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Financing public education when altruistic agents have retirement concerns

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Abstract

El capital humà i, per tant, l'educació tenen un impacte en el benestar futur de la societat. En aquest treball s'estudia la connexió entre el suport dels votants per una educació pública i les preocupacions sobre la seva jubilació. Mostrem com els votants anticipen els efectes positius que té l'educació sobre les pensions futures. Així, el recolzament a un sistema educatiu públic s'incrementa si el sistema de pensions és més redistributiu, cosa que també es dóna entre els ciutadans que prefereixen una educació privada. També mostrem que el tipus d'equilibri "ends against the middle" pot ocórrer fins i tot quan la taxa impositiva preferida pels votants és decreixent amb la renda.

Human capital and, therefore, education have an impact on the society's future welfare. In this paper we study the connection between the voters' support to public education and the retirement concerns. We show that voters anticipate the positive effect of education on future pensions. The support for a publicly financed education system increases, the more redistributive the pension system is, and this is true also amongst citizens preferring a private school. We also show that the "ends against the middle" equilibrium can occur even when the voters' preferred tax rate is decreasing in income.

JEL Classification number: D72, H31, H42, H52, H55
Keywords: Education, Pension System, Voting, Altruism, OLG

1 Introduction

Public debt concerns call for a reduction of the deficit and a reform of the welfare system. This paper contributes to the rich literature on the political economy of publicly provided goods, combining the voters' concerns for the provision of public education and of pensions.

Indeed, education and the pension system share several features, for instance, both are publicly financed/provided private goods[1] that are often used for inter- and intra-generational redistribution purposes. Both are deeply scrutinised, which limits the policy space of a politician with re-election concerns. Few papers consider simultaneously the provision of these two goods,[2] and they all conclude that, at least in the long run, education has a positive effect on growth and, consequently, a more generous pension system is sustainable.[3] In this paper, we explicitly model the demand for public education, when this has an impact on future welfare, and in particular the average income of the future generation depends on the current investment in education.

The microeconomic justification for agents' desire for a publicly financed education relates to a willingness of agents to guarantee a minimum level of education for all the citizens.[4] In part this is explained as intergenerational altruism[5] (parents care about their own children's education); another explanation is that agents care about the level of education of the whole society [6] In our model we account for both types of altruism and we add a third component: assuming a positive relationship between education and future incomes, we obtain a link between future pensions and the current level of education. Agents internalise the effect of

[1]Among others, Lochner and Moretti (2004), Checchi (2003), and Cai and Treisman (2004) provide some justifications for the public financing of private goods such as education.

[2]Kaganovich and Zilcha (1999) focus on the optimal (in terms of growth) allocation of tax proceeds between education and social security. Similarly, Pecchenino and Pollard (2002) and Zhang and Zhang (2004) study the impact of social security and education on growth. Soares (2003) uses an OLG model of growth, in which agents choose the public investment in education and allocate their time between investing in human capital and working. Section IV in Pirttila and Tuomala (2002) uses the investment on education as a determinant of productivity and studies the long run impact of public education and the use of education and pensions to have transfers among the population. Boldrin and Montes (2005) and Boldrin and Montes (2009) use an OLG model to study education and pensions. While our approach is positive, these are normative studies. The first one describes the theoretical design to accomplish the optimal intergenerational transfer scheme through pensions and education. The second one tests the theoretical model in the presence of demographic shocks, using Spanish data.

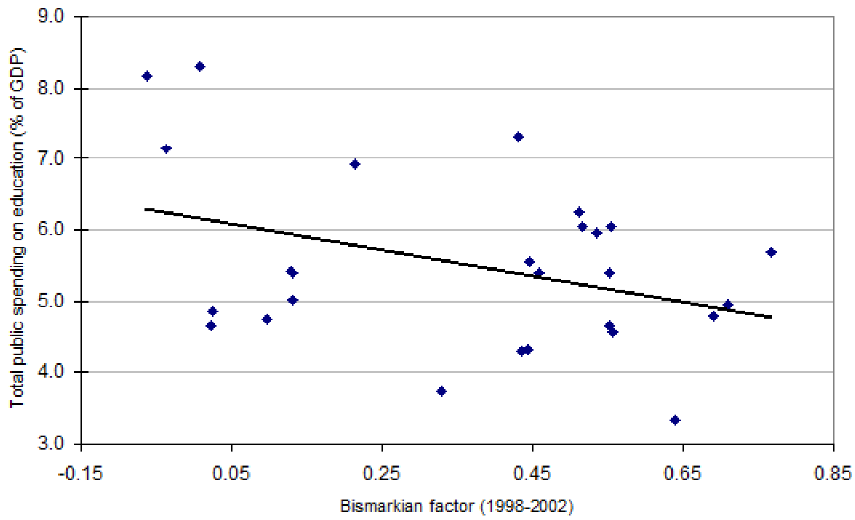
[3]The idea of education having an impact on growth and productivity has been also broadly used in the growth literature (Romer 1986, Lucas 1988, Gradstein and Justman 1997).

[4]From a microeconomic perspective, education is sometimes treated as a consumption good (Dur and Glazer 2008), or it is consumed for its signaling role on the labour market (Spence 1973) and, more generally, to increase own future expected income. These model are explain the consumption of education, but do not explain why it should be publicly financed.

[5]See Epple and Romano (1996a), (1996b), Glomm and Ravikumar (1998), Chen and West (2000), Cohen-Zada and Justman (2003), Gradstein and Justman (2005), and Piolatto (2010).

[6]This may happen for pure altruism or because of some positive externalities, such as the reduction in social conflicts, as in Lott 1990, Usher 1997 and Gradstein 2000). With a little abuse of notation, we define this behaviour as "altruistic", even if it may be the consequence of some selfish decisions.

education on pensions. If pensions are redistributive, therefore, agents care about others' level of education, and the more redistributive is the pension system, the more voters care about others' education. Therefore, we expect higher support for public spending in education where the pension system is more redistributive. Indeed, empirical evidence confirms a negative correlation between the Bismark Factor (a measure of the degree of redistribution, decreasing when redistribution increases) and total (Figures 1) and per-capita (Figure 2) public expenditure in education.[7]



Note: Data from World Bank. Correlation = -0.38

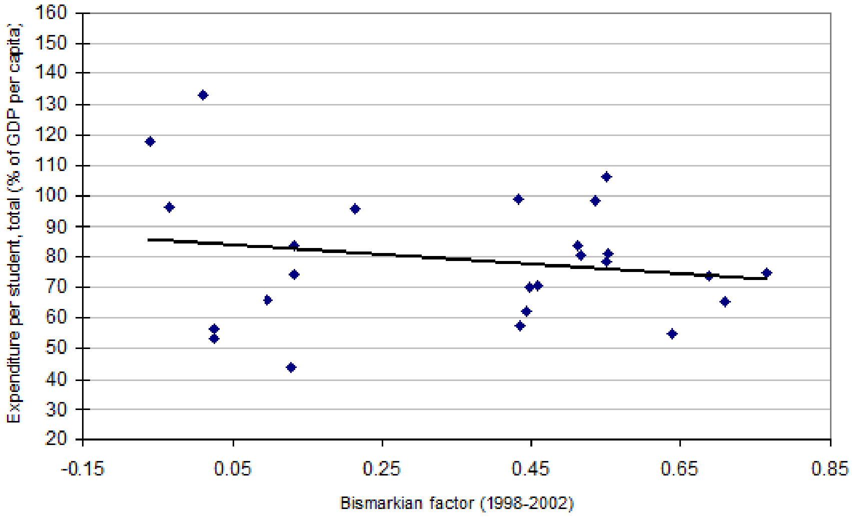
Figure 1: Bismarkian factor vs total public spending on eduction (% of GDP) in OECD countries

In our model, agents attend either a publicly financed school or a (costly) private one and vote over the tax rate to finance public education. Voters' decision depends also on the effect of education on pensions. Agents know and account for the fact that the average level of education of society has an impact on the pension system, with the impact's magnitude depending on the degree of redistribution of the pension scheme. In order to disentangle this private benefit accrued from others' education from any possible altruistic behaviour, we account separately for both in the agents' utility function. We model an overlapping generations (OLG) society in which agents, on top of the altruistic component, perceive a pension in their last period of life that depends on the population's average instruction.[8]

A notable result of the model is that it reconciles two results in the literature that were

[7]See appendix A.1 for more details on data sources and definition of variables.

[8]We explicitly analyse the case of pensions, but minor modifications allow to reinterpret the model in terms of the impact of average education on citizens' welfare via any social security service implying some income redistribution.



