Sequential detuning of a drumhead with a lock-in amplifier

Author: Ricard Sánchez Elias
Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.

Advisor: Joan Manel Hernàndez Ferràs
Dept. de Física de la Matèria Condensada, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.

Abstract: Whenever a drumhead is tuned, a perturbation in the tension of the membrane induces the splitting of frequencies for the degenerated asymmetric modes (m, n), with $m \neq 0$. With the use of a lock-in amplifier and piezoelectric sensors and actuators attached at the head, the amplitude of the signal generated by the sensors is recorded as the membrane is forced to vibrate over a range of frequencies, answering noticeably when mode's harmonics are reached. The progression from uniform conditions where the modes are accessible, to the measure of frequency splitting when the drum's lugs are symmetrically tightened demonstrates the functionality of the system and the behavior of non-uniform tension for a vibrating membrane.
Keywords: Acoustics, wave equation, normal modes, degeneracy and frequency splitting.
SDGs: SDG 4, Quality education and SDG 9, Industry, Innovation and Infrastructure.

I. INTRODUCTION

By the end of the 19th century, it became known that whenever the head of a drum is struck, a discrete collection of frequencies can be excited. In order to give a numerical value to these frequencies, which are strictly associated with the zeros of the Bessel functions, the wave equation is ideally solved by assuming an uniform tension and density.

However, this is at best an ideal approximation. There are examples and studies that have been conducted of membranophones that are far form an ideal uniformity. When the design of the drumhead is discussed, there is the case of some Indian drumheads, the *mridangam* and *tabla*, which are intentionally built (or fabricated or built) to have a central part of a heavier density material. This might help to cancel some harmonic modes and induce a stronger sense of pitch [1]. A good example of intentionally non-uniformity tension, by design, is the Japanese musical drum, *kotsuzumi*, which uses tension ropes (shirabeo) that are pulled by the player's during performance, modifying the tension of the membrane at will. In addition, as it is exposed in [2], variations in the volume of air inside the drum or the complex coupling effect that can be achieved by the interaction of two membranes are important phenomena that must also be considered.

Independently of these multiple real factors, most drums require a process of tuning its head (or heads in case we are dealing with two) by adjusting the applied tension via some lugs normally distributed symmetrically trough the rim of the membrane. The aim of this paper is study the way an uniform density drumhead goes out of tune, listing the splitting of frequencies in asymmetrical modes and the way they change by little perturbations of the tension with respect to the tuned membrane. In Section II and III, the normal modes for a uniform membrane are described theoretically, as the way the frequencies of asymmetrically modes split, making it useful for the tuning of heads. Section IV is destined to expose the setup used in order to record these frequencies, just as the different measures taken. Lastly, Section VI is destined to conclusions.

II. UNIFORM CIRCULAR MEMBRANE

Assuming no air loading nor membrane stiffness, the ideal drumhead is considered as a circular membrane of radius a with uniform mass density σ and uniform tension T, over a rigid circular supporting frame, that fixes its perimeter. By striking its surface, the membrane vibrates obeying the wave equation (in polar coordinates)

$$T\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 F}{\partial \theta^2}\right)=\sigma\frac{\partial^2 F}{\partial t^2} \tag{1}$$

where $F(r,\theta,t)$ describes the transverse displacements perpendicular to the surface in time. By [3], the solutions of the modes have the form

$$F(r,\theta,t)=J_m(k_{mn}r)\left\{\begin{array}{l} A\cos(m\theta+\phi_{mn}) \\ B\sin(m\theta+\phi_{mn})\end{array}\right\}e^{i\omega t}. \tag{2}$$

where $J_m(k_{mn}r)$ are the first kind Bessel functions of order m. The modes of vibration of a uniform membrane are labeled by the integers m and n, which correspond to the number of diametrical and circular nodal lines, respectively. Nodal lines represent the points where there is no vertical displacement.The constants ϕ_{mn} are phase angles that come from integration and can be fixed by initial conditions as they define the orientations of the modes that have nodal lines. The eigenfrequencies $\omega_{mn}=k_{mn}\sqrt{T/\sigma}$ are determined by the Dirichlet boundary condition that the displacement must be null at the rim or $J_m(k_{mn}a)=0$. In general, if we denote as β_{mn} as the n-th zero of the Bessel function of order m, the frequencies of a uniform drumhead are given by

$$\omega_{mn}=(1/2\pi a)\,(T/\sigma)^{1/2}\beta_{mn} \tag{3}$$

The associated nodal lines and circles of the first 12 modes are shown in Fig.1 with their (m,n) designation.

