Fractional charge and statistics in few-body lattice quantum Hall states

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Abstract: The Laughlin fractional quantum Hall effect (FQHE) is one of the paradigmatic strongly-correlated systems that has captivated physicists for decades. Arguably, its most defining property is the emergence of collective excitations with fractional charge and statistics. In this project, we report the observation of these signatures in one of the most well-studied lattice analogs to the FQH: the Hofstadter model. Moreover, we discuss whether variational neural networks can be valid ansätze of the FQH ground state.
Keywords: Condensed matter, topological order, anyons, neural quantum states, Chern number
SDGs: Good health and well-being; Quality education; Industry, Innovation and infrastrstructure

I. INTRODUCTION

Until the 1980s the physical description of phase transitions had successfully been explained by Landau's symmetry-breaking theory. It postulates that a phase is essentially an equivalence relation between states that share the same symmetry, and that any phase transition involves a spontaneous breaking of that symmetry (SSB). For instance, the para-ferro magnetic transition in spin systems involves the SSB of the rotational symmetry.

However, the discovery of the FQH effect showed the limitations of Landau's symmetry-breaking theory. During the 1990s multiple Hall states were being reported with different characteristic "fractions"; while sharing all the same symmetries [1, 2]. Thus, the FQHE became the first *topologically ordered phase of matter* to be discovered, a new class of materials outside Landau's paradigm, which are characterized by the emergence of collective excitations with fractional charge and statistics (anyons):

$$|\Psi(\mathbf{r}_1, \mathbf{r}_2)\rangle = e^{i\varphi}|\Psi(\mathbf{r}_2, \mathbf{r}_1)\rangle \qquad \varphi/\pi \in \mathbb{Q} \tag{1}$$

The FQH effect describes the behavior of particles (initially fermions, but the description was later generalized for bosons too) under the effect of a magnetic flux Φ and with arbitrary 2-body repulsive potentials:

$$H = \frac{1}{2m_p}\sum_i (\mathbf{p}_i^2 + e\mathbf{A}(\mathbf{x}_i))^2 + \sum_{i<j} V(|\mathbf{x}_i - \mathbf{x}_j|) \tag{2}$$

One of the most significant parameters in the description of the quantum Hall states is the filling fraction, which describes the proportion between the number of particles (N) and the number of flux quanta $(n_\phi)^\dagger$: $\nu = N/n_\phi$. For example, when the interactions are neglected, each particle under the magnetic field describes a quantized cyclotron orbit, which gives a filling fraction of $\nu = 1$. This system is called the integer quantum Hall effect (IQHE).

Although this Hamiltonian has not yet been exactly solved (neither exactly nor perturbatively) for general non-negligible interactions; Laughlin proposed an ansatz that captures the properties of the FQHE [3]:

$$\Psi_m(z_1, .., z_N) = \prod_{i<j}(z_i - z_j)^m e^{-\sum_i |z_i|^2/4l_B^2} \tag{3}$$

The Laughlin wavefunction describes matter with fractional filling $\nu = \frac{1}{m}$ and constant density $\rho = \nu n_\phi / A$. It also describes the ground state of a gapped energy spectrum, which makes it resistant to small perturbations. Finally, the Laughlin wavefunction predicts the signature property of the FQHE; i.e. the system can reach its low-energy excited states by introducing localized quasiholes with charge $q = e/m$ and anyonic phase $\varphi = \pi/m$.

The goal of this thesis is to observe these anyonic excitations in a lattice analog of the FQHE: the Hofstadter model. Before presenting this model, we are going to introduce the composite fermion theory, which explains the physical origin of the fractionalization.

II. COMPOSITE FERMION THEORY

In condensed matter physics, especially in highly correlated systems, it is usually the case that the right degrees of freedom to describe the system are not the same as the original ones. In fact, strongly interacting particles may reorganize themselves so that weakly coupled, composite quasiparticles may emerge. One of the most well-known examples are the Cooper pairs, a composite quasiparticle that emerges in superconductors. These ideas are what motivated the description of fermionic FQH states using composite particles [4, 5].

We start by analyzing the Laughlin wavefunction which can be divided into two parts:

$$\Psi_m(z) = \psi(z)\prod_{i<j}(z_i - z_j)^{m-1} \tag{4}$$

where $z_j = x_j - iy_j$ denotes the position of the jth particle as a complex number. The first term in this decomposition is a Slater determinant, which corresponds

[†] $n_\phi \equiv \Phi/\Phi_0$, where $\Phi_0 \equiv 2\pi\hbar/e$ is the magnetic flux quantum and Φ, the total flux through the system.

