Isospin symmetry breaking in infinite nuclear matter

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Abstract: Isospin symmetry, a fundamental concept in nuclear physics, is known to be only approximate in real nuclear systems. In this thesis, we investigate isospin symmetry breaking by quantifying charge symmetry breaking and charge independence breaking effects using the Argonne v_{18} nucleon-nucleon potential. The analysis is conducted within the Hartree-Fock approximation, modeling nucleon wave functions in infinite nuclear matter as plane waves. Expectation values of the relevant interaction terms are computed for both symmetric nuclear matter and pure neutron matter, obtaining repulsive energy corrections of approximately 0.283 MeV and 1.008 MeV per nucleon at saturation density ($\rho_0 = 0.16$ fm^{-3}), respectively. For symmetric nuclear matter, the results obtained align well with the literature, and for pure neutron matter, they are partially consistent. Overall, the study highlights the value of realistic potentials and the usefulness of Hartree-Fock in addressing isospin symmetry breaking in nuclear matter.
Keywords: Isospin symmetry breaking, infinite nuclear matter, Hartree-Fock approximation, nucleon-nucleon interactions
SDGs: This work is aligned with the UN Sustainable Development Goals 4, 7 and 9

I. INTRODUCTION

The concept of isospin, originally introduced by Heisenberg [1] and Wigner [2], has proven to be a powerful tool in understanding the strong nuclear force and the structure of atomic nuclei. It treats the proton and neutron as two states of the same particle, differing only in their third component of isospin [3]. In this idealized picture, the nuclear strong interaction is invariant under rotations in isospin space, leading to what is known as isospin symmetry. However, this symmetry is not exact in nature. Small but measurable deviations from isospin symmetry [4], collectively referred to as isospin symmetry breaking (ISB), provide deep insights into the underlying dynamics of nuclear forces [5].

Isospin symmetry breaking (ISB) is typically categorized into two types: charge symmetry breaking (CSB) and charge independence breaking (CIB). Charge symmetry breaking refers to the difference in the nuclear interaction between two protons and two neutrons, while charge independence breaking accounts for the difference between the neutron-proton interaction and the average of the proton-proton and neutron-neutron interactions. Defining v_{NN} as the interaction between two nucleons, the CSB and CIB terms are

$$v_{\rm CSB} = v_{pp} - v_{nn}\,, \tag{1}$$

$$v_{\rm CIB} = \frac{1}{2}(v_{pp} + v_{nn}) - v_{np}\,. \tag{2}$$

These effects, although subtle, play a significant role in precision nuclear physics and are essential for explaining phenomena such as the binding energy differences in mirror nuclei [6] and isobaric analog states (IAS) [7], where theoretical predictions that don't consider ISB underestimate the experimental values by about 3–9% (Okamoto-Nolen-Schiffer (ONS) anomaly) [8, 9].

The aim of this work is to evaluate isospin symmetry breaking in infinite nuclear matter in the framework of the Hartree-Fock method [10], employing the realistic Argonne v_{18} nucleon-nucleon (NN) potential [11]. This potential is particularly well-suited for our investigation, as it includes explicit charge-dependent (CIB) and charge-asymmetric (CSB) terms, allowing a detailed exploration of ISB effects on the state equation of nuclear matter.

In Sec. II we present the Argonne v_{18} potential and the terms we are working with. The mean-field theory used to solve the present many-body problem is introduced in Sec. III. Section IV discusses the CIB and CSB energetic corrections per particle obtained, comparing them to other calculations [7, 12]. Finally, Sec. V contains the conclusions and an outlook on the study.

II. ARGONNE v_{18} POTENTIAL

Argonne v_{18} is a high-quality non-relativistic potential that accurately fits *np*, *pp* and low-energy *nn* scattering data. This model incorporates 14 spin-isospin operational components (an updated version of the Argonne v_{14} [13], which is charge-independent); and three additional CIB and one CSB operational components.

The NN potential is written as a sum of an elecromagnetic (EM) part (v^{EM}), a one-pion-exchange (OPE) part (v^{π}), and an intermediate and short-range phenomenological part (v^{R}):

$$v(NN) = v^{EM}(NN) + v^{\pi}(NN) + v^{R}(NN)\,. \tag{3}$$

In this thesis only the explicit CSB and CIB contributions of the OPE potential and the phenomenological part are considered.