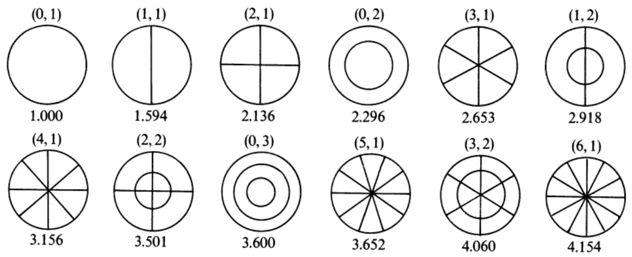


FIG. 1: The first 12 modes of an uniform circular membrane where it is represented both nodal lines and frequencies relatives to the fundamental first mode (0,1)(Fletcher and Rossing, [4]). Given one frequency, for instance, w_{11}, all the other are defined $w_{mn} = w_{11} \cdot \beta_{mn}/\beta_{11}$

Looking to the solutions of the modes in eq. (2), any mode that contains at least one nodal diameter ($m > 0$) will be degenerate, as modes with the sine and cosine are both solutions of the wave equation. Therefore, for each pair (m, n) wit $m > 0$, there will be two orthogonal modes with the same frequency, w_{mn}. For this reason, each of the modes with $m > 0$ depicted in Fig.1 has a pair with the exact same frequency and shape, although its nodal lines will be rotated by an angle $\pi/2m$, which means that nodes an antinodes are interchanged.

III. TUNING AND FREQUENCY SPLITTING

Typical drums, such as snares, tom-toms or bass drums have their heads in a non-uniform tension condition, because their membranes are held taut by a set of symmetrically distributed tuning lugs. Tightening each of these rods forces the membrane to stretch locally where the lug is placed, although the effect on the average tension is general and it usually translates in a raise of the pitch of the drum.

Tuning a drumhead is a process that aims to get the tension as uniform as possible. When it is done by ear, the head is tapped near each tuning lug. Then, the tension of each one is adjusted whenever there are audible differences between each lug.

While the tuning of a guitar's string is based on changing the tension to reach the desired fundamental frequency, in the case of drums, the fundamental mode is not a helpful guide to tune. Firstly, tapping near the edge of the head can be explained by the fact that strikes near the rim excites the asymmetric modes ($m > 0$) , such as the first overtone mode $(1, 1)$, which is the lowest frequency degenerated mode and the most prominent of the splitting modes. As it is been shown, for an ideal membrane, the mode $(1, 1)$ produces a single frequency and can appear with its nodal line oriented in any direction, as it represents a linear combination of the two orthogonal states described in Eq. (2). However, the place where the head is tapped actually influences the mode's orientation by forcing an antinode near the strike location. Thus, if the symmetry is broken due to a non-uniform perturbation when two opposed lugs are tighten, orthogonal axes are created in the membrane, one being faster and the other slower. This modification produces a frequency splitting of the orthogonal modes, breaking degeneracy. A drum will be tuned, for this reason, when this frequency splitting is minimized.

Worland [5] suggests a model of the $(1, 1)$ mode's response to the simplest tension perturbation. An increased tension across a single diameter of the membrane, from top to bottom, will cause the wave to propagate faster in the vertical direction than in the horizontal direction, resulting in two different frequencies w_{11}^+ and w_{11}^- for the orthogonal $(1, 1)$ mode. Since the placement of tapping determines the distribution of the nodal lines, Fig. 2 shows which of these orthogonal $(1, 1)_+$ and $(1, 1)_-$ modes are excited depending on where the head is struck.

