Quantum Jump Approach to Driven-Dissipative Atom Dynamics

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Abstract: Spontaneous photon emission by atoms is a fundamental quantum process with significant implications for quantum technologies. We study the dynamics of N laser-driven two-level atoms interacting with the electromagnetic vacuum and spontaneously emitting photons. Although the framework of master equations for the density matrix provides a rigorous description of this process, its computational cost scales exponentially as $\sim 2^{2N}$. As an alternative, we implement the quantum jump method, a stochastic approach based on averaging over quantum trajectories. We begin with the case of a single atom and then extend the analysis to a chain of closely spaced atoms, focusing on the error scaling in the stochastic method. By comparing computational performance, we find that the quantum jump approach becomes increasingly advantageous for larger systems. These results establish quantum trajectories as a reliable and efficient tool for simulating collective spontaneous emission in complex quantum systems.
Keywords: Atom-light interactions, electric dipole moment, decay rate, Hamiltonian, Monte Carlo simulation, density matrix
SDGs: Quality education; Affordable and Clean Energy; Industry, innovation and infrastructure

I. INTRODUCTION

Spontaneous photon emission is one of the most fundamental quantum processes, arising from the interaction between a quantum emitter and the vacuum fluctuations of the electromagnetic field. Control over this process is crucial in quantum technologies. For instance, tailoring decay rates enables the creation of on-demand single-photon sources for quantum communication, and extends qubit coherence in quantum computing [1].

The rate at which an excited atom decays depends on several factors, including the strength of its electric dipole transition and the density of available electromagnetic modes at the transition frequency. When a second atom is placed nearby, the decay dynamics can be significantly altered due to the indirect interaction mediated by the shared vacuum field. This interaction leads to collective radiative effects such as superradiance (enhanced emission) and subradiance (suppressed emission) [2, 3].

Modeling the dynamics of such systems, especially when extended to many atoms, is highly nontrivial, due to the high number of photonic and atomic degrees of freedom involved. For many-body dipole–dipole coupled systems, one typically employs a density-matrix master equation which rigorously describes the open-system evolution of the atomic ensemble [4]. However, even under the simple assumption of atoms at fixed positions (neglecting motion), the size of the Hilbert increases exponentially with the number of atoms. This scaling severely limits the applicability of exact numerical methods based on the density matrix formalism.

To address this challenge, we explore an alternative approach based on the quantum jump (or quantum trajectories) formalism. This method, developed in parallel by several groups [5, 6], provides a stochastic unraveling of the master equation in terms of individual quantum trajectories, offering a conceptually intuitive and computationally efficient framework. In particular, quantum jump simulations scale more favorably for systems with many atoms and can reveal dynamical features, such as intermittent emission and correlations, that are obscured in ensemble-averaged treatments.

In this work, we apply the quantum jump formalism to obtain the dynamics of a system of atoms undergoing spontaneous photon emission and driven by a laser field. We first analyse the case of a single atom, and then extend the study to an atomic chain where dipole–dipole interactions play a significant role. We focus on analysing the error performed by the stochastic unraveling compared to exact integration of the master equation.

II. THEORETICAL FRAMEWORK

A. Lindblad Master Equation for a Single Atom

We consider a quantum system consisting of a single atom that can only occupy two energy levels (a *two-level atom*): the ground state $|g\rangle$ and the excited state $|e\rangle$. This atom interacts both with the set of modes of the quantized electromagnetic field in its ground state and with a classical, monochromatic laser field. On the one hand, the interaction with the vacuum is responsible for the phenomenon of spontaneous emission and therefore introduces dissipation and irreversibility into the system. On the other hand, the classical field can excite the atom. It is the fact that the laser has a well-defined frequency, close to a single atomic transition, that justifies the *two-level atom* approximation.

The light field (or more precisely, the continuum of vacuum photon modes) has an effectively infinite number of degrees of freedom, making it impossible to track its

dynamics in detail. A standard approach in quantum optics is to treat the atomic system as an open quantum system, coupled to an external bath corresponding to the light field. In this case, the system does not follow a unitary evolution governed by the Schrödinger equation. Instead, a more general description based on the atomic density matrix must be adopted.

