Chirped laser pulses for rapid adiabatic passage in trapped ions

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Abstract: This work investigates the use of chirped picosecond laser pulses to induce rapid adiabatic passage (RAP) in a single trapped ion, aiming for population inversion between two quantum states. A theoretical framework for RAP linearly chirped pulses is developed, and a novel method to characterise ultrashort pulses is introduced. Numerical simulations solving the Schrödinger equation confirm that RAP leads to state transitions with 100% probability in the resonant regime, independent of the pulse area. The effects of a detuning between the transition and pulse frequencies on the transition probability are also analysed, revealing a linear dependence between the transition probability curve size and the pulse width.
Keywords: Trapped ion, ultrashort laser pulses, rapid adiabatic passage.
SDGs: Affordable and clear energy; industry, innovation and infrastrucutre.

I. INTRODUCTION

Well-isolated quantum systems are excellent sensors of electromagnetic fields. Specifically, chirped picosecond laser pulses can resonantly cause a dipole transition on a single trapped ion via rapid adiabatic passage (RAP) [1]. This is a population inversion method consisting of the interaction between the qubit and an ultrashort pulse with a carrier frequency near the electronic transition that has been modified linearly through time. If the process is performed adiabatically, it can produce a transition between the two quantum states with a 100% probability [2][3].

This project focuses on two main objectives. The first is to study the dynamics of a two-level quantum system (qubit) interacting with a chirped laser pulse, which does not produce crossing of energies, to achieve a transition between its states through a RAP [3]. The second objective is to investigate how this interaction can be used as a quantum sensor to characterise the temporal shape and properties of the driving pulse. This is interesting because there are no similar protocols. Measuring ultra-short pulses usually compares the pulse with a shifted version of itself. [1]

The study begins with defining the system's Hamiltonian, which models the ion as a two-level quantum system and its interaction with a pulse. Then, the numerical method used to solve the time-dependent Schrödinger equation is introduced, along with two validation tests that confirm its reliability. The project focuses on analysing rapid adiabatic passage with linearly chirped laser pulses, both without and with a detuning. It is verified that a transition between the ground and the excited state of a trapped ion can be driven via RAPs. Finally, a novel method is proposed to estimate the pulse envelope from the ion's response to it.

II. DEVELOPING SECTIONS

A qubit is the smallest useful amount of quantum information. It is described by a two-level system, meaning two orthogonal and physically distinguishable states $|0\rangle$ and $|1\rangle$, which form a two-dimensional Hilbert space $\mathcal{H}$. The qubit can adopt any possible state in this Hilbert space, either the basis states or an arbitrary superposition of them $\alpha|0\rangle + \beta|1\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$ [4].

A. Qubit hamiltonian

In this project, an intrinsically anharmonic energy spectrum, such as the Ca^+ ion, describes the qubit. In this scene, the ground and first excited and long-lived states represent $|0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, respectively. When the ion is illuminated with chirped microwaves $E(t) = \Omega(t)\cos(\omega(t)t + \phi)$, it induces transitions between the eigenstates. The dynamics of this qubit, considering $\hbar = 1$, is described by the Hamiltonian

$$H = E_0\mathbf{1} + \frac{\Delta}{2}\boldsymbol{\sigma^z} + \Omega(t)\cos(\omega(t)t+\phi)\boldsymbol{\sigma^x} = E_0\mathbf{1} + H_0 + H' \quad , \tag{1}$$

where the energy offset E_0 can be ignored and the parameter Δ is the qubit gap. It is convenient to eliminate the qubit's free evolution at frequency Δ by working in the *interaction picture* [4].

Assuming that the correction is small compared with the qubit's free evolution, the temporal unitary operator is defined as $U(t) = U_0(t)V(t)$, where $U_0(t) = \exp\left(-i\frac{\Delta}{2}\sigma^z t\right)$. Then, applying the Schrödinger equation,

$$i\frac{dU}{dt} = i\frac{dU_0}{dt}V + iU_0\frac{dV}{dt} = HU \quad .$$

So finally,

$$i\frac{dV}{dt} = \left[U_0^{-1}HU_0 - iU_0^{-1}\frac{dU_0}{dt}\right]V = H_I V \quad . \tag{2}$$