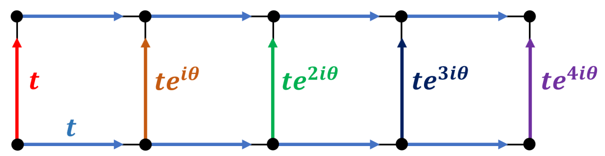

Figure 1: Hopping terms in a Hofstadter lattice in the Landau gauge ($\mathbf{A} = Bx\mathbf{e}_y$), where $\theta \equiv 2\pi\Phi/\Phi_0$.

to the wavefunction for non-interacting particles, i.e., the lowest Landau level in the IQHE ($\psi(z) \equiv \Psi_{m=1}(z)$). The second term is equivalent to adding $m-1$ *vortices* in the same positions as the particles. In this context, we are going to understand a *vortex* $(z-\eta)$ centered in η as a structure that introduces a phase factor of $\varphi_{vortex} = 2\pi$ to the wavefunction when a particle moves around it.

Now, let us illustrate what is the effect of binding these $m-1$ vortices to the particles. The probability that two particles come within distance r from each other rapidly drops like r^{2m}, much faster than in the non-interacting Slater determinants, which decrease like r^2. We can see that these vortices are very effective at keeping particles apart from each other and screen much of the Coulomb repulsion.

Thus, the composite fermion theory proposes that particles minimize their repulsion by forming bound states with $m-1$ vortex-like excitations. Although this theory was initially developed for fermionic systems, the arguments presented also can be applied for bosonic FQH states. In this case, however, composite bosons are formed. This theory explains two of the main features of the FQHE:

1. Charge of quasihole-like excitations: A quasihole can be produced by introducing a vortex $\prod_i(z_i-\eta)$ in FQH matter. The Ahanorov-Bohm phase[‡] that the vortex will pick after encircling an area that contains a flux Φ_A and $N_A = \nu\Phi_A/\Phi_0$ particles is:

$$\gamma_{AB} \equiv 2\pi\frac{q\Phi_A}{e\Phi_0} = N_A\varphi_{vortex} \quad \rightarrow \quad \boxed{q = \ e/m} \tag{5}$$

2. Anyonic statistics of the quasiholes: Now let's repeat the same scheme but introducing a second quasihole (of charge e/m) in the area that is encircled by the first quasihole. This new quasihole depletes some of the particles in the area: $N_A = \nu\Phi_A/\Phi_0 - 1/m$. The extra contribution to the Aharonov-Bohm phase that this change produces, can be associated to the braiding phase of the quasiholes ($\gamma_{\text{br}} \equiv 2\varphi$):

$$\boxed{\gamma_{\text{br}} = \frac{2\pi}{m}} \tag{6}$$

[‡] When a quantum particle describes a closed loop, it will pick a phase γ_{AB} proportional to the encircled magnetic flux Φ.

III. HOFSTADTER MODEL

The Hofstadter model [6] describes bosonic particles hopping in a two-dimensional lattice and subjected to an external magnetic field. The Hamiltonian reads:

$$H = -t\left(\sum_{\langle i,j\rangle} e^{i\theta_{ij}}a_i^\dagger a_j + h.c\right) + \frac{U}{2}\sum_i n_i(n_i-1) + \sum_i V_i(\mathbf{r})n_i \tag{7}$$

where $a_i^\dagger$ (a_i) is the creation (annihilation) operator for a boson at site i and n_i is the corresponding number operator. In this expression, three different terms can be identified:

- A term describing the hopping of bosons between nearest-neighbour sites with strength t and with a dynamical phase factor $e^{\theta_{ij}}$ that mimics the effect of magnetic field on the particles (Fig. 1).
- A term describing on-site interactions with strength U. In this project, we will mostly consider the hard-core boson limit ($U \to \infty$), as it simplifies the calculations. This particular case forbids states with $n_i > 1$ occupation numbers, which dramatically decreases the Hilbert's space dimension.
- A term describing possible external energy offsets $V_i(\mathbf{r})$, which we will use to either encode trapping harmonic potentials or localized impurities to pin the quasiholes.