By tracing out the photonic degrees of freedom and under the Born-Markov approximation [4], it is possible to arrive to a differential equation only depending on the atomic reduced density matrix, known as the Lindblad master equation (from now on $\hbar = 1$):

$$\partial_t \rho = -i\,[\mathcal{H}_0, \rho] + \mathcal{L}(\rho). \tag{1}$$

Here, the first term is unitary and describes the coherent dynamics of the system (it is essentially the Schrödinger–von Neumann equation). The Hamiltonian $\mathcal{H}_0$ describes the atom–laser coupling, which can be written in the rotating wave approximation as:

$$\mathcal{H}_0 = -\delta\,\sigma^+\sigma^- + \frac{\Omega}{2}(\sigma^+ + \sigma^-), \tag{2}$$

where $\sigma^+ = |e\rangle\langle g|$ and $\sigma^- = (\sigma^+)^\dagger = |g\rangle\langle e|$ correspond, respecively, to the creation and anihilation operators of an atomic excitation. The parameter $\delta = \omega_L - \omega_A$ is the detuning between the laser and atomic frequencies and $\Omega = -d\mathcal{E}_0$ is the Rabi frequency and characterizes the coupling between the atomic dipole d and the laser electric field (the strength of the atom–field coupling).

In contrast, the second term of the master equation corresponds to the Lindblad dissipator, which renders the dynamics non-unitary and represents the interaction with the external photonic bath. For a single atom in free space and interacting with the electromagnetic vacuum it acquires the following form:

$$\mathcal{L}(\rho) = \frac{\Gamma}{2}\left(2\sigma^-\rho\sigma^+ - \sigma^+\sigma^-\rho - \rho\sigma^+\sigma^-\right), \tag{3}$$

where $\Gamma = d^2\omega_0^3/3\pi c^3\epsilon_0$ is the *spontaneous decay rate*, i.e., the probability per unit time that the system undergoes a spontaneous transition from $|e\rangle$ to $|g\rangle$.

B. Lindblad Master Equation for N Atoms

When several atoms couple to the same radiation mode, this induces coherent dipole-dipole interactions between the atoms and collective spontaneous photon emission. Following the same procedure as for a single atom, a master equation of the Lindblad form Eq. (1) can be derived [7], now with:

$$\mathcal{H}_0 = -\delta\sum_{i=1}^{N}\sigma_i^+\sigma_i^- + \frac{\Omega}{2}\sum_{i=1}^{N}(\sigma_i^+ + \sigma_i^-) + \sum_{\substack{i,j=1\\ i\neq j}}^{N} J_{ij}\sigma_i^+\sigma_j^-,$$

$$\mathcal{L}(\rho) = \sum_{i,j=1}^{N}\frac{\Gamma_{ij}}{2}\left(2\sigma_j^-\rho\sigma_i^+ - \sigma_i^+\sigma_j^-\rho - \rho\sigma_i^+\sigma_j^-\right). \tag{4}$$

Here the operators σ_i^+ and σ_i^- correspond to the creation and anihilation operators of a single excitation of the i-th atom. The couplings $J_{ij} = \Re[G_{ij}]$ and $\Gamma_{ij} = -2\Im[G_{ij}]$ are determined by the real and imaginary parts of the Green's function propagator, which dictates how the photon propagates between two atoms at distance r_{ij}. In the case of a chain of atoms transversally polarized with electric dipole moment transverse to the chain axis, it takes the simple form $G(r) = (3\Gamma e^{ikr}/4k^3r^3)(k^2r^2 + 3ikr - 3)$.

C. Quantum Jump Approach

We present here an alternative approach using a wave function treatment to describe the atomic system. The apparent incompatibility between such a wave function approach and the inherent irreversibility of the spontaneous process we are dealing with is lifted by introducing repeated *gedanken measurements* on the atomic system simulating the detection of the spontaneously emitted photons. The random result of each of these measurements determines the atomic state afterward and is at the origin of the irreversibility.

Let us begin by rewriting Eq. (1) in the following form:

$$\partial_t \rho = -i\,[\mathcal{H}_{\text{eff}}, \rho]_* + \Gamma\sigma^-\rho\sigma^+, \tag{5}$$

where we define the effective non-Hermitian Hamiltonian as $\mathcal{H}_{\text{eff}} \equiv \mathcal{H}_0 - i\frac{\Gamma}{2}\sigma^+\sigma^-$ and the commutator $[A, B]_* \equiv A\cdot B - B^\dagger\cdot A^\dagger$.

