any case, the mid second century BC represents a *terminus post quem* (cf. Section 1.4 below).

2 Tables were surprisingly rare in astrological treatises (in contrast with astronomy): I discuss a columnar list in Ptolemy's *Tetrabiblos* I 21 (at Section 1.2 below) – there is another one at II 3; and a ruled table in Valens VIII, stemming from Critodemus (at Section 2.4 below), but cf. Valens' criticisms of Critodemus' excessive use of tables in Critodemus F 3 Tolsa (=Valens III 9.1–6). In Valens V 8.1–100 there is also a simple, unruled two-column list with numbers in the first column, also derived from Critodemus: cf. Tolsa 2024, 157–176.

3 Ptolemy is the most explicit author. See, for example, his comments on the dimensions of the table of chords and his instructions for its reconstruction in case of scribal corruption at the end of *Syntaxis* I 10 (Heiberg 47).

4 Another case is that of the tables in Ptolemy's *Harmonics* II 14–15. In the manuscripts (e.g. Vat. gr. 191), all the tables are of five columns, while only those corresponding to the enharmonic and those of II 15 should be of this type. Furthermore, there is always an empty space the size of a column between the first two columns and the other three, which could have served to express a division between the enharmonic scales of two oldest and paradigmatic scholars (Archytas and Aristoxenus) on the one side and the three newer ones on the other. Concerning the table of Critodemus (cf. below, Section 2. 4), it is possible that it was completed with extra columns by later copyists: cf. Tolsa 2024, 196.

5 Cf. Netz 2004, 8–9.

1 Unruled and Ruled Columnar Lists

First, it will be necessary to define what will be considered a table. The simplest and broadest definition of a table is that of a matrix of columns and rows. This would in principle include virtually any kind of grid structure, even grid-like drawings without any information in the squares, or those of *stoikhedon* inscriptions in which the letters are carved in the center of every square without leaving any empty squares. The grid itself may be visible or not: we can accept tables without rulings, too. But a more specific definition would be desirable, taking both our own aims and our material into account. For example, based on the Mesopotamian documents, Eleanor Robson considers only numerical tables, differentiating between formal (with rulings) and informal tables (without rulings), and leaving out as not-tables what she calls prosaic accounts, in which each item begins a new line, but each line is structured differently from the other.[6] Robson's choice to consider only numerical tables and not to propose more strict definitions is influenced by two factors: (1) the vast majority of tablets with rulings contain numerical data, and: (2) these tablets mostly contain accounts structured in a similar way, namely, in two axes of which the horizontal one categorizes different types of numerical data into columns, and the vertical one displays the data attributed to different individuals or items.

This format seems to be shared among several ancient Mediterranean and Near Eastern cultures, even if only in Mesopotamia do we find a relatively significant use in the preserved documents. Robson has exhaustively studied the evidence, identifying concrete periods and places in which tables were used with some frequency; all in all, outside the domain of astronomy the use of tables seems to have been irregular, and not circumscribed to a single period or place, or diffused from a single discovery.[7] For Egypt, an in-depth study is lacking, but from a superficial examination it seems that the format was not unknown, but not frequent either. For example, there are two tables carved on the Amon temple of Karnak specifying rations of offerings decreed by Tuthmosis III, in which the content is neatly divided by type of offering (in the columns) and by the yearly festivals (rows).[8] Also related to temple administration, a papyrus in the Abusir archive, from the fifth Dynasty, records inventories of furniture of the Soknopaios temple made by different crews of workers.[9]

6 Robson 2022, 20.

7 Some examples are known from the 3rd millennium, but only in the Old Babylonian period tables enjoyed some presence, although they were still a rarity influenced by the individual choices of scribes. The peak is in the 13th/14th centuries in Nippur, where a third of ca. 600 administrative documents is tabular: Robson 2004, 126, 137.

8 Gardiner 1952, plate VIII and Sparling 1995, 279. I thank Marina Escolano-Poveda for her kind and informative guidance through the bibliography on these tables.

9 British Museum EA 10735/12; cf. Spencer 2010, 259.

Moving on to the material in Latin, Andrew M. Riggsby looks for tables using a stricter definition, according to which the two axes should not impose a hierarchy in the reading of the table. To use his own example, a phonebook displaying two columns of addresses and telephone numbers would rather be called a list, since it is designed to be read downward.[10] On the other hand, he does not require numerical content or rulings, so that he for example accepts displays of grammatical cases in Varro's *De lingua latina*.[11] Using this strict definition, he barely finds examples of tables, even in fields where one would expect them such as grammatical works – Varro's is an exception for its Pythagoreanizing tendencies. The most relevant examples are two fragmentary duty rosters from Roman Egypt (P. Gen. lat. inv. 1, and O. Claud. 208) in which rows present soldiers' names and columns are days.[12] Of course, with a more generous definition he could have included much more documentary evidence that he rather classified as lists of the phonebook type.

A critical review by Dolganov reveals a certain inconsistency on the part of Riggsby, who seems to accept Mesopotamian tables as true tables but on the other hand classifies Latin columnar lists as mere lists.[13] The unspecified distinction may be one of complexity: Mesopotamian tables usually have more than two columns. More complexity seems to be transferred to the graphical level via the rulings, which separate more effectively the space when there is a significant number of columns. It may be the case that Riggsby is biased by the types of tables he finds in the Latin evidence: duty rosters are a very particular kind of table in which the two axes seem to have a similar degree of hierarchy, in that columns do not serve to separate different kinds of numerical information as most Mesopotamian tables do (they are just different days). The other example presented by Riggsby, that of Varro, fits more easily into what Greek writers call πλινθίον, the symmetric table. As we will see, this category of table has less to do with displaying real-life information, and instead is linked to magic squares and other graphical representations perceived as curiously symmetric.

In the Greek evidence, we will have all the types mentioned so far, from accounts structured in simple unruled tables to the more complex astronomical tables, including symmetric tables. The first two types correspond to Robson's tables: no symmetry between the axes, and to be read from top to down. I call this kind *columnar list*, since columns neatly divide the types of information (and hence are or may be headed), while rows represent individuals. There is a minimum of two columns, and there is no need of rulings.

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10 Riggsby 2019, 48.

11 Riggsby 2019, 51–52.

12 Reproductions in Riggsby 2019, 53–54.

13 Dolganov 2023: see also the ensuing responses by author and reviewer.

With respect to the contents, the simple columnar lists – unruled and with few columns, and called ἀναγραφαί or ὑπογραφαί – are mostly accounts found in documentary evidence, but they were also used in some types of technical treatises. More complex columnar lists were generally ruled (and thus I am more inclined to call them tables) and of astronomical content (κανόνες or κανόνια).

1.1 Unruled Columnar Lists: Accounts

We do not find ruled tables for administrative purposes in the epigraphical documents or papyri. Greek temple accounts are either prosaic lists – that is, new items do not start new lines – such as the Delian inventories (IG XI 2), or unruled columnar lists, in which the numbers are situated at the right of the column of text, such as in this unprovenanced papyrus (P. Lund IV 4):

λυχνίαι χαλκ(αῖ)	α
ποτήρια χαλκ(αῖ)	ε
. . .	
bronze lampstands	1
bronze jars	5
. . .	

This is the most typical format of the Greek account.[14] There is no grid and no column headers. The content is understood to be self-explanatory. In epigraphical records such as the lists of Athenian revenues from taxes on allied polis, we may find the numbers on the left column.[15] We may have up to two numerical columns: for example, in the inscriptions from the Athenian agora detailing properties confiscated for mutilating herms, one column displays the sales tax and another the total amounts.[16]

1.2 Unruled Columnar Lists: Mathematical Texts

Columnar lists were used in texts dealing with numbers. It is thus not surprising that they appear several times in the works of Claudius Ptolemy. To begin with, we

14 There are countless examples. Cf. e.g. the account of working days and wages in the temple of Soknopaios in P. Berol. inv. 6848, for which see Balamoshev and Claytor 2022; or the inventory list BGU XIII 2359.

15 IG II3 270, from 442/1. See also e.g. the accounts for the building of the Parthenon, IG I3 449, from 434/3 BC. Most Mesopotamian tables also reserve the right-most column for the descriptors: see Robson 2022, 23.

16 IG I3 426, from 414 BC.

can consider the list of phases of the fixed stars that forms the central part of *Phases* a format in between the prosaic and the columnar list. Here we have four kinds of information: the day number within the month, the latitude for which the concerned phase is meant (expressed as the maximum hours of daylight), the information on the astronomical phase, and meteorological information for the day. The situation is complicated by the fact that for a single day we might have phases for more than one latitude, as in the entry for the third day of month Thoth in the passage I copy below (Heiberg 14). In such circumstances, it would make little sense to strictly divide the text in different columns apart from the day number.

ὑποτάξομεν ἤδη τὴν ἀναγραφὴν ἔχουσαν οὕτως·

ΘΩΘ

α. ὡρῶν ιδ <· ὁ ἐπὶ τῆς οὐρᾶς τοῦ Λέοντος ἐπιτέλλει. Ἱππάρχῳ ἐτησίαι παύονται. Εὐδόξῳ ὑετία, βρονταί, ἐτησίαι παύονται.
β. ὡρῶν ιδ· ὁ ἐπὶ τῆς οὐρᾶς τοῦ Λέοντος ἐπιτέλλει, καὶ Στάχυς κρύπτεται. Ἱππάρχῳ ἐπισημαίνει.
γ. ὡρῶν ιγ <· ὁ ἐπὶ τῆς οὐρᾶς τοῦ Λέοντος ἐπιτέλλει. ὡρῶν ιε· ὁ καλούμενος Αἲξ ἑσπέριος ἀνατέλλει. Αἰγυπτίοις ἐτησίαι παύονται. Εὐδόξῳ ἄνεμοι μεταπίπτοντες. Καίσαρι ἄνεμος, ὑετός, βρονταί. Ἱππάρχῳ ἀπηλιώτης πνεῖ.

We will now append here the register in the following way:

Thoth

1. 14 ½ hours: the star on the tail of Leo rises. According to Hipparchus, Etesian winds stop. For Eudoxus, rain, showers, Etesian winds stop.
2. 14 hours: the star on the tail of Leo rises, and Spica disappears. For Hipparchus, a change of weather.
3. 13 ½ hours: the star on the tail of Leo rises. 15 hours: the so-called Western goat rises. For the Egyptians, Etesian winds stop. For Eudoxus, the winds change. For Caesar, wind, rain, showers. For Hipparchus, Eastern wind.

Even if the structure is only vaguely tabular, Ptolemy seems to have sought a certain standardization in that, unlike in other calendars, for every day in every month there is at least one latitude containing predictions, thus not leaving any day of the year empty. We do not have ancient copies of this work, but the oldest manuscripts (Vat. gr. 1594, 9th c. and BNF 2390, 13th c.) consistently show different entries for different days and the day numbers separated from the text quasi forming a different column. In fact, in Vat. gr. 1594 the separation is reinforced by placing the day numbers alternatingly on the left and on the right throughout the successive columns of text.

