# Spectral density of Wishart matrices and the replica method

Author: Pol Martínez Ortega, pmartior8@alumnes.ub.edu
*Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.*

Advisor: Matteo Palassini, palassini@ub.edu

**Abstract:** Random matrix theory provides a powerful framework for understanding universal features of complex systems, especially in the large-size limit. In this work, we study the spectral density of Wishart-Laguerre random matrices using the Edwards-Jones formula. We reinterpret the averaged quantity in the Edwards-Jones formula as the partition function of a disordered system and apply tools from statistical physics. Our results illustrate how techniques from statistical physics of disordered systems naturally extend to random matrix theory, offering physical insight and analytical methods for exploring spectral properties in complex systems.
**Keywords:** Disordered systems, eigenvalues, partition function
**SDGs:** SDG 4 - Quality education, SDG 9 - Industry, innovation and infrastructure

## I. INTRODUCTION

A random matrix is a matrix whose elements are random variables sampled from a given probability distribution. The interest in random matrices within physics began in the 1950s with the work of E.Wigner regarding the high energy spectrum of heavy nuclei [1].

Wigner's idea was to model the Hamiltonian of the nuclei as a random matrix, given the impossibility of an exact treatment for the system of nucleons due to the complexity of the interactions between them. This idea proved to be successful in describing certain universal properties of the heavy nuclei spectrum, such as the level repulsion or the global distribution of levels for large enough nuclei.

This approach later extended to a wide range of physical disciplines [2]. This broader perspective revealed that many problems in physics can be reformulated as questions about the spectral properties of random matrices – particularly in the limit where the matrix size becomes large, in which analytic results and universal features often become accessible. In this framework, the key physical quantities of interest are directly related to the distribution of eigenvalues of suitably defined random matrices.

The study of disordered systems offers a great deal of examples where this interplay between randomness and physical observables becomes central, as the disorder can effectively be modeled by randomness in the system's Hamiltonian. A notable example within this field is the random-energy model, introduced by B.Derrida as a simple case of solvable disordered system, which can be solved using the replica method [3].

In this work, we explore such connections by applying the Edwards-Jones formula to compute the spectral density of a Wishart matrix, both in the annealed and quenched regimes. The quenched calculation is performed using the replica method, illustrating how techniques from disordered physics systems naturally extend to the realm of random matrix theory.

## II. GENERAL CONSIDERATIONS

### A. The Edwards-Jones formula

The spectral density, $\rho(x)$, of a random matrix $H \in \mathbb{R}^{N\times N}$, is the probability density function for a single eigenvalue of $H$, namely $\rho(x) = \langle \sum_{i=1}^{N} \delta(x - x_i)\rangle / N$, where $x_i$ are the $N$ eigenvalues of $H$. The Edwards-Jones formula [4] allows to obtain this spectral density for a real symmetric random matrix $H$, knowing only the joint probability distribution function of the $N(N-1)/2$ independent elements of the upper triangle of the matrix, $\rho[H] \equiv \rho(H_{ij}, i \le j)$. The Edwards-Jones formula reads:

$$\rho(x) = \frac{-2}{\pi N} \lim_{\epsilon \to 0^+} \Im \frac{\partial}{\partial x} \Big\langle \log Z(x) \Big\rangle, \tag{1}$$

where:

$$Z(x) = \int_{\mathbb{R}^N} d\mathbf{y}\, \exp\left[-\frac{i}{2}\mathbf{y}^T[x_\epsilon \mathbb{I}_N - H]\mathbf{y}\right] \; ; \; x_\epsilon \doteq x - i\epsilon. \tag{2}$$

In Eqs. (1) and (2), $\langle(\cdot)\rangle$ denotes the expectation with respect to $\rho[H]$, $\Im$ denotes the imaginary part of a complex number and $\mathbb{I}_N$ denotes the $N$-dimensional identity matrix. A major difficulty when computing the spectral density with Eq. (1) is computing the integral of $\log Z(x)$, in which $Z(x)$ is itself another integral, as shown by Eq. (2). This can make calculations rather difficult, even in the $N \to \infty$ limit, where some simplifications take place and analytic results become available.