We now assume that all spontaneously emitted photons are detected with a perfect photon counter, and we perform a measurement at time $t + dt$ to determine whether a photon has been emitted. Depending on the outcome of this measurement (either 0 or 1 detected photons), the system's wave function $|\Psi(t + dt)\rangle$ is projected accordingly, resulting either in a quantum jump to the ground state or a no-jump evolution.

The probability that a spontaneous emission occurs during the infinitesimal time interval dt is given by $dp = \Gamma|c_e|^2 dt$, where c_e is the excited-state amplitude of the wave function $|\Psi(t)\rangle$. Numerically, in order to mimic the randomness of the measurement result, we choose a random number ϵ uniformly distributed between 0 and 1. The two possible cases $\epsilon > dp$ and $\epsilon < dp$ correspond, respectively, to the detection of 0 and 1 photon.

In the case where no photon is detected (that is, no quantum jump occurs), the system evolves under the action of the non-Hermitian Hamiltonian $\mathcal{H}_{\text{eff}}$. This non-unitary evolution leads to a gradual decrease in the norm of the wave function, reflecting the probability loss due to possible emission. The state evolves as $|\Psi(t + dt)\rangle = \exp\left(-i\mathcal{H}_{\text{eff}}dt\right)|\Psi(t)\rangle$, which must then be normalized. On the other hand, if a photon is detected during dt, this implies that the atom has spontaneously decayed from the excited state to the ground state. In this case, the wave function undergoes a quantum jump and collapses to $|\Psi(t + dt)\rangle = |g\rangle$. These two possible evolutions define the stochastic dynamics of the system

under continuous photodetection, forming the basis of the quantum jump formalism.

This method can be directly generalized to the N atom case, replacing $\mathcal{H}_0$ by the expression in Eq. (4). Also, there are now N independent decay channels described by the operators $O_k = \sum_i V_i^k \sigma_i^-$, $k \in \{1, \cdots, N\}$, where V_i^k are the coefficients of the eigenstates of the Γ_{ij} matrix. Once the photon emission takes place, the atoms are projected to one of the states $|\Psi(t+\delta t)\rangle = O_k |\Psi(t)\rangle$ with probability proportional to $p_k = \langle\Psi(t)| O_k^\dagger O_k |\Psi(t)\rangle$, and normalized to 1 [8].

Error quantification.– In order to compare both methods, we can use the trace distance, a quantitative measure of the distinguishability between two quantum states. Given two density matrices ρ and σ, the trace distance is defined as

$$T(\rho, \sigma) = \frac{1}{2}\mathrm{Tr}\left[\sqrt{(\rho-\sigma)^\dagger(\rho-\sigma)}\right]. \tag{6}$$

Physically, the trace distance quantifies the maximum difference in measurement outcome probabilities between the two states under optimal measurement. A value of $T(\rho, \sigma)$ close to 0 indicates that the states are experimentally indistinguishable, while a value of $T(\rho, \sigma)$ close to 1 means that they are perfectly distinguishable.

We choose $\rho = \rho_{\mathrm{jump}} = \sum_{i=1}^{N_{\mathrm{traj}}} |\Psi\rangle_i \langle\Psi|_i / N_{\mathrm{traj}}$ as the state obtained over many quantum jump trajectories (here $|\Psi\rangle_i$ denotes the state obtained within a single trajectory i), while $\sigma = \rho_{\mathrm{exact}}$ is the state computed by direct integration of the master equation. Hence, a small trace distance between ρ_{jump} and ρ_{exact} proves that the quantum trajectories method faithfully reproduces the dynamics predicted by the master equation.

III. RESULTS

A. Single-atom Population Dynamics

The Lindblad master equation Eq. (1) yields for each density matrix component a set of differential equations, commonly known as the Optical Bloch Equations (OBE):

$$\partial_t \rho_{ee} = i\frac{\Omega}{2}(\rho_{ge} - \rho_{eg}) + \Gamma\rho_{ee}, \tag{7}$$

$$\partial_t \rho_{ge} = i\frac{\Omega}{2}(\rho_{gg} - \rho_{ee}) - \left(i\delta + \frac{\Gamma}{2}\right)\rho_{ge}, \tag{8}$$

and $\partial_t \rho_{gg} = -\partial_t \rho_{ee}$, $\partial_t \rho_{eg} = (\partial_t \rho_{ge})^*$.