In this particular case, $i\frac{dU_0}{dt} = H_0U_0$, so the interaction Hamiltonian is redefinited as

$$H_I = U_0^{-1}H'U_0 \tag{3}$$

and the wavefunctions become

$$|\psi_I(t)\rangle = U_0(t)^\dagger|\psi(t)\rangle \quad [4].$$

Considering these changes and defining $\sigma^+ = |1\rangle\langle 0|$ and $\sigma^- = |0\rangle\langle 1|$,

$$\begin{aligned} H_I &= e^{i\frac{\Delta}{2}\sigma^z t}\Omega(t)\left[e^{i(\omega(t)t+\phi)} + e^{-i(\omega(t)t+\phi)}\right]\sigma^x e^{-i\frac{\Delta}{2}\sigma^z t} \\ &= \Omega(t)\left[e^{i(\omega(t)t+\phi)} + e^{-i(\omega(t)t+\phi)}\right]\left[e^{i\Delta t}\sigma^+ + e^{-i\Delta t}\sigma^-\right] \\ &\approx \Omega(t)\left[e^{-i(\omega(t)-\Delta)t-i\phi}\sigma^+ + e^{i(\omega(t)-\Delta)t+i\phi}\sigma^-\right] \quad . \end{aligned}$$

In the last step, the *rotating-wave approximation* (RWA) has been used to neglect the rapid oscillations of the terms $(\omega(t)+\Delta)$. This is an acceptable estimation as long as the pulse amplitude is small compared to the drive frequency ω.

In order to eliminate the time-dependent exponentials, we need to make a second change of variables. Copying the above methodology, the temporal operator can be defined as $V(t) = U_{back}(t)R(t)$ and using (2), the new effective Hamiltonian becomes

$$H_{\text{eff}} = U_{\text{back}}^{-1}H_I U_{\text{back}} - i\,U_{\text{back}}^{-1}\frac{\partial U_{\text{back}}}{\partial t} \quad . \tag{4}$$

Supposing that $U_{\text{back}} = e^{i\theta(t)\sigma^z}$, the first term leads to

$$\begin{aligned} U_{\text{back}}^{-1}H_I U_{\text{back}} &= e^{-i\theta(t)\sigma^z}\, H_I\, e^{i\theta(t)\sigma^z} \\ = \Omega(t)&\left[e^{i(\omega(t)-\Delta)t+i\phi}e^{2i\theta(t)}\sigma^- + e^{-i(\omega(t)-\Delta)t-i\phi}e^{-2i\theta(t)}\sigma^+\right] \end{aligned}$$

and taking $(\,\omega(t)-\Delta\,)\,t = -2\theta(t)$,

$$U_{\text{back}}^{-1}H_I U_{\text{back}} = \Omega(t)[\cos\phi\,\sigma^x + \sin\phi\,\sigma^y].$$

Considering the second term in (4),

$$\begin{aligned} -i\,U_{\text{back}}^{-1}\frac{\partial U_{\text{back}}}{\partial t} &= -ie^{-i\theta(t)\sigma^z}i\sigma^z\frac{d\theta(t)}{dt}e^{i\theta(t)\sigma^z} \\ = \sigma^z\frac{d\theta(t)}{dt} &= \sigma^z\left(-\frac{\omega(t)-\Delta}{2} - \frac{d\omega(t)}{dt}\frac{t}{2}\right) \quad . \end{aligned}$$

Joining both solutions at $\phi = 0$,

$$H_{\text{eff}} = \sigma^z\left(\frac{\Delta-\omega(t)}{2} - \frac{d\omega(t)}{dt}\frac{t}{2}\right) + \Omega(t)\sigma^x \quad .$$

Defining the detuning δ as the difference between the laser carrier frequency and the transition frequency, and the chirp factor D as the frequency changing velocity (or phase acceleration) [5][3];

$$\omega(t)t = \xi(t) = (\Delta+\delta)t + \frac{1}{2}Dt^2 \tag{5}$$

Finally, the Hamiltonian that is going to be used, from now on called H, is expressed as

$$H = -\frac{\delta + Dt}{2}\sigma^z + \Omega(t)\sigma^x \quad . \tag{6}$$

The above Hamiltonian has the following eigenvalues

$$E_\pm = \pm\sqrt{\left(\frac{\delta+Dt}{2}\right)^2 + \left(\frac{\Omega(t)}{4}\right)^2} \quad . \tag{7}$$

To be able to work with (6), a Schrödinger equation solver function for a time-dependent Hamiltonian has been developed.