Note: Data from World Bank. Correlation = -0.19

Figure 2: Bismarkian factor vs public expenditure per student (% GDP per capita) in OECD countries

previously considered as incompatible. In a dual (public-private) educational system, preferences are not single peaked and the median voter theorem does not apply (Stiglitz 1974). In the theoretical literature, the *Slope Rising in Income* (SRI) and *Slope Decreasing in Income* (SDI) assumptions guarantee the existence of a voting equilibrium.[9] The intuition behind the alternatives is that an increase in the tax rate implies both more redistribution and better quality of the public service. When (in terms of voters' preferences) the former element dominates the latter, preferences show the SDI property and viceversa. The standard result under SDI (Gradstein, Justman, and Meier 2005) is that a coalition of the wealthier voters opposes a coalition of the poorest in determining the preferred tax to finance education. Under SRI (Epple and Romano 1998, and Epple et al. 2004), instead, the *ends against the middle* equilibrium (middle class versus the wealthy and poor voters) prevails. There is no strong evidence suggesting that one assumption is more realistic. We show that the *ends against the middle* equilibrium can occur even within the SDI framework. We also conclude that the common result of the median voter being pivotal under SDI is not robust to the introduction of education externalities.

The structure of the paper is as follows: section 2 presents the model, section 3 describes

[9]These two, mutually exclusive, assumptions (widely used in the literature) introduce a monotonicity restriction, implying a Single Crossing Property analogous to the Spence-Mirrlees condition. The preferred tax rate (conditional on attending public school) is assumed to be respectively increasing or decreasing in income. Technically, it is a condition on the sign of the derivative (with respect to income) of the marginal rate of substitution between education and the numeraire: non-negative under SRI and non-positive under SDI.

the agents' behaviour, and section 4 shows the possible equilibria. Section 5 illustrates the model using a specific functional form and provides some comparative statics results, while section 6 concludes.

2 The model

Three cohorts (students, adults and retirees) live at the same time. There is no population growth: each adult has one child. At the end of each period, children become adults and adults retire. Compulsory education is both publicly and privately provided: the two are mutually exclusive. The quality of education, modelled as per-student expenditure, is denoted by X_P for the public sector and X_R for the private one. $\overline{X}$ is the average quality of education, measured in terms of the cohort's average spending for instruction.

Public education is financed through a proportional tax (t) on income (ω) and its access is free. We assume the quality of public education to be homogeneous amongst schools. Private schools are costly, and students choose the level of quality to buy.[10]

Adults vote over the universal tax rate to finance public education, and choose the instruction that their child receives (public or private). For private school students, the decision includes the share of budget devoted to education; the remaining is used to consume the numeraire b. When deciding, they are concerned by their children's education and their own consumption of the numeraire (both in the current and in the next period of their life) as in the models of "intergenerational altruism". Adults care also about others' education: we separate here the two components presented in the introduction. On the one hand, we consider the "direct effect", or "altruism" (agents care about the average level of education, for any possible reason, including social stability), and the "indirect effect", or "selfish component" that accounts for the effect (through pensions) of others' education on the own future consumption.

Altruism enters the utility function through $V : \Re^2 \rightarrow \Re$ depending on both $\overline{X}$ and individuals' income ω.[11]

The average income of each generation is assumed to be a positive function of the education received: increasing the tax rate increases the average educational level and also agents' future income. This affects retirees' pension, via the public pension system.

[10] One can think of a person choosing amongst different schools, or a single school proposing extra services à la carte (e.g., elective classes, gym, library, extra-curricular activities).

[11] There are several rationales for altruism (as we broadly defined it) to depend on income and in particular to be increasing in it. As discussed in Dur and Teulings (2001): richer people's marginal utility of consumption is lower; education is often positively correlated both with income and with the importance that people attribute to others' education; richer people care more about social stability (and education has an impact on it); finally education can be a cheaper and preferable way to redistribute amongst social classes.

In period z, the life-time utility function, W_z, assumed to be separable, is

$$W_z(X, b_z, b_{z+1}, \overline{X}, \omega_z) = U(X, b_z) + G(b_{z+1}) + V(\overline{X}, \omega_z) \tag{1}$$

where U is the utility during adulthood (depending on consumption and owns children's school quality), G is the utility of consumption when retired, and V is the altruistic component. Adults face three trade-offs: 1) consumption of the numeraire versus their child's education, 2) current versus future consumption, and 3) own consumption versus average education of society (that comes from the altruistic component of the utility function).

For the sake of simplicity, people do not discount the future. Income, endogenous and time dependent, is distributed on the interval $(\omega_{\min}, \omega_{\max})_z$, with density f_z and cumulative distribution F_z. The average income is $\overline{\omega}_z = \int_{\omega_{\min}}^{\omega_{\max}} \omega_z f(\omega_z) d\omega_z$. Besides the tax to finance public education, a tax of rate s is levied to finance pensions: adults' disposable income is $\beta\omega$, where $\beta = [1 - t - s]$). Retirees' only source of income is given by their pension, which is not taxed.

The pension system is of the Pay-As-You-Go type consisting, similarly to Casamatta et al. (2000b), of a contributory and a noncontributory part. The pension system is mixed: a retiree in period $z + 1$ receives $\alpha s \cdot \omega_z + [1 - \alpha] s \cdot \overline{\omega}_{z+1}$. That is: he receives a fraction $\alpha \in [0, 1]$ of the tax for pension $(s \cdot \omega_z)$ paid in period z (contributory or Bismarkian component), and a share $[1 - \alpha]$ of the average contribution $s \cdot \overline{\omega}_{z+1}$ paid by current workers (noncontributory component); both s and α are exogenous.[12]

We assume the usual regularity conditions. Goods are normal (income elasticity takes values in the interval $(0, 1)$), functions U, G and V are of class C^2 or higher, U is increasing and concave both in X and b, and there are no cross effects between X and ω $\left(\text{i.e., } \frac{\partial^2 U}{\partial \omega \partial X} = 0\right)$. G is increasing and concave in consumption and V is increasing and concave in average education quality.

In order to capture the presence of some positive externalities of agents' education on income, we assume that adults' average income $\bar{\omega}$ is an increasing and concave function of the average level of education: $\frac{\partial \bar{\omega}}{\partial \overline{X}} \geq 0$ and $\frac{\partial^2 \bar{\omega}}{\partial \overline{X}^2} \leq 0$.

Lemma 1 (Aggregate expenditure for education)**.** *The aggregate educational expenditure is increasing in the tax rate t:* $\frac{\partial}{\partial t}\left(t\overline{\omega}_z + \int_{\omega_{\min}}^{\omega_{\max}} X_R(\omega_z) d\omega_z\right) \geq 0$.

Proof. A rise in the tax rate implies a reduction in disposable income for all agents. Those attending private school adjust their consumption of education according to their income elasticity of demand. Under the assumption of normality of X, the reduction in consumption is lower than the extra tax collected. Since all the collected tax is used to finance public education, aggregate expenditure in education is positively correlated with the tax rate t. □

[12]See Casamatta et al. (2000b) for a model on retirements with vote over s.

Corollary. *Because education is measured in terms of expenditure, lemma 1 implies that average education is increasing in the tax rate.*

Corollary. *Given the relation between average income and average education, and the previous corollary, we conclude that* $\overline{\omega}_z$ *is a function of* t_{z-1}*, with* $\partial\overline{\omega}_z/\partial t_{z-1} > 0$ *and* $\partial^2\overline{\omega}_z/\partial t_{z-1}^2 \leq 0$*. Henceforth, we replace the notation* $\overline{\omega}_z$ *by* $\overline{\omega}(t_{z-1})$*, to stress the intertemporal relation occurring with a one period lag.*

Corollary. *Altruism is increasing in the tax rate:* $\frac{dV(t_z,\omega)}{dt_z} \geq 0$*: by the chain rule,* V *increases in average educational expenditure and thus in the tax level.*

Remark: Income, depending on t, is endogenous. We do not compute the steady state level of t. Therefore, we do not need to specify the density function f_z nor do we need to study how income changes through time. Even though the density function might change over time, and so its extremes $(\omega_{\min}, \omega_{\max})$, the model's results are not affected, since they focus on the way coalitions are formed and not on the tax rate's absolute value. The *ceteris paribus* consequences on the tax rate of a generalised increase in income are *a priori* unknown. The assumptions on the preferred tax (SRI and SDI) only concern an agent's preferred tax, when others' contributions are constant. When others' income levels change, for a constant tax rate, tax proceeds and per student expenditure rise. The preferred tax rate remains constant only if utility functions are homothetic, otherwise it might increase or decrease through time.