Using the fourth-order Runge-Kutta method, we can numerically solve the OBEs exactly and observe the evolution of, for example, the excited-state population $\rho_{ee} = \langle\hat{n}\rangle$, where $\hat{n} = \sigma^+\sigma^-$ is the excitation number operator. This is shown in Fig. 1, for different values of the system parameters δ/Ω and Γ/Ω.

To begin with, we consider the role of the detuning δ. As the system moves away from resonance ($\delta = 0$),

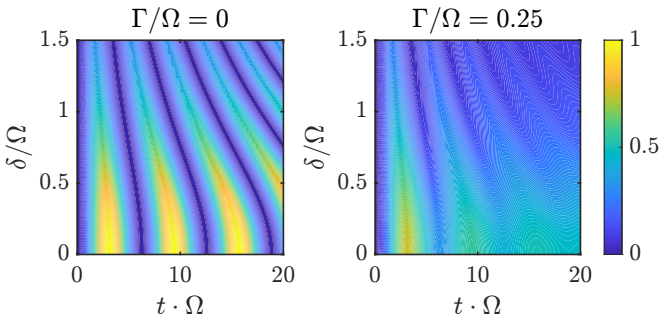

FIG. 1: Time evolution of the excited state population ρ_{ee} for different values of detuning δ and decay rate Γ in units of Rabi frequency Ω.

the excitation probability is expected to decrease, since the laser becomes less effective at driving transitions. In addition, the population oscillates with a renormalized frequency $\sqrt{\Omega^2 + \delta^2/4}$. In parallel, we also explore the influence of the decay rate Γ, which leads to a damping in the oscillations. Increasing Γ reduces the population of the excited state and leads to faster relaxation towards the ground state.

Next, we may ask whether the wave function approach yields similar behavior. To investigate this, we employ a Monte Carlo method; that is, we generate a large number of individual stochastic trajectories of the system, as explained in Sec. II C. We define the average excitation number over N_{traj} different simulated trajectories as:

$$\langle\hat{n}\rangle_{\mathrm{av}} = \frac{1}{N_{\mathrm{traj}}}\sum_{i=1}^{N_{\mathrm{traj}}} \langle\hat{n}\rangle_i, \tag{9}$$

and evaluate the estandard deviation of the mean as:

$$\Delta\langle\hat{n}\rangle_{\mathrm{av}} = \frac{1}{\sqrt{N_{\mathrm{traj}}(N_{\mathrm{traj}}-1)}}\sum_{i=1}^{N_{\mathrm{traj}}}\left[\langle\hat{n}\rangle_i^2 - \langle\hat{n}\rangle_{\mathrm{av}}^2\right]. \tag{10}$$

This corresponds to the standard deviation of a single trajectory divided by $\sqrt{N_{\mathrm{traj}}}$.

Having qualitatively verified that the wave function approach reproduces the same behavior as the solution of the master equation, we now aim to quantify the error introduced by this alternative method and how it depends on the number of trajectories used. Since the exact time-dependent density matrix is known from solving the OBEs, we can compute the trace distance $T(\rho_{\mathrm{jump}}, \rho_{\mathrm{exact}})$ as defined before and for different number of trajectories.

In Fig. 4, we show $T(\rho_{\mathrm{jump}}, \rho_{\mathrm{exact}})$ for a single atom as a function of the number of trajectories (solid red circles), in logarithmic scale for both axis, together with the many-body case results that will be studied later. The results clearly exhibit a power-law decrease in error with the number of trajectories, following a $1/\sqrt{N_{\mathrm{traj}}}$ scaling. This behavior is expected due to the statistical nature of the wave function approach. Assuming each trajectory