A. One-pion exchange potential

The charge-dependent structure of the OPE potential reads

$$v^\pi(pp) = f_{pp}^2 v_\pi(m_{\pi^0})\,, \tag{4}$$
$$v^\pi(np) = f_{pp}f_{nn}v_\pi(m_{\pi^0}) + (-1)^{T+1}2f_c^2 v_\pi(m_{\pi^\pm})\,, \tag{5}$$
$$v^\pi(nn) = f_{nn}^2 v_\pi(m_{\pi^0})\,, \tag{6}$$

where T is the total isospin of the np coupling and can take the values $T = 0, 1$. In addition,

$$v_\pi(m) = \left(\frac{m}{m_s}\right)^2 \frac{mc^2}{3}\left[Y_\mu(r)\boldsymbol{\sigma_i}\cdot\boldsymbol{\sigma_j} + T_\mu(r)S_{ij}\right]\,, \tag{7}$$

where $S_{ij} = (3\sigma_{zi}\sigma_{zj} - \boldsymbol{\sigma_i}\cdot\boldsymbol{\sigma_j})$, $\boldsymbol{\sigma_i}$ and σ_{zi} being the spin operator and its corresponding z component. $Y_\mu(r)$ and $T_\mu(r)$ are radial functions whose expressions can be found in Appendix A. The parameter μ is the average of the pion masses,

$$\mu = \frac{1}{3}(m_{\pi^0} + 2m_{\pi^\pm})c/\hbar\,. \tag{8}$$

The charged-pion mass is taken as the scaling mass ($m_s = m_{\pi^\pm}$), and the coupling constants are taken charge-independent ($f_{pp} = -f_{nn} = f_c \equiv f$, with $f^2 = 0.075$). Hence, ISB will be exclusively due to the mass difference between neutral and charged pions.

From these terms, we can build the CSB and CIB contributions of the OPE potential according to (1) and (2):

$$v^\pi_{\text{CSB}} = v^\pi(pp) - v^\pi(nn) = 0\,, \tag{9}$$
$$v^\pi_{\text{CIB}} = \frac{1}{2}[v^\pi(pp) + v^\pi(nn)] - v^\pi(np)|_{T=1} \tag{10}$$
$$= \frac{2f^2c^2}{3m_{\pi^\pm}^2}(m_{\pi^0}^3 - m_{\pi^\pm}^3)\left[Y_\mu(r)\boldsymbol{\sigma_i}\cdot\boldsymbol{\sigma_j} + T_\mu(r)S_{ij}\right]\,.$$

B. Intermediate- and short-range phenomenological potential

The strong interaction potential can be projected into an operator format with 18 terms [11],

$$v_{ij} = \sum_{p=1}^{18} v_p(r_{ij})O_{ij}^p\,. \tag{11}$$

The first 14 terms have the same charge-independent operators as the Argonne v_{14} potential, so we will focus on the four additional terms, whose operators explicitly break isospin symmetry.

The CIB and CSB operators are

$$O_{ij}^{p=15,18} = T_{ij}, (\boldsymbol{\sigma_i}\cdot\boldsymbol{\sigma_j})T_{ij}, S_{ij}T_{ij}, (\tau_{zi}+\tau_{zj})\,. \tag{12}$$

Here $T_{ij} = (3\tau_{zi}\tau_{zj} - \boldsymbol{\tau_i}\cdot\boldsymbol{\tau_j})$, and $\boldsymbol{\tau_i}$ and τ_{zi} are the isospin operator and its z component. These terms are abbreviated as T, σT, tT and τz. The corresponding radial functions are

$$v_T(r) = -0.02510T_\mu^2(r) + \\ + (9.0905 + 4.9290\mu r)W(r)\,, \tag{13}$$
$$v_{\sigma T}(r) = 0.02510T_\mu^2(r) + \\ - (9.0905 + 4.5831\mu r)W(r)\,, \tag{14}$$
$$v_{tT}(r) = 0\,, \tag{15}$$
$$v_{\tau z}(r) = (0.9803 + 0.5315\mu r)W(r)\,, \tag{16}$$

all expressed in MeV. $W(r)$ is a radial function that can be found in Appendix A.