The band structure of the Hofstadter model shares many similarities to that of the FQHE, which is our main motivation for studying this system. It belongs to a class of systems with *topological bands*, which means that the bands have associated a non-zero *Chern number*. The Chern numbers are topological integer invariants that characterize the "winding" of the electronic bands $|u_{\mathbf{k}}^{(n)}\rangle$. A Chern number $\mathcal{C} = 1$ means that the particles gain an extra phase of 2π (*winding*) when encircling the whole Brillouin Zone (BZ):

$$\mathcal{C}^{(n)} = \frac{1}{2\pi}\int_{BZ} i\epsilon_{ij}\frac{\partial}{\partial k_i}\langle u_{\mathbf{k}}^{(n)} \,|\, \frac{\partial}{\partial k_j}u_{\mathbf{k}}^{(n)}\rangle d^2\mathbf{k} \in \mathbb{Z} \tag{8}$$

As part of this thesis, we have also computed the band structure of the Hofstadter model (Fig. 2), which shows that it also has a topological non-trivial nature.

The fact that the Chern number is a topological invariant means that it cannot change under smooth adiabatic transformations. A band cannot modify its winding unless a discontinuous (non-smooth) transformation occurs, such as when a band gap closes. This is why it is only possible to draw equivalence relations between systems with the same Chern number. A nice analogy can be made using the Möbius strip: its winding cannot be changed, no matter how we deform it, unless we make destructive changes, such as cutting the strip and rejoining the ends.

Motivated by this idea, Scaffidi and Möller [7], showed that the states in the bottom-left tail of the Hofstadter

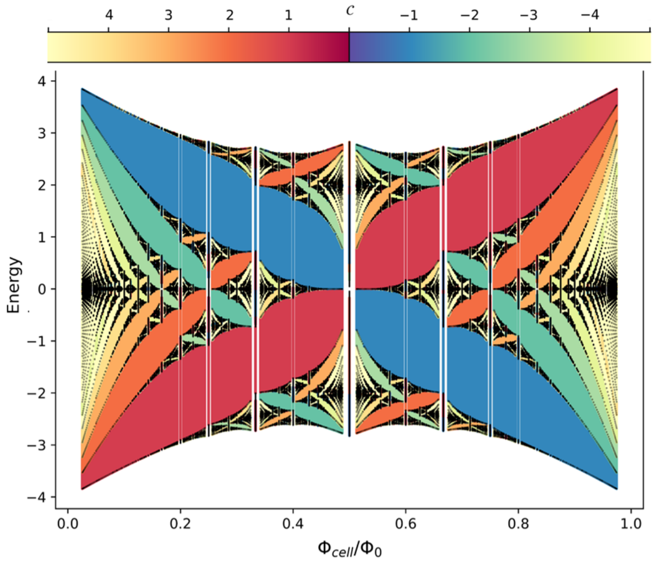

Figure 2: Band structure of the Hofstadter model (in units of t) as a function of the magnetic flux through a unit cell Φ_{cell}. The colors represent the Chern numbers associated with each gap.

butterfly can be adiabatically deformed to the lowest Landau level of the FQH effect (both with $\mathcal{C} = 1$) . This evidence further proves the equivalence between these two systems, which motivates us to look for fractional states in the Hofstadter model.

IV. NUMERICAL METHODS

In order to calculate the main physical observables (i.e. braiding phase, density depletions) we will use the following methods:

- Exact diagonalization of the hamiltonian.
- Variational neural quantum states, particularly Restricted Boltzmann Machines (RBM)

The usage of neural quantum states (NQS) is closely linked with the variational principle of quantum mechanics. The idea is that the expected value of the hamiltonian, calculated with a variational wavefunction dependent on a series of parameters, will always be greater than the energy of the ground state of the system.

$$\langle\psi(\alpha_1, ..., \alpha_N)|H|\psi(\alpha_1, ..., \alpha_N)\rangle \geq E_{GS} \tag{9}$$

This is why NQSs, if properly optimized, can be good ansätze of the ground state of any many-body quantum system. In this thesis, we have used as a variational a function a RBM, provided by the software package NetKet [8]. The RBM can be expressed as:

$$\Psi(n_i) = e^{\sum_i a_i n_i} \left[\prod_{i=1}^{M} 2\cosh\left(b_i + \sum_j^N W_{ij} n_j \right) \right] \tag{10}$$

where W_{ij}, b_i are variational parameters that have to be optimized through Monte Carlo stochastic reconfiguration and n_i the occupation of each site.