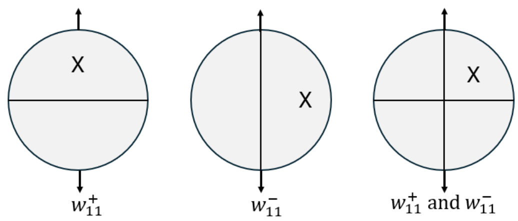


FIG. 2: Model of the $(1, 1)$ mode's answer to an increase of tension from top to bottom and how striking the head at different locations (marked by an "X") will excite the $(1, 1)_+$ and $(1, 1)_-$ modes, or even both.

IV. EXPERIMENTAL SETUP

A. Setup justification

Most studies related with vibration modes use a microphone to record the sound generated by the membrane once it is struck with a drumstick. Despite its accessibility, there are several potential issues that can arise. Firstly, microphones register sound from a specific point in space, so the recorded signal depends heavily on their placement. Secondly, there exists different types of microphones (dynamic, ribbon, condenser, etc.) which frequency responses and sensitivity properties might have notable variance. For instance, while a dynamic microphone may emphasize mid-range or low frequencies, a condenser microphone may have a higher precision with higher frequencies.

In any case, the microphone converts the sound wave, which is a superposition of all the normal modes of the drumhead, in a digital audio recording which can be visualized via a waveform that shows the amplitude of the sound (or loudness) in time. To analyze it, the signal must be processed with software for frequency analysis (e.g., FFT analysis) which will compute the discrete Fourier transform of the audio track [6].

Lastly, the way the membrane is struck is an important aspect of the experimentation. Cahoon [7] proved that for large deflections of the membrane (stronger hits) there was an increase in the effective tension. Moreover, he showed that the restoring force was not directly proportional to the deflection of the membrane. Then, the fundamental frequency corresponding to the first mode

(0,1) could be rewritten as

$$\omega_{01} = (1/2\pi a)\left[(T_0 + kd^2)/\sigma\right]^{1/2}\beta_{01} \qquad (4)$$

where T_0 is the tension of the head at equilibrium, and k is a constant that relates the increase in tension to the deflection amplitude d. Due to this correction, it can be proved that the frequency of each mode will change with time, decreasing from the instant the head is hit and having more relevance when the drum is struck a hard blow [8].

All things considered, an experimental setup has been designed to overcome these problems, from the inconvenience of using a microphone and not working directly with frequencies to the non-uniformity way of striking the membrane, which causes a variance in frequency modes in time.

B. System Overview and Signal Flow

As no electronic speckle pattern interferometer is used to visualize each mode [9], to localize the vibrational modes of a drumhead a system based on a computer-controlled lock-in amplifier in conjunction with piezoelectric transducers is put together. The lock-in amplifier simultaneously generates the excitation signal and measures the amplitude of the system's response, enabling precise detection of resonant frequencies. The setup is shown schematically in Fig. 3. The components are the following:

1. **Lock-in Amplifier:** a EG&G INSTRUMENTS Lock-in Amplifier is used. A lock-in Amplifier (LIA) is a device able to detect very small AC signals all the way down to a few nanovolts, making it really useful when they are obscured by noise sources many thousands of times larger. They use phase-sensitive detection to isolate the signal at a specific reference frequency and phase, rejecting noise at other frequencies. In this experiment, it also functions as a signal generator.

2. **Piezoelectric Actuator:** Receives the excitation signal from the LIA and induces vibrations on the drumhead with the same frequency as the generated signal.

3. **Drumhead:** 14 in. Yamaha *Gigmaker* snare with eight lugs (radius 17.78 cm). To avoid the influence of air and the coupling effect, the snare head and the snares have been both disassembled, leaving the batter head only. The batter head is a Remo Ambassador X14 single layer 14-mil Mylar Film. The National Institute of Standards and Technology (NIST) lists the density of Mylar (polyethylene terephthalate or PET) as $\rho = 1.40$ g/cm^3[10]. With this value, $\sigma = 0.04978$ g/cm^2.

4. **Piezoelectric Sensor:** A second piezoelectric element is attached to the drumhead to sense the mechanical vibrations. This element converts the mechanical motion back into an electrical signal, which is a superposition of the resonant response and external noise.

5. **Computer-Controlled Frequency Sweep:** The lock-in amplifier is interfaced with a computer, which with the program LabView the frequency can be swept in a desired range of the excitation signal. At each frequency step, the amplitude of the response is recorded. This data is used to construct a frequency response curve of the drumhead, from which resonant frequencies and mode shapes can be identified.