The T, σT and tT terms are charge-dependent (CIB), and the term τz is charge-asymmetric (CSB) [14].

III. HARTREE-FOCK THEORY

The Hartree-Fock (HF) approximation is a mean-field theory that describes an interacting fermion system using two-body interactions. The wave function of the system Φ is written as a Slater determinant, a totally antisymmetric product of single-particle wave functions ϕ_i, found by minimizing the total energy of the system [10].

In this work, the HF theory is used to calculate the potential energy per particle of each term discussed in the previous section, for infinite nuclear matter.

Given a mean-field potential v, its Hartree-Fock expectation value in a system with A particles is

$$\langle\Phi|v|\Phi\rangle = \frac{1}{2}\sum_{i=1}^{A}\sum_{j=1}^{A}\langle\phi_i\phi_j|v|\phi_i\phi_j\rangle_{\text{AS}}\,, \tag{17}$$

i.e., the summation of the anti-symmetrized expectation value of the mean-field potential over all the possible pairs of particles ij $(i, j = 1, ..., A)$. Here, the summations over all the particles are calculated integrating the potential and wave functions on their spatial domain.

A. Plane waves

Because we are dealing with an infinite system, we can take plane waves as the single-particle wave functions [15]:

$$\phi_i \to \phi_{\mathbf{k}} = \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{\Omega}}\,; \quad \sum_i \to \frac{g\Omega}{(2\pi)^3}\int_{k_F} d^3k\,, \tag{18}$$

where g is the degeneracy, equal to 4 in symmetric nuclear matter, a uniform system with the same neutron and proton densities; Ω is the quantisation volume, and k_F represents the Fermi shpere's volume in the momentum space (of radius k_F). When differentiating between

protons and neutrons one must take $g_n = g_p = 2$.

In this formulation, the state density of the system is

$$\rho = \frac{k_{F_n}^3 + k_{F_p}^3}{3\pi^2} \equiv \rho_n + \rho_p \,, \tag{19}$$

k_{F_n} and k_{F_p} being the Fermi momentums for neutrons and protons, and ρ_n and ρ_p their corresponding densities.

B. Antisymmetrization

In the Hartree-Fock formalism, the anti-symmetrized matrix elements can be found by anti-symmetrizing the interaction, which simplifies the calculation of expectation values and is particularly convenient for this work. So, in equation (17), the anti-symmetrized matrix elements can be written as

$$\langle \phi_i \phi_j | v | \phi_i \phi_j \rangle_{\text{AS}} = \langle \phi_i \phi_j | v(1 - P_{ij}) | \phi_i \phi_j \rangle \,, \tag{20}$$

where $P_{ij} = P_\sigma P_\tau P_r$ is the operator that exchanges paricles i and j: P_σ exchanges their spin, P_τ their isospin and P_r exchanges their position. The first two operators are defined as $P_\sigma = \frac{1+\boldsymbol{\sigma_i}\cdot\boldsymbol{\sigma_j}}{2}$ and $P_\tau = \frac{1+\boldsymbol{\tau_i}\cdot\boldsymbol{\tau_j}}{2}$, but within the HF theory, since it only considers the trace of the matrices, $\langle \boldsymbol{\sigma_i} \cdot \boldsymbol{\sigma_j} \rangle \to \langle \sigma_{z,i}\sigma_{z,j} \rangle$ and $\langle \boldsymbol{\tau_i} \cdot \boldsymbol{\tau_j} \rangle \to \langle \tau_{z,i}\tau_{z,j} \rangle$. Because we are dealing with spin-symmetric matter, in all calculations P_σ can be replaced by $1/2$ [16].

IV. RESULTS

The expectation values per particle of all the ISB terms discussed in Sec. II are calculated using the methodology explained in the previous section. The results are presented separating the charge-dependent terms (the CIB contribution of the OPE potential, T, σT and tT) and the charge-asymmetric terms (the CSB contribution of the OPE potential and τz). For more specific details on the mathematical expressions obtained, see Appendix B.