### B. Statistical physics of disordered systems

In statistical physics, a disordered system is one where certain parameters of the Hamiltonian, such as couplings or fields, are quenched random variables, i.e. they do not equilibrate. A central object of interest is the partition function, which depends on the realization of the disorder:

$$Z_D = \int d\mathbf{y} \exp\Big( - \beta \mathcal{H}_D\left(\mathbf{y}\right) \Big),$$

where $\mathcal{H}_D(\mathbf{y})$ is the Hamiltonian of the system for a given realization of the disorder $D$, and $\mathbf{y}$ are the dynamical variables. The disorder variables $D$ are considered fixed in the timescale of the thermal fluctuation of the dynamical variables $\mathbf{y}$. The quenched free energy of the system is then obtained as the expectation of $\log Z_D$ over the disorder $D$, $F_{quenched} = -\frac{1}{\beta}\langle \log Z_D \rangle$.

In the annealed approximation, the disorder variables $D$ are considered to fluctuate thermally on the same timescale as the dynamical variables $\mathbf{y}$, so the disorder can be considered in the Hamiltonian on the same level as the dynamical variables. The free energy is then obtained as $F_{ann} = -\frac{1}{\beta}\log\langle Z_D \rangle$.

Computing the quenched free energy directly is difficult due to the logarithm inside the average. A powerful trick from statistical mechanics of disordered systems is the replica method, based on the identity:

$$\langle \log Z_D \rangle = \lim_{n\to 0} \frac{1}{n} \log \langle Z_D^n \rangle. \tag{3}$$

This allows to treat the problem by computing integer powers of the partition function, analytically continuing the result to $n \to 0$. The resulting system of $n$ coupled replicas allows us to reduce the problem of averaging the logarithm to averaging an exponential, at the cost of introducing interactions between replicas.

In the analysis of this interactions, an important assumption is that of *replica symmetry* (RS): that all replicas behave identically and are statistically equivalent. However, in many physically relevant cases – such as in spin glasses like the Sherrington-Kirkpatrick model – this symmetry breaks down, leading to *replica symmetry breaking* (RSB) [5].

A simple illustrative model is the random-energy model, in which one can compute explicitly both the annealed and quenched free energy, and observe a phase transition associated with RSB in the low-temperature regime [6].

### C. Quenched disorder and the replica trick

In the context of the Edwards-Jones formula, we can interpret $Z(x)$ as a (complex) partition function for a fictitious system with dynamical variables $\mathbf{y}$ and quenched disorder in $H$. Then, the average in Eq. (1) corresponds to a quenched average of the free energy.

One possible simplification is to treat the disorder as annealed, leading to:

$$\langle \log Z(x) \rangle \approx \log \langle Z(x) \rangle.$$

This is not exact in general, but turns out to give correct results in some ensembles, like the Gaussian Orthogonal Ensemble (GOE), as can be seen in [4].

The more rigorous approach is to apply the replica method as introduced above. This strategy has been successfully used in random matrix theory to compute spectral densities and has strong parallels with disordered spin systems.

### D. The Wishart-Laguerre ensemble

We now apply these ideas to the Wishart ensemble, defined as the ensemble of matrices of the form $W = HH^T$, with $H \in \mathbb{R}^{N\times M}$; $M \geq N$. The elements of $H$ are independent identically distributed random Gaussian variables, with $\langle H_{ij} \rangle = 0$ and $\mathrm{Var}(H_{ij}) = 1$. We express this as $H_{ij} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1)$.

Wishart matrices appear naturally in various areas of physics, such as in quantum information theory [7]. The spectral properties of these matrices are well studied, and their eigenvalue density is known to converge (in the large $N$ limit) to the Marčenko-Pastur distribution. This result can be derived using the Stieltjes transform or the resolvent method, as in [4].

To compute the spectral density of $W$ using the Edwards-Jones formula, we need the *joint probability density function*, or jpdf, of the independent matrix elements $W_{ij}$ for $i \leq j$, which is given by [4]:

$$\rho[W] = \int \prod_{i,j} dH_{ij}\, \rho_H[H] \delta(W - HH^T),$$

where $\rho_H(H)$ is the jpdf of the matrix elements of $H$ stated above. Therefore given an arbitrary function of $W$, $F(W)$, such as the partition function in Eq. (2), we have:

$$\int \prod_{i\leq j} dW_{ij} \rho[W] F(W) = \int \prod_{i,j} dH_{ij} \rho_H[H] F(HH^T). \tag{4}$$