Ptolemy's calendar can be conceived as a deluxe version of stone parapegmata, the public calendars of star-phases and meteorological predictions in which holes were drilled for each entry, for the purpose of marking the current day by moving a peg.[17] See, for example, the Miletus *parapegma* (I reproduce the beginning of the part corresponding to Aquarius):

- ἐν ὑδροχόω[ι ὁ] ἥλιος
- [.....] ἑῷιος ἄρχεται δύνων
καὶ λύρα δύνει
- •
- ὄρνις ἀ[κ]ρόνυχτος ἄρχεται δύ[νων]
- • • • • • • • •
- ἀνδρόμεδα ἄρχεται ἑῷια ἐ[πι]
τέλλειν

- The sun is in Aquarius
- [.....] begins to set in the evening
and the Lyre sets
- •
- Cygnus begins to set in the evening
- • • • • • • • •
- Andromeda begins to rise in the morning

This is the standard form of the epigraphical *parapegma*. It has in common with Ptolemy the fact that the holes for the days (here unnumbered) are situated at the left, but it has even less of a tabular structure, since quite frequently several continuous days have no information and the holes are just placed horizontally one after the other, breaking the quasi-columnar structure. *Parapegmata* that are not designed for public exhibition, such as the one on papyrus P. Hibeh 27, do not even start at different lines for different days. On the other hand, Geminus' *parapegma*, which has been preserved in the manuscript tradition, has different rows for each entry, but the text is completely prosaic (I reproduce the beginning of Leo):

Τὸν δὲ Λέοντα διαπορεύεται ὁ ἥλιος ἐν ἡμέραις <λα>.
Ἐν μὲν οὖν τῇ α$^{\eta}$ ἡμέρᾳ Εὐκτήμονι Κύων μὲν ἐκφανής, πνῖγος δὲ ἐπιγίνεται· ἐπισημαίνει.
Ἐν δὲ τῇ ε$^{\eta}$ Εὐδόξῳ Ἀετὸς ἑῷος δύνει.

The sun travels Leo in 31 days.
In the first day according to Euctemon Sirius is shining, it is stifling hot; change of weather.
In the fifth day according to Eudoxus Aquila sets in the morning.

17 The Miletus parapegmata is edited and translated in Lehoux 2005.

In the *Geography*, Ptolemy undertakes the huge enterprise to provide coordinates for every significant city or natural accident in the known world. The large central portion of the work, covering books II–VII, consists in the list of such places with their coordinates, interrupted by titles. The contents are grouped first by region, then by type of place, for example the Greek mountains here (III 14.11):

Ὄρη δὲ εἰσὶν ἐν τῷ εἰρημένῳ τμήματι τό τε Καλλίδρομον ὄρος, οὗ τὸ μέσον

ἐπέχει μοίρας	μθ	λη δ´
καὶ ὁ Κόραξ ὄρος	μθ γ´	λη
καὶ ὁ Παρνασσὸς ὄρος	ν γ´	λη
καὶ ὁ Ἑλικὼν ὄρος	να	λζ <´ δ´

There are mountains in the said region: mount Kallidromos, whose center has the

coordinates	49	38 ¼
and mount Korax	49 1/3	38
and mount Parnassus	50 1/3	38
and mount Helicon	51	37 ¾

The features of this list are different. In contrast with the case of the calendar in the *Phases*, the structure here is completely regular: name of the place and coordinates (longitude and latitude). Furthermore, there is a special feature which allows for ulterior improvements, namely the ample space left in between, which is not only evident in the manuscripts, but also signaled in Ptolemy's presentation (II 1.3):

> Διὸ καὶ τὰς παραθέσεις τῶν μοιρῶν ἐφ' ἑκάστου τοῖς ἐκτὸς μέρεσι τῶν σελιδίων παρεθήκαμεν κανονίων τρόπον, προτάσσοντες μέντοι τὰς τοῦ μήκους τῶν τοῦ πλάτους, ὅπως, ἐάν τινες ἐμπίπτωσι διορθώσεις ἀπὸ τῆς πλείονος ἱστορίας, ἐνῇ ἐν τοῖς ἐχομένοις διαλείμμασι τῶν σελιδίων ποιεῖσθαι τὰς παραθέσεις αὐτῶν.
>
> So we have written beside in the extreme part of the columns, in the manner of tables, the coordinates of every place, putting the longitude before the latitude, so that, if anyone finds corrections from a deeper investigation, it would be possible to write their coordinates.

Here "columns" (σελίδια) refers to the papyrus columns. Ptolemy's observation ensures that the manuscripts reflect the original shape of these columnar lists. It is worth noting that Ptolemy does not say that he puts the information in tables (κανόνια, κανόνες) as elsewhere when ruled tables are used, but "in the manner of tables" (κανονίων τρόπον).[18] The disclaimer likely refers to the absence of rulings

18 The exceptions are the tables in the *Tetrabiblos* and *Harmonics* which were transformed and inserted in the ancient manuscript tradition (see below). In relation with Ptolemy's phrasing here, Ptolemy often says that he has written tables for such and such theory "in the following manner" (e.g. *Synt*. 1.461.20 Heiberg: ἐπραγματευσάμεθα πρὸς τὴν τοιαύτην ἐπίσκεψιν κανόνια περιέχοντα τὸν τρόπον τοῦτον), but this is not quite the same as "in the manner of tables" (κανονίων τρόπον).

(as we have it, again, in the manuscripts). It would make little sense to draw rulings for such simple presentation which, in addition, takes the most part of this large work. Contrary to Riggsby's requirement of symmetry, in this case the table has an extremely predominant vertical format.

Greek arithmetical tables preserved on papyrus show the same format: several columns of text divided in subcolumns, without rulings. Specifically, the two numbers that are to be added or multiplied are written very close together at the left, and the result is given at the right.[19] See for example P. Mich. inv. 5663:

Col. I		Col. II		//	Col. I		Col. II	
ε ς	λ	η δ	λβ		5 6	30	8 4	32
ε ζ	λε	η ε	μ		5 7	35	8 5	40
ε η	μ	η ς	μη		5 8	40	8 6	48
ε θ	με	η ζ	νς		5 9	45	8 7	56
ε ι	ν	η η	ξδ		5 10	50	8 8	64
ς α	ς	η θ	οβ		6 1	6	8 9	72
ς β	ιβ	η ι	π		6 2	12	8 10	80
ς γ	ιη	θ α	θ		6 3	18	9 1	9
. . .					. . .			

It is clear that the papyrus columns are not conceived as the columns of a table, since the numbers do not correspond with each other across columns: for example, the first preserved column begins by 5 × 6, while the second column at the same level has 8 × 4. In fact, my rendering is not completely faithful, since the lines of column I and those of column II are not aligned. This disposition clearly reflects the act of speech, and likely derives therefrom, that is, from the recitation of the times table from top to bottom. For comparison, the Mesopotamian and the Egyptian arithmetical tables are structured in the same unruled format by entries. It is difficult to assess the extent to which these textual traditions might have influenced one another: the influence of Egyptian arithmetic in Greek arithmetic is well-known in concrete aspects such as the use of unit-fractions, but the columnar-list formatting seems too general to be an indicator of a possible transmission.[20]

19 Division tables are similarly presented in two columns, featuring an expression such as τῶν μ ("from 40") in the left column, and the result κ ("20") in the right column; unlike the factor in multiplication tables, the divisor is not expressed in the entries. Cf. e.g. P. Rain. Unterricht 154.

20 For a collection of Babylonian arithmetical tables, including multiplication tables, see Friberg 2007, 45–98 (arithmetical tables) and 101–121 (metrological tables). Many images are available at http://www.ams.org/publicoutreach/feature-column/fc-2012-05 (last accessed 4/1/2024). For Egyptian tables, see Imhausen 2016, 84–101. For a broad overview of Greek arithmetical tables, see Azzarello

Even if this simple kind of table found many uses, we have the impression that it was little employed in scientific treatises. Thus, we have found some unruled columnar lists in Ptolemy, but they are sometimes explicitly avoided in some places of his corpus where it would probably seem natural to use them. For example, in his astrological treatise Ptolemy refuses to provide a list for what he calls the Chaldean terms, on the grounds that his explanation of their working is sufficient (*Tetrabiblos* I 21.12 Hübner):

> Ὁ δὲ Χαλδαϊκὸς τρόπος ἁπλῆν μέν τινα ἔχει, οὐχ οὕτως δὲ αὐτάρκη […], ὥστε μέντοι καὶ χωρὶς ἀναγραφῆς δύνασθαι ῥᾳδίως τινὰ διαβάλλειν αὐτούς.
>
> The Chaldaean method involves a simple sequence, although not self-evident […], so that, nevertheless, one could easily understand them even without any recording.

There were several different systems for this doctrine. The astrological terms were subdivisions within each of the zodiacal signs ruled by the planets; within each sign intervals corresponding to each of the planets were defined comprising each of them a different number of degrees. The unruled columnar list for what Ptolemy calls the "Chaldean terms" would have looked like this:

Aries	Jup 8	Ven 7	Sat 6	Mer 5	Mar 4
Taurus	Ven 8	Sat 7	Mer 6	Mar 5	Jup 4
Gemini	Sat 8	Mer 7	Mar 6	Jup 5	Ven 4
Cancer	Mar 8	Jup 7	Ven 6	Sat 5	Mer 4

. . . *(from here on for the next signs the structure repeats itself)*

Here the structure is repetitive enough for us to reconstruct the list with Ptolemy's indications, but for the more typical system of terms, which Ptolemy calls the "Egyptian," he has no other option than to register them in full, since they did not seem to follow a simple set of rules (a circumstance that Ptolemy finds fault with): here the manuscripts contain a ruled table comprising the signs as columns, and showing partial sums of the terms, which were useful to astrologers for discovering to what term-ruler belonged a concrete degree. However, more probably Ptolemy displayed the terms in unruled format without the partial sums, as in the table reconstructed above for the "Chaldean terms." Otherwise, considering his practice elsewhere, we would expect him to mention the partial sums and the ruled format. This is supported by the fact that in the oldest Arabic translations of

2021. For a catalogue of Greek multiplication tables, see Sesiano 2021. We may note that, even if from the Greek side only specimens from Egypt survive, the Greek and the ancient Egyptian traditions are distinct in significant ways, for example in the fact that multiplication tables are almost absent in the ancient Egyptian documents.

the *Tetrabiblos* this table appears as an unruled columnar list, without the partial sums.[21]

1.3 Unruled Columnar Lists without Numbers: Philosophy and Grammar

Aristotle used the columnar list for non-numerical contents. He has a 3-column list in the *Eudemian Ethics* (1221a), containing affects (πάθη) of the soul ordered according to the categories of excess, deficiency, and adequacy:

ὀργιλότης	ἀναλγησία	πραότης
θρασύτης	δειλία	ἀνδρεία
ἀναισχυντία	κατάπληξις	αἰδώς
. . .		

irascibility	lack of feeling	gentleness
foolhardiness	cowardice	bravery
shamelessness	shyness	modesty
. . .		