To illustrate the nature of the ISB energetic corrections, we have considered the cases of symmetric nuclear matter (SNM), with the same density of protons and neutrons ($\rho_p = \rho_n$); and pure neutron matter (PNM), made out entirely of neutrons ($\rho_p = 0$).

The results from this work, that we have labelled Av18 HF, have been compared with previous calculations done using different interactions and/or methods: the results obtained from Brueckner-Hartree-Fock (BHF) calculations[1] of the Argonne v_{18} potential in Ref. [12] (labelled Av18 BHF); and from the Hartree-Fock calculations of the SAMi-ISB interaction [7], which has been fit to reproduce the results of Ref. [12] and the IAS energy in ^{208}Pb (labelled SAMi-ISB HF).

A. Symmetric nuclear matter

The energetic corrections induced by CIB and CSB in symmetric nuclear matter (SMN) for different values of the total state density are plotted in Fig. 1, together with the corresponding contributions of the models Av18 BHF [12] (only CIB), and SAMi-ISB HF [7].

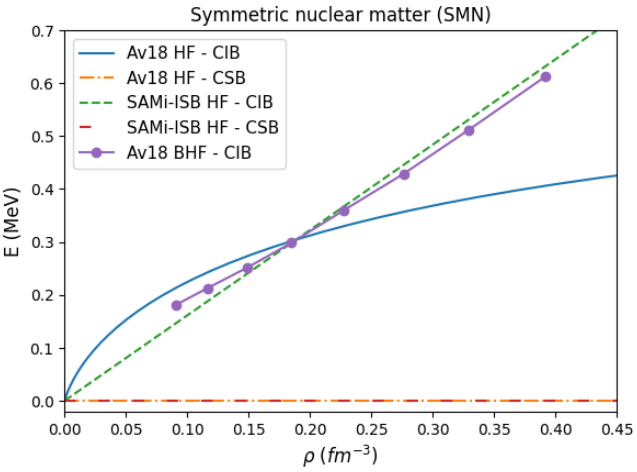

FIG. 1: Contributions to the energy per particle of the CSB and CIB terms as a function of the state density, for SMN. Three different models have been included: Av18 HF (the results from this work), SAMi-ISB (from Ref. [7]) and Av18 BHF (from Ref. [12]).

As seen in Fig. 1, in SMN, the CSB terms of all the models considered are zero, indicating that there is no charge-asymmetric correction to the system's potential energy.

For the CIB part, comparing the repulsive contribution obtained in this work (Av18 HF) with the other two models considered (Av18 BHF and SAMi-ISB HF), it can be seen that, although the dependency of the energy per particle with the density is clearly different, all the values are of the same order of magnitude. It is interesting to notice that all the curves cross approximately at $\rho \approx 0.19$ fm^{-3}.

At the saturation density $\rho_0 = 0.16$ fm^{-3}, we can analyze the nn, pp, np, and pn contributions (the latter

[1] A refinement of the Hartree-Fock method that includes short-range correlations between nucleons, going beyond the mean-field approximation [17].

two being identical by definition) of the model studied in this thesis for each term individually, understanding the nucleon-nucleon contribution in a given term as its dependency on ρ_N and $\rho_{N'}$ (see Appendix B for a more detailed definition).

v	$\langle v\rangle_{nn}$	$\langle v\rangle_{np}$	$\langle v\rangle_{pn}$	$\langle v\rangle_{pp}$	$\langle v\rangle$
v_T	0.044	-0.044	-0.044	0.044	0.0
$v_{\sigma T}$	0.158	-0.158	-0.158	0.158	0.0
v_{tT}	0.0	0.0	0.0	0.0	0.0
$v_{\tau z}$	-0.042	0.0	0.0	0.042	0.0
v^{π}_{CIB}	0.141	0.0	0.0	0.141	0.283
v^{π}_{CSB}	0.0	0.0	0.0	0.0	0.0

TABLE I: Energetic contributions of the *nn*, *np*, *pn* and *pp* interactions per nucleon, and their sum (last column), of the CIB terms (T, σT, tT and the OPE CIB share) and the CSB terms (τz and the OPE CSB portion) in MeV, in SMN at saturation.