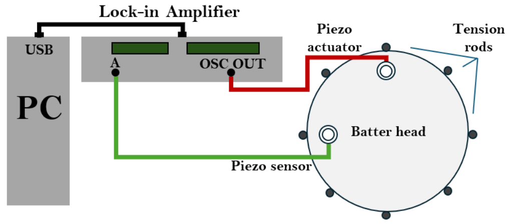


FIG. 3: Schematic drawing of the experimental setup.

V. RESULTS

A. Measurements on a tuned head

To test this setup, the head has been previously tuned, with respect to the second mode or first overtone, to a frequency of $w_{11} = 226$ Hz with the use of a frequency analiser EQ plugin in *Ableton Live*. Although the objective is, precisely, not using this kind of technology, in order to prove the system functionality, the more uniform conditions on the membrane are required. The drumhead has been tuned with respect to the first overtone mode $(1,1)$ due to the tuning process presented in Section III that relies on the frequency splitting of the asymmetric modes. For this first measure, both piezos, the actuator and sensor, are closely placed to the center of the membrane, since the modes (m,n) with $m = 0$ are excited much easier. More over, a LabView program, via the lock-in, is supposed to register the amplitude of the signal converted by the piezoelectric sensor for a large range of frequencies, from 110 Hz to 1010 Hz, in order to detect the most amount of modes.

Before starting the listing, the system is firstly calibrated attaching both piezoelectric sensor and actuator, without the drum, and doing a first measurement. The amplitude curve concludes that the system is more sensitive for high frequencies and the expected peaks for each mode might increase progressively its amplitude. As peaks can be easily found with or without this correction, all measurements that are shown do not have this consideration.

The amplitude of the signal created by the sensor and processed by the lock-in for a large amount of frequencies is plotted in Fig. 4, where each peak represent that the membrane has gone into resonance or similarly, the system has reached a normal mode with a characteristic frequency w_{mn}. To localize just the first modes, the plotted frequencies reach 650 Hz.

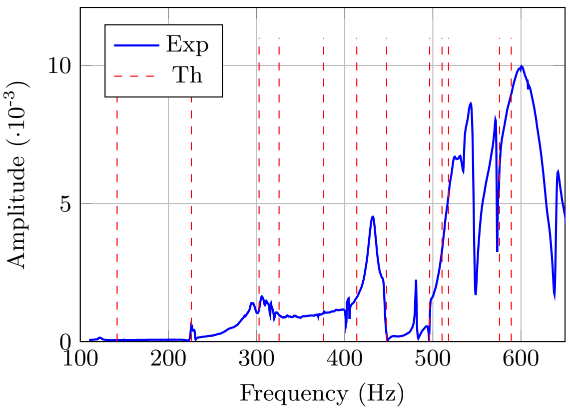


FIG. 4: Amplitude of the signal (Exp) as a function of the frequency of vibration (Hz). The red vertical lines (Th) mark the first twelve theoretical frequencies of the tuned head.

The first twelve peaks and its frequencies, representing the first twelve normal modes of vibration of the head, are collected in Tab. I. It also includes relative frequencies, with respect to the first overtone, a comparison with the theoretical values and the relative error (ε_r) of the measurements.

Mode	w^t_{mn} (Hz)	w^{exp}_{mn} (Hz)	w^t_{mn}/w_{11}	w^{exp}_{mn}/w_{11}	ε_r
(0,1)	141.782	122.5	0.627	0.542	0.136
(1,1)	226.000	226	1.000	1.000	0.000
(2,1)	302.846	306	1.340	1.354	0.010
(0,2)	325.531	323	1.440	1.429	0.008
(3,1)	376.147	404.5	1.664	1.790	0.075
(1,2)	413.719	432.5	1.831	1.914	0.045
(4,1)	447.463	481	1.980	2.128	0.075
(2,2)	496.378	493.5	2.196	2.184	0.006
(0,3)	510.414	531.5	2.258	2.352	0.041
(5,1)	517.787	543	2.291	2.403	0.049
(3,2)	575.634	571.5	2.547	2.529	0.007
(6,1)	588.961	602	2.606	2.664	0.022

TABLE I: Theoretical and experimental frequencies of the first twelve normal modes for the batter head when the $(1,1)$ is tuned at 226 Hz. The theoretical and experimental values for the relative frequencies are also considered, apart from the relative error.