V. RESULTS

As described in the theoretical section, the aim of this thesis is to simulate emergent phenomena from a topologically ordered system, such as the Hofstadter model, with special focus on its fractional excitations. Furthermore, we propose ourselves to analyze whether the RBM can be a good ansätze for topologically ordered systems, by checking if they can properly reproduce these excitations. In this section we describe two theoretical setups that reveal specific features about the FQHE, such as charge fractionalization and anyonic excitations.

In particular, we focus on describing a fractional Hall state with a filling number of $\nu = 1/2$, which has the following properties:

- The Laughlin wavefunction is even (which is why the bosonic Hofstadter model is a suitable lattice analog).
- The fractional excitations (quasiholes) have electric charge $q = e/2$ and anyonic phase $\varphi = \pi/2$ (which corresponds to a braiding phase of $\gamma_{\text{br}} = 2\varphi = \pi$).

A. Fractional charge redistribution through potential offsets

For our first simulations we will use the setup described in [9]. The idea is to introduce two spatially-separated localized potential offsets with opposite sign to pin quasiparticles and quasiholes. By plotting the integrated particle density (which is proportional to the electric charge) near each defect, we expect to observe the following properties of the FQHE.

- First, considering that the FQH states have many-body gapped energy spectra, we expect to see that the system is resistant against small perturbations.
- Second, we expect the potential dip to attract a quasiparticle (QP), while the potential bump, a quasihole (QH) with a quantized charge of $\pm e/2$.

The system we will describe consists of a rectangular lattice with $N_x \times N_y$ sites and open boundary conditions, which contains $(N_x - 1) \times (N_y - 1)$ cells. We fill the system with N bosons, then apply a magnetic flux Φ_{cell} on each cell and perturb the system by applying localized and opposite potential offsets V_i on two separate regions.

The filling number is the ratio between the number of bosons that each cell provides, and the number of flux quanta that fit into each cell.

$$\nu = \frac{N/N_{\text{cell}}}{\Phi_{cell}/\Phi_0} \tag{11}$$

In order to simulate a state with $\nu = 1/2$, we choose a suitable shape and number of particles. We calculate

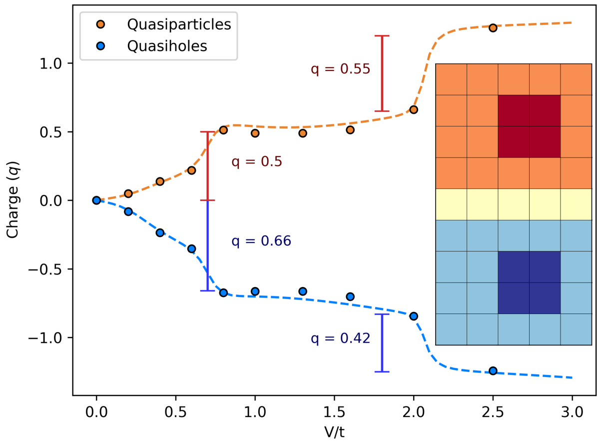

Figure 3: Results for a 9x5 lattice (right) with $N = 4$ and $\Phi_{cell}/\Phi_0 = 1/4$. We introduce offsets in the darker cells and compute the charge accumulation/depletion in the red/blue regions (as a function of the offsets strength). The points and dashed lines correspond to the results given by the RBM and exact diagonalization, respectively.

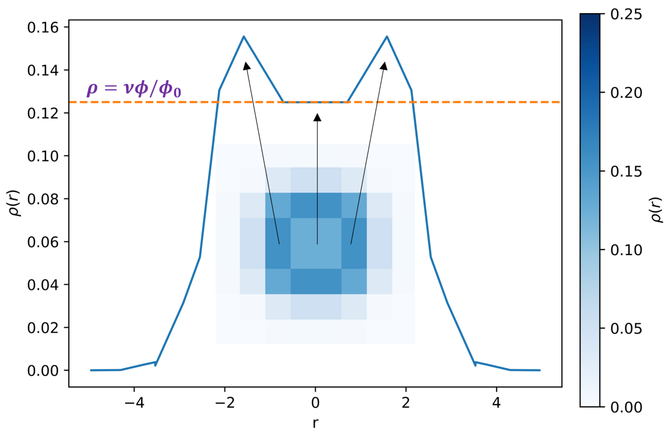

Figure 4: Density profile in a system with $N = 3$ bosons in an 8x8 lattice and under the effect of a confining harmonic potential with constant $V_0 = 0.02t$. We observe a central plateau with density $\rho = \nu\Phi_{cell}/\Phi_0$.

the groundstate of this system as a function of V using both exact diagonalization and the RBM and plot the integrated particle density near the impurities (Fig. 3).