From table I, we observe that the sum of all the values $\langle v\rangle_{NN}$ for all the individual terms is null –except for the CIB component of the OPE potential–, but the NN interactions exhibit competing attractive and repulsive effects that largely cancel out. Coinciding with the nature of CSB and CIB contributions (see eqs. (1) and (2)), the τz term has equally attractive *nn* and repulsive *pp* interactions, and the T and σT terms have repulsive *nn* and *pp* interactions that are cancelled by the attractive *np* and *pn* contributions. The tT term is zero by definition. The OPE potential doesn't have a CSB term, and the CIB component is made of repulsive same-nucleon couplings, but lacks the different-nucleon attractive interaction one would expect from looking at (10). This is probably a consequence of doing Hartree-Fock calculations, as it is only an approximate method for solving many-body problems.

The total isospin symmetry breaking correction per nucleon in symmetric nuclear matter at saturation is around 0.283 MeV. Looking at Fig. 1 it can be seen that this value is coherent with the results from Refs. [7, 12].

B. Pure neutron matter

The CIB and CSB energetic contributions of the models Av18 HF and SAMi-ISB HF [7], for systems made out entirely of neutrons are plotted in Fig. 2.

The results displayed in Fig. 2 show that in the case of pure neutron matter (PNM), the CSB and CIB contributions in both models considered have opposite signs: the charge-asymmetric terms introduce attractive corrections that become more negative with increasing density; in contrast, the charge-dependent terms are repulsive and increase with neutron density.

When comparing the CIB and CSB contributions of

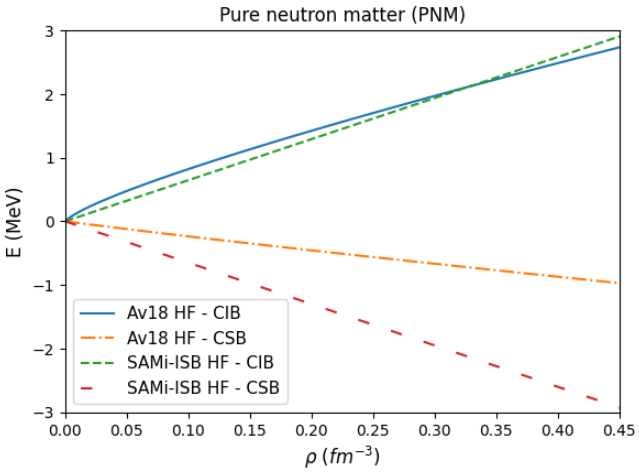

FIG. 2: Contributions to the energy per particle of the CSB and CIB terms as a function of the state density, for PNM. Two different models have been included: Av18 HF (the results from this work) and SAMi-ISB (from Ref. [7]).

each model we must take into account that, in SAMi-ISB, the CIB component has been fit to the Argonne v_{18} potential, while the CSB part is phenomenological. So, it is coherent that the CIB contribution obtained in this thesis is very similar to the one in the literature, although the linear dependency in SAMi-ISB HF can't be seen in Av18 HF; and the CSB terms differ significantly.

Looking into Av18 HF, the repulsive CIB contribution exceeds the attractive CSB contribution, resulting in a net positive isospin symmetry breaking correction, which grows with the neutron density. For SAMi-ISB, the charge-dependent and charge-asymmetric terms are exactly the opposite of each other, leaving the total ISB correction to 0.

We examine the individual contributions of each interaction term at saturation density ($\rho_0 = 0.16$ fm^{-3}) for PNM.

v	$\langle v\rangle_{nn}$	$\langle v\rangle_{np}$	$\langle v\rangle_{pn}$	$\langle v\rangle_{pp}$	$\langle v\rangle$
v_T	0.180	0.0	0.0	0.0	0.180
$v_{\sigma T}$	0.632	0.0	0.0	0.0	0.632
v_{tT}	0.0	0.0	0.0	0.0	0.0
$v_{\tau z}$	-0.182	0.0	0.0	0.0	-0.182
v^{π}_{CIB}	0.378	0.0	0.0	0.0	0.378
v^{π}_{CSB}	0.0	0.0	0.0	0.0	0.0

TABLE II: Contribution of the *nn*, *np*, *pn* and *pp* interactions, and their sum (last column), of the CIB terms (T, σT, tT and the OPE CIB share) and the CSB terms (τz and the OPE CSB portion) in MeV, for PNM at saturation.