B. Measurements of the (1,1) mode during a sequential detuning

Once the system is proved for uniform conditions, the membrane is induced to a perturbation when two opposed lugs are tightened. In order to measure the resulting frequency splitting of the $(1,1)$ orthogonal modes, Worland's model is put to the test. However, since the membrane needs to be tapped, normally with a drumstick, so the modes are determined, the model is forced to be adjusted and proved with the described setup.

For a new set of measurements, the batter head is "retuned" with respect to the first overtone, to a frequency of $w_{11} = 220$ Hz. Apart from the piezoelectric actuator, three different piezos acting as sensors are placed in the membrane as it can be seen in Fig.5: the first one (H) is attached close to the lug that will be tightened. Perpendicular to the tensioned diameter originated by the tuning and close to a lug, the second piezo (L) is attached. Finally, the third and the last sensor (B) is found between them. They are not all connected to the lock-in at the same time, so a measurement in each position is required. For each sensor, the amplitude of its converted signal is listed for a lower range of frequencies, from 210 Hz to 250 Hz, since the $(1,1)$ mode is well-localized. The amplitude curve recorded for each sensor is plotted in Fig.6a. Even though the head is previously tuned, and the conditions are as uniform as possible, there are little differences between the peaks recorded by the sensor "H" and sensor "L", being the first one higher as the model expected. In this case, $\Delta w_{1,1} = 4$ Hz ($\Delta w_{1,1}/w_{1,1} \sim 2\%$).

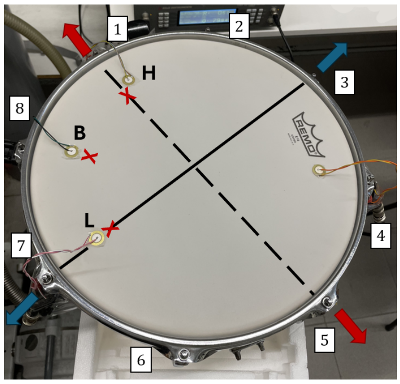
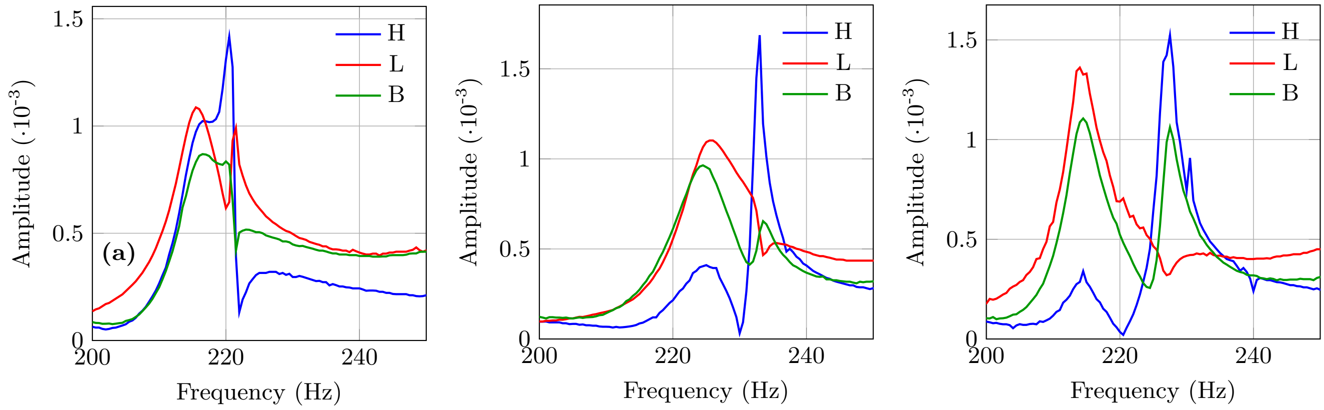


FIG. 5: Experimental setup in order to measure the $(1,1)$ splitted modes when the the 1-5 lugs have been tightened 1/4 turn. It also indicates the lugs 3-7 that are lightened in the final measure. Three piezoelectric sensors marked with and "H", "B" and "L" are placed in consideration of the model described in Fig. 2.