He calls this layout ὑπογραφή, the same word that he applies to an anatomical lettered diagram which appears in *History of animals* (510a). In *De interpretatione* 22a he uses the same term[22] to refer to a similar columnar list for his theory of opposites in four columns, which is shown in its original form only in some manuscripts:[23]

21 It is the same formatting used in the English translation by Egleston-Robbins. An edition of the translations by ʻUmar ibn al-Farrukhān and Ḥunayn ibn Isḥāq by Keiji Yamamoto and Taro Mimura is in preparation (https://ptolemaeus.badw.de/work/192). The partial sums do not appear either in the 13th-century Latin translation by William of Moerbecke (direct from the Greek): cf. Vuillemin-Diem and Steel (2015), 190. For the problems associated with the table of Ptolemy's own terms, cf. Tolsa 2018. For the ruled table appearing in the Greek manuscripts displaying the familiarities of the world's regions with the signs (II 4.1) we probably have a similar evolution from an original columnar list, since Ptolemy notes that these familiar countries will be "merely noted against each of the signs" (ἐφ' ἑκάστου τῶν δωδεκατημορίων κατὰ ψιλὴν παράθεσιν).

22 The same usage across several types of treatises ensures that these were not later additions as in the case of Ptolemy's *Harmonics* (see below).

23 E.g. in Laur. Plut. 72.4, f. 46v. That this is the original format and not the diagram with four quadrants that is found in the margin of most manuscripts (as in the same folio of Laur. Plut. 72.4) can be gleaned from Ammonius' discussion of the passage, in which he identifies the first column as the column of "the possible" (δυνατόν), whereas the blocks of the four-quadrant diagram are formed by the elements of each row of the columnar list as displayed above. Ammonius, *In Ar. int.* 232, κατὰ τὴν αὐτὴν ὥσπερ σελίδα τοῦ διαγράμματος, παραλαβὼν τὴν ἐκ μεταθέσεως κατάφασιν τοῦ δυνατοῦ, ταύτῃ πάλιν ἐφεξῆς καταλέγει τὰς ἀκολουθούσας κατὰ τοὺς λοιποὺς τῶν τρόπων προτάσεις, εἶτα ἐφεξῆς μέτεισιν ἐπὶ τὸ ἀπαριθμεῖσθαι τὰς κατὰ τὴν ἑτέραν σελίδα τοῦ διαγράμματος προτάσεις.

δυνατὸν εἶναι	ἐνδεχόμενον εἶναι	οὐκ ἀδύνατον εἶναι	οὐκ ἀναγκαῖον εἶναι
δυνατὸν μὴ εἶναι	ἐνδεχόμενον μὴ εἶναι	οὐκ αδύνατον μὴ εἶναι	οὐκ ἀναγκαῖον μὴ εἶναι
οὐ δυνατὸν εἶναι	οὐ ἐνδεχόμενον εἶναι	ἀδύνατον εἶναι	ἀναγκαῖον εἶναι
οὐ δυνατὸν μὴ εἶναι	οὐ ἐνδεχόμενον μὴ εἶναι	ἀδύνατον μὴ εἶναι	ἀναγκαῖον μὴ εἶναι

possible to be	admissible to be	not impossible to be	not necessary to be
possible not to be	admissible not to be	not impossible not to be	not necessary not to be
not possible to be	not admissible to be	impossible to be	necessary to be
not possible not to be	not admissible not to be	impossible not to be	necessary not to be

Even if he does not use the same word, Aristotle may have been influenced by the layout of what he calls the συστοιχίαι of the Pythagoreans, literally “elements belonging together” that were probably structured in two columns. For example, in *Metaph.* 986a we read:

> ἕτεροι δὲ τῶν αὐτῶν τούτων τὰς ἀρχὰς δέκα λέγουσιν εἶναι τὰς κατὰ συστοιχίαν λεγομένας, πέρας ἄπειρον, περιττὸν ἄρτιον, ἓν πλῆθος.
>
> Others from the same group say that the principles are ten, those that are called “according to the *systoichiai*”: limit-unlimited, odd-even, one-multiple.

The deduction that these may have been distributed into two columns comes from assertions like *Metaph.* 1093b (τῆς συστοιχίας ἐστὶ τῆς τοῦ καλοῦ τὸ περιττόν, τὸ εὐθύ, “the even, the straight, belong to the συστοιχία of the good”), in which Aristotle puts together as one συστοιχία the first elements of the pairs. Furthermore, the fact that in many of the manuscripts there is no conjunction between the elements of the pairs suggests that a space was left in between in the Pythagorean writings.

The unruled columnar lists of the *Eudemian ethics* and of *De interpretatione* also often appear in prosaic style in the manuscripts.[24] Such textual evolution reveals how foreign was this format even to ancient Greek philosophical treatises. As for why we find it in Aristotle, it is worth recalling that his surviving treatises were of internal circulation within the Lyceum: thus, it would probably be more admissible for them than for other, more literary texts, to transgress the rigid standards of papyrus-writing etiquette.

This format must have been common, however, in less formal texts, such as grammatical lists. See, for example, two second-century-AD papyri showing two-

24 Cf. e.g. Laur. Plut. 81.12, f. 62v for the former and Laur. Plut. 72.05, f. 63r for the latter.

subcolumn formatting, the first for nominal form and case name, and the second one for Greek verbal form and Latin translation (in Greek letters):

P. Oxy. 86.5538:

τὸ βέλος	ὀρθ[ή
τοῦ βέλος	γεν[ική
τῷ βέλει	δοτ[ική

. . .

P. Oxy. 82.5302:

ζωγράφουσιν	πινγουντ
ζωγράψ[ουσιν]	πινγηντ
ζεύγνυσι	ιουνγιτ
ζεύγν[υμι]	ιουνγω
ζεύξω	ιουνγαμ

. . .

Again as in the case of arithmetical tables, these tables are not ruled, and the pairs of subcolumns do not correspond to each other; rather, the whole papyrus is conceived as a long list of two columns.[25]

1.4 Ruled Columnar Lists: Astronomical Tables

This type is characterized by having multiple columns (sometimes including subcolumns) typically extending beyond the regular breadth of the papyrus column. Their complex structure thus naturally requires rulings, but similarly to the unruled type, the vertical dimension is predominant and often requires these tables to split into several partitions due to their great number of rows.

Such tables are for the most part astronomical. Out of the roughly 5,500 Oxyrhynchus papyri which have hitherto been published, approximately 100 are astronomical tables which have been edited, translated and commented in a dedicated monograph (they are more than half the total number of published astronomical papyri).[26] Surviving astronomical tables by known authors are mostly limited to Ptolemy. The production of tables is one of the main goals throughout Ptolemy's *Almagest*, and his *Handy Tables* consist in a more convenient presentation of the tables of the *Almagest*, without the theoretical

25 In the first papyrus, to the left of τὸ βέλος we have δοτική corresponding to an illegible word in the previous subcolumn. Similarly, in the second papyrus we have three forms of the same verb next to the three first rows (corresponding to two verbs).

26 Jones 1999. The volume also contains theoretical and procedure texts, as well as horoscopes (mostly just lists of planetary positions). Cf. *ibid.*, 7 for the number of papyri per century, Oxyrhynchus and not Oxyrhynchus. Demotic astronomical papyri are also contemplated.

development and in more practical formats for astrologers. These tables will be crucial to astrologers from the third century onward for the calculation of the planetary positions.[27] The end-products of Ptolemy's *Analemma* and the *Harmonics* are, once again, tables. The tables of the *Harmonics* are of course not astronomical, but it is clear that Ptolemy adapts the format of the astronomical table (along with the sexagesimal-style fractions) to music theory, after having advocated for similar principles for both the study of the motions of the heavenly bodies and harmonics (I 2).

Astronomical tables existed at least from the late Hellenistic age, a time of substantial transmission of astronomical and astrological lore from Mesopotamia. The details of such transmission are for the most part obscure to us due to the lack of surviving texts. We know that Hipparchus in the mid-second century BC used Babylonian observations of eclipses for his theory of the motion of the Sun and the Moon.[28] Hipparchus' age also witnessed the birth of Greek astrology as we know it from later times.[29] As for tables, we have several indications suggesting their use by astronomers and astrologers at that time: for example, it is accepted that Hipparchus had constructed a table of chords (an equivalent of a sine table) for similar purposes as Ptolemy's (*Almagest* I 11);[30] and the treatise of Hypsicles on the rising times seems establish the foundations for the construction of the so-called *anaphorical* table, a table of rising times for the calculation of the ascendant point. An Oxyrhynchus papyrus (POxy 4276, 2nd/3rd c. AD) in fact mentions a table of rising times attributed to Hipparchus (ἀναφορικόν), and Vettius Valens (IX 12) refers to tables for the motion of the Sun and the Moon which he attributes to Hipparchus, Sudines, Kydenas, and Apollonius. It is often supposed that Sudines is the expert on exstipicy (entrails divination) mentioned by Polyaenus as accompanying king Attalus I of Pergamon and that Apollonius is the geometer from Perge (both 3rd c. BC), but historians of astronomy disagree with these identifications, and place these astronomers close to or later than Hipparchus' age.[31] This period coincides with a greater Hellenization in

27 See the tables on papyri deriving from them: Jones 1999, I:160–165 and II:118–150.

28 Toomer 1988.

29 Heilen 2011, 24 argues that Nechepsos and Petosiris F 6 Riess refers to wars between Greek states, and that therefore the oldest version of this astrological manual predates the defeat of Corinth in 146 BC.

30 There is no precise mention of a table, but Theon of Alexandria *In Ptol. Synt.* I 10 refers to Hipparchus' treatise on chords.

31 Jones 1990, 13 argues that at least the second time that Valens mentions Apollonius should be amended to Apollinarius, an astronomer of Ptolemy's age or slightly earlier, and that Apollonius is probably Apollonius of Myndus, an astronomer of the late 2nd/early 1st c. BC. Rochberg 2008b dismisses the connection Sudines-Attalus on the grounds that astronomers are rarely also entrail-

Mesopotamia, which no doubt fostered the transmission of Babylonian astronomical practices and ideas.[32] Of course, some form of Greek astronomy existed before that time, but most theories and methods necessarily involving precise calculations with tables seem to have started by then.[33]

A specimen from around 100 BC, the Keskintos astronomical inscription, may be indicative of the evolution from simple columnar list to astronomical table. The table is unruled (but this may be due to its epigraphic nature) and it provides period relations of different kinds for the planets, in a manner which is not particularly space-effective and which is, as is often the case in unruled columnar lists, reminiscent of verbal accounts (I reproduce Jones' translation of the first well-preserved entries):[34]

Mars	in longitude	zodiacals	15,492	Mars	in longitude	zodiacals	154,920
Mars	in latitude	tropicals	15,436	Mars	in latitude	tropicals	154,360
Mars	in depth	revolutions	4,096x	Mars	in depth	revolutions	401,650
Mars	in rel. pos.	passages	13,648	Mars	in rel. pos.	passages	136,480
Jupiter	in longitude	zodiacals	2,450	Jupiter	in longitude	zodiacals	24,500

In fact, the whole structure consists of two tables side by side showing integer specific periods for each of the planets (four different kinds of periods for each) in 29,140 and 291,400 solar years respectively. Each table, comprising in the present state (if it were complete) $4 \times 5 = 20$ lines (for the five planets) and four columns, could be reduced to just 7 rows (one for each planet) and 5 columns (headed as: planets, zodiacal periods in longitude, tropical periods in latitude, revolutions in depth; passages in relative position).