From looking at this figure, we can make two observations that seem to indicate the presence of fractional states in the Hofstadter lattice. First of all, the system is mostly insensitive towards these potential offsets (impurities) throughout most of the simulation, which can be attributed to the gapped nature of the FQHE spectrum.

Moreover, we can observe abrupt changes in the integrated particle densities near each defect with values generally close to $\pm e/2$. This result can be explained, as the system obtains enough energy to be in many-body excited states by generating pairs of QP-QH. Then, they are transferred respectively to the potential dips and bumps generated by the defects in the bulk, which is why this fractional charge separation can be observed. The deviations from the expected value can be attributed to finite-size effects.

B. Use of a potential offset with a confining harmonic potential

With the former method, we have checked that the RBM, trained accordingly with the Hofstadter model, can properly describe two important signatures of the FQHE: its gapped spectrum and the charge fractionalization. However, it could be argued that the most defining feature of the FQHE, which is linked to its topologically ordered nature, are its anyonic excitations. To calculate the braiding phase $\varphi_{\rm br}$ associated with the exchange of two quasiholes, we will use a setup similar to that described in [10]. We consider a system with $N = 3$ bosons in an 8x8 lattice with open boundary conditions. We apply a background magnetic flux $\Phi_{cell}/\Phi_0 = 0.25$ and we also introduce an additional harmonic confining potential in the form $V_i = V_0|\vec{r}_i - \vec{r}_{\rm center}|^2$.

However, we cannot guarantee that for any arbitrary value of the trapping strength V_0 the system will stabilize in the $\nu = 1/2$ FQHE. In fact, one of the defining features of the FQHE is a plateau of constant particle density $\rho = \nu\Phi_{cell}/\Phi_0$. Thus, the first step we take is to identify the value of V_0, with which we obtain a central plateau with particle density $\rho = 0.125$. We find that this value is $V_0 = 0.02t$ (Fig. 4).

Afterwards, we identify the pinning potential V_i, for which the system places 1 and 2 QH in the center of the lattice, respectively. We follow a very similar procedure to the one explained in section A: we compute the density depletion around the potential offset and look for sharp changes with magnitude around $\Delta\rho \approx 0.5$. We find that these values are $V_i = 0.4$ and $V_i = 6$ (Fig. 5).

1. *Quasihole statistics through density depletion profiles*

In previous works, a new method was developed to calculate the braiding phase through the density depletion profiles ($d_{1\rm QH}(\vec{r})$, $d_{2\rm QH}(\vec{r})$) generated by 1 and 2 quasiholes, respectively [11]:

$$\frac{\varphi_{\rm br}}{2\pi} = \pi\frac{\phi}{\phi_0}\sum_{j\in\rm QH}[d_{2\rm QH}(\vec{r}_j) - 2d_{2\rm QH}(\vec{r}_j)]|\vec{r}_j|^2 \qquad (12)$$

where the depletions are defined as:

$$d_{k\rm QH}(\vec{r}) = \langle n(\vec{r})\rangle_{0\rm QH} - \langle n(\vec{r})\rangle_{k\rm QH} \qquad (13)$$

This scheme for determining $\varphi_{\rm br}$ is particularly advantageous for small systems, compared to more traditional schemes. The latter involve the explicit computation of the braiding phase by separating the quasiholes and rotating one around the other. In contrast, the method we use in this thesis has no need for this process, which saves a considerable amount of computational time and can work on even small lattices, such as the one we use.

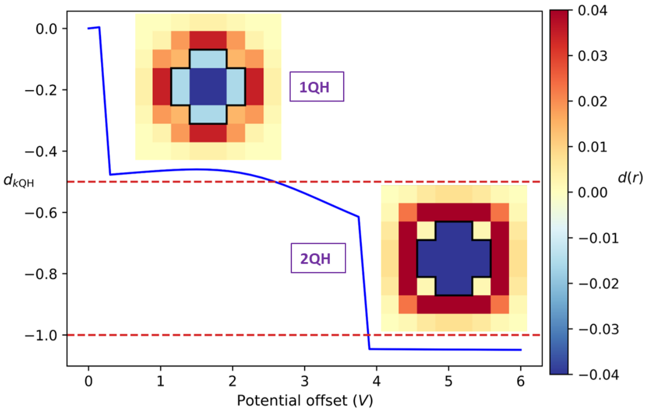

Figure 5: Charge depletion around the potential offset as a function of its intensity. Attached, are the depletion profiles $d(r)$ of the situations where 1 and 2 QHs are introduced, respectively. We consider that the QHs only occupy the highlighted area.