B. Schrödinger equation solver

A code has been developed to solve numerically the time-dependent Scrödinger equation for an arbitrary time-dependent Hamiltonian. It uses the fifth-order Runge-Kutta integration method to resolve the differential equation

$$\frac{dU(t)}{dt} = -iH(t)U(t) \quad , \tag{8}$$

where $U(t)$ is the unitary matrix and $H(t)$ is the Hamiltonian of our quantum system. The designed function begins with an initial condition $U(t_0) = \mathbf{1}$ and evolves until a final time t_f. This iterative solver saves the time and the unitary matrix calculated at each iteration and saves them in two vectors that are returned at the end.

To be sure that the solver works correctly, two verifications can be made. The first check is to represent the temporal evolution of a known quantum system and compare it with the known exact solution. Taking a constant pulse, $H = \Omega_0\sigma^x$, the temporal operator takes the form

$$U(t) = \exp(-i\Omega_0 t\sigma^x) = \cos(\Omega_0 t)\mathbf{1} - i\sin(\Omega_0 t)\sigma^x \quad . \tag{9}$$

Both solutions, the one using the solver function and the one using (9), are illustrated in Fig. (1) with solid line and dashed line, respectively. As it is seen, both representations coincide, supporting the use of the solver.

The second check is to compare the evolution using a Gaussian pulse $H = \Omega(t)\sigma^x$, with $\Omega(t) = \Omega_0\exp(-0.5(t/w)^2)$ being Ω_0 the amplitude and w the width of the Gaussian pulse. By aproximating $U(t) \approx \exp(-i\theta(t)\sigma^x) = \cos(\theta(t))\mathbf{1} - i\sin(\theta(t))\sigma^x$ and using the Schrödinger equation (8),

$$-i\dot{\theta}\sigma^x U = -i\Omega(t)\sigma^x U \quad ,$$

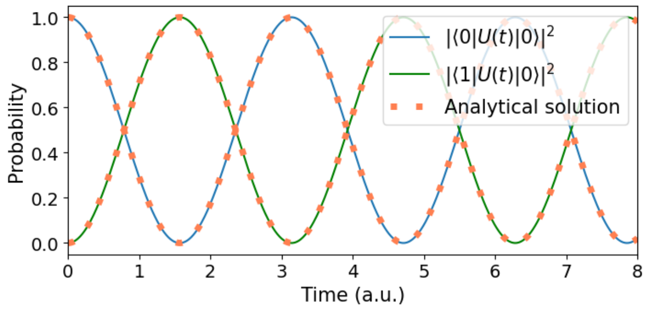

FIG. 1: Time evolution of the qubit state $|0\rangle$ under a constant pulse with amplitude $\Omega_0 = 1.0$. Probability of remaining in $|0\rangle$ (blue) or transitioning to $|1\rangle$ (green) and the known solution (9).

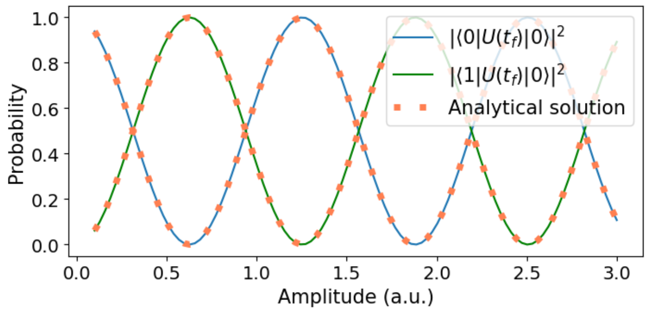

FIG. 2: Time evolution of the quantum state $|0\rangle$ under a Gaussian pulse with width $w = 1.0$. Probability of remaining in $|0\rangle$ (blue) or transitioning to $|1\rangle$ (green) and the solution calculated from the rotation angle (10).

a relation between the pulse area and the rotation angle can be found. In particular,

$$\theta = \int_{t_0}^{t_f} \Omega(t')dt' \quad . \tag{10}$$

This equation shows that the transition probability depends on the pulse intensity, duration and shape only through one parameter, θ [2]. When the integrated area of the pulse is equal to π, a complete transition occurs from one pure state to the other [5].

Using (10) to calculate U as described, the solution is represented -in dashed lines- with the solver output -in solid lines- in Fig. (2). This figure shows that, by picking a proper pulse amplitude, the transition probability can be consciously modified. [2] Again, it is clear that the solver is working as expected.