A natural question is whether Greek astronomical tables were a Babylonian import. I will argue that this is a fair possibility. Babylonian astronomy had many coexisting branches, comprising texts both in the form of prosaic account and in fully tabular layout.[35] These are extremely formalized types of texts, the products of a well-established institutional setting.[36] In the tabular format, a standard set of parameters is often given in columns against different times represented by the rows.

diviners, which leaves us with the evidence of mentions of Sudines as an astronomer by Pliny and possibly Posidonius (late 2nd/early 1st BC). Rochberg 2008a also tentatively places Kidenas in the period 150–50 BC.

32 See Tolsa 2024, 3.

33 See, for example, Evans 1998, 75–83 for the practices of sphere-making since classical times.

34 Jones 2006, 13.

35 Steele 2017, 11 identifies observational astronomy, goal-year astronomy, mathematical astronomy, and schematic astronomy.

36 For some evidence of the context related the Esagila temple of Babylon, see Rochberg 1998, 6 and 80.

For example, planetary ephemerides list in two columns synodic phenomena of the planets (stations, conjunctions, and so on) giving the dates in the first column and the zodiacal position in the second one.[37] The Greek counterparts, representing a significant subset of Greek astronomical tables, are the so-called planetary epoch tables. These give longitudes and dates of planetary synodic phenomena in practically the same format as the Babylonian tablets, in addition to using the same mathematical procedures (before Ptolemaic tables) – zone and zig-zag functions.[38] In other cases Greek tables differ widely from their Babylonian predecessors.[39] In the end, probably the best way to put it is that both arrived together: astronomical theory frequently in the form of astronomical tables.[40]

1.4.1 Explanatory Tables in Ptolemy's *Harmonics* and Porphyry's Commentary

The tables in this section are also ruled and read from up to down, but they have a special feature in the matter of content which we have not seen so far: they offer in a tabular presentation numerical information which has been already provided in prose. This use, which seems natural to us, is in no way typical of ancient texts, and in fact all the tables of this kind which I have been able to identify are related to Ptolemy's *Harmonics.* The first table that we encounter in Porphyry's commentary on this work (136 Düring) is introduced with these words, when commenting on Ptolemy's chapter I 12:[41]

---

37 Steele 2008, 58–59.

38 The corresponding papyri are only from the second century AD onward, but this is an accident of transmission: see Jones 1999, 121–150.

39 An illustrative case is that of the Babylonian system of rising times, which was also mainly set up in calendrical entries, using the so-called schematic calendar of 360 days, featuring fixed dates for solstices and equinoxes: cf. Steele 2017, 3–5 and chs. 2 and 3. In this case, the Greek counterparts were very different, even if the underlying theory was derived from Babylonian practice. Whereas Babylonian texts display the culmination of certain stars at sunrise/sunset and use the schematic calendar for the entries, Greek tables simply provide the time for every sign (or part thereof) to ascend from the horizon.

40 Sidoli 2004, 33 also suggests that tables (referring to astronomical tables) had their origin in Babylonian astronomy.

41 The table can be found, for example, in Vat. gr. 187 f. 154r, where it also carries extra information taken from chapter I 15 (in Ptolemy and Porphyry): I assume with Düring and Barker (Düring 136, Barker 412) that the pitch-values for the notes were not originally in Porphyry's table, since they are not discussed in the present chapter, and because from Porphyry's expression it seems that he is just willing to present "clearly" information that has already been given.

τοῦ δὲ σαφοῦς ἕνεκα καὶ τοὺς ἀριθμοὺς ὑπέταξα τῶν ἓξ τετραχόρδων ἔχοντας οὕτως.

For the sake of clarity, I have set down the numbers of the six tetrachords, which are the following.

And this is the table in translation: it is a ruled table, with column headings.

enharmonic	soft chromatic	tense chromatic	soft diatonic	tonic diatonic	tense diatonic
over[43] 4	over 6	over 6	over 7	over 8	over 9
over 23	over 14	over 11	over 9	over 7	over 8
over 45	over 27	over 21	over 20	over 27	over 15

[a]ἐπὶ δ in the original, which can be translated into modern notation as 5:4. All these ratios are of the kind called superparticular, that is, *(n + 1):n*.

Porphyry gives the ratios that are proposed later by Ptolemy for the six different musical genera (at I 15), which he has given just before in prose format. Then, still in the discussion of I 12, he writes another table for the ratios of Aristoxenus,[42] and in the comments to I 13 he provides another one for the ratios of Archytas. These two tables are identical in content (but not in shape) with the ones found in Ptolemy's manuscripts in the corresponding chapters, which were probably copied (together with their introductory sentences) into Ptolemy's manuscripts from Porphyry's.[43]

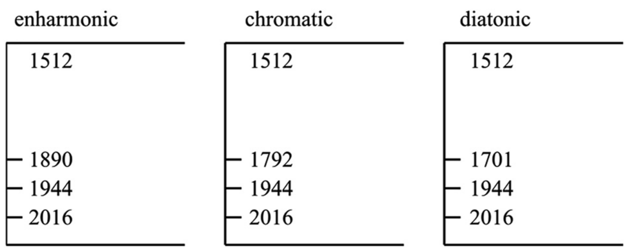
As for the tables in Ptolemy *Harmonics* I 15 containing his own proposed ratios, they are again probably not original by Ptolemy, since they are of the same explanatory type as Porphyry's, which is never used elsewhere in Ptolemy's

42 The diagram (extant for example in Vat. gr. 187, f. 155) is wrongly said to be absent in Porphyry's manuscripts in Barker 2015, 417 n. 630, probably due to Düring's omission in his edition.

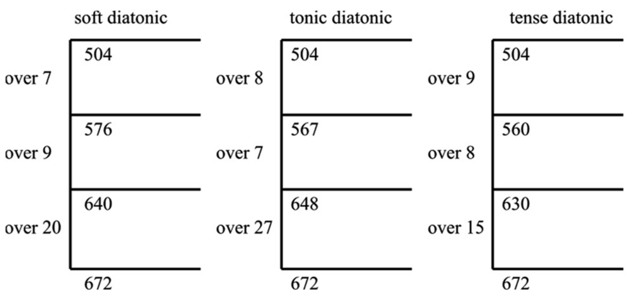
43 The reason why Porphyry wrote this kind of tables is probably that he was influenced by the fact that he was commenting on a work that already contained tables of this kind, namely Ptolemy's *Harmonics*. The situation is similar with other kinds of visual explanatory devices. For example, Ammonius drew a diagram in his commentary on Aristotle's *De interpretatione* (93 Busse, referred to as διάγραμμα in 109) probably in imitation of Aristotle's mentioned columnar table in *De interpretatione* 22a (for which Ammonius uses the same word, 232). Other explanatory diagrams in the Greek manuscript tradition are difficult to date. For example, in the manuscripts of music theory there are numerous marginal diagrams in the shape of arches (denoting intervals), but they are not signalled by the text, and the manuscripts themselves are not earlier than the 13th century. Branch diagrams were introduced in Western scholarship in the 12th century from the Arabic tradition, where they had been used since the 9th century. The so-called Porphyrian trees appear first in a 9th-century manuscript of the *Etymologies* of Isidorus of Seville, but they were very probably first developed in relation with Porphyry's *Isagoge* in Boethius' translation, since the tree-like structure with the trunk and left and right branches is very appropriate for describing the steps of the *dihairesis* in two *differentiae* from a common *genus*. See Savage-Smith 2002, 122 on branch diagrams in relation with the so-called Alexandrian summaries.

works.[44] However, it seems that Porphyry could see them in his copy of Ptolemy's treatise, since he copies part of the introductory phrases in his *lemmata* (they were inserted in the manuscript tradition of the *Harmonics* sometime between Ptolemy and Porphyry). Inspection of the principal manuscripts shows that these tables show a consistently different appearance from the tables copied from Porphyry (in I 12 and 13), which is a further point supporting a different origin:

Ptolemy I 13 (e.g. Vat. gr. 191, f. 328v):



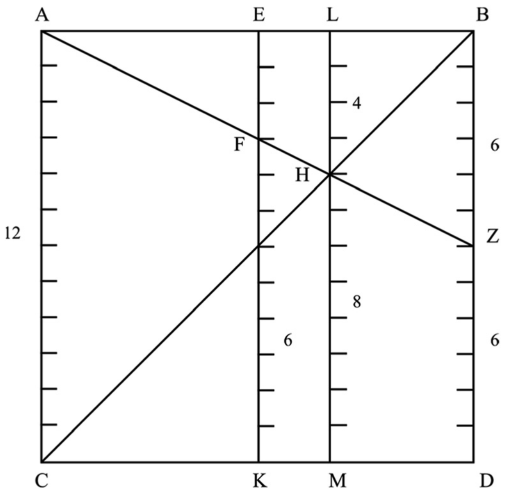
Ptolemy I 15 (e.g. Vat. gr. 191, f. 330v):



As can be seen in the drawings, neither set of tables had a complete grid.[45] The explanatory tables in Ptolemy I 12 and 13 resemble musical diagrams like the ones in chapter II 3 of the same work, depicting the instrument called *helikon*:

44 Another modification, perhaps occurred at the same time, is the chapter division of Ptolemy's *Harmonics* in 16 chapters each, which inevitably recalls Porphyry's arrangement of Plotinus' treatises into enneads. Porphyry himself justifies how he was pleased by the "perfection of the number six and the nines" for his edition of Plotinus: οὕτω δὴ καὶ ἐγὼ νδ ὄντα ἔχων τὰ τοῦ Πλωτίνου βιβλία διεῖλον μὲν εἰς ἓξ ἐννεάδας τῇ τελειότητι τοῦ ἓξ ἀριθμοῦ καὶ ταῖς ἐννεάσιν ἀσμένως ἐπιτυχών (*Vit. Plot.* 24). The number 16 is a square, like the number 9. Did Porphyry find inspiration in the chapter division he found in the *Harmonics*?

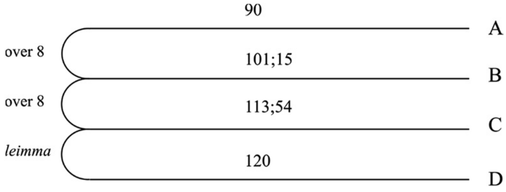
45 They are completely ruled as regular tables in the manuscripts of Porphyry's commentary, though. See Vat. gr. 187, f. 153r–155v, and Vat. gr. 198, f. 58v–59v.