	$q_{\rm QH}/e$	$\varphi_{\rm br}/(2\pi)$
ED	0.475	0.482
RBM	0.483	0.446

Table I: Charge and braiding phase of the quasiholes generated in a Hofstadter lattice and calculated through exact diagonalization (ED) and a variational RBM. Both results are close to the expected value for the $\nu = 1/2$ state: 0.5.

With the results in the previous section, we calculate the integrated charge of the quasiholes and the braiding phase (eq. 12). The results are summarized in Tab. I. We can observe that both the RBM and the exact diagonalization predict that the lattice will generate anyonic excitations with charge $e/2$ and statistical phase $\varphi = \varphi_{br}/2 = \pi/2$).

VI. CONCLUSIONS

In this work, we have successfully studied the topological nature of the Hofstadter model and we have observed defining features of fractional quantum Hall states. The conclusions derived from this work are the following.

The Hofstadter model presents a band structure with non-trivial topological Chern numbers. In particular, in the limit of low fluxes, the ground state presents a Chern number of 1, which is why the Hofstadter model can be mapped to FQH states.

Moreover, we have observed that the Hofstadter model can hold quasihole-like excitations with fractional charge and statistics, which are the most characteristic properties of topologically-ordered phases such as the FQH effect. These properties are also corroborated by our calculations with the neural networks, which seems to indicate that the RBM can be good variational functions for the ground state of the FQHE.

The study of highly correlated materials has been experimenting nowadays notable success. Thus, possible continuations of this work could be the study of more complex materials with topological bands, such as the twisted bilayer graphene (TBG).

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Càrrega i estadística fraccionària en estats Hall quàntics reticulars de pocs cossos

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Resum: L'efecte Hall quàntic fraccionari de Laughlin (FQHE) és un dels sistemes fortament correlacionats que més ha captivat als físics durant dècades. Es podria dir que la seva propietat més característica és l'emergència d'excitacions col·lectives amb càrrega i estadística fraccionària. En aquest projecte, observem aquestes excitacions distintives en un dels models reticulars anàlegs al FQH més estudiats: el model de Hofstadter. A més, discutim si les xarxes neuronals variacionals poden ser uns ansätze vàlids de l'estat fonamental del FQH.
Paraules clau: Matèria condensada, ordre topològic, anyons, estats quàntics neuronals, número de Chern
ODSs: Salut i benestar; Educació de qualitat; Indústria innvocació i infraestructures

Objectius de Desenvolupament Sostenible (ODSs o SDGs)

1. Fi de la es desigualtats		10. Reducció de les desigualtats	
2. Fam zero		11. Ciutats i comunitats sostenibles	
3. Salut i benestar	X	12. Consum i producció responsables	
4. Educació de qualitat	X	13. Acció climàtica	
5. Igualtat de gènere		14. Vida submarina	
6. Aigua neta i sanejament		15. Vida terrestre	
7. Energia neta i sostenible		16. Pau, justícia i institucions sòlides	
8. Treball digne i creixement econòmic		17. Aliança pels objectius	
9. Indústria, innovació, infraestructures	X		

Una de les principals aplicacions de l'estudi dels anyons és la implementació pràctica d'ordinadors quàntics. El principal obstacle en la seva realització experimental és la decoherència dels estats quàntics: petites pertorbacions poden fàcilment ocasionar que els qubits perdin la seva coherència i que l'ordinador deixi de funcionar. Els anyons, d'altra banda, són uns defectes topològics que apareixen en sistemes com el FQHE que són resistents a perturbacions locals i contínues. Aquesta estabilitat és la raó per la qual els *ordinadors quàntics topològics* amb anyons són una de les implementacions pràctiques més prometedores.

D'aquesta manera, aquest TFG està relacionat amb l'ODS 9, ja que contribueix en l'estudi d'una de les tecnologies del futur. A més, una de les aplicacions de la computació quàntica més prometedora és la simulació de les propietats quàntiques de molècules i, en especial, fàrmacs. Per tant, aquest TFG pot contribuir a l'ODS 3, que busca millorar la salut i el benestar de la població. Finalment, aquest treball forma part d'un grau universitari de Física i creiem que pot ajudar a motivar acadèmicament a futurs alumnes que passin per la facultat (ODS 4, educació de qualitat).