We restrict our attention to when the monotonicity condition Slope Decreasing in Income (SDI) holds for function U: the marginal rate of substitution between education and the numeraire is decreasing in income $\left(\frac{\partial MRS_{X,b}}{\partial\omega} \leq 0\right)$.[13]

Population is normalised to one and the share of students attending a public school is n_p. From the government budget constraint, the quality of public school X_P is equal to

$$X_P = \frac{t\overline{\omega}}{n_p} \tag{2}$$

Adults' consumption of the numeraire is equal to $b_z = \beta\omega - X_R$ (disposable income net of the expenditure for private education). To stress that numeraire's consumption is different for agents attending a private school, for the numeraire consumption level in period z we use the notation $b_z = \beta\omega$ when $X_R = 0$ (agents attending public school) and $b_{z,R} = \beta\omega - X_R$ when $X_R \neq 0$.

[13] The only role of the SDI assumption in this paper is to allow easy comparisons with similar papers. The main model results are not affected.

Equation (1), defining the lifetime utility of an agent, can be rewritten as follows

$$W_z =$$
$$U(X_P, b_z) + \quad G(b_{z+1}) + \quad V(t_z, \omega) \text{ for } X_R = 0 \tag{3a}$$
$$U(X_R, b_{z,R}) + \quad G(b_{z+1}) + \quad V(t_z, \omega) \text{ for } X_R \neq 0, \tag{3b}$$

where Equation (3a) represents the utility of an agent whose child attends a public school, while Equation (3b) is for the others.

All relevant decisions are taken by adults. They choose their offsprings' school (public or private), they vote over the tax rate t and (for the case of private education) they choose how to share their budget between children's school tuition and the numeraire. Retirees consume all their pension to buy the numeraire good, which implies that a retiree's consumption of numeraire in period $z+1$ is given by $b_{z+1} = \alpha s \cdot \omega_z + [1-\alpha] s \cdot \overline{\omega}(t_z)$.

Lemma 2. *The third derivatives of functions U and G with respect to their arguments are positive, that is: $\frac{\partial^3 U}{\partial X^3} \geq 0$, $\frac{\partial^3 U}{\partial b^3} \geq 0$, $\frac{\partial^3 G}{\partial b^3} \geq 0$.*

Proof. See Appendix B.1. □

3 Voters' behaviour

We analyse here voters' behaviour, separating those preferring public education subsection 3.1) from the others (subsection 3.2). To do that, we first introduce two known results that we use subsequently: i) an income threshold $\hat{\omega}(t)$ exists, such that agents prefer public school if and only if $\omega < \hat{\omega}(t)$, while for $\omega = \hat{\omega}(t)$ they are indifferent; ii) the threshold $\hat{\omega}(t)$ determines a unique equilibrium. Note that adults' choice between public and private education only depends on the first part of the utility function (U), which shows the same properties as the utility function in Epple and Romano (1996a) and Glomm and Ravikumar (1998).

The maximisation problem of an agent whose child attends a private school is:

$$\max_{X_R} W_z = U(X_R, b_{z,R}) + G(b_{z+1}) + V(t_z, \omega) \tag{4}$$
$$s.t.\ b_{z,R} = (1-t-s)\omega - X_R$$
$$b_{z+1} = \alpha s \cdot \omega_z + [1-\alpha] s \cdot \overline{\omega}_{z+1}$$

The level of X_R maximising Equation (4) depends on t and it solves the equation: $\frac{\partial U}{\partial X_R} = \frac{\partial U}{\partial b}$. We define $X_R^* = \arg\max(W_z)$. The value for t also defines the numeraire consumption of adults choosing public education, this being all their disposable income.

Lemma 3 (From Epple and Romano, 1996a)**.** *Agents prefer private schooling, if and only if $U(X_R^*, b_{z,R}) > U(X_P, b_z)$. Given t, the level of income $\hat{\omega}(t)$ that makes an agent indifferent* *between the alternatives is unique. For any income* $\omega > \hat{\omega}(t)$, $U(X_R^*, b_{z,R}) > U(X_P, b_z)$*; vice versa, if* $\omega < \hat{\omega}(t)$*, then* $U(X_R^*, b_{z,R}) < U(X_P, b_z)$.

Proof. Since Equation (4) depends on X_R only through its first component, the framework is the same as in Epple and Romano (1996a). See Epple and Romano (1996a), pp. 300-304 and in particular lemma 1 (and its corollary 1) and lemma 2 for the proof. □

The share of students in public school, n_p, depends on the distribution of income:[14] $n_p = F(\hat{\omega}(t))$.

Lemma 4 (From Glomm and Ravikumar, 1998)**.** *For all* $t \in (0,1)$*, it exists a unique* n_p *that solves* $n_p = F(\hat{\omega}(t))$*. It can be observed that* $\hat{\omega}(t)$ *is decreasing in* n_p*, while increasing in* $\overline{\omega}$ *and in* t*.*

Proof. See proposition 2 and lemma 1 in Glomm and Ravikumar (1998). □

The income level for which an agent is indifferent between public and private education decreases in the number of public school students $\left(\frac{\partial \hat{\omega}}{\partial n_p} < 0\right)$, this comes from a congestion effect: the per-student available resources decrease when more students choose public education (given the tax rate), this makes public education less attractive and, therefore, the income cut-off level to be indifferent between the public and private education decreases. A change in the tax rate influences $\hat{\omega}(t)$ through two channels: it increases the total resources for public education, and it reduces agents' disposable income: numeraire's consumption falls and its marginal utility increases. Out of equilibrium, a further (indirect) effect of a tax increase is that it provokes an increase in n_p, which causes a reduction of $\hat{\omega}(t)$.

3.1 The behaviour of voters preferring public education ($\omega < \hat{\omega}$)

The utility function of agents whose child attends public school (i.e., $\omega < \hat{\omega}$) is given by Equation (3a), which can be rewritten as:

$$W_z = U\left(\frac{t\overline{\omega}}{n_p}, \beta\omega\right) + G(\alpha s \cdot \omega_z + [1-\alpha]s \cdot \overline{\omega}(t_z)) + V(t_z, \omega) \tag{5}$$

The maximisation of this expression with respect to t_z yields the following first order condition:

$$\omega_z \frac{\partial U}{\partial b} = \frac{\overline{\omega}_z}{n_p}\frac{\partial U}{\partial X} + k\frac{\partial \overline{\omega}(t_z)}{\partial t_z}\frac{\partial G}{\partial b} + \frac{\partial V}{\partial t_z} \tag{6}$$

where factor $k = s[1-\alpha]$ relates retirees' pension in time $z+1$ to adults' average income in $z+1$, which depends on the current tax rate t_z. It is an equity measure, i.e., it computes how much the society is redistributing, and it is equal to 0 under a purely Bismarckian/contributory

[14]See Glomm and Ravikumar (1998) for more details and properties of n_p.

system ($\alpha = 1$). A decrease in k implies a reduction in adults' marginal benefits of increasing the current education tax. Equation (6) implicitly defines $t^*(\omega)$, the preferred tax rate of an agent of income ω, equating the marginal cost and the marginal benefit of increasing the tax. The left hand side represents the loss of utility caused by a reduction in the consumption of the numeraire during the current period. The right hand side includes the additional utility generated by i) the higher quality of education that the own child receives (first term), ii) the increase in future consumption, through the increase in pensions (second term), iii) the altruistic attitude component, for which the average education level matters (last term).

Note that 1) if pensions are not related to the educational level (i.e., the pension system is purely contributory or education does not affect wages) and 2) agents are not altruistic ($\frac{\partial V}{\partial t_z} = 0$), then we are back to the model of Epple and Romano (1996a).

Equation (6) can be rewritten as:

$$\omega_z \frac{\partial U}{\partial b} - \frac{\overline{\omega}_z}{n_p} \frac{\partial U}{\partial X} = k \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \frac{\partial G}{\partial b} + \frac{\partial V}{\partial t_z} \tag{7}$$

The right hand side accounts for the second period consumption and the altruistic component. It is a function of the Bismarckian factor, the impact of t on average income, the importance of the second period consumption and altruism, and it is always positive. The left hand side regroups the effects of a tax rate change on the first-period utility. It takes negative values in the unrealistic case of $\frac{\partial U/\partial X}{\partial U/\partial b} > \frac{n_p \omega_z}{\overline{\omega}}$, which is a condition on the marginal rate of substitution between education and current consumption, occurring if the marginal utility of education is so large that agents prefer not to consume the numeraire in the first period of life.