(Vat. gr. 191, f. 334r)

In this instrument, the strings are represented by the vertical lines, which appear divided by short horizontal strokes to mark possible bridge positions dividing the string and thus producing different notes. Similarly, the short horizontal strokes in the tables of I 12 and 13 suggest different bridge positions along a string represented by a vertical line.

In turn, the tables of Ptolemy I 15 resemble the diagrams of *Harmonics* II 1:



(Vat. gr. 191, f. 333r)

Here it is the horizontal lines which represent different strings attuned to different pitches, also ordered in a musical scale. The resemblance is reinforced by the fact that in both diagrams we find the same kind of expressions (e.g. ἐπὶ η, “over 8”) between the horizontal lines on the side to denote the ratio of the corresponding interval. Ultimately, the implication may be that ruled tables were not at all an obvious element in treatises of music theory by the time when these ones were introduced, an issue which was tackled by disguising them in the shape of musical diagrams.

1.4.2 Terms for Astronomical Tables and Relation with Music Theory

Let us now deal with the terms related to astronomical tables: κανών, κανόνιον, and ὄργανον. The first one seems to be the most regular term, and the earliest one attested (Plutarch, *Soll. an.* 979C ἀστρολογικῶν κανόνων). In the mid-second century AD, Claudius Ptolemy used κανών and its diminutive κανόνιον, and Vettius Valens κανών and ὄργανον (but not κανόνιον); Pappus and Theon of Alexandria in their commentaries on the *Almagest* and the *Handy Tables* maintain Ptolemy's terminology, reserving ὄργανον for a physical astronomical instrument. Both Ptolemy and Valens have compounds for table-making, an activity which seems to have been established as a quasi-synonym of astronomy.[46] It is perhaps surprising that there is no other mention of astronomical tables in authors from before the fourth century AD: nothing in Geminus, in Theon of Smyrna, or in Cleomedes (to mention the most obvious candidates); nothing in Vitruvius, Cicero, or Pliny, although both Cleomedes and Pliny the Elder use the intriguing term κανονικοί (*canonici*) for astronomers (Cleomedes, *De motu* 222; Pliny *NH* II 73).[47] Probably the oldest attestation is the term κανόνιον applied to Critodemus' table in Valens VIII (see below), which can be traced back to the early fist century AD.

Since astronomical tables were used at least from the mid-second century BC, it is possible that they did not receive a special name before. Indeed, if they began to be called κανών around the turn of the first century BC/AD, this could plausibly be related to the popularity of the science of harmonics about that time. The term κανωνική practically came to replace ἁρμονική for the discipline of music theory, appearing as early as Geminus, Vitruvius (both 1st c. BC), or Philo of Alexandria (early 1st c. AD). The κανών was an instrument consisting of one string with a movable bridge placed at measurable distances, which was used by music theorists to ascertain and try the intervals built from arithmetical ratios (such as 2:1 for the octave). Although the claim by Ptolemaïs of Cyrene (late 1st c. BC or early 1st AD[48]) that the name of the discipline did not come from the instrument is very dubious,[49] she probably had a point in her argument that it came from the straightness of the

46 Ptolemy: κανονοποιία, κανονογραφία, κανόνος ψηφοφορία; Valens: κανονικαὶ συμπήξεις, κανονογραφία, κανονοποιία, ὀργανοποιία, (κανονικὴ) ὀργανοθεσία.

47 In the case of Pliny, the term is probably his own, and not derived from his regular sources for the astronomical content (mostly Varro), because here he expresses the astronomers' impossibility to explain a phenomenon (this is not Varro's style). Furthermore, Pliny uses a similar expression elsewhere in a different context: *immo vero, sed ratio canonicos fallit*/cf. XXXIII 126 [on metals]: *mirabili ratione non fallente.*

48 Barker 1989, 239.

49 Creese 2010, 76.

discipline.[50] The argument recalls those by other theorists explaining the name of the instrument. Thus, Panaitios claimed that the instrument κανών received its name from the fact that it is the κριτήριον of the discipline (Porph. 66.22–23). Of course, the term κανών had the meaning of a model or rule, as in Polycleitus' famous work. Ptolemy's double explanation is probably the best (*Harmonics* II 1):

> Τὸ μὲν οὖν ὄργανον τῆς τοιαύτης ἐφόδου καλεῖται κανὼν ἁρμονικός, ἀπὸ τῆς κοινῆς κατηγορίας καὶ τοῦ κανονίζειν τὰ ταῖς αἰσθήσεσιν ἐνδέοντα πρὸς τὴν ἀλήθειαν παρειλημμένος.
>
> The instrument of this method is called *harmonic canon*, borrowed from common usage and from straightening what is lacking in the senses toward the truth.

By common usage of κανών Ptolemy refers to the most obvious meaning of the word: the tool for measuring straightness in construction works, namely a mason's line.[51] The image of the tensed string on the κανών is too remarkably similar to that of the tensed mason's line to deny an origin of the naming in this worker's tool. Secondarily, the abstract notion of straightness and model must have played a role as well.

Now, how did astronomical tables come to be called κανόνες, κανόνια, and ὄργανα? It is unlikely that the notion came from tables of music theory, because, as we have seen, the term was in use before Ptolemy, and at the same time it is doubtful that authors of music theory before Ptolemy used tables of the type that Ptolemy used. Rather, the tables in the *Harmonics* which are originally Ptolemy's, those at the end of book II, seem to be influenced by Ptolemy's experience with astronomical tables, since he presents them as a project wholly of his own which supersedes difficulties in his predecessors' works (end of II 13). Porphyry does not mention tables by other authors in his commentary. Ptolemy's tables even feature the fractional parts of the bridge positions in sexagesimals, as in astronomy.

50 Porph. *In Ptol. Harm.* 22.27–30: οὐχ ὡς ἔνιοι νομίζουσι ἀπὸ τοῦ κανόνος ὀργάνου παρονομασθεῖσαν, ἀλλ' ἀπὸ τῆς εὐθύτητος ὡς διὰ ταύτης τῆς πραγματείας τὸ ὀρθὸν τοῦ λόγου εὑρόντος καὶ τὰ τοῦ ἡρμοσμένου παραπήγματα.

51 I think this is a more likely inspiration than the measuring rod, another possible meaning of κανών. It is true that the κανών as Ptolemy describes it had a small ruler attached to it (I 8: ἐφαρμόσαντες δὴ τῇ χορδῇ κανόνιον) to measure the bridge-positions, but it is the notion of straightness that prevails in all ancient literature, not that of measuring. See e.g. the uses in Euripides in which it is not a norm or rule, *Herc.* 814 & 945 and *Troades* 6. Besides, it seems likely that the abstract meaning "rule" also came from the construction tool and not the other way around: an interesting parallel is the use of παράπηγμα to denote a general rule: cf. Philo Alex. *Sacr. Abel et Cain.* 60 κανόνων καὶ παραπηγμάτων, and *Spec. leg.* III 164 κανόνες καὶ ὅροι. A παράπηγμα was a meteorological calendar (in which pegs were moved from day to day). Creese 2010, 76 wrongly asserts that it was a term for table because of its equation with κανών in *Suda* pi 410, but no author uses the term for a table (the text in the *Suda* entry must mean "rule").

However, it is plausible, and even likely, that the name of the instrument κανών and its significance for the self-presentation of the discipline of harmonics influenced the naming of astronomical tables. Indeed, in astronomy we witness a similar naming scheme as in harmonics: from κανών the mason's instrument to κανών an instrument of the discipline (hence too Valens' term ὄργανον, instrument), which because of its centrality in the definition of the science – tables established the models of the heavenly motions in the same way as the harmonic κανών established the right intervals – comes to designate the practitioners themselves (κανονικοί). The use of κανών as a metaphor is not entirely exclusive of these two sciences: as is well known, already Epicurus had used it for the foundations of his epistemology. However, in no other science the metaphor came to be equivalent to the whole discipline; furthermore, astronomy and music theory were conceived as sister sciences since as early as Archytas (fr. 1 Huffmann), a kinship which Plato had powerfully divulged (*Rep.* 530d) and which was very present in the intellectual atmosphere of late Hellenistic times, deeply influenced by Middle Platonism. Such perceived closeness between the two disciplines would have fostered the appropriation of the metaphor from music theory.

In astronomical tables, the grid could easily be perceived as an orthogonal pattern of mason's lines. More specifically, mason's lines were crucial to define vertical directions, and astronomical tables had a pronounced verticality. This does not mean that the metaphor did not work for the horizontal lines,[52] only that it more clearly signaled the vertical direction, making the term a very appropriate metaphor for astronomical tables.[53] In this regard, it is tempting to hypothesize that the medieval Arabic term for a set of astronomical tables, *zīj* (a Middle Persian borrowing), was in fact a very informed translation of the Greek, since another attested meaning of *zīj* is a mason's line.[54]

2 Symmetric Tables

The tables that I present in this section are *markedly* symmetric, that is, they are built and described in such a way in the texts in which they are inserted that their properties of

52 See for example the texts in the Moschion stele, where the term κανόνες refers to all the lines of the grid: Mairs 2017, 237 (text B).

53 In the Oxyrhynchus astronomical tables whose astronomical content is datable we see that, with time, fewer horizontal lines were used. This was probably the result of increasing familiarity with the layout and of the prominence of verticality. In the tables from before the 2nd c. AD, 4 out of 5 have horizontal rulings for every row; in the 2nd c., only 1 out of 10, while 6 have rulings every two rows. In the tables from the 3rd c., only 1 out of 16 has rulings for every row, 5 have them every two rows, and 5 have no horizontal rulings at all. And out of them all, only one has no vertical rulings.

54 Cf. a standard account in Kennedy 1956, 123: "[from a mason's line] the word came to stand for the set of parallel threads making up the warp of a fabric," but this latter meaning has, to my knowledge, not been attested.

symmetry, mostly based on numerical principles, are made apparent. That this works as a category for the ancients is confirmed by the fact that such tables are consistently called a different name (πλινθίον, πλινθίς). The table described by Varro (*LL* X 43–44) discussed by Riggsby (cf. beginning of Section 1 above) would be an example of this kind:[55]

1	2	4
10	20	40
100	200	400

albus	albo	albi
alba	albae	albae
album	albo	albi

Varro draws a parallel between a table of nominal declension and a table of multiples, remarking on the fact that the same two directions that can be found in the table of numbers can be found in the table of declension: thus, horizontally we go from the nominative to the other cases, while vertically we remain in the same case (*similiter in verborum declinationibus est bivium, quod et ab recto casu declinantur in obliquos et ab recto casu in rectum*), just as in the table of numbers horizontally we go from the base to the multiples and vertically we 'see' the same base. This double direction suggests an absence of hierarchy: the table could be flipped diagonally, and the symmetry is reinforced by the fact that Varro does not pick the rest of cases, but just three to make the table square.

We will find similar claims all along our survey of the Greek evidence. We should say from the outset that it is not at all surprising that in the Latin evidence it is precisely Varro who draws this kind of table, because such tables were mostly used by Pythagoreanizing authors like him.