## III. SPECTRAL DENSITY OF WL MATRICES

### A. Annealed calculation

We proceed now to compute the spectral density for the Wishart-Laguerre ensemble in the limit $N, M \to \infty$, keeping $c = N/M$ fixed ($c \leq 1$), for the annealed approximation. The annealed partition function is:

$$\langle Z(x) \rangle = \int_{\mathbb{R}^N} d\mathbf{y} \exp\left( \frac{-ix_\epsilon}{2} \mathbf{y}^\mathbf{T} \mathbf{y} \right) \left\langle \exp\left( \frac{i}{2} \mathbf{y}^\mathbf{T} W \mathbf{y} \right) \right\rangle_W . \tag{5}$$

Using Eq. (4), we can compute the last factor in Eq. (5)

as:

$$\left\langle \exp\left(\frac{i}{2}\mathbf{y}^{\mathbf{T}}W\mathbf{y}\right)\right\rangle_W = \left\langle \exp\left(\frac{i}{2}\mathbf{y}^{\mathbf{T}}HH^T\mathbf{y}\right)\right\rangle_H =$$
$$= \left\langle \exp\left[\frac{i}{2}\sum_{i=1}^{M}\left(\sum_{j=1}^{N}H_{ji}y_j\right)^2\right]\right\rangle_H =$$
$$= \left\langle \exp\left(\frac{i}{2}\sum_{i=1}^{M}A_i^2\right)\right\rangle_H ,$$

where we have defined the $M$-dimensional Gaussian random vector $A$ as: $A_i = \sum_{j=1}^{N} H_{ji}y_j, \forall i = 1, ..., M$. Since each component is a linear combination of different and independent Gaussian random variables with distribution $\mathcal{N}(0,1)$, every $A_i$ itself is a Gaussian random variable, with $\langle A_i\rangle = 0$ and $\mathrm{Var}(A_i) = \|\mathbf{y}\|^2$. This means that $A_i \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \|\mathbf{y}\|^2)$, therefore:

$$\left\langle \exp\left(\frac{i}{2}\mathbf{y}^{\mathbf{T}}W\mathbf{y}\right)\right\rangle_W = \prod_{i=1}^{M}\left\langle \exp\left(\frac{i}{2}A_i^2\right)\right\rangle =$$
$$= \left\langle \exp\left(\frac{i}{2}A_1^2\right)\right\rangle^M = \left(1 - i\|\mathbf{y}\|^2\right)^{-M/2}, \quad (6)$$

were in the last equality we have used the know result for the characteristic function for the square of a Gaussian variable. Inserting the result in Eq. (6) into the equation for $\langle Z(x)\rangle$ in Eq. (5) we get:

$$\langle Z(x)\rangle = \int_{\mathbb{R}^N} d\mathbf{y}\exp\left(\frac{-ix_\epsilon}{2}\|\mathbf{y}\|^2\right)\left(1 - i\|\mathbf{y}\|^2\right)^{-M/2}. \quad (7)$$

We can convert the last factor in Eq. (7) into an exponential using the following identity derived from the Gamma function definition:

$$\left(1 - i\|\mathbf{y}\|^2\right)^{-M/2} \propto \int_0^\infty u^{\frac{M}{2}-1}e^{-u\left(1 - i\|\mathbf{y}\|^2\right)}du. \quad (8)$$

Inserting Eq. (8) into Eq. (7) we get, after some manipulation, the following expression:

$$\langle Z(x)\rangle \propto \int_0^\infty u^{\frac{N}{2c}-1}e^{-u}\left[i\left(\frac{x_\epsilon}{2} - u\right)\right]^{-N/2}du =$$
$$= \int_0^\infty e^{N\mathcal{S}[u;x]}du, \quad (9)$$

where we identified the *effective action*, $\mathcal{S}[u;x]$, as:

$$N\mathcal{S}[u;x] = \left(\frac{N}{2} - 1\right)ln(u) - u - \frac{N}{2}\log\left[i\left(\frac{x_\epsilon}{2} - u\right)\right].$$