In table II, as expected, there are no *pp*, *np* and *pn* contributions (since the proton density is 0). The *nn*

interactions are repulsive in all the non-zero CIB terms, and attractive for the non-zero CSB term.

The individual contributions in this case are significantly larger than in symmetric matter, resulting in a total isospin symmetry breaking correction of 1.008 MeV.

V. CONCLUSIONS

In the present work we have studied the ISB energetic corrections of the Argonne v_{18} potential. The calculations have been done using the Hartree-Fock approximation for infinite nuclear matter, and we have analysed the charge symmetry breaking and charge independence breaking effects for two different systems: symmetric nuclear matter (SMN) and pure neutron matter (PNM).

In the first case, we have found null CSB and positive CIB contributions, making the total ISB correction repulsive. From the comparison of the results with the CIB and CSB components from Refs. [7, 12] we can conclude that the CSB is rightfully zero and, for the CIB term, although the dependency of the correction with the density isn't very similar to the ones from the literature, the values obtained stay within the same order of magnitude.

By fixing the density at its saturation value, we have been able to study the contribution of each term individually: for T, σT and τz, the individual NN components have the correct structure of CIB and CSB terms, and their sum cancels out. The charge-dependent term of the OPE potential doesn't have the theoretical CIB structure, which could be an indication of the limitations of the Hartree-Fock approximation. The global ISB correction obtained is of 0.283 MeV, very close to the ones given by the models in Refs. [7, 12].

For PNM, all the non-zero CIB and CSB terms give repulsive and attractive corrections, respectively. Comparing the results with the ones from Ref. [7], the CIB contributions are very similar, but the CSB components have a clear deviation, due to the definition of the latter to fit specific experimental data.

In this case, when considering the saturation density, we only find attractive and repulsive nn contributions for the CSB and CIB terms, since the proton density is zero. These contributions add up to a repulsive ISB correction of 1.008 MeV.

Looking at the global repulsive ISB corrections obtained in both cases, the results found for pure neutron matter are far larger than the ones for symmetric nuclear matter. For SNM it is coherent to say that the results are physically realistic, since they are close to other results previously obtained, but for PNM, although the CIB component found in this work is very similar to the results from Ref. [7], the CSB correction is very different, making the total ISB correction not compatible with literature.

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[1] W. Heisenberg. *Z. Phys.*, 77:1–11, Jan 1932.
[2] E. Wigner. *Phys. Rev.*, 51(2):106–119, 1937.
[3] *Nuclear and Particle Physics*, chapter 3. John Wiley & Sons Ltd, 2006.
[4] R. Machleidt. *Phys. Rev. C*, 63(2):024001, Jan 2001.
[5] R. N. Ali N. Rather C. Sarma J. A. Sheikh, S. P. Rouoof and P. C. Srivastava. *Symmetry*, 16(6), 2024.
[6] M. Konieczka W. Satula T. Nakatsukasa P. Baczyk, J. Dobaczewski and K. Sato. *Phys. Lett. B*, 778:178–183, 2018.
[7] G. Colò X. Roca-Maza and H. Sagawa. *Phys. Rev. Lett.*, 120(20):202501, May 2018.
[8] K. Okamoto. *Phys. Lett.*, 11(2):150–153, 1964.
[9] J. A. Nolen Jr. and J. P. Schiffer. Coulomb energies. *Annu. Rev. Nucl. Sci.*, 19:471–526, 1969.
[10] *Many-Particle Theory*, chapter 7. Adam Hilger, 1991.
[11] R. Schiavilla R. B. Wiringa, V. G. J. Stoks. *Phys. Rev. C*, 51(1):38–51, Jan 1995.
[12] R. Machleidt H. Müther, A. Polls. *Phys. Lett. B*, 445(3):259–264, Jan 1999.
[13] R. A. Smith R. B. Wiringa and T. L. Ainsworth. *Phys. Rev. C*, 29(4):1207–1221, Apr 1984.
[14] *Charge Symmetry, Quarks and Mesons*, chapter 2. Definitions, relations and classifications, pages 15–17. North Holland, 1990.
[15] *Many-Particle Theory*, chapter 9. Adam Hilger, 1991.
[16] G. Colò. Nuclear physics. Dipartimento di Fisica Università degli Studi di Milano.
[17] *TheNuclear Many-Body Problem*, chapter 5. Springer-Verlag, 1941.
[18] I.E. Lagaris and V.R. Pandharipande. *Nucl. Phys. A*, 359(2):331–348, 1981.