2.1 An Arithmetical Table in a Funerary Stele

A table of multiples 10 × 10 engraved on the funerary stele of a teacher (Gen. MAH inv. 27937) may well be the first attested table of this kind. It forms the atrezzo of a learning room along with a cithara hanging on the wall, the teacher, and a small-scaled boy.[56] The inscription under the relief identifies the deceased as Πτολεμαῖος ὁ γεωμέτρης. The piece is unprovenanced, and it has tentatively been dated to the 3rd century BC, but a more realistic estimate would be between 200 and 50 BC.[57] In this case we have a full-fledged table with rulings:

55 The tables are not drawn in the manuscript.

56 The papyrus P.Gen. inv. 432 also contains part of a 10 × 10 table exactly like this one (only the last two columns are preserved in this case). Both the stele and this papyrus are mentioned in Sesiano 2021, 87 and classified as multiplication tables.

57 The date in the 3rd c. BC is suggested in Chamay and Schärlig 1998. My proposal for a later dating is backed by the epigraphist Paul Iversen (personal communication). Firstly, the letter shapes of the inscription, which appear at the beginning of the Hellenistic age, are still in use toward its end. See

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Since multiplication tables adopt a different form (see above), I believe that we should consider this specimen a table of multiples. We will see below that Nicomachus presents this same table as a table of multiples. Of course, as a table of multiples it would also serve the purposes of multiplication. It is easy to perceive that the medium here underscores and reflects the square grid. Indeed, the stele is designed as a sort of *mise-en-abîme*: the whole scene is set in a deeply incised square, and the square table is balanced by another square formed by the teacher's seat at the other extreme. If the multiplication tables that we commonly find on papyrus reflect the oral, repetitive learning of the operations – in the same way as school kids learn today, the multiplicand is repeated in each row – the inscriptional milieu privileges the synoptic presentation and a symmetric and compact presentation.

2.2 Nicomachus of Gerasa and Theon of Smyrna (Late First Century AD)

These authors do not use the words πλινθίς/πλινθίον, but rather the more generic term διάγραμμα.[58] Nicomachus sets up four tables in his *Introduction to Arithmetic*,

Guarducci 1987, 81–82 for the "apicature" embellishing the letters from the mid 3rd century BC. Secondly, practices of self-representation in funerary art are more individualized in mid- to late Hellenismus, especially in Asia Minor: see Ridgway 2000, 190. See images of the funerary stele at https://www.mahmah.ch/collection/oeuvres/stele-funeraire/027937 (last accessed 21/09/2024).

58 Other than the tables, Nicomachus also calls διάγραμμα a musical scale he sets up in *Enchiridion* 12, as Theon Smyrna in his *De utilitate* (64); Theon also calls by this name a diagram in the form of the while Theon just one in his only surviving work, *On useful things for understanding the mathematics of Plato*. I will briefly describe them, together with Nicomachus' and Theon's remarks.

(a) Nicomachus I 19. A 10 × 10 table presented as a sequence of series of multiples, exactly like the table on the Geneva stele (Section 2.1 above). Nicomachus signals several properties of the table: in the first place, the second row minus the first gives 1 2 3 …, the third minus the first gives 2 4 6 …; etc. Secondly, the relation between the elements of the third and the second column give the so-called sesquialter ratios, which are crucial in music theory (3:2, 6:4, 9:6 …); and those of the fourth to those of the third give the sesquitertian ratios (4:3, 8:6, 12:9 …). Then, the corner cells of the table are all units, and in particular 10 times 10 equals 100; finally, the diagonal starting from 1 is formed by squares.

(b) Nicomachus I 10. A 7 × 7 table of odd-times even numbers obtained through multiplication of the numbers of the series of odd numbers from 3 (3, 5, 7 …) by the numbers of the even-times even series from 4 (4, 8, 16, 32 …). In relation to this table, Nicomachus presents as idiosyncratic of odd-times even numbers (that is, numbers obtained through a product of an even and an odd number) characteristics which are in fact properties of arithmetic and geometric progressions. Indeed, the rows are geometric progressions, and the columns are arithmetic; the observations are related to middle elements and their relationship with the extremes.

(c) Nicomachus II 3. Two 7 × 7 triangular tables (upper right half with the main diagonal) containing sequences of doubles and triples in the rows, respectively. The first numbers of the first rows in the first table are the following (the subsequent rows are built so that the diagonal is formed by the triples):

1	2	4	8
	3	6	12
		9	18
			27

Nicomachus mentions the fact that in this table the ratios formed by subsequent numbers in vertical are always sesquialter (3:2 and equivalent). A similar

letter Λ (*De ut.* 93), and a regular geometric lettered diagram (181). Boethius in his Latin translation of Nicomachus' *Intr. arithm.* uses the terms *descriptio* (probably translating διάγραμμα) and *dispositio* (ἔκθεσις, "layout") for tables; cf. Nicom. *Intr. arithm.* II 12.4 ἔκθεσις for the layout of table (d), a common word also in Ptolemy in the context of tables.

property with the ratio 4:3 is found in the second table, which presents a diagonal formed by quadruples.

(d) Nicomachus II 12. A table containing the number of dots (or pebbles) that form polygonal numbers, ordered in rows by type of polygon (triangles, squares, pentagons, etc.). These figures are extensions of the classical *tetraktys* image of an equilateral triangle formed by four rows of 1, 2, 3, and 4 dots respectively. Then the first row (triangles) is 1, 3, 6, 10, etc. (that is, the number of dots in each triangle, in increasing order). Nicomachus notes a couple of curious relations across different rows of the table.

(e) Theon 101 Hiller. Theon constructs a 3 × 3 square containing the first nine natural numbers in order, by columns:

1	4	7
2	5	8
3	6	9

He notes that in all straight lines through the center (the second row, the second column, and the two diagonals), 5 is the arithmetic mean of the extremes.[59]

In all five tables, most of them square, we see that both authors remark on relatively surprising characteristics. Nicomachus even employs the term θαυμαστῶς ("amazing"). It is clear that Nicomachus and Theon attempt to seduce their readers with curious relationships between numbers found in many possible directions, mainly horizontally and vertically, but also diagonally. It is significant, for example, that Nicomachus speaks of στίχοι (usually "rows") for both rows and columns.[60]

2.3 Dispute-resolving πλινθίς in a Pseudo-Pythagorean Letter and Related Tables

Several manuscripts[61] contain this table introduced by a letter allegedly penned by Pythagoras, addressed to his son Telauges and providing instructions for using it in the resolution of disputes between two individuals, beginning in this way:

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59 See Vinel 2005 for an analysis of the presentation of this square here and in a reference by Iamblichus, together with a speculation about whether there are hints of an otherwise unattested tradition of Greek magic squares.

60 In relation with table (a), in I 19: οὐκοῦν τῶν μὲν πρώτων στίχων ἀρχομένων ἀπὸ μονάδος ἐπί τε πλάτος καὶ ἐπὶ βάθος γαμμοειδῶς.

61 E.g. BNF gr. 2009, f. 2v; BNF gr. 2256, f. 593v; BNE 4616, f. 82v, etc. Text edited in Zuretti 1934 [*CCAG* 11.2], 139–140.

Πυθαγόρας Τηλαύγῃ χαίρειν
Πολλὰ παθὼν καὶ πολλὰ πειράσας ἐπέσταλκά σοι τόδε βιβλίον ἔχον ἐν ἑαυτῷ πλινθίδα πάνυ χαριεστάτην· ἐντυχὼν γὰρ εἰς αὐτὴν διὰ τῶν ὑποκειμένων γραμμάτων εἴσεται τά τε ἐνεστῶτα καὶ τὰ προγεγονότα καὶ τὰ αὖθις ἐσόμενα. ὑπέταξα οὖν πλινθίδα ἐννεάδος δοκιμαζομένην τρόπῳ τοιῷδε . . .

Pythagoras to Telauges,
After much toil and effort, I have sent you this book containing a most graceful table. For when you get hold of it you will have access to the present, past and future through its lines. I have placed below the table of the ennead, to be tested in the following way . . .

	W	W			W	W			W	W			W	W		acc'r	
9	1	9	2	9	3	9	4	9	5	9	6	9	7	9	8	9	9
W			W	W			W	W			W	W		acc'd			W
8	1	8	2	8	3	8	4	8	5	8	6	8	7	8	8	8	9
	W	W			W	W			W	W		acc'r			W	W	
7	1	7	2	7	3	7	4	7	5	7	6	7	7	7	8	7	9
W			W	W			W	W		acc'd			W	W			W
6	1	6	2	6	3	6	4	6	5	6	6	6	7	6	8	6	9
	W	W			W	W		acc'r			W	W			W	W	
5	1	5	2	5	3	5	4	5	5	5	6	5	7	5	8	5	9
W			W	W		acc'r			W	W			W	W			W
4	1	4	2	4	3	4	4	4	5	4	6	4	7	4	8	4	9
	W	W		acc'd			W	W			W	W			W	W	
3	1	3	2	3	3	3	4	3	5	3	6	3	7	3	8	3	9
W		acc'r			W	W			W	W			W	W			W
2	1	2	2	2	3	2	4	2	5	2	6	2	7	2	8	2	9
acc'd			W	W			W	W			W	W			W	W	
1	1	1	2	1	3	1	4	1	5	1	6	1	7	1	8	1	9

W, "wins"; acc'r, "the accuser wins"; acc'd, "the accused wins"

In the instructions, the text tells the reader to reduce the name of the contestants to numbers from 1 to 9, by adding up the numbers corresponding to the letters of their names in a modified alphanumerical system (α, ι, ρ = 1; β, κ, σ = 2, etc.) and subtracting so many sets of nine (enneads) as possible. Then we look for the cell in the table featuring the numbers of the two contestants and whoever has the N above wins (νικᾷ) (W in our version). In case of equality, the text alternately says the accuser or the accused wins.

As we can see, the table is a square displaying all the possibilities (9 × 9 = 81). It is also evident that the N's are perfectly balanced, as if in a chess board, except in the diagonal which represents a sort of parenthesis in which the result is obtained in the mentioned fashion (but even the diagonal itself is balanced).

The date of this document is unknown: we only have the *terminus ante quem* of Hippolytus (around 200 AD), who explains the numerological procedure in his *Refutatio* (IV 14); it has been tentatively dated as late Hellenistic because Pythagorean pseudepigraphs are generally thought to have been composed around that time.[62]

2.3.1 Alphanumerical Number System in Byzantine Anonymous Arithmetical Treatise

Toward the end of this treatise preserved in a Vatican manuscript (Barb. gr. 4) we find a table displaying the regular Greek alphanumerical system, in which the first column, entitled μονάδες, contains letters α to θ; the second (δεκάδες) ι to ϙ, and so on, up to a ninth column (μυριοντάκις μυριάδος i.e. 100.000.000's) α″ to θ″. The column headings are oddly placed at the bottom, probably because the numbers run from down to top. The table itself appears under the title πλινθὶς ἀρίστη τῆς τῶν ἀριθμητικῶν στοιχείων καταγραφῆς (“Magnificent table regarding the layout of the arithmetical elements”), and the text describes its columns in the following way:

> εἰσὶ δὴ τῶν ἀριθμῶν τάξεις ἐννέα, ἐκ τῆς ὑπερκοσμίου καὶ νοερᾶς ἐννάδος τὴν μίμησιν ἔχουσαι
>
> There are nine categories of numbers, imitating the supramundane and noetic ennead.