A set of a sequential detuning follows. Firstly, to increase the perturbation, two opposite lugs are tightened 1/4 turn (the tension rods are shown in Fig.5) and the proceeding is repeated. The amplitude curve recorded for each sensor is plotted in Fig.6b. Noticeably, the peak differences between the sensor "H" and sensor "L" widen, in comparison with the uniformly-tuned drum. In this case, there is an increase in $\Delta w_{1,1} = 9$ Hz ($\Delta w_{1,1}/w_{1,1} \sim 4\%$). Apart from this, the sensor "B" agrees well with Worland's model, as there are two peaks in its curve matching the values in "L" and "H", respectively. In other words, "B" registers a superposition of the splitted modes $(1,1)_+$ and $(1,1)_-$.

Lastly, having measured the splitting of frequencies of the first overtone when the tension of a diameter increases, the perpendicular diameter is lightened the same amount (the lugs 3-7 of Fig.5 are loosened 1/4 of a turn). A first theory would be that as there have been a symmetrically increase of tension in a diameter and a re-



(a) Tuned head. **(b)** Lugs 1-5 tightened 1/4 turn. **(c)** Lugs 1-5 tightened and lugs 3-7 loosened.

FIG. 6: Amplitude of the signal measured by the three piezoelectric sensors as a function of the vibration frequency (Hz) of the drumhead, under three different tension configurations. (a) Tuned head with respect to the first overtone, $w_{11} = 220$ Hz. Respectively to "H" and "L", $w_{11}^{+} = 220.5$ Hz and $w_{11}^{-} = 215.5$ Hz. (b) Lugs 1-5 have been tightened 1/4 turn. Respectively to "H" and "L", $w_{11}^{+} = 233.5$ Hz and $w_{11}^{-} = 224.5$ Hz. (c) Lugs 1-5 have been tightened 1/4 turn and the lugs 3-7 lightened 1/4 turn. Respectively to "H" and "L", $w_{11}^{+} = 227.5$ Hz and $w_{11}^{-} = 214.5$ Hz.

duction of the same amount in the orthogonal diameter, then, the forces might cancel bringing the tuned situation back. However, if the experiment is repeated, the plotted curves, summarized in Fig.6c, manifest not only the persistence of the frequency splitting but a growth in the difference between peaks of the "L" sensor and the "H" sensor, being $\Delta w_{1,1} = 13$ Hz ($\Delta w_{1,1}/w_{1,1} \sim 6\%$). This variation is originated because when the perpendicular diameter suffers a reduction in tension, generally in the batter head, the average tension decreases, causing a little drop in the frequency registered by "H", but since the wave will travel slower locally where the tension is reduced, the diminishment of frequency that is obtained in "L" is more significant.

VI. Conclusions

After the written analysis of the different measurements, there are several outcomes that come to light. Firstly, the system that has been designed using a lock-in amplifier and piezoelectric actuators and sensors to capture the frequencies in which the batter head is excited works with precision. Regardless of being in a uniform context, when the membrane is tuned, or when the head suffers perturbations in the tension, the lock-in is able to register the way the mode $(1, 1)$ splits its degeneracy into two distinct frequencies. Lastly, not only Worland's model for the frequency splitting agrees with the result presented, but the revision in order to adapt it to the setup is successful.

Acknowledgments

I would like to express my gratitude to my advisor, Joan Manel Hernàndez, for always being attentive and coming up with the brilliant idea for the setup. Thanks to Pau and Joan for lending me a snare and helping me change the membrane and tune it. Finally, this would have been much harder without the ideas of Julia Vilageliu that allowed me to find a thematic for this work.