Trencament de la simetria d'isospí en matèria nuclear infinita

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Resum: La simetria d'isospí, malgrat ser un concepte fonamental en la física nuclear, és tan sols una aproximació en els sistemes nuclears reals. En aquest treball s'estudia el trencament d'aquesta simetria, quantificant els efectes del trencament de la simetria de càrrega (CSB) i de la independència de càrrega (CIB) mitjançant el potencial nucleó-nucleó Argonne v_{18}. L'anàlisi es duu a terme emprant l'aproximació de Hartree-Fock, modelitzant les funcions d'ona dels nucleons amb ones planes en matèria nuclear infinita. S'avaluen els valors esperats dels termes rellevants per a matèria nuclear simètrica i matèria neutrònica, obtenint correccions repulsives d'aproximadament 0,2826 MeV i 1,0080 MeV per a la densitat de saturació ($\rho_0 = 0,16$ fm^{-3}), respectivament. Aquests resultats concorden amb altres càlculs previs, malgrat que la dependència amb la densitat d'estats no coincideix del tot. Aquest estudi destaca la importància de models realistes per avaluar els efectes del trencament de la simetria d'isospí, així com també posa a prova la validesa de l'aproximació de Hartree-Fock en el tractament d'aquests sistemes de molts cossos.
Paraules clau: Trencament de simetria d'isospí, matèria nuclear infinita, aproximació Hartree-Fock, interaccions nucleó-nucleó
ODSs: Aquest TFG està relacionat amb els Objectius de Desenvolupament Sostenible 4, 7 i 9

Objectius de Desenvolupament Sostenible (ODSs o SDGs)

1. Fi de la es desigualtats		10. Reducció de les desigualtats	
2. Fam zero		11. Ciutats i comunitats sostenibles	
3. Salut i benestar		12. Consum i producció responsables	
4. Educació de qualitat	X	13. Acció climàtica	
5. Igualtat de gènere		14. Vida submarina	
6. Aigua neta i sanejament		15. Vida terrestre	
7. Energia neta i sostenible	X	16. Pau, justícia i institucions sòlides	
8. Treball digne i creixement econòmic		17. Aliança pels objectius	
9. Indústria, innovació, infraestructures	X		

El contingut d'aquest TFG es relaciona amb l'ODS 7 (energi neta i sostenible), en particular amb la fita 7.3, sobre la millora de l'eficiència energètica, donat que les correccions energètiques calculades són aplicables a totes les reaccions nuclears, de manera que poden contribuir directament l'optimització de l'eficiència energèticañ nuclear. De la mateixa manera, també es pot vincular amb l'ODS 9 (indústria, innovació, infraestructures), amb les fites 9.4, augmentant l'eficàcia de l'ús dels recursos, i 9.5, contribuint a l'investigació científica i millorant la capacitat tecnològica del sector d'energia nuclear. Per altra banda, es pot associar amb la fita 4.4 de l'ODS 4 (educació de qualitat) ja que, com a part d'un procés de formació superior en física nuclear teòrica, millora les competències tècniques dels joves per a accedir a l'ocupació, el treball digne i l'emprenedoria.

Appendix A: Numerical values of the parameters used in the calculations

$Y_\mu(r)$ and $T_\mu(r)$ are the usual Yukawa tensor functions with the exponential cutoff of the Urbana [18] and Argonne v_{14} models, and $W(r)$ is a Woods-Saxon function which provides the short-range core:

$$Y_\mu(r) = \frac{e^{-\mu r}}{\mu r}\left(1 - e^{-cr^2}\right), \tag{A1}$$

$$T_\mu(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right)\frac{e^{-\mu r}}{\mu r}\left(1 - e^{-cr^2}\right)^2, \tag{A2}$$

$$W(r) = \left[1 + e^{(r-r_0)/a}\right]^{-1}. \tag{A3}$$

As already mentioned, μ is the average of the pion masses, $\mu = \frac{1}{3}(m_{\pi^0} + 2m_{\pi^\pm})c/\hbar$.