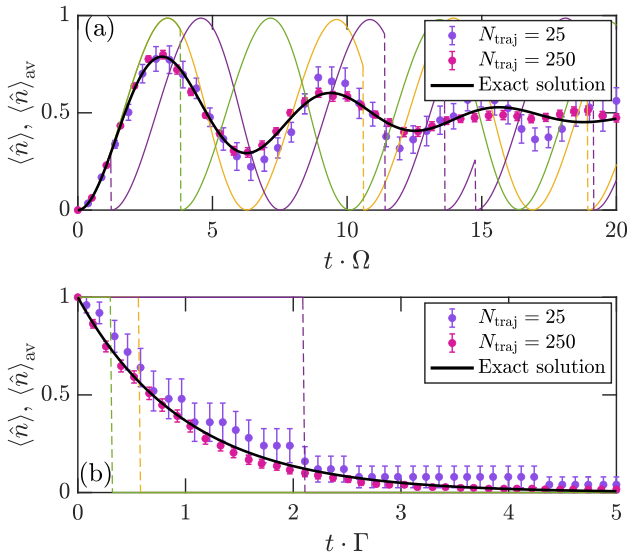

FIG. 2: Time evolution of the excited state population of a two-level atom: (a) driven and initially in the ground state ($\Gamma/\Omega = 0.2$, $\delta/\Omega = 0.1$), and (b) undriven and initially in the excited state ($\Omega = 0$). The excited state population $\langle \hat{n} \rangle = \rho_{ee}$ from the master equation (solid black line) is compared with the trajectories average $\langle \hat{n} \rangle_{\text{av}}$ (full circles), for the number of trajectories N_{traj} indicated in the legend. The error bars represent the calculated standard deviation of the mean. Additionally, three sample trajectories are shown in each subplot, illustrating individual quantum jumps.

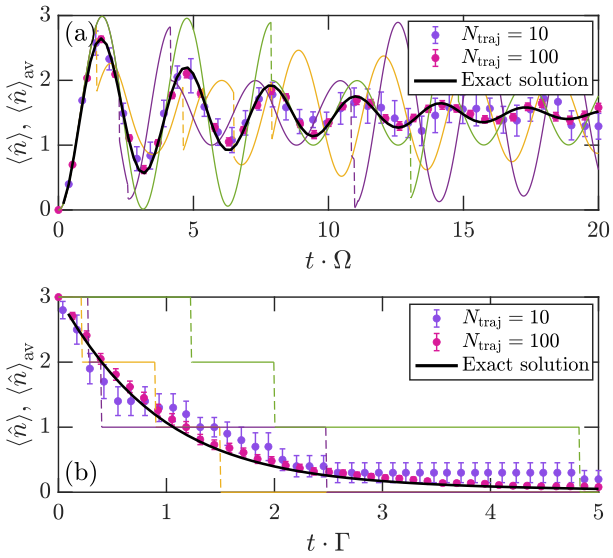

FIG. 3: Time evolution of the average excitation number for $N = 3$ two-level atoms: (a) driven and initially all atoms in the ground state ($\Gamma/\Omega = 0.2$, $\delta/\Omega = 0.1$), and (b) undriven and initially all atoms in the excited state ($\Omega = 0$). Results from the master equation $\langle \hat{n} \rangle$ (solid black line) are compared with the trajectories average $\langle \hat{n} \rangle_{\text{av}}$ (full circles), for the number of trajectories N_{traj} indicated in the legend. The error bars represent the calculated standard deviation of the mean. Additionally, three sample trajectories are shown in each subplot, illustrating individual quantum jumps.

is independent, and according to the central limit theorem, the error in the mean value of an observable must decrease proportionally to $1/\sqrt{N_{\text{traj}}}$, as in typical Monte Carlo methods.

B. Many-body Population Dynamics

Similarly to the single-atom case, we can verify if the wave function approach accurately captures the time evolution of the system in the many-body case. In Fig. 3, we plot over time the ensemble average excitation number, $\langle \hat{n} \rangle_{\text{av}}$, defined in Eq. (9) now with $\hat{n} = \sum_{j=1}^{N} \sigma_j^+ \sigma_j^-$. This represents the average over all trajectories of the quantum mechanically expected value of the number of excited atoms. We can see that the quantum jump method successfully reproduces the expected behavior of the system. In contrast to the single-atom results shown in Fig. 2, where quantum jumps manifest as discrete transitions between just two levels, here each event signals the decay of one atom from the excited to the ground state, and the overall dynamics arise from the accumulation of these individual processes.