Lemma 5. *For a voter whose child is attending a public school, the preferred tax changes with income as follows:*

$$\frac{\partial t^*}{\partial \omega} = \frac{-\frac{\partial U}{\partial b} - \beta \omega_z \frac{\partial^2 U}{\partial b^2} + k \alpha s \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \frac{\partial^2 G}{\partial b^2} + \frac{\partial^2 V}{\partial t \partial \omega}}{-\left[\omega_z^2 \frac{\partial^2 U}{\partial b^2} + \left[\frac{\overline{\omega}_z}{n_p}\right]^2 \frac{\partial^2 U}{\partial X^2} + k \frac{\partial^2 \overline{\omega}(t_z)}{\partial t_z^2} \frac{\partial G}{\partial b} + \left[k \frac{\partial \overline{\omega}(t_z)}{\partial t_z}\right]^2 \frac{\partial^2 G}{\partial b^2} + \frac{\partial^2 V}{\partial t^2}\right]}. \tag{8}$$

The sign of Equation (8) is given by

$$sign\left(\frac{\partial t^*}{\partial \omega}\right) = sign\left(-\frac{\partial U}{\partial b} - \beta \omega_z \frac{\partial^2 U}{\partial b^2} + k \alpha s \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \frac{\partial^2 G}{\partial b^2} + \frac{\partial^2 V}{\partial t \partial \omega}\right), \tag{9}$$

and it depends on the last term (the derivative of the altruism component V), absent which Equation (8) is always negative.

Proof. Equation 8 is obtained by implicitly deriving Equation 6 with respect to ω. Its sign only depends on the numerator: the denominator is always negative and becomes positive, being preceded by the minus sign. Furthermore, $k \alpha s \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \frac{\partial^2 G}{\partial b^2}$ is always negative and so does also $-\frac{\partial U}{\partial b} - \beta\omega_z \frac{\partial^2 U}{\partial b^2}$ by the SDI assumption. Consequently, only $\frac{\partial^2 V}{\partial t \partial \omega}$ may reverse the sign of the equation, if positive and sufficiently large. $\square$

In Epple and Romano (1996a), the SDI assumption (implying $-\frac{\partial U}{\partial b} - \beta\omega \frac{\partial^2 U}{\partial b^2} \leq 0$) ensures that the preferred tax is decreasing in income. Here two additional terms contribute to determine if the preferred tax is increasing in income, which occurs if and only if $\frac{\partial^2 V}{\partial t \partial \omega} \geq \left(\frac{\partial U}{\partial b} + \beta\omega_z \frac{\partial^2 U}{\partial b^2}\right) - k\alpha s \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \frac{\partial^2 G}{\partial b^2}$. On the right hand side, both the bracketed term and the remaining term are positive: the intuition is that the first period dis-utility from an increase in t is increasing in income (elements in the bracket) and the second period utility variation for a tax increases is decreasing in income (element outside the bracket).

Define $\tilde{\omega}$ as the inflexion point for which Equation (8) equals 0, that is $\frac{\partial t^*(\omega)}{\partial \omega} |_{\tilde{\omega}} = 0$. Then, further deriving Equation (8) with respect to ω determines if the preferred tax rate is concave or convex in income, thus if $\tilde{\omega}$ is a minimum or a maximum. The shape of the preferred tax depends on: i) the impact on agents utility of the numeraire consumption (both in their first and second period of life) and ii) the shape of function V (how income affects altruism). Unfortunately, we cannot determine the concavity of $t^*(\omega)$ (nor the sign of Equation 8) without adding some additional assumptions on the shape of the third derivatives of U, G, and V.

As a way to reduce the possible cases to study, we present empirical evidence aimed at finding a justification for focusing on the convex case. Indeed, our evidence (described in Appendix A.2) suggests that the preferred tax rate is convex in income; almost everywhere decreasing, although we cannot exclude that it may increase for sufficiently large levels of income. Limitations in the available data and in the possible proxies for the preferred tax rate made it impossible to have unquestionable results. Nevertheless, based on what empirical evidence there is, we proceed under the assumption that the preferred tax is convex in income.[15]

3.2 The behaviour of voters preferring private education ($\omega > \hat{\omega}$)

In Epple and Romano (1996a), parents of private school students are always in favour of no tax. The additional factors introduced (pension concerns and altruism) affect their behaviour. The utility function of agents whose children (at equilibrium) attend private school (i.e., $\omega > \hat{\omega}$) is given by Equation (3b), which can be rewritten as:

$$W_z = U(X_R^*, \beta\omega - X_R^*) + G(\alpha s \cdot \omega_z + [1-\alpha] s \cdot \overline{\omega}(t_z)) + V(t_z, \omega_z) \tag{10}$$

[15] Note that the results for the concave case were qualitatively similar to those of the convex case. Readers interested in the analysis of the concave case can refer to a previous and preliminary version of this study, published as working paper Piolatto (2011).

From the F.O.C. we derive the preferred tax rate, defined implicitly as:

$$\omega_z \frac{\partial U}{\partial b} = k \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \frac{\partial G}{\partial b} + \frac{\partial V}{\partial t_z}. \tag{11}$$

In Equation (11), the marginal cost of decreasing consumption in the first period equals the sum of consumption benefits in the second period and the additional utility connected with benevolence. A change in t does not affect X_R^*. Given income, the preferred tax is smaller for voters choosing a private school: indeed here, compared to Equation (6), the $\frac{\overline{\omega}}{n_p}\frac{\partial U}{\partial X}$ term is missing, which means that people do not care about the quality of public school *per se*. Moreover, numeraire consumption is lower (because of the school tuition to be paid), thus the marginal cost of an additional reduction in consumption is larger. In fact, $\beta\omega - X_R^* < \beta\omega$ (where $\beta\omega$ is the consumption of numeraire of people attending public school), thus the marginal cost of the tax ($\omega\frac{\partial U}{\partial b}$) for adults choosing a private school for their offsprings is larger than if they had chosen a public school.

Lemma 6. *For voters whose child attends private school, the preferred tax changes with income as follows:*

$$\frac{\partial t^*}{\partial \omega} = -\frac{-\frac{\partial U}{\partial b} - \beta\omega_z \frac{\partial^2 U}{\partial b^2} + k\alpha s \frac{\partial \overline{\omega}(t_z)}{\partial t_z}\frac{\partial^2 G}{\partial b^2} + \frac{\partial^2 V}{\partial t \partial \omega}}{\omega_z^2 \frac{\partial^2 U}{\partial b^2} + k\frac{\partial^2 \overline{\omega}(t_z)}{\partial t_z^2}\frac{\partial G}{\partial b} + \left[k\frac{\partial \overline{\omega}(t_z)}{\partial t_z}\right]^2 \frac{\partial^2 G}{\partial b^2} + \frac{\partial^2 V}{\partial^2 t}}. \tag{12}$$

Altruism absent, the preferred tax is always decreasing in income.

Proof. Use the implicit function theorem on Equation 11 to compute the derivative of the optimal tax with respect to income. The denominator is always negative. The sign, as in the previous section, depends only on the numerator. The SDI assumption is sufficient to ensure that the sign of the two first terms in the numerator is always negative. □

Qualitatively, the shape of t^* is the same for agents preferring private ($\omega > \hat{\omega}$) and public education ($\omega < \hat{\omega}$).

4 Equilibrium

When comparing the behaviour of voters preferring either type of education, we obtain the following result, stated in Lemma 7, concerning the preferred tax rate:

Lemma 7. *The sign of Equations (8) and (12) is the same. This means that, although the preferred tax rate depends on voters school choice, the way it changes with income (sign of the derivative) is the same. More importantly, the stationary point $\tilde{\omega}$ is the same regardless of the chosen type of education. As we concentrate on the convex case, the stationary point $\tilde{\omega}$ represents the income of the agent for which the preferred tax rate is the lowest amongst all the agents that have chosen the same type of education.*

Proof. Because the numerator of both Equations (8) and (12) is the same; differences in the denominator determine a change in the value of t^* but not in the sign. The stationary point $\tilde{\omega}$ coincides with the point in which the numerator equals zero, thus it is the same for both types of agent. □

The previous lemma states that the stationary point $\tilde{\omega}$ (which under the convexity assumption is a minimum) does not depend on the school choice. Instead, the value of the preferred tax does depend on the school choice, as stressed by the following Proposition 1.

Proposition 1. *The preferred tax rate is always larger for agents if their children attend public school.*

Proof. See Appendix B.2. □

Figure 3 represents a possible shape of the preferred tax rate, for the convex case, distinguishing between the case of public and private education.

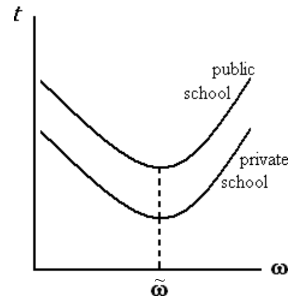


Figure 3: The preferred tax rate (convex case) depends on the chosen type of education

Under the same assumptions, in Epple and Romano (1996a) the median voter is always decisive. In terms of coalition formation, their equilibrium seems not to be robust to the introduction of altruism and pension concerns. We derive here some general properties on the type of equilibrium and in particular on the type of coalitions that may form, without using specific functional forms.[16] We conclude that coalitions, contrary to what predicted in Epple and Romano (1996a), are not always homogeneous in income.