C. RAP pulse

A RAP pulse is when a chirped pulse is applied to an atom to do a Rapid Adiabatic Passage. This technique achieves a transition between the two quantum states $|0\rangle$ and $|1\rangle$ while the system remains in the instantaneous eigenstate of the Hamiltonian, provided the interaction changes are sufficiently slow relative to the energy gap [3].

To illustrate the atom's behaviour under a RAP pulse, the instantaneous eigenenergies and the atom's excited state population are plotted in Fig. (3) as a function of time. In the resonant limit (zero detuning $\delta = 0$), when $\Omega_0 = 0$, the energies of both eigenstates follow straight lines that cross at $t = 0$, as expected from equation (7). However, when $\Omega_0 \neq 0$, the light pulse opens a gap between the eigenstates' energies as shown in Fig. (3)(top, blue). If the speed at which the eigenenergies change is small compared to the energy gap, the starting states will evolve adiabatically and follow the represented blue line, completing a smooth transition between them. In other words, the atom transitions from the eigenstate at $t \to -\infty$, which is $|0\rangle$ ($|1\rangle$), to the eigenstate at long times, $|1\rangle$ ($|0\rangle$), remaining at all times on the instantaneous eigenstate subspace. This passage is observed in Fig. (3)(bottom, blue), characterised by having almost no oscillations.

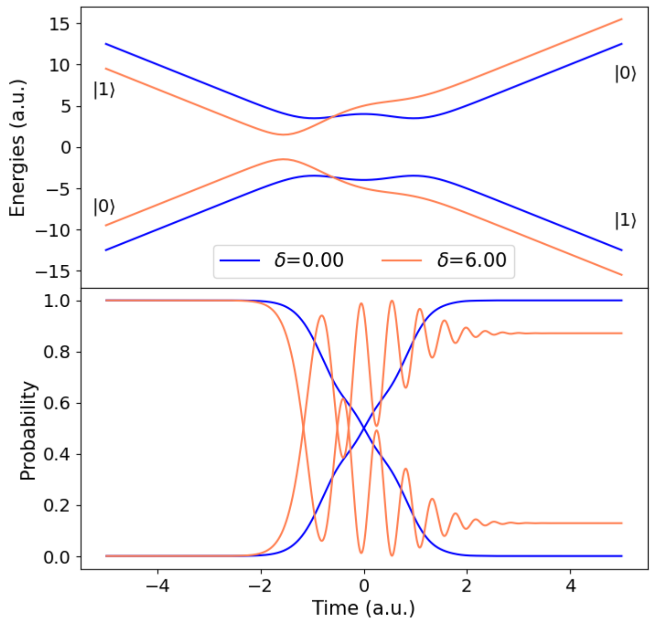

FIG. 3: Eigenstates energies and transition probabilities of the Hamiltonian described in (6) with amplitude $\Omega_0 = 4.0$, width $w = 1.0$ and chirp $D = 5.0$. Results obtained for no detuning and $\delta = 6.0$ are represented in blue and orange, respectively. In the graphic below, the top lines measure the probability of staying in the same state, while the other lines indicate the transition probability.

In the off-resonant limit, a detuning between the transition and pulse frequencies ($\delta \neq 0$) modifies the eigenenergies and state populations through time. Fig. (3)(top, orange) shows a displacement of the point where the two energies would cross without a pulse. So, when the pulse is applied, its maximum effect does not coincide with the "cross point", and the energy gap becomes smaller at higher detuning values. Consequently, the probability for a population inversion is lower than unity, as confirmed in Fig. (3)(bottom, orange). Also, the unadiabaticity produces oscillations in the projections of $U(t)|0\rangle$ over the two initial quantum states.

Based on the discussion for $\delta = 0$, the first hypothesis to be worked on is the existence of an adiabatic regime where the RAP pulse produces a transition from $|0\rangle$ to $|1\rangle$ under resonant conditions. Given that an adiabatic transition requires the system to evolve more slowly than the energy gap, it is reasonable to use the chirp factor D, previously described as the frequency sweep rate, as a parameter determining the adiabacity. Its dependence is plotted in Fig. (4), showing there is a limit value that D should take to be able to produce the adiabatic transition. Once it reaches this value, it has been confirmed that $U(t)|0\rangle$ is still an instantaneous eigenvalue of $H(t)$ throughout the time. Moreover, the pulse area has almost no influence on the transition success: it has to take a minimum value to make the transition possible; however, once it reaches it, the transition probability does not depend on the area. This can be seen in Fig. (5).