We can apply the Laplace approximation to the last integral in Eq. (9), which consists in approximating the integral by the leading term, which is the one that maximizes $\mathcal{S}$. Since we are considering the limit $N \to \infty$, the Laplace approximation yields the exact result. After some manipulation, we get the following expression for Eq. (1):

$$\rho(x) = \frac{-2}{\pi}\lim_{\epsilon\to 0^+}\Im\frac{\partial}{\partial x}\mathcal{S}[u_*;x] =$$
$$= \frac{-2}{\pi}\lim_{\epsilon\to 0^+}\Im\left[\frac{-1}{4}\left(\frac{1}{x_\epsilon/2 - u_*}\right)\right], \quad (10)$$

where $u_*$ is the value which makes the action $\mathcal{S}$ maximum. Solving $0 = \partial_u\mathcal{S}$, neglecting terms of order lower than $\mathcal{O}(N)$, we get $u_* = \frac{-x+i\epsilon}{2(1-c)}$. Inserting this value into Eq. (10) we obtain:

$$\rho(x) \propto \frac{1}{\pi}\lim_{\epsilon\to 0^+}\Im\left(\frac{x + i\epsilon}{x^2+\epsilon^2}\right) = \frac{1}{\pi}\lim_{\epsilon\to 0^+}\frac{\epsilon}{x^2+\epsilon^2}.$$

Finally, imposing the normalization condition for $\rho(x)$, since it is a probability density function, and considering that $\lim_{\epsilon\to 0^+}\frac{\epsilon}{x^2+\epsilon^2} = \pi\delta(x)$, we obtain the desired spectral density for the annealed approximation:

$$\rho(x) = \delta(x), \quad (11)$$

which is not the correct asymptotic spectral density for the WL ensemble, i.e. the Marčenko-Pastur density. Therefore, the annealed approximation does not yield the correct result for the Wishart-Laguerre ensemble, and a more rigorous approach is needed.

### B. Replica-symmetric quenched calculation

Using the replica method described in Eq. (3), we compute the spectral density in a more rigorous approach. The replicated partition function $Z^n(x)$ corresponds to the partition function for a system of $n$ identical independent copies, i.e. replicas, of our original system, and it is given by:

$$\langle Z^n(x)\rangle = \int\prod_{a=1}^{n}d\mathbf{y}^a\exp\left(-\frac{ix_\epsilon}{2}\sum_{a=1}^{n}\|\mathbf{y}^a\|^2\right)$$
$$\times\left\langle \exp\left(\frac{i}{2}\sum_{a=1}^{n}\mathbf{y}^{a^T}W\mathbf{y}^a\right)\right\rangle_W, \quad (12)$$

where $a = 1, ..., n$ is the so-called replica index. Using Eq. (4), and denoting the trace of a matrix as Tr, we can express the last factor in Eq. (12) as:

$$\left\langle \exp\left(\frac{i}{2}\sum_{a=1}^{n}\mathbf{y}^{a^T}W\mathbf{y}^a\right)\right\rangle_W =$$
$$= \left\langle \exp\left[\frac{i}{2}\mathrm{Tr}\left(H^T\left(\sum_{a=1}^{n}\mathbf{y}^a\mathbf{y}^{a^T}\right)H\right)\right]\right\rangle_H =$$
$$= \left\langle \exp\left(\frac{i}{2}\mathrm{Tr}\left(H^TYY^TH\right)\right)\right\rangle_H. \quad (13)$$

In Eq. (13) we have defined the matrix of replica vectors $Y = (\mathbf{y}^1, ..., \mathbf{y}^n) \in \mathbf{R}^{N\times n}$. Since each of the $M$

columns of $H$ is a Gaussian random vector, $H_\mu \overset{\text{i.i.d}}{\sim} \mathcal{N}(0, \mathbb{I}_N)$, we can use the following identity:

$$\left\langle \exp\left(\frac{i}{2}\mathrm{Tr}\left(H^T YY^T H\right)\right)\right\rangle_H = \left[\mathrm{Det}\left(\mathbb{I}_N - iYY^T\right)\right]^{-M/2}.$$

Then, Eq. (12) reads:

$$\langle Z^n(x)\rangle = \int \prod_{a=1}^{n} d\mathbf{y}^a \exp\left(-\frac{ix_\epsilon}{2}\sum_{a=1}^{n}\|\mathbf{y}^a\|^2\right) \times \left[\mathrm{Det}\left(\mathbb{I}_N - iYY^T\right)\right]^{-M/2}. \qquad (14)$$

Proceeding as in [6], we introduce the *replica overlap matrix*, $Q \in \mathbb{R}^{n\times n}$, as:

$$Q_{ab} \doteq \frac{1}{N}\left(Y^T Y\right)_{ab} = \frac{1}{N}\mathbf{y}^{a^T}\mathbf{y}^b. \qquad (15)$$