For agents with income $\omega > \hat{\omega}$ (private school costumers), the direct effect of a tax is always negative (it is only a cost). Indirect effects are positive through the change on the average level of education and income. An increase in t induces a rise in consumption when retired; the redistributive effect of pensions has a larger impact on agents with a lower income, who will thus be more prone to an increase in the tax rate. If the pension system is highly

[16]We cannot solve the model in closed form without introducing some further assumptions and using specific functional forms.

redistributive (that is, when α is small), then all agents consumption in the last period of life (i.e., retirement) will heavily depend on average income and thus on average education. As a consequence, the support for a larger tax rate increase also amongst agents with income above the average (it is the only way to increase the level of consumption in retirement). Altruism induces all voters to be in favour of positive tax rates. The willingness to smooth consumption over time (intertemporal elasticity of substitution) and to balance the marginal utility of the numeraire and of education (intra-goods elasticity of substitution), and the degree of benevolence of the society, all contribute to determine the degree of convexity of the utility function.

We turn now to the political game in which the tax rate to finance public education, t, is chosen. An equilibrium consists of a tax rate t such that half of the population would prefer a higher tax rate, while the other half would prefer a lower one. We solve the model graphically, and we identify the types of coalition that can form, when people are asked to choose the equilibrium tax. For our graphical analysis of the equilibrium we use Figure 4 to distinguish all possible coalitions that may form. Since we observe a discontinuity in the preferred tax rate for the income level $\omega = \hat{\omega}$ (the income of the agent indifferent between public and private school), the equilibrium might depend on the position of $\hat{\omega}$ relative to $\tilde{\omega}$.

We assemble all agents with the highest preferred tax rate, up to form a group of half of the population. The light-grey areas in Figure 4 correspond to levels of income for which households prefer a reduction in the tax rate. The dashed lines identify the equilibrium tax rate, that by construction is such that half of the population (dark-grey area) would prefer a larger tax rate, and the corresponding Condorcet winner(s) that, a priori, is different from the preferred tax rate by the voter who is indifferent between public and private education ($\hat{\omega}$), the median voter, and $\tilde{\omega}$. The preferred tax rate is decreasing for the poorest agents in society, who always belong to the coalition asking for an increase in the tax rate. Instead, t^* may be increasing or decreasing for the richest agent.[17]

Note that the low-income agents always belong to the coalition in favour of an increase of the tax rate. This coalition (regrouping half of the population) may be entirely formed by the poorest agents in the society (equilibrium *low income against high income*), in which case the median voter is decisive; it is also possible that the richest agents in the society also join the coalition, therefore we observe a *ends against the middle* coalition. It is also possible that amongst the agents who prefer public education those with the largest income join the coalition in favour of an increase in the tax rate. Overall we observe, therefore, four possible coalitions (in terms of income), illustrated in Figure 4 (respectively in the top-left,

[17]Figure 4 depicts the case of an increasing right tail of the curve, while it is also possible that the function $t^*(\omega)$ is convex and decreasing over all the interval of existence.

bottom-left, top-right and bottom-right charts): 1) *low income against high income*, 2) *low income against high income* with some middle income agents joining the low income ones, 3) *ends against the middle* and 4) *ends against the middle* with some middle income agents joining the "very low and very high income" coalition. Note that equilibria 2) and 4) are only possible when $\hat{\omega} > \tilde{\omega}$. Further analytical results on the thresholds and conditions determining the different equilibria require introducing additional restrictions on the utility function that we consider and on the density of the population for different levels of income.

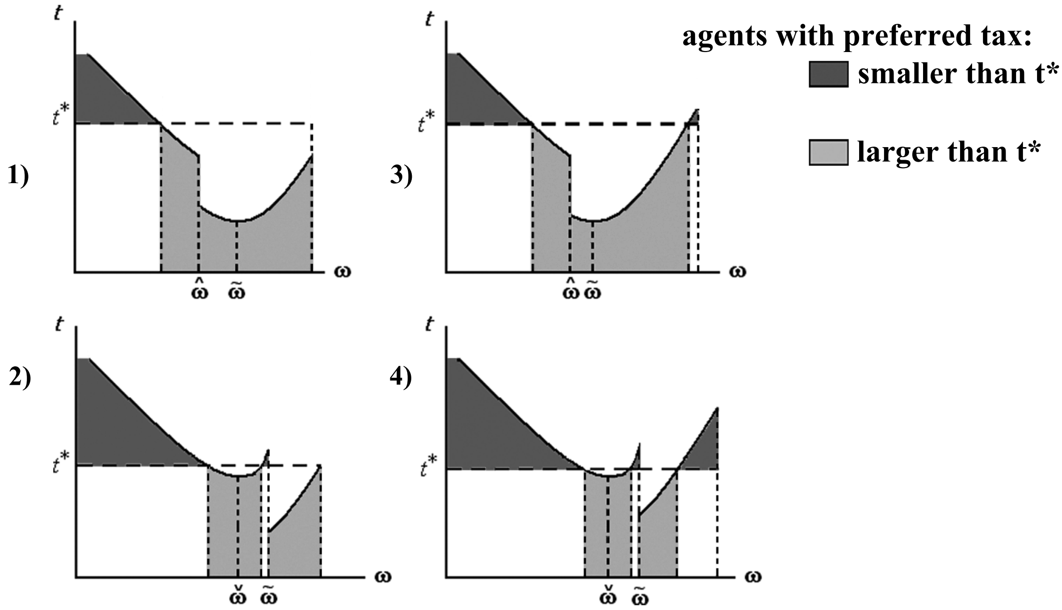


Figure 4: The different equilibria

Proposition 2. *Despite the SDI assumption, both the* low income against high income *equilibrium, and the* ends against the middle *one (opposing the middle class to the rest of the population) can be observed. For each of the described equilibria a variant may occur, in which the richest voters amongst those choosing public schools decide to join the coalition asking for an increase of the tax rate.*

Proof. Figure 4 and its explanation serve as a proof. The idea is that the preferred tax rate function is defined as two discontinuous segments, each convex. According to the population density function and the value of the function near the discontinuity point, the coalition may include the poorest agents in the population as well as the richest agents attending public school and the richest agents attending private school. □

When comparing our results with those in Epple and Romano (1996a), the additional concern for pensions, as well as altruism, will induce larger equilibrium tax rate. What is interesting to notice is that a further increase in the tax rate may occur in equilibrium, that depends on the way the coalition is formed and not directly on the additional concerns that we introduced in our model, as stated in Proposition (3).

Proposition 3. *Any deviation from the* low income against high income *equilibrium (equivalent to the Epple and Romano 1996a result under SDI), results in a higher equilibrium tax rate.*

Proof. Under the SDI assumption, the agents with the lower income have a decreasing preferred tax rate, with the median voter that is decisive and whose preferred tax rate is the lowest amongst the agents in the coalition. The two possible deviations from this equilibrium occur when some agents with income larger than the median voter (either the richest agents in the society or the richest agents amongst those choosing public education or both) prefer a tax rate larger than the one preferred by the median voter. In this case, those agents in the coalition with the lowest preferred tax rate leave the coalition and are replaced by others with a larger preferred tax rate. The tax rate corresponding to the new decisive voter must therefore be larger than before. □

Corollary. *The increase in the equilibrium tax rate necessarily induces,* ceteris paribus*, an increase in the per-capita expenditure for public education. This implies that the quality of public education increases. Part of this increase is offset (* arbitrage *effect) by the increase in the number of students choosing public education as a consequence of it becoming more attractive.*

Agents' motivation to lobby for an increase in the tax rate may be very different. Low income voters desire a good public service and profit of the (intra-temporal) redistribution between social classes. Both the highest-income voters within the public school and the highest income agents in the whole population are moved either by pure altruism or because of the inter-temporal redistribution effect of education, which implies that a larger current tax rate implies more consumption in the future. Those agents tend to loose from intra-temporal redistribution, moreover those choosing private education also suffer from a low consumption of the numeraire in the first period of life, which would further increase in case of a tax increase.

5 An illustration of the model

In this section we use simple functional forms to illustrate some properties of the model. We introduce two sets of assumptions. The first and less restrictive, allows us to obtain some general results. The second includes stronger assumptions and we need it to obtain closed-form solutions for more profound analysis of the problem. We are aware that some of the assumptions depart from those used in the education literature that calibrate models to fit the data. The complexity of the model requires simple functional forms to make the model tractable. We believe that this example helps clarify the results of the model.