[1] B. S. Ramakrishnaand , M. M. Sondhi, "Vibrations of Indian musical drums regarded as composite membranes," *J.Acoust. Soc.Am.*, **26**, 523–529 (1954).
[2] Rossing, Thomas D. "The Physics of Kettledrums." *Scientific American*, vol. 247, **5**, 172-179, (1982).
[3] P.M. Morse, *Vibration and Sound, 2nd ed.* (McGraw Hill, New York, 1948) (reprinted by Acoust. Soc. Am., 1981).
[4] N. H. Fletcher andT. D. Rossing, *The Physics of Musical Instruments* (Springer-Verlag,NewYork, 1998), Chap.3.3.
[5] R.Worland, "Normal modes of a musical drumhead under non-uniform tension". *J Acoust Soc Am.* **127(1)**, 525-33, (2010).
[6] T. D. Rossing, I. Bork, H. Zhao, and D. O. Fystrom, "Acoustics of snare drums", *J. Acoust. Soc. Am.*, **92** , 284–294 (1992).
[7] D. E. Cahoon, "Frequency-time analysis of the bass drum sound," Masters degree thesis, Brigham Young U., Provo, UT (1970) (unpublished).
[8] H. Fletcher and I. Basse, "Some experiments with the bass drum", *J. Acoust. Soc. Am.* **64**, 1570-76 (1978).
[9] T. R. Moore, "A simple design for an electronic speckle pattern interferometer," *Am. J. Phys.*, **72**, 1380–1384, (2004).
[10] National Institute of Standards and Technology, "ESTAR database", `https://pml.nist.gov/PhysRefData/Star/Text/ESTAR-t.html`
[11] P. M. Morse and K. U. Ingard, *Theoretical Acoustics*, (Princeton University Press, Princeton, NJ, 1968), 209–213.

Desafinació seqüencial d'una membrana de tambor amb un amplificador lock-in

Author: Ricard Sánchez Elias
Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.

Advisor: Joan Manel Hernàndez Ferràs
Dept. de Física de la Matèria Condensada, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.

Resum: Sempre que s'afina la membrana d'un tambor, una pertorbació en la tensió de la membrana indueix la divisió de les freqüències dels modes asimètrics degenerats (m, n), amb $m \neq 0$. Tot i que les diferències es poden detectar a l'oïda —fet clau per als bateries—, enregistrar-les numèricament pot ser més complicat. Per analitzar-les, s'ha dissenyat un muntatge que es posarà a prova. Mitjançant l'ús d'un amplificador lock-in i sensors i actuadors piezoelèctrics col·locats a la membrana, s'ha registrat l'amplitud del senyal produït pels sensors quan la membrana és excitada per vibrar en un rang de freqüències, responent de manera notable quan s'assoleixen les freqüències pròpies dels modes. Des de condicions uniformes, on els modes són accessibles, fins a la mesura de la divisió de freqüències quan es tensen simètricament els cargols del tambor, es demostra la funcionalitat del sistema i el comportament d'una membrana vibrant quan la tensió no és uniforme.
Paraules clau: Acústica, equació d'ona, modes normals de vibració, degeneració i divisió de freqüències.
ODSs: Aquest TFG està relacionat amb els Objectius de Desenvolupament Sostenible (SDGs), concretament l'ODS 4, Educació de qualitat i l'ODS 9, Indústria, innovació, infraestructures.

Objectius de Desenvolupament Sostenible (ODSs o SDGs)

1. Fi de la es desigualtats		10. Reducció de les desigualtats	
2. Fam zero		11. Ciutats i comunitats sostenibles	
3. Salut i benestar		12. Consum i producció responsables	
4. Educació de qualitat	X	13. Acció climàtica	
5. Igualtat de gènere		14. Vida submarina	
6. Aigua neta i sanejament		15. Vida terrestre	
7. Energia neta i sostenible		16. Pau, justícia i institucions sòlides	
8. Treball digne i creixement econòmic		17. Aliança pels objectius	
9. Indústria, innovació, infraestructures	X		

Aquest TFG s'alinea amb l'ODS 9 – Indústria, Innovació i Infraestructura, ja que proposa un sistema de mesura precís i avançat per analitzar les freqüències pròpies de la membrana d'un tambor mitjançant sensors piezoelèctrics i un amplificador lock-in, evitant les limitacions dels mètodes acústics convencionals. També contribueix a l'ODS 4, Educació de Qualitat, ja que afavoreix la millora de coneixements procediments experimentals.