We treat this variables as the new independent integration variables, and we enforce the definition in Eq. (15) with the use of the following Dirac delta functional representation for the identity:

$$\mathbb{I} = \int dQ d\hat{Q}\, \exp\left[\frac{iN}{2}\mathrm{Tr}(Q\hat{Q}) - \frac{i}{2}\sum_{a,b=1}^{n}\hat{Q}_{ab}\mathbf{y}^{a^T}\mathbf{y}^b\right], \qquad (16)$$

where $\hat{Q} \in \mathbb{R}^{n\times n}$ is the conjugate variable of $Q$. After some manipulation, inserting Eq. (16) into Eq. (14) yields:

$$\langle Z^n(x)\rangle = \int dQ d\hat{Q}\, \exp\left\{-N\mathcal{S}[Q,\hat{Q};x]\right\}, \qquad (17)$$

where we identified the *effective action*, $\mathcal{S}$, as:

$$\mathcal{S}\left[Q,\hat{Q};x\right] = \frac{-i}{2}\mathrm{Tr}(Q\hat{Q}) + \frac{M}{2N}\log\mathrm{Det}(\mathbb{I}_n - iNQ) + \frac{1}{2}\log\mathrm{Det}(x_\epsilon\mathbb{I}_n + \hat{Q}). \qquad (18)$$

We can compute the integral in Eq. (17) in the $N \to \infty$ limit with the Laplace approximation. In principle, we should find the extremum of $\mathcal{S}$ with respect to all possible matrices $Q$ and $\hat{Q}$. Therefore, in order to continue, it is necessary to make an assumption about the behavior of these matrices. We assume the so-called *replica symmetric ansatz*:

$$Q_{ab} = q\,\delta_{ab} \quad \hat{Q}_{ab} = \hat{q}\,\delta_{ab}.$$

With this assumption, the action in Eq. (18) becomes simply $\mathcal{S}[q,\hat{q};x] = ns[q,\hat{q};x]$, with $s$ being:

$$s[q,\hat{q};x] = \frac{-i}{2}q\hat{q} + \frac{1}{2c}\log(1-iNq) + \frac{1}{2}\log(x_\epsilon + \hat{q}). \qquad (19)$$

The expectation value for the replicated partition function is, performing the Laplace approximation in Eq. (17):

$$\langle Z^n(x)\rangle = \exp\left[-Nns(q^*,\hat{q}^*;x)\right], \qquad (20)$$

where $q^*$ and $\hat{q}^*$ are the values that maximize the function $s$ given in Eq. (19). Applying the replica identity in Eq. (3) to the expression for $\langle Z^n(x)\rangle$ given by Eq. (20), we get:

$$\langle \log Z(x)\rangle = \lim_{n\to 0}\frac{1}{n}\log\langle Z^n(x)\rangle = -Ns(q^*,\hat{q}^*;x),$$

and inserting this into Eq. (1) reads:

$$\rho(x) = \frac{2}{\pi}\lim_{\epsilon\to 0^+}\Im\frac{\partial s(q^*,\hat{q}^*;x)}{\partial x} = \frac{2}{\pi}\lim_{\epsilon\to 0^+}\Im\left(\frac{1}{2(x_\epsilon + \hat{q}^*)}\right). \qquad (21)$$

Solving the system of equations $\partial_q s = \partial_{\hat{q}} s = 0$, dropping the superscript $^*$ from now on, we get:

$$\hat{q} = \frac{-N}{c(1-iNq)} \quad q = \frac{-i}{x_\epsilon + \hat{q}}. \qquad (22)$$

Examining the expression for $q$ in Eq. (22) and the expression inside $\Im$ in Eq. (21), we can write the spectral density as:

$$\rho(x) = \frac{2}{\pi}\lim_{\epsilon\to 0^+}\Im\left(\frac{iq(x)}{2}\right) = \frac{1}{\pi}\lim_{\epsilon\to 0^+}\Re\, q(x), \qquad (23)$$

where $\Re$ denotes the real part of a complex number. The real part of $q(x)$ can be computed solving for $q$ in Eq. (22). After some algebraic manipulations, $\Re\, q(x)$ can be expressed as:

$$\Re\, q(x) = \frac{-1}{2Nc(x^2+\epsilon^2)}\left[\epsilon N(1-c) \pm (xd + \epsilon p)\right], \qquad (24)$$

where $d$ and $p$ are expressions derived during the algebraic manipulation. We only give the expression for $d$ in the $\epsilon \to 0^+$ limit, since it is the only one we will need:

$$d = \sqrt{\frac{|a_0| - a_0}{2}}, \quad \text{with} \quad a_0 = c^2(x-\gamma^+)(x-\gamma^-);\ \ \gamma^{\pm} = N(1 \pm c^{-1/2})^2. \qquad (25)$$