The first set of assumptions requires the utility function of households (Equation 1) to take the form:

$$W_0(X, b_0, b_1, \overline{X}, \omega_0) = U(X) + U(b_0) + G(b_1) + V(\nu(\omega)\overline{X}). \tag{13}$$

Moreover, we assume function U to be homogeneous of degree d, and the income in period 0 to be distributed on the interval $[0, 1]$. The second group of assumption, that we will use only later in this section, includes that $F_0(\omega) = \omega^{0.2}$, $U(\cdot) = \frac{1}{0.1}(\cdot)^{0.1}$, $\nu(\omega) = g^2\omega^2$, $G(b_1) = b_1$ and $\overline{\omega}_1 = \overline{\omega}_0 + q\overline{X}$, where the last assumption means that in period 1 the average income is an affine function of the education received by period 1 workers, with the average income in the previous period as the constant term; g and q in the previous formulas are numerical parameters.

Using the first set of assumptions we can derive the optimal consumption of education and numeraire for the agents choosing the private education system, which are identical: $X_R = \frac{(1-t-s)\omega}{2} = b_{0,R}$. Given that people choosing public education consume $X_P = \frac{t\overline{\omega}}{n_p}$ and $b_0 = (1 - t - s)\omega$, we can compute the income of the indifferent agent $\hat{\omega}$, which is the income that solves the equation $F(\omega)\omega = \frac{t\overline{\omega}}{(1-t-s)} \frac{1}{U^{-1}(2^{1-d}-1)}$. The average consumption of education is therefore $\overline{X} = t\overline{\omega} + \int_{\hat{\omega}}^{1} \frac{1-t-s}{2}\omega f(\omega)d\omega$, which can be rewritten as

$$\overline{X} = \frac{1-s}{2}\int_{\hat{\omega}}^{1} \omega f(\omega)d\omega + \left(\int_{0}^{\hat{\omega}} \omega f(\omega)d\omega + \frac{1}{2}\int_{\hat{\omega}}^{1} \omega f(\omega)d\omega\right)t. \tag{14}$$

Finally, from the maximisation problem of agents, we can obtain the first order condition that implicitly defines the preferred tax by an agent attending either kind of education. Equation (6) becomes then

$$\frac{\overline{\omega}}{n_p}U'\left(\frac{t\overline{\omega}}{n_p}\right) - \omega U'\left((1 - t - s)\omega\right) + s(1 - \alpha)\frac{\partial\overline{\omega}_1}{\partial\overline{X}}\frac{\partial\overline{X}}{\partial t}G'(b_1) + \nu(\omega)\frac{\partial\overline{X}}{\partial t}V'\left(\nu(\omega)\overline{X}\right) = 0 \tag{15}$$

and Equation (11) becomes

$$\omega 2^{1-d}U'\left((1 - t - s)\omega\right) = s(1 - \alpha)\frac{\partial\overline{\omega}_1}{\partial\overline{X}}\frac{\partial\overline{X}}{\partial t}G'(b_1) + \nu(\omega)\frac{\partial\overline{X}}{\partial t}V'\left(\nu(\omega)\overline{X}\right). \tag{16}$$

Using the second group of assumptions, the two previous equations become:

$$\Upsilon \stackrel{\text{def}}{=} \left(\frac{\overline{\omega}}{n_p}\right)^{0.1} t^{-0.9} - \omega^{0.1}(1 - t - s)^{-0.9} + (sq(1 - \alpha) + g^2\omega^2)\kappa = 0 \tag{17}$$

and

$$t = 1 - s - \frac{2\omega^{\frac{1}{9}}}{(sq(1-\alpha) + g^2\omega^2)\kappa} \tag{18}$$

where $\kappa \stackrel{\text{def}}{=} \frac{\partial \overline{X}}{\partial t} \approx 0.0497$ is the effect on average education of a marginal change in the tax rate, under the second set of assumptions, obtained from Equation (14) that now becomes $\overline{X} = \frac{1}{12}(1-s) + \frac{1}{6}\left(1 - \frac{1}{6(2^{0.9}-1)^{10}}\right)t$.

We can now study the role of the parameter α and how it affects the preferred tax rate of an agent. For agents choosing public education, we can derive the impact of α as $\frac{\partial t*}{\partial \alpha} = -\frac{\frac{\partial \Upsilon}{\partial \alpha}}{\frac{\partial \Upsilon}{\partial t}}$ implicitly. Note that the sign only depends on $\frac{\partial \Upsilon}{\partial \alpha}$, as the denominator is always negative by the second order condition from the maximisation problem. The result of the derivation is $\frac{\partial t}{\partial \alpha} = -\frac{-qs\kappa}{-0.9\left(\left(\frac{\varpi}{np}\right)t^{-1.9} - (1-t-s)^{-1.9}\right)}$ and, as we expected, the sign of the derivative is negative. For the agents choosing private education the sign of the derivative is also negative: $\frac{\partial t*}{\partial \alpha} = -\frac{2qs\omega^{1/9}}{(sq(1-\alpha)+g^2\omega^2)^2\kappa}$.

Giving some numerical values to the parameters s, α, q, and g, the charts in Figure 5 help understanding the way the coalition forms when voting over the tax rate.

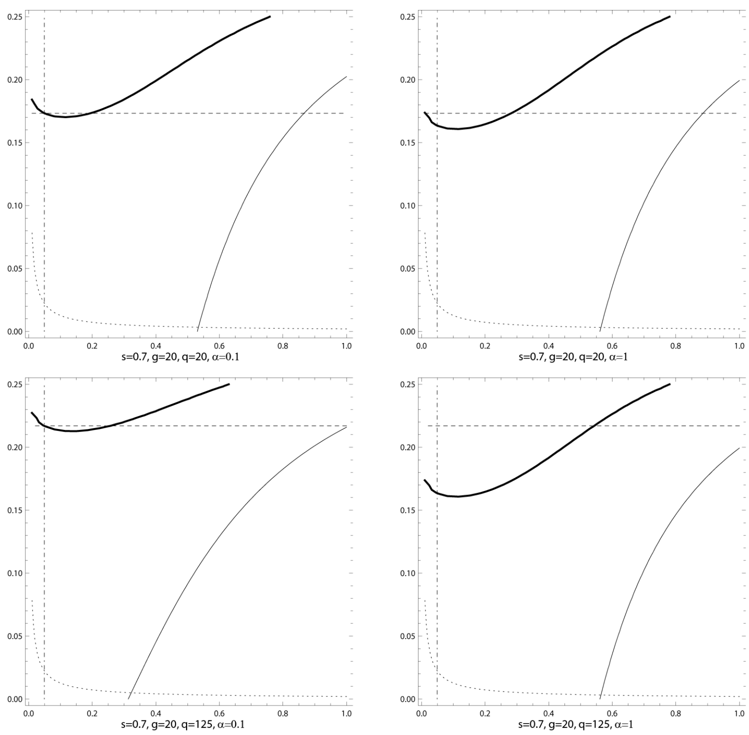


Figure 5: The coalition formation

In Figure 5 the thick line represents the preferred tax by households choosing public education, and the thin one is for those choosing private education. The vertical dashed-dotted line represents the median income agent, and the horizontal dashed line is his preferred tax rate for $\alpha = 0.1$ (Bismarkian factor), which is also the value of the parameter for the two charts on the left side of Figure 5, whereas for the right column we set $\alpha = 1$. The dotted line represents the distribution $F(\omega)$ of the population (using a 1:100 scale to fit into the picture). Comparing the top two charts, we notice that passing from $\alpha = 0.1$ to $\alpha = 1$ the preferred tax rate decreases for all agents, that is, the less redistributive is the social security system, the lower the preferred tax. Notice that we locate the horizontal dashed line at $\alpha = 0.1$, so that it is easier to compare the left charts with the right ones. The two bottom charts differs from the two previous ones by the level of q, that passes from $q = 20$ (top charts) to $q = 125$ (bottom charts). As you can notice, both α and q have an impact on the type of coalition that forms in equilibrium. While α measures the distributive role of the pension system (a larger α indicates a larger impact of the contributory part, and thus greater redistribution), q determines the impact of average education on average income. As is made clear in figure 5, the median income voter is not always decisive. Here, in particular, the agents with the largest income join the coalition of the ones with the lowest income, and the equilibrium is of the type "ends against the middle" for both values of α. Instead, the bottom-left chart shows a case in which the preferred tax rate of all the richest agents in the economy is below the one of the median voter, but when we move to the case of $\alpha = 1$ (bottom-right figure) we obtain again that some agents choosing private education prefer a larger tax rate than the median voter.