Because of its similar emphasis on the concept of the ennead, the same 9 × 9 format, and the same labelling (πλινθίς), we can deduce that the Pythagorean dispute-resolving table influenced the creation of this one.[63]

2.3.2 Lunar Days in Pseudepigraphic Letter from Nechepsos to Petosiris

This is a curious specimen also related to the Pythagorean dispute-resolving table. It is similarly found in a pseudepigraphic letter, now from Nechepsos to Petosiris, a well-known pseudonymous pair of Graeco-Egyptian astrological authors. In this case, we do not have a ruled table with an equal number of columns and rows, but a composite structure of four columnar lists reproducing the general square

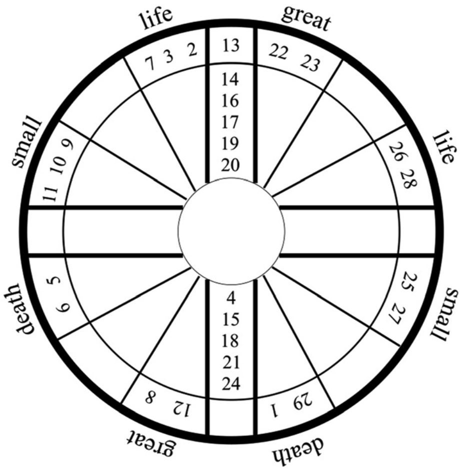
62 Neugebauer and Saliba 1989, 89. The article also explores the material related to the tables (Section 2.3.2 below).

63 For an edition of this treatise and an examination of its codicological context, see Acerbi et al. 2018.

shape that is typical of symmetric tables (in 2 × 2 blocks). The two upper blocs run from the first day of the month to the fifteenth, and the two lower blocks from the 16th to the 30th, in both cases oddly jumping from the left columnar list to the right. Again, this is a numerological procedure, now for discovering the outcome of an illness in a single individual. Basically, one looks up the number corresponding to the day of the month – obtained from the alphanumerical sum of the letters of the day, e.g. πρώτη = 80 + 100+800 + 300+8 = 1,288 – then adds the number of the personal name as in the Pythagorean table, then reduces *modulo* 30, and finally finds the resulting number in the so-called circle of Petosiris. The latter is another diagram presenting four quadrants divided by a cross, with the lower part labelled as "death" and the upper as "life," with a further subdivision in "great" and "small" for both categories, and the 30 numbers of the days of the month distributed in an apparently irregular fashion in the diagram. Depending on the quadrant or the division where your number falls you will be better or worse off. I reproduce first the table of lunar days and then the circle of Petosiris.

Moon the first 1288[64]			
second	815	third	718
fourth	1014	fifth	518
sixth	333	seventh	129
eighth	155	ninth	364
tenth	338	eleventh	393
twelfth	1142	thirteenth	979
fourteenth	1380	fifteenth	809
+To these days add first 138			
sixteenth	434	seventeenth	755
eighteenth	1559	nineteenth	480
twentieth	613	twenty-first	1901
twenty-second	1428	twenty-third	1331
twenty-fourth	1627	twenty-fifth	1126
twenty-sixth	946	twenty-seventh	742
twenty-eighth	748	twenty-ninth	977
thirtieth	1009		

64 See below for an explanation of the numbers. The bold initial and the + symbol reproduce the appearance of the manuscript. The ordinals represent the days of the month. In the second heading, the number 138 corresponds to the word κοίλη, referring to the waning Moon: see Neugebauer and Saliba 1989, 200. See *ibid.*, 190 for a list of manuscripts. I reproduce the text and formatting of Vat. gr. 952, f. 186r.



(Vat. gr. 952, f. 176r[65])

The four parts of the table of lunar days are called πλινθίον in the text that precedes the diagram. What seems to be at stake here is the influence of the label πλινθίς for the Pythagorean table, plus the influence of the quadrants (πλινθία) of the circle of Petosiris – πλινθίον was also used for a celestial square (see Section 2.5 below).[66] It looks as if the days of the month had been distributed using a similar tetradic pattern as in this circle, and at the same time in a formatting vaguely resembling a symmetric table. Furthermore, it is worth noting that the circle of Petosiris – which is variously called ὄργανον, ὄργανον κανονικόν, and κανόνιον in the manuscripts[67] – shows remarkable numerical symmetries:

- The vertical rectangles above and below the central circle add up 86 and 88 respectively.
- Two diagonals add up the same quantity: (22 + 23) + (12 + 8) = 65 = (26 + 28) + (6 + 5).
- The other two diagonals add up 82 and 42, but their arithmetical mean is 62, very close to the 65 of the other diagonals.

65 I have only drawn the numbers and written the information discussed in the text: there are other labels in the circle such as the four traditional elements.
66 For the dating of the circle of Petosiris the prominent position of number 13 might be relevant, since it is probably a Christian influence. Unlike the Pythagorean table for disputes, the lunar numbers and the Petosiris circle are not described by Hippolytus, so they may well be of a later date.
67 Specifically, ὄργανον κανονικόν Vat. gr. 952, f. 175v; ὄργανον BNF 4246, f. 16r; κανόνιον BNF Suppl. gr. 446, f. 44r.

2.4 Tables for the Length of Life by Critodemus (in Valens VIII)

In book VIII of his *Anthologies*, the astrologer Vettius Valens (second century AD) deals with and reproduces two tables that come from the work of Critodemus (probably end of second/early first century BC) but which were probably appended to Critodemus' work at a later stage, probably in the early first century AD.[68] Both tables are enigmatically called κανόνιον καὶ πλινθίον. Indeed, on the one hand they shows the typical structure of an astronomical table, with columns representing the zodiacal signs and the rows corresponding to the 30 degrees of each zodiacal sign. This is, for example, the same structure as Ptolemy's tables of rising times in the *Handy Tables*. On the other hand, Critodemus' tables are structured in vertical sets of numbers (not explicitly but by construction) arranged in a symmetric square (the second table is a variation of the first). They are used to obtain the length of life entering the table in the degree corresponding to the ascendant degree of the concerned person, then multiplying a certain quantity (dependent on the ascendant degree) by the number corresponding to the relevant sign-column.

The numbers in the sign-column can be grouped vertically in sets of 6 numbers, so that in one column there are only 5 different sets (there are 30 degree-rows) and the sets are never repeated in one column or in one row, taking the first 5 signs (then the structure repeats itself). This type of balanced arrangement is called a Latin square (Euler's denomination). If we designate the five consecutive sets of the first column by A B C D E, then we have:[69]

Lib	Sco	Sag	Cap	Aqu
A	D	B	E	C
B	E	C	A	D
C	A	D	B	E
D	B	E	C	A
E	C	A	D	B

68 For a complete analysis of these tables, see Tolsa 2024, ch. 8. See also King 1989.
69 In the first table, the numbers are even integers from 2 to 30. Specifically, A = (2 4 6 8 10 12) and B = (26 28 30 2 4 6), etc. To form the next set of 6, the first three elements of the preceding set are repeated as the second half of the next one, and then the first three elements are completed counting backwards with the convention 0 = 30. See both tables in their complete form in Tolsa 2024, 178–185 (the second one is a variation of the first).

2.5 On the Terms πλινθίς and πλινθίον

Now, what are symmetric tables (πλινθία and πλινθίδες)? They are ruled tables mostly displaying simple, integer numbers that maintain some kind of symmetry or balance. They are frequently square, and they rather belong to the genres of arithmology and recreative arithmetic than to science. Some instances recall the famous magic squares that were transmitted from China to the Islamic world via India, but there is no evidence that anything exactly like them (where numbers in all rows, columns, and main diagonals add up the same value) was developed or adopted in ancient Greek culture.[70]

Where do the terms for them come from? Unlike κανόνες and κανόνια, πλινθία and πλινθίδες could simply refer to grids without contents. Our first datable evidence of the term is the Jewish exegete Philo (early 1st c. AD), who refers to a diagram which he calls πλινθίον in the following way (*De opificio mundi* 107):

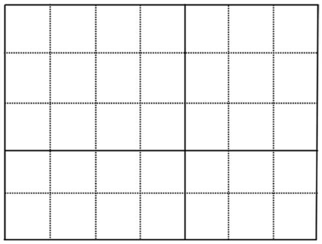
> Ἔστι δὲ οὐ τελεσφόρος μόνον, ἀλλὰ καὶ ὡς ἔπος εἰπεῖν ἁρμονικωτάτη καὶ τρόπον τινὰ πηγὴ τοῦ καλλίστου διαγράμματος, ὃ πάσας μὲν τὰς ἁρμονίας, τὴν διὰ τεττάρων, τὴν διὰ πέντε, τὴν διὰ πασῶν, πάσας δὲ τὰς ἀναλογίας, τὴν ἀριθμητικήν, τὴν γεωμετρικήν, ἔτι δὲ τὴν ἁρμονικὴν περιέχει. τὸ δὲ πλινθίον συνέστηκεν ἐκ τῶνδε τῶν ἀριθμῶν, ἓξ ὀκτὼ ἐννέα δώδεκα· (…) τὸ διάγραμμα ἢ πλινθίον ἢ ὅ τι χρὴ καλεῖν . . .

> There is such a thing as the source of the most beautiful diagram which is not only perfect, but, so to speak, the most harmonic. This diagram comprises all harmonies: the fourth, the fifth, the octave; and all means: the arithmetic, the geometric, and also the harmonic. The table (πλινθίον) consists of these numbers: six eight nine twelve. (…) The diagram or table (πλινθίον) or whatever we should call it . . .

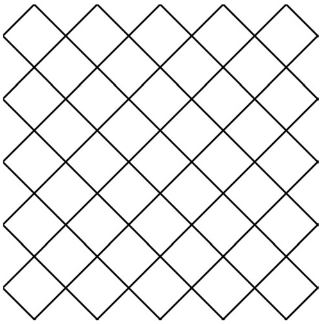
The four numbers apparently contained in this grid (περιέχει) were used to display the different ratios defining the basic concords of ancient Greek music and the three basic means. They are found, for example, in the diagram of the *helikon* in Ptolemy's *Harmonics* (cf. Section 1.4.2 above). No doubt Philo referred to the same grid that Plutarch mentions in *De anima procreatione in Timaeo* 2018A, and which was drawn by Aristides Quintilianus in his treatise on music theory (III 4) – the four numbers are found in the sum of small squares in each of the four rectangle-divisions:[71]

70 See Tolsa 2021 for the transmission of magic squares from India to Islam; Vinel 2005 examined a balanced table in Theon (see above in Section 2.3) as a possible instance of a Greek magic square; more recently Sesiano 2020 wrongly claimed that an Arabic treatise on magic squares was a translation from the Greek: see the review by Oaks 2022, 102.