Inserting Eq. (24) into Eq. (23), taking the limit $\epsilon \to 0^+$, and considering the expression for $d$ in Eq. (25), we obtain the following expression for the spectral density:

$$\rho(x) = \frac{1}{2\pi N x}\sqrt{(\gamma^+ - x)(x-\gamma^-)}; \quad \gamma^{\pm} = N(1\pm c^{-1/2})^2,$$

which, after reescaling the eigenvalues as $x = Ny$, yields the Marčenko-Pastur density:

$$\rho(y) = \frac{1}{2\pi y}\sqrt{(\zeta^+ - y)(y-\zeta^-)}; \quad \zeta^{\pm} = \left(1 \pm c^{-1/2}\right)^2 \qquad (26)$$

## IV. NUMERICAL VERIFICATION

The annealed and quenched spectral densities differ markedly. While the annealed approximation in Eq. (12)

yields a Dirac delta, the quenched result in Eq. (26) correctly reproduces the Marčenko–Pastur distribution, which describes the asymptotic eigenvalue density of Wishart matrices.

The failure of the annealed approximation stems from persistent fluctuations in the partition function $Z(x)$ from Eq. (2), even as $N \to \infty$. As a result, $\langle \log Z(x) \rangle$ and $\log\langle Z(x) \rangle$ diverge significantly, consistent with Jensen's inequality. For GOE matrices, such fluctuations vanish in the large $N$ limit, causing both expressions to coincide asymptotically and making the annealed approximation accurate.

The correct asymptotic spectral density for the WL ensemble can also be derived using methods from the physics of disordered systems beyond the replica trick, such as the *cavity method*, which is illustrated in [6].

Finally, we confirm this behavior numerically by diagonalizing large Wishart matrices and comparing the empirical spectral density with theoretical predictions. All linear algebra computations were performed using NumPy's *linalg* module, which provides a high-level interface to LAPACK routines in Python.

Figure 1 shows the results for $N = 150$ and two values of $c$, illustrating excellent agreement with the Marčenko–Pastur law. Figures 2 and 3 highlight convergence near the spectral edges as $N$ increases.

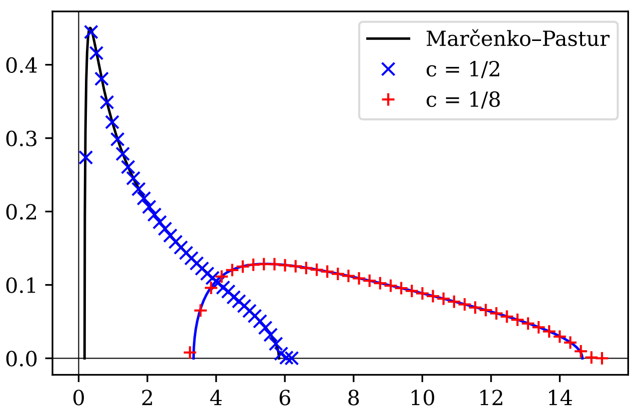


FIG. 1: Empirical spectral density for $N = 150$ and $c = 1/2$, 1/8, vs. Marčenko-Pastur law.

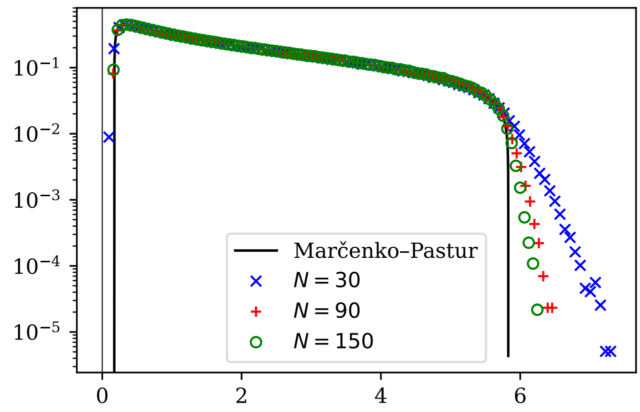


FIG. 2: Convergence to the theoretical curve for $c = 1/2$. Lin-log scale highlights convergence near spectral edges for increasing $N$.