6 Conclusions

Adults transfer part of their income to new generations to provide education. Under the *Slope Decreasing in Income* (SDI) assumption, the marginal effect on agents' utility of an increase in the tax rate is larger through the channel of redistribution than through the channel of goods consumption. Therefore, the poorest voters (that benefit from redistribution) are in favour of large tax rates even at the price of low levels of consumption of the numeraire. On the opposite, richer voters (amongst those attending public school) prefer a smaller level of consumption of education since it implies also less redistribution and larger levels of consumption of the numeraire. Under this setting, the model reproduces the standard results in Epple and Romano (1996a). The median-income voter is pivotal, his most preferred tax rate is the majority voting equilibrium, and two coalitions opposes in equilibrium: one composed by the poorest agents in the economy and the other by the richest ones.

We add to this basic framework two additional reasons, namely pure altruism and self interest, why a voter may be in favour of publicly financed education, regardless of having children attending a public school. The self interest takes the form of intergenerational redistribution of wealth through pensions. Pensions in period $t + 1$ depend on the level of education, and thus taxes, in period t. Actually there may be several additional reasons why even people without children attending public school are interested in a more educated society: for instance, this implies less social problems, more technological and scientific progress (eventually leading to more infrastructures, services for the elders, new medical treatments, etc.).

Although the introduction of altruism and self interest reduce tractability, it is possible to solve the model graphically, reducing possible equilibria to four. In addition, section 5 provides a more profound analytical solution of the model, using some specific functional forms. We conclude that, under this new framework, the median voter is no longer always the Condorcet winner, this occurs only in one out of four possible cases. Otherwise, a middle-class coalition opposes, at equilibrium, a coalition of the poorest and richest voters ("ends against the middle" equilibrium). In both cases, we observe a possible deviation from equilibrium, with the wealthier agents in public school that may join the coalition calling for an increase in the tax rate. Those voters are middle-class agents, with an income close to the one of the agent indifferent between public and private education. They are very sensitive to a change in the tax rate, so that, according to the level of redistribution and their level of altruism, they may modify their behaviour being in favour of either an increase or a decrease of the equilibrium tax rate. The result of the basic model is affected by the introduction of the two additional elements that we proposed, in terms of i) the number of people attending public school: n_p weakly increases; ii) the equilibrium value of the tax rate (t^*), which (weakly) increases too; iii) the type of equilibrium: we show that some middle and high income agents may join the coalition in favour of an increase in the tax rate; and iv) the identity of the pivotal voter, which is no longer the median voter. We leave for future research, the interesting question of how the political decision over the tax rate to finance public school affects the voters preferences over the pension system. In particular, it could be interesting to allow agents (as in Casamatta et al. 2000a) to vote also over the tax s that finances pensions and over the Bismarkian factor α.

Appendix

A Empirical appendix

A.1 Empirical evidence: relation between public spending in education and the pension system

Figures 1 and 2 are constructed using two sources of information. We use the World Development Indicators (WDI) from the World Bank on public expenditure on education for a sample of OECD countries. World Bank data for the WDI contains annual data for the period 1998-2007.

To account for the redistributive component of the pension system (the Bismarkian factor) we use the estimates of Krieger and Traub (2009) who present the estimation using microdata (at a household level) taken from the Luxembourg Income Study (Luxembourg Income Study 2008) of the Bismarkian factor. The Bismarkian factor divides the pension benefit into a flat component (such as a basic or minimum pension) and into an earnings-related component. Their results are reproduced in Table 1.

Note from Table 1 that the estimates of the Bismarkian factor are 5 years averages and only for the period 1998-2002 we have an estimate for all the countries. This has been the period selected to construct Figures 1 and 2 presented in the text.

In a final effort to obtain an estimated coefficient relating the degree of redistribution of the pension system and the public expenditure on education we have constructed an unbalanced dataset to estimate a simple model[18] that relates public education spending (real expenditure on public primary and secondary education) with a series of possible determinants such as income (log of real per capita GDP), demographics (fraction of population at school age, fraction of population over 65 and fraction of population less than 15), other economic variables (log of the share of trade, imports and exports over GDP), revenue side of governments (tax revenues as a % of GDP) and the pension system (Bismarkian factor).

As previously explained, the fact of having the Bismarkian factor for 5-years average reduces the number of time observations available to use (although it has the advantage of avoiding business cycles). Given these data limitations the OLS estimates for the 5-year period (1988-2002) are not very robust and the estimates of the Bismarkian factor are only statistically significant, and negative as expected, when regressed alone against the public expenditure variables. The significance vanished when we introduce other possible determinants of public education expenditure, although the negative sign always remains.[19]

[18] For a similar econometric framework see for instance Baqir (2002), Verbina and Chowdhury (2004) for Russia or Fernández and Rogerson (2001) for US states.

[19] Econometric results are not reported but available upon request.

Table 1: Bismarkian factor for selected OECD countries

	0	1	2	3	4	5	6
Country		(1979-	(1983-	(1988-	(1993-	(1998-	(2003-
	1978	1983)	1987)	1992)	1997)	2002)	2004)
Australia	–	-0.019	-0.093	0.007	0.146	0.025	0.024
Austria	–	–	–	–	0.492	0.535	–
Belgium	–	–	0.413	0.586	0.598	0.516	–
Canada	0.004	0.036	0.048	0.066	0.116	0.129	–
Denmark	–	–	0.181	0.173	-0.021	0.009	-0.061
Finland	–	–	-0.065	-0.112	-0.029	0.553	0.511
France	–	0.697	0.722	0.686	0.778	0.765	–
Germany	0.299	0.409	0.398	0.278	0.491	0.556	–
Greece	–	–	–	–	0.694	0.639	–
Ireland	–	–	0.140	–	0.426	0.436	–
Italy	–	–	0.378	0.362	0.492	0.551	–
Luxemburg	–	–	0.382	0.353	0.296	0.328	0.508
Mexico	–	–	0.539	0.511	0.522	0.689	0.709
Netherlands	–	0.156	0.159	0.161	0.236	0.132	–
Norway	–	0.301	0.194	0.226	0.151	-0.035	–
Spain	–	–	–	0.427	0.409	0.444	–
Sweden	0.477	0.412	0.476	0.553	0.423	0.432	0.214
Switzerland	–	0.257	–	0.185	–	0.551	–
UK	-0.019	0.038	0.027	0.047	0.067	0.097	0.132
USA	0.223	0.342	0.296	0.310	0.360	0.458	0.447

A.2 In support of the assumption that the preferred tax rate is convex

In our theoretical model, for the sake of simplicity, we present only the results for the case in which the preferred tax rate is convex (see Figure 6 and below for its interpretation).[20] To the best of our knowledge, there is no direct way to estimate that shape given that, for instance, there are no surveys providing such information on taxes. Nevertheless, to approximate the shape of the individuals preferred tax rate and, hence, to solve the model relying on the convex case we provide an explanation and empirical evidence that, at least for the US case, the tax rate seems indeed to be convex.

For this purpose we used data from the PSID (the Panel Study of Income Dynamics), the longest running longitudinal household survey in the world (directed by faculty at the University of Michigan). The PSID is a US survey covering a representative sample of over 18,000 individuals living in 5,000 families in the United States. The data set contains information on employment, income, wealth, expenditures, health, marriage, childbearing, child development, philanthropy, education, and numerous other topics. We make use of the PSID for 2005.

For our purposes we make use of data from the PSID regarding donations to educational

[20]The results are qualitatively similar for the concave case.

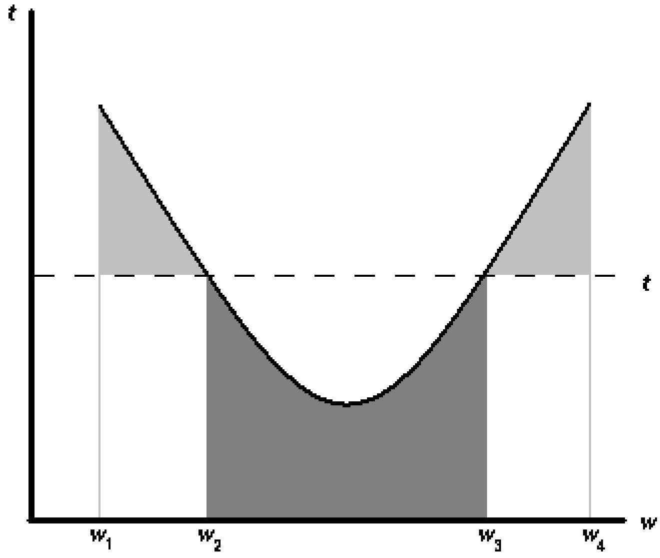


Figure 6: The preferred tax rate

institutions, from which we can grasp important information. Before moving to the data, having a look at Figure 6 we can point out the kind of information that we may obtain from the data.