71 Cf. the footnotes to the translation of Aristides Quintilianus in Barker 1989, 500.



The lexicon by the grammarian Julius Pollux (2nd/3rd c. AD) furnishes another instance, calling πλινθίον the grid on which a game similar to checkers (πόλις) is played.[72] The terms πλινθία and πλινθίδες are diminutives of the word πλίνθος, “brick”. Did their meaning as grids derive from the fact that the grid resembles a brick construction? This is very likely, especially if we think of *opus reticulatum*, in which bricks are square-shaped and disposed in an orthogonal grid, in contrast with the other types of masonry techniques where the coincidence of vertical and horizontal joints must be avoided.



Structure of *opus reticulatum*

This style of brick construction was in vogue between the first century BC/AD in central and southern Italy. This does not mean that the symmetric table was born there and by that time, but it may suggest that in that chronological and geographical context it was more fashionable than elsewhere or at other times. We have seen that the dispute-resolving Pythagorean table was called πλινθίς (Section 2.3 above). In general, the date of the texts attributed to Pythagoras can only be approximately established between the Hellenistic and the early Imperial age, and

72 ἡ δὲ διὰ πολλῶν ψήφων παιδιὰ πλινθίον ἐστί, χώρας ἐν γραμμαῖς ἔχον διακειμένας· καὶ τὸ μὲν πλινθίον καλεῖται πόλις, τῶν δὲ ψήφων ἑκάστη κύων (“The game of many counters is a grid [πλινθίον], which has spaces lying between the lines. And the grid is called polis, and each of the counters a dog”). See Austin 1940, 261–266. The game was probably related to Roman *ludus latrunculorum*: cf. Austin 1934, 26.

their provenance is similarly difficult to track down, but Rome is a fair possibility.[73] As we have seen, Nicomachus, who was probably connected to the Roman ruling class, and who contextualized his arithmetical work in a Platonizing-Pythagorean setting, used this type of tables, too.[74]

Vitruvius also knew the term πλινθίον (IX 8.1), since he applied it to a type of sundial displaying the heavens as a coordinate grid.[75] The question is not straightforward here, since we know that πλινθίον also designated a square in the heavens. It was used in this sense several times by Hipparchus in his commentary on Aratus, for single squares defined by constellations (I 5.2, 5.4, 5.5, 5.6, etc.). Ptolemy takes up this terminology in his catalogue of stars in *Almagest* VII 5 (2.38 Heiberg), and Hippolytus (early 3rd century AD) used it for the four quadrants of the heavens in *Refutatio* IV 44.[76] In the pseudoepigraphic letter attributed to the Egyptian astrologer-king Nechepsos (Section 2.3.2 above), it is clear that the author, probably from Egypt, was not thoroughly familiar with the concept as a symmetric table and mixed it with the tradition of πλινθίον as a heavenly quadrant. Therefore, there seem to have existed two different naming traditions, colliding or coinciding in some instances, both derived from the idea of brick: an earlier one emphasizing the idea of spherical rectangle, in the fields of mechanics and astronomy (starting as early as the third century BC); and a later one, perhaps originating in Italy (from

73 A good case can be made for Alexandria in the case of some of the philosophical pseudo-Pythagorean writings: cf. the studies by Bonazzi 2007 for the ones attributed to Archytas, and Varoli 2021 for the Timaeus Locri, both of which underline the similarities with doctrines of the middle Platonist Eudorus of Alexandria.

74 Nicomachus dedicated his *Manual of harmonics* to a noble lady who is addressed (without name) as ἀρίστη καὶ σεμνοτάτη γυναικῶν (1.1), an epiclesis probably adequate only for the Roman elite. See the hypothesis that she could have been Plotina, the wife of emperor Hadrian, based on her known interests, in McDermott 1977. Quick dissemination of Nicomachus' work in the West, probably owing to his Roman connections, may have been a factor in the relatively early translation into Latin of his *Introduction into arithmetic* by Apuleius in the 2nd century AD (Cassiodorus *Inst*. II 7).

75 Vitruvius mentions a sundial which he defines as *plinthium sive lacunar*, "which is also placed in the Circus Flaminius" (*quod etiam in circo Flaminio est positum*), allegedly invented by a certain Scopinas of Syracuse. The Latin term *lacunar* is used for panel-ceilings like the one in the Roman Pantheon, so there is little doubt that this type must be the one incised with squares, in which not only the hours of the day are measured, but also the declination of the sun, revealing the season of the year.

76 πλινθίον is also the front rectangle of the catapult (since as early as the 3rd c. BC, cf. Philo of Byzantium *passim*): cf. Thomas Magister <Πλινθίον> οὐδεὶς Ἀττικίζων εἶπεν, ἀλλὰ πλαίσιον (πλαισίον = a rectangular frame). It is also a rectangle of land, as in numerous papyri of the Ptolemaic and imperial age up to the 3rd century AD: BGU 14.2385 (3rd c. BC, Oxyrhynchites?), BGU 10.1925 (2nd c. BC, Thebes), etc. A square of troops is also called a πλινθίον because of its grid structure (from the 1st c. AD onward). πλινθίς was also a geometric figure in the shape of a square brick and the corresponding arithmetical number formed as $a \times a \times b$ so that $a>b$ (the definition is used in Nic. *Intr. arithm.* II 17, Theon of Smyrna 41.18, and Hero *Def.* 113).

the first century BC onward), denoting an orthogonal grid. That the origin of both traditions in the vocabulary is in building and manufacturing should not be surprising, since this field was extremely productive in the creation of terms for mathematical objects and textual layouts: we have already seen the example of κανών, but there are numerous examples: to name just a few, τραπέζιον from the shape of a table (τράπεζα); γνώμων (an L-shaped area) from the carpenter's square; σελίς (column of text on papyrus) from σέλμα (deck of a ship and hence block of parallel rowing benches).

As for the symmetric tables themselves, their established status as a category of table can be contextualized together with better-known visual devices in Greek culture. One element is the *stoikhedon* epigraphical style, which was not in vogue anymore by the time these tables appeared, but which must have been in display in numerous surviving inscriptions.[77] The text of such inscriptions was disposed in a rectangular grid of small square cells each containing one letter. In most cases, especially in Attica, the grid itself was not incised or very shallowly incised, but elsewhere it was often clearly seen.[78]

Then, we have several archaeological artifacts dated in the early centuries of the Roman Imperial age (thus coinciding with the emergence of symmetric tables), which contain visual games using letters in square grids or in a square frame. In Greek, there are two examples of square grids containing a phrase that can be read jumping from the center in any direction to one of the four corners (without going backwards). One is from the reverse of the *Tabulae iliacae* (1st c. AD), forming a sentence with the title of the work and the signature of the author.[79] It may be relevant that the production context of this relatively early instance is Rome. Another square grid of the same type appears in the so-called Moschion stele (2nd or 3rd c. AD), where it is accompanied by bilingual texts in Demotic and Greek, providing instructions to the reader; in the Greek texts the square is referred to as πλινθίς.[80] A similar kind of crossword game is represented by the famous SATOR and ROMA OLIM squares, in which the text can be read from either corner to the opposite (upper left to lower right or the other way around), be it in rows or

77 This format was especially in vogue for any type of official inscriptions for around two centuries, from ca. 500 to 300 BC. Cf. Butz 2010, 41.

78 Butz 2010, 50; Austin 1938, 27.

79 For a complete reproduction and photographs, see Squire 2010, 79–80.

80 See the complete reconstruction of the Moschion square in Austin 1939, 131. Here there are no empty squares and as a consequence the text cannot be read diagonally. By the way, in the inscriptions of this stele we find the only ancient instance of the word στοιχηδόν in ancient texts. For the Greek text and English translation, see Mairs 2017, 237–238.

columns. Both are found in Pompeii – again, a Roman context.[81] These kinds of visual wordplays found a continuation in the Latin poems of Publilius Optatianus (early fourth century), which were set up in *stoikhedon* format, often in the shape of a square, and containing symmetric *intexti*. The square is not the only shape in which such inscriptional games appear – for example, some of the *Tabulae iliacae* contain a crossword in a different shape[82] – but it is undeniable that the square shape was the most frequent one.[83]

3 Conclusions

Ruled tables were not a usual textual format in ancient Greek documents. Not surprisingly, they were frequently used in astronomy. It is more interesting to discover that they were in fact not much used out of this domain. Unruled tables, on the other hand, appear with some frequency in non-literary texts in the form of lists – columnar lists, as I have called them – displaying accounts, or in scientific treatises presenting data apt for distribution in multiple rows and a few columns separating different types of information. Astronomical tables (κανόνες, κανόνια, or ὄργανα) can be conceived as more complex tables (comprising more columns) of the same kind. On the other hand, ruled tables presenting no hierarchy between columns and rows (πλινθίδες and πλινθία) seem to have been mostly used for displaying surprising or playful numerical relationships in Pythagorean and Pythagorean-Platonic *milieux*.

The two kinds of ruled tables may have first appeared in the second century BC, but our evidence is not clear enough to confirm this hypothesis. On the one hand, it is believed that Hipparchus wrote astronomical tables, and that Greek astrology (whose practitioners employed astronomical tables) started off by that time, too. On

81 Specifically, the SATOR square has been found in a number of other locations. There is a similar square in Greek, but which is not a palindrome: it is a graffito in the agora of Smyrna: ΜΗΛΟΝ ΗΔΟΝΗ ΛΟΓΟΣ ΟΝΟΜΑ ΝΗΣΑΣ. Cf. Bagnall et al. 2016, 122–123.

82 It pictures the altar in the shield of Achilles: Squire 2010, 85.

83 Also, even if the letters cannot form a grid in this case, the rhythmic poem λέγουσιν ἃ θέλουσιν appearing in numerous objects dated to the early Roman empire, is mostly displayed with the text forming a square. In fact, the symmetries in it are underlined in most exemplars in such a way that the first letters of the first three and the last three lines are pictured very alike (ΛΕΓ ΑΘΕ ΛΕΓ with an extremely round E, ΟΥ CΥ CΥ with an extremely round C), and similarly for the final letters (CIN CIN CAN, OI – OI). Cf. Whitmarsh 2021, 135–163 for an analysis of the rhythm of the poem and a catalogue of the objects. In Egypt, we have examples of similar crossword inscriptions in ruled grids that are not squares: one is an impressive hymn to Mut which can be read both horizontally (in rows) and vertically (in columns), forming two different coherent texts. See Stewart 1971, esp. 88: it is from the New Kingdom, like the five other known examples.

the other, our earliest example of a symmetric table is difficult to date precisely, but it is certainly from late-Hellenistic period. Since Greek astronomical tables likely received the external influence of Babylonian tables, the simplest scenario would be that symmetric tables were born after them and were influenced by them.

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