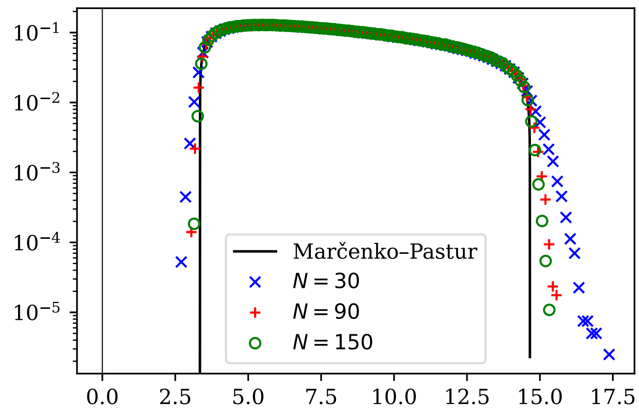


FIG. 3: Convergence to the theoretical curve for $c = 1/8$. Lin-log scale highlights convergence near spectral edges for increasing $N$.

## Acknowledgments

I would like to express my gratitude to my advisor, Dr. Matteo Palassini, for his guidance and dedication. I also thank my partner and family for their support and encouragement during all these years.

[1] Wigner, E. P. "On the statistical distribution of the widths and spacings of nuclear resonance levels". *Proc. Cambridge Philos. Soc.* **47**: 790–798 (1951).
[2] Guhr, T., Müller-Groeling, A. and Weidenmüller, H. A. "Random-matrix theories in quantum physics: common concepts". *Phys. Rep.* **299**: 189–425 (1998).
[3] Derrida, B. "Random-energy model: An exactly solvable model of disordered systems". *Phys. Rev. B* **24**: 2613–2626 (1981).
[4] Livan, G. Novaes, M. and Vivo, P. *Introduction to Random Matrices: Theory and Practice*. Springer, 2018.
[5] Parisi, G. "Infinite Number of Order Parameters for Spin-Glasses". *Phys. Rev. Lett.* **43**: 1754–1756 (1980).
[6] Mézard, M. and Montanari, A. *Information, Physics, and Computation*. Oxford University Press, 2009.
[7] Majumdar, S. N. "Extreme eigenvalues of Wishart matrices: Application to entangled bipartite systems". *The Oxford Handbook of Random Matrix Theory*, Oxford University Press (2011).

## Densitat espectral de matrius de Wishart i el mètode de les rèpliques

Author: Pol Martínez Ortega, pmartior8@alumnes.ub.edu
*Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.*

Advisor: Matteo Palassini, palassini@ub.edu

**Resum:** La teoria de matrius aleatòries proporciona un marc potent per comprendre les característiques dels sistemes complexos, especialment en el límit de grans dimensions. En aquest treball, estudiem la densitat espectral de les matrius aleatòries de Wishart-Laguerre mitjançant la fórmula d'Edwards-Jones. Reinterpretem la quantitat mitjanada a la fórmula d'Edwards-Jones com la funció de partició d'un sistema desordenat, i apliquem tècniques de la física estadística. El resultat il·lustra com les eines de la física estadística de sistemes desordenats s'apliquen de forma natural a la teoria de matrius aleatòries, oferint una comprensió física i mètodes analítics per estudiar les propietats espectrals en sistemes complexos.
**Paraules clau:** Sistemes desordenats, autovalors, funcions de partició
**ODSs:** ODS 4 - Eduació de qualitat, ODS 9 - Indústria, innovació i infraestructures

**Objectius de Desenvolupament Sostenible (ODSs o SDGs)**

| | | | |
|---|---|---|---|
| 1. Fi de la es desigualtats | | 10. Reducció de les desigualtats | |
| 2. Fam zero | | 11. Ciutats i comunitats sostenibles | |
| 3. Salut i benestar | | 12. Consum i producció responsables | |
| 4. Educació de qualitat | X | 13. Acció climàtica | |
| 5. Igualtat de gènere | | 14. Vida submarina | |
| 6. Aigua neta i sanejament | | 15. Vida terrestre | |
| 7. Energia neta i sostenible | | 16. Pau, justícia i institucions sòlides | |
| 8. Treball digne i creixement econòmic | | 17. Aliança pels objectius | |
| 9. Indústria, innovació, infraestructures | X | | |