As explained, Figure 6 presents the shape of the agents' preferred tax rate, under the convexity assumption, and the current equilibrium tax rate t. Suppose that an agent has a preferred tax rate that is below the equilibrium one (therefore, agents with income $\omega \in [\omega_2, \omega_3]$). This means that, given his utility function, he would prefer to have everyone contributing to the educational system less than what they currently do, therefore we expect (rational) agents with income $\omega \in [\omega_2, \omega_3]$ not to donate anything to any educational institution. If we consider now agents with a preferred tax rate above the equilibrium one (agents with income $\omega \in [\omega_1, \omega_2]$ or $\omega \in [\omega_3, \omega_4]$), those agents would prefer to have everyone contributing more, so as to improve the quality of the educational system. Those agents may want to contribute more, through donations for instance, or they may contribute more only if everyone would do so (through an increase in the tax rate) but they prefer not to pay more for education if only part of the population does so. From the previous considerations, we can infer the following: i) if we observe a donation, by the weak axiom of revealed preferences, we expect the agent to be in favour of an increase in the tax rate (if he voluntarily reduces his disposable income in order to improve the quality of education, he should be also in favour of a generalised increase in the tax rate, implying that everyone would contribute more); ii) if we do not observe a donation, we cannot infer that the agent would prefer a lower tax rate, as he may be willing to contribute more only if everyone does so.

If the preferred tax rate is convex, we should therefore expect to observe more donations amongst agents in the tails ($\omega \in [\omega_1, \omega_2]$ or $\omega \in [\omega_3, \omega_4]$) and very few for middle-income agents. Note that in our theoretical model we support the idea that agents with income below the average have an additional incentive to support high tax rates (compared to agents with income above the average) which is related to the redistributive effect of education. This component should not affect donations, therefore the predictions that we can make when we observe the data about donation may underestimate the willingness for a larger tax rate for the agents with income below the average.

Once we have derived the implications of a convex shape of the preferred tax rate and its relation with donations to education we can observe in the data if we find such pattern. From the PSDI we use questions ER27449 "Donations (>25$) to charity last year", [21] ER27474 "Donations to organization for education",[22] ER27475 "Dollar amount of education donations" and ER28037 "Total family income in 2004". Using the income variable we sort individuals by income (in deciles); Table 2 presents the main figures of the variables used.

Table 2: Donations descriptive statistics

	all indiv.	# total donat.	%/ all indiv.	# donat. to educ	%/ all indiv.	%/ total donat.
D1 income	799	203	25,4%	19	2,4%	9,4%
D2 income	798	300	37,6%	31	3,9%	10,3%
D3 income	814	377	46,3%	61	7,5%	16,2%
D4 income	780	384	49,2%	54	6,9%	14,1%
D5 income	796	466	58,5%	78	9,8%	16,7%
D6 income	808	515	63,7%	93	11,5%	18,1%
D7 income	782	554	70,8%	112	14,3%	20,2%
D8 income	797	647	81,2%	140	17,6%	21,6%
D9 income	804	684	85,1%	182	22,6%	26,6%
D10 income	788	713	90,5%	306	38,8%	42,9%
Totals	*7966*	*4843*	*60,8%*	*1076*	*13,5%*	*22,2%*
	Note: rows show data for individuals sorted by income deciles.					

As expected the number of donations increases with income, this is especially true for donations to education (except for the 4^{th} decile of income that decreases). However, for our purposes we compute the ratio between the amount of donations to education and the

[21] The survey includes the following definition: "Charitable organizations include religious or non-profit organizations that help those in need or that serve and support the public interest. They range in size from national organizations like the United Way and the American Red Cross down to local community organizations. They serve a variety of purposes such as religious activity, helping people in need, health care and medical research, education, arts, environment, and international aid. Our definition of charity does not include political contributions".

[22] "For example, colleges, grade schools, PTAs, libraries, or scholarship funds. Please do not include direct tuition payments for you or other family members".

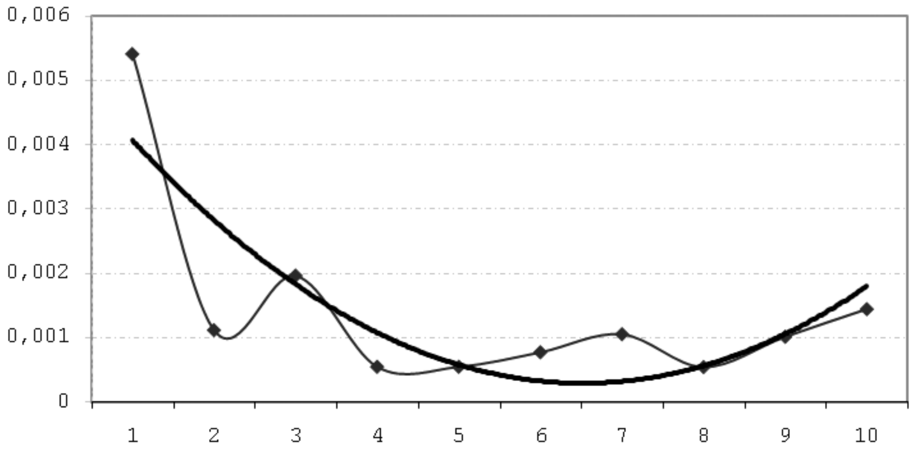


Figure 7: Total amount of donations to education over total income (by income deciles)

total income to capture the relative effort of each household when donating to education, see Figure 7.

Note that the shape presented in Figure 7 resembles a convex curve indicating that low and high income levels devote a higher share of its income to donations to education (they relatively donate more to education). Following the previous argument (see Figure 6) we can infer that those individuals have a higher tax rate and, hence, are more prone to want more spending in education in society. As a result of this evidence, we solve our theoretical model in the case of a convex preferred tax rate.

B Proofs

B.1 Proof of Lemma 2

Given that the first derivatives are positive ($\frac{\partial U}{\partial X} \geq 0$, $\frac{\partial U}{\partial b} \geq 0$, $\frac{\partial G}{\partial b} \geq 0$) and the second derivatives are negative ($\frac{\partial^2 U}{\partial X^2} \leq 0$, $\frac{\partial^2 U}{\partial b^2} \leq 0$, $\frac{\partial^2 G}{\partial b^2} \leq 0$), third derivatives have to be positive. To prove that, consider that a negative second derivative implies that the first derivative is a decreasing function. Then, a decreasing and concave function with an unbounded domain necessarily crosses, at some point, the horizontal ax, taking then negative values. This contradicts the fact that, for any value of the variable, the first derivative is always positive. Then a necessary condition for the first derivative to be always positive, when the second derivative is negative, is that the third derivative is positive (thus the first derivative is a decreasing and convex function).

B.2 Proof of Proposition 1

Denote t_1 the tax that maximises the utility of an agent of income ω_1 when his child attends a private school; by Equation 11 it has to be that $\omega \frac{\partial U}{\partial b} = k \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \frac{\partial G}{\partial b} + \frac{\partial V}{\partial t}\Big|_{\omega=\omega_1; t=t_1}$ (i.e., the marginal cost and benefit of the tax are equal). Suppose that an agent with the same income prefers his child to attend a public school. Compared to the previous agent, the larger consumption of numeraire implies that the left hand side (LHS) of the equation (the marginal cost) decreases. Instead, the marginal benefit (right hand side - RHS) is larger, because it includes, for people whose children attend a public school, the extra term corresponding to the marginal benefit in the first period of life of an increase in the tax rate. As a consequence, t_1 cannot be the equilibrium tax for this other agent and $\omega \frac{\partial U}{\partial b} < k \frac{\partial \overline{\omega}(t_z)}{\partial t_z} \frac{\partial G}{\partial b} + \frac{\partial V}{\partial t} + \frac{\overline{\omega}}{n_p} \frac{\partial U}{\partial X}\Big|_{\omega=\omega_1; t=t_1}$. Since LHS is increasing in t and RHS is decreasing, the equilibrium tax must be higher (see Figure 8).

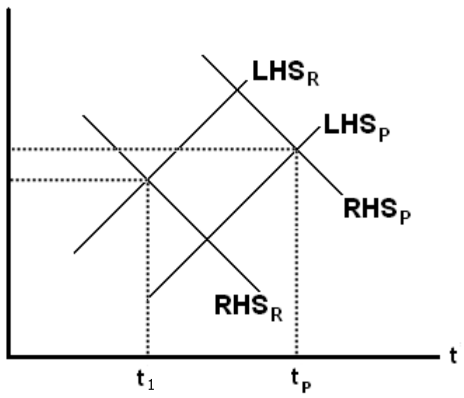


Figure 8: Preferred tax under public and private regime

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