In addition, we investigate how the error evolves as a function of the number of trajectories N_{traj} used, and if this dependence is influenced by the atom number N. Fig. 4 shows that, as N increases, the value of the trace distance also increases. While we still observe an approximate power-law scaling $\sim 1/\sqrt{N_{\text{traj}}}$ in the error, deviations from this trend become more pronounced as the system size increases. This is possibly due to the numerical inaccuracy of the solver for the unitary evolution (Runge-Kutta method), as we expect the error will increase as the Hilbert space dimension increases. It could in principle be reduced by decreasing the tolerance error in the integration method.

Another comparison that can be made between the two approaches concerns the computation time required by each method as a function of the number of atoms in the system. In Fig. 5 we observe that initially, the quantum jump method is significantly slower than the exact approach. However, for a system of N two-level atoms, the Hilbert space has dimension 2^N, and thus the density matrix involves 2^{2N} elements. This leads to a computational cost that scales exponentially with $\sim 2^{2N}$ for the exact master equation approach. In contrast, each individual quantum trajectory requires the time evolution of a wave function of size 2^N, resulting in a milder scaling, $\sim 2^N$. Therefore, as the number of atoms increases, the difference in computational cost between the two methods decreases, eventually reaching a point where the quantum jump approach becomes computationally more efficient than the exact method. However, we note that the number of trajectories N_{traj}

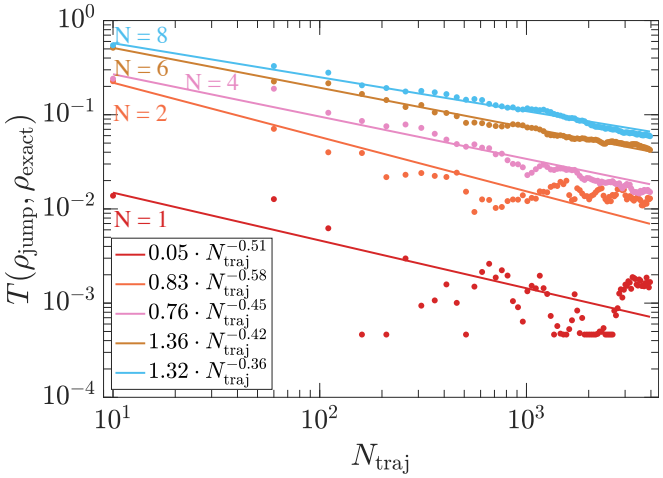

FIG. 4: Trace distance $T(\rho_{\text{jump}}, \rho_{\text{exact}})$ between the exact density matrix and the one reconstructed from quantum jump simulations, as a function of the number of trajectories, for different N. A clear power-law decrease ($\sim N_{\text{traj}}^{-1/2}$) in the error is observed as the number of trajectories increases, while the trace distance increases with larger N.

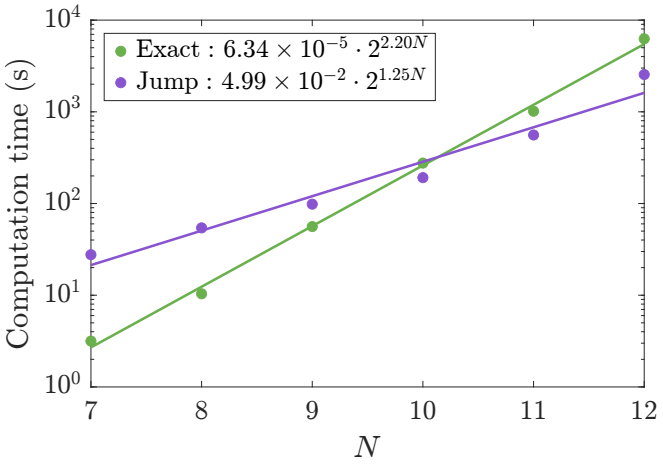

FIG. 5: Scaling of computation time with atom number N using the density matrix formalism and the wave function approach. Both methods were simulated using 50 time steps; 100 trajectories were used for the wave function approach. Simulations were performed on a 4-core machine.

required to achieve a given level of accuracy also affects the overall cost of the jump approach. As the trace distance decreases as $\sim 1/\sqrt{N_{\text{traj}}}$, improving precision demands more simulations. Consequently, the crossover point where the quantum jump method becomes advantageous shifts towards higher atom numbers as stricter accuracy requirements are imposed.