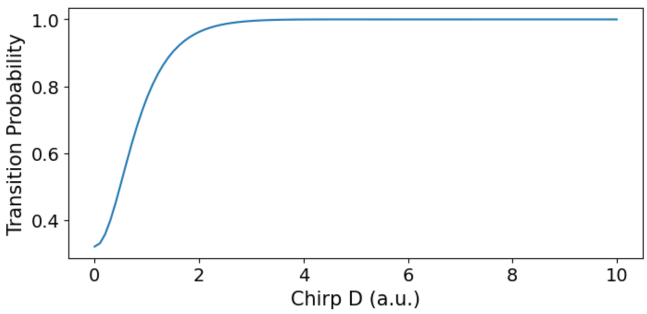

FIG. 4: Transition probability for a RAP pulse as a function of the chirp factor D under resonant conditions with amplitude $\Omega_0 = 4.0$ and width $w = 1.0$.

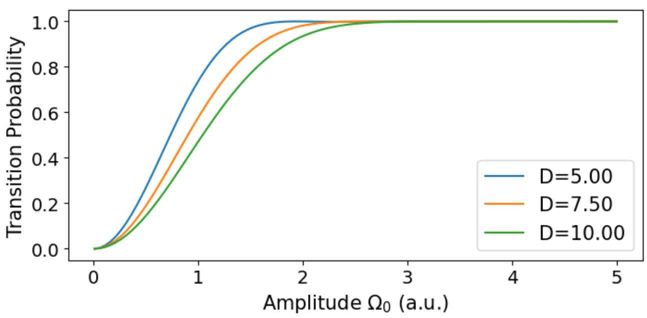

FIG. 5: Transition probability for a RAP pulse as a function of the amplitude Ω_0 under resonant conditions with width $w = 1.0$.

At this point, the first hypothesis has been proved. There is indeed an adiabatic regime where the chirped pulse drives a RAP in an atom. From Fig. (4) and Fig. (5), one set of parameters to produce the transition is: $\Omega_0 = 4.0$, $D = 5.0$, $w = 1.0$ and $\delta = 0.0$. These are the thoughtfully chosen values used in Fig. (3)(blue).

From this point, a new look at the matter is going to be discussed.

D. Novelties

In Fig. (3), it is observed that applying a detuning modifies the energy diagram and reduces the transition probability. Taking a deeper look into that second effect, Fig. (6) compares the probability of a transition between the qubit states depending on the detuning. It shows a symmetrical relation that tends asymptotically to one general curve, which is flattened on the top at unit probability and decays on both sides. Based on this behaviour, the second hypothesis is that the response $P_{0\to1}(\delta)$ is related to the pulse width. To confirm it, a graphic showing the relation between the FWHM (full width at half maximum) of the curves $P_{0\to1}(\delta)$ in Fig. (6) and the pulse width is plotted in Fig. (7).

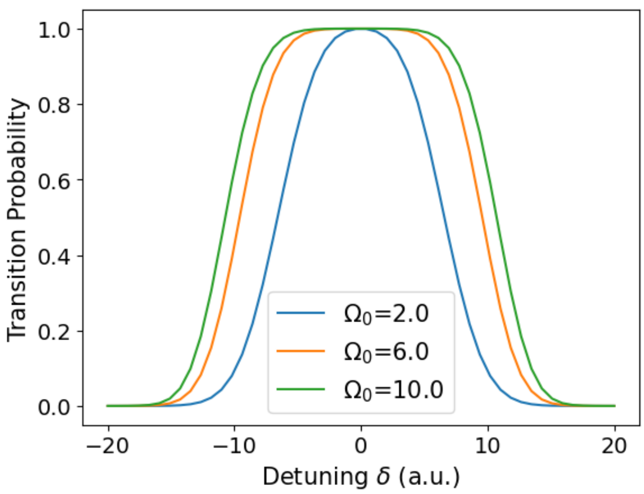

FIG. 6: Transition probability for a RAP pulse as a function of the detuning δ with chirp $D = 5.0$ and width $w = 1.0$.