The values of the parameters and fundamental constants used in this work are the ones provided in [11].

c [fm^{-2}]	r_0 [fm]	a [fm]	μ [fm^{-1}]
2.1	0.5	0.2	0.69953

TABLE III: Values of the parameters in $T_\mu(r)$, $Y_\mu(r)$ and $W(r)$. The average pion mass has been calculated with $m_{\pi^0} = 134.9739$ MeV/c^2 and $m_{\pi^\pm} = 139.5675$ MeV/c^2, and the conversion constant $\hbar c = 197.32705$ MeV·fm

Appendix B: Results of the calculations

The expectation values per nucleon of all the terms considered in Sec. II are:

$$\langle v^\pi_{\rm CIB}\rangle = \frac{f^2c^2}{m^2_{\pi^\pm}}(m^3_{\pi^\pm} - m^3_{\pi^0})(F_{nn}(Y_\mu) + F_{pp}(Y_\mu)), \tag{B1}$$

$$\langle v^\pi_{\rm CSB}\rangle = 0, \tag{B2}$$

$$\langle v_T(r)T\rangle = \frac{1}{\rho}\left[(\rho_p - \rho_n)^2 J(v_T) + F_{np}(v_T) + F_{pn}(v_T) - F_{nn}(v_T) - F_{pp}(v_T)\right], \tag{B3}$$

$$\langle v_{\sigma T}(r)\sigma T\rangle = \frac{3}{\rho}\left[F_{np}(v_{\sigma T}) + F_{pn}(v_{\sigma T}) - F_{nn}(v_{\sigma T}) - F_{pp}(v_{\sigma T})\right], \tag{B4}$$

$$\langle v_{tT}(r)tT\rangle = 0, \tag{B5}$$

$$\langle v_{\tau z}(r)\tau z\rangle = \frac{1}{\rho}\left[(\rho_p^2 - \rho_n^2)J(v_{\tau z}) + F_{nn}(v_{\tau z}) - F_{pp}(v_{\tau z})\right]. \tag{B6}$$

The radial integral $F_{ab}(v)$ is

$$F_{ab}(v) = \frac{1}{2\pi^4}\int_0^\infty r v(r)\hat{I}(k_{F_a}, k_{F_b}, r)dr,$$

with

$$\hat{I}(k_{F_a}, k_{F_b}, r) = \pi\left[\frac{k^4_{F_b} + 2k^2_{F_a}k^2_{F_b} - 3k^4_{F_a}}{4}\hat{i}_+ + \frac{2k^3_{F_a} + 3k^2_{F_a}k_{F_b} - k^3_{F_b}}{3}\hat{i}_1 - \frac{3k^2_{F_a} + k^2_{F_b}}{4}\hat{i}_2 + \frac{1}{12}\hat{i}_4 + \frac{(k^2_{F_a} - k^2_{F_b})^2}{12}\hat{i}_{-2}\right],$$

defining $\hat{i}_\alpha = \int_{|k_{F_a}+k_{F_b}|}^{k_{F_a}+k_{F_b}} \sin(q'r)q'^\alpha dq'$.

Also, the integrals $J(v)$ are

$$J(v) = 4\pi\int_0^\infty r^2 v(r)dr.$$

The numerical values of these results for different Fermi momentums are presented in Fig. 1 (SMN: $k_{F_p} = k_{F_n}$) and Fig. 2 (PNM: $k_{F_p} = 0$).

The contributions $\langle v\rangle_{NN}$ of each term presented in Tabs. I and II refer to the components of that term that depend on $\rho_N\rho_N$ and F_{NN}. For a given term (B1-B6), the nn contribution is the sum of the components linear to ρ_n^2 and F_{nn}, pp of the terms ρ_p^2 and F_{pp}, and np of the terms $\rho_n\rho_p$ and F_{np}.