IV. CONCLUSIONS

In this work, we explored the dynamics of open quantum systems through the case of laser driven two-level atoms coupled to the electromagnetic vacuum. Using the Lindblad master equation formalism, we modeled the dissipative and coherent dynamics of the system both in the single-atom case and in the many-body regime, where collective effects became relevant. We also implemented and analyzed the quantum jump approach as an alternative numerical method to simulate open quantum systems. Despite its stochastic nature, we showed that the trajectory-averaged dynamics obtained via quantum jumps reproduced the results of the exact master equation with high fidelity. Specifically, we used the trace distance as a quantitative measure of agreement between the two methods, and confirmed that the error decreased as $1/\sqrt{N_{\text{traj}}}$, consistent with the expected scaling of Monte Carlo methods. We also found that the trace distance between the quantum trajectories and the exact result increased with the number of atoms, but the $1/\sqrt{N_{\text{traj}}}$ scaling remained approximately valid.

Finally, we compared the computational efficiency of both approaches. Although direct integration of the master equation was initially more efficient, we find that for sufficiently large systems, the quantum jump approach became computationally advantageous, offering a scalable method for simulating the dynamics of open many-body quantum systems. These results validated the quantum jump method as a reliable and efficient alternative for solving Lindblad-type master equations.

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Mètode de Salts Quàntics per a la Dinàmica Atòmica Induïda i Dissipativa

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Resum: L'emissió espontània de fotons per part dels àtoms és un procés quàntic fonamental amb implicacions importants per a les tecnologies quàntiques. Estudiem la dinàmica de N àtoms de dos nivells excitats per làser que interactuen amb el buit electromagnètic. Tot i que el formalisme d'equacions mestra per a la matriu de densitat proporciona una descripció rigorosa d'aquest procés, el seu cost computacional escala exponencialment com $\sim 2^{2N}$. Com a alternativa, implementem el mètode de salts quàntics, un enfocament estocàstic basat en la mitjana sobre trajectòries quàntiques. Comencem pel cas d'un sol àtom i després estenem l'anàlisi a una cadena d'àtoms pròxims entre si, centrant-nos en l'escalat de l'error del mètode estocàstic. En comparar el rendiment computacional, trobem que l'enfocament de salts quàntics esdevé progressivament més avantatjós per a sistemes més grans. Aquests resultats estableixen les trajectòries quàntiques com una eina fiable i eficient per simular l'emissió espontània col·lectiva en sistemes quàntics complexos.
Paraules clau: Interaccions entre àtoms i llum, moment dipolar elèctric, ritme de decaïment, Hamiltonià, simulació de Monte Carlo, matriu densitat
ODSs: Educació de qualitat; Energia neta i sostenible; Indústria, innovació, infraestructures

Objectius de Desenvolupament Sostenible (ODSs o SDGs)

1. Fi de la es desigualtats		10. Reducció de les desigualtats	
2. Fam zero		11. Ciutats i comunitats sostenibles	
3. Salut i benestar		12. Consum i producció responsables	
4. Educació de qualitat	X	13. Acció climàtica	
5. Igualtat de gènere		14. Vida submarina	
6. Aigua neta i sanejament		15. Vida terrestre	
7. Energia neta i sostenible	X	16. Pau, justícia i institucions sòlides	
8. Treball digne i creixement econòmic		17. Aliança pels objectius	
9. Indústria, innovació, infraestructures	X		

En primer lloc, aquest treball contribueix a l'ODS 4: Educació de qualitat, ja que fomenta la formació en ciència fonamental i, per tant, contribueix a l'educació a nivell universitari. En segon lloc, la recerca sobre dinàmiques d'emissió espontània i control quàntic pot tenir aplicacions futures en el desenvolupament de sistemes de comunicació i computació més eficients energèticament, en sintonia amb l'ODS 7: Energia neta i sostenible. Finalment, també es pot relacionar amb l'ODS 9: Indústria, innovació i infraestructures, ja que podria contribuir al desenvolupament d'eines computacionals avançades que podrien ser essencials en el disseny de noves tecnologies quàntiques.