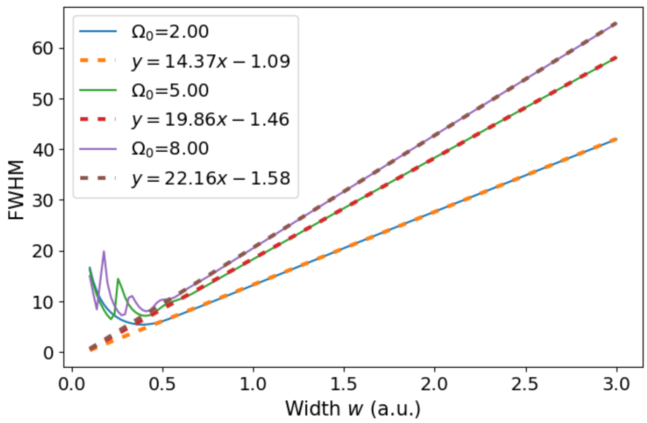

FIG. 7: Relation between the probability size in Fig. (6) and the pulse width for $D = 5.0$. The linear regression line is also plotted for each amplitude.

Fig. (7) shows a linear relation between the FWHM of the transition probability curve and the pulse width in the adiabatic regime, demonstrating the second hypothesis. The irregularities for small width values are a consequence of not being under adiabatic conditions.

The perfect linear relation between the transition probability curve and the pulse sizes leads to the third hypothesis: the pulse shape can be reconstructed from the relation $P_{0\to 1}(\delta) = F(\Omega(-\delta/D))$, where F is an unknown function. This association has not been found, but its existence has been demonstrated.

III. CONCLUSIONS

- A function that solves numerically the Schrödinger equation for a time-dependent Hamiltonian described in (8) has been developed, and its correct functioning has been verified.
- The rapid adiabatic passage (RAP) for an atom has been described and observed in Fig. (3). It has been confirmed that the chirp factor D determines the adiabatic and resonant regime in which the RAP pulse produces a transition between the two quantum states of a qubit (Fig. (4)). Also, from Fig. (5), it has been observed that the transition probability does not depend on the pulse area.
- Applying a detuning δ between the pulse and transition frequencies modifies the energy spectrum, and the transition probability decreases with increasing δ. These effects are shown in Fig. (3).
- Under a finite detuning, the response $P_{0\to 1}(\delta)$ size is linearly dependent on the pulse width, as demonstrated in Fig. (7).
- The pulse envelope can be reconstructed from the relation $P_{0\to 1}(\delta) = F(\Omega(-\delta/D))$, where F is an unknown function.

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Pols làser amb "chirp" per a una transició adiabàtica ràpida en ions atrapats

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Resum: Aquest projecte elabora una investigació en l'ús de pols làser amb "chirp" de picosegons de duració per a produir una transició adiabàtica ràpida (RAP) en un ió atrapat, amb l'objectiu de produir un intercanvi de població entre dos estats quàntics. S'ha desenvolupat un marc teòric per a pols lineals en el "chirp" i s'ha introduït un nou mètode per a caracteritzar aquests pols. Simulacions numèriques de l'equació de Schrödinger confirmen que el pols RAP produeix transicions entre els estats quàntics amb un 100% de probabilitat dins d'un règim adiabàtic, independentment de l'àrea del pols. També s'han estudiat els efectes d'afegir un desajust entre les freqüències de transició i del pols elèctric, indicant una dependència lineal entre la mida de la corba de probabilitat de transició i l'amplada del pols.
Paraules clau: Ió atrapat, pols làser ultracurts, transició adiabàtica ràpida.
ODSs: Aquest TFG està relacionat amb els Objectius de Desenvolupament Sostenible (SDGs).

Objectius de Desenvolupament Sostenible (ODSs o SDGs)

1. Fi de la es desigualtats		10. Reducció de les desigualtats	
2. Fam zero		11. Ciutats i comunitats sostenibles	
3. Salut i benestar		12. Consum i producció responsables	
4. Educació de qualitat		13. Acció climàtica	
5. Igualtat de gènere		14. Vida submarina	
6. Aigua neta i sanejament		15. Vida terrestre	
7. Energia neta i sostenible	X	16. Pau, justícia i institucions sòlides	
8. Treball digne i creixement econòmic		17. Aliança pels objectius	
9. Indústria, innovació, infraestructures	X		

El contingut d'aquest TFG, part d'un grau universitari de Física, es relaciona amb l'ODS 7, i en particular amb la fita 7.1, ja que contribueix a la modernització de les tecnologies energètiques. També es pot relacionar amb l'ODS 9, fita 9.4, perquè presenta una tecnologia per a aumentar l'eficiència dels recursos de la indústria. Les dues fites, les aborda des d'una perspectiva de reduir el temps necessari per a una manipulació dels estats quàntics.