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A theoretical and practical study on linear reforms of dual taxes

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A theoretical and practical study on linear reforms of dual taxe

Abstract: We extend the linear reforms introduced by Pfähler (1984) to the case of dual taxes. We study the relative effect that linear dual tax cuts have on the inequality of income distribution -a symmetrical study can be made for dual linear tax hikes-. We also introduce measures of the degree of progressivity for dual taxes and show that they can be connected to the Lorenz dominance criterion. Additionally, we study the tax liability elasticity of each of the reforms proposed. Finally, by means of a microsimulation model and a considerably large data set of taxpayers drawn from 2004 Spanish Income Tax Return population, 1) we compare different yield-equivalent tax cuts applied to the Spanish dual income tax and 2) we investigate how much income redistribution the dual tax reform (Act '35/2006') introduced with respect to the previous tax.

Keywords: Dual taxes, linear reforms, Lorenz domination, lattices

JEL: E60, E62

Resum: En aquest treball extenem les reformes lineals introduïdes per Pfähler (1984) al cas d'impostos duals. Estudiem l'efecte relatiu que els retalls lineals duals d'un impost dual tenen sobre la distribució de la desigualtat -es pot fer un estudi simètric per al cas d'augments d'impostos-. Tambe introduïm mesures del grau de progressivitat d'impostos duals i mostrem que estan connectades amb el criteri de dominaci de Lorenz. Addicionalment, estudiem l'elasticitat de la càrrega fiscal de cadascuna de les reformes proposades. Finalment, gràcies a un model de microsimulació i una gran base de dades que conté informació sobre l'IRPF espanyol de l'any 2004, 1) comparem l'efecte que diferents reformes tindrien sobre l'impost dual espanyol i 2) estudiem quina redistribució de la riquesa va suposar la reforma dual de l'IRPF (Llei '35/2006') respecte l'anterior impost.

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1 Introduction

In the last decades, there have been trends to reform tax systems in OECD countries. One of the implicit goals of tax reforms has been the reduction of the individual tax burden as a policy measure to boost economy and to promote incentives. In this context, Personal Income Tax reforms have had a prominent role in the political agenda.

In the case of the Spanish Income Tax, three reforms have been applied since 1998. The latest one introduced the Dual Income Tax (Act 35/2006). Tax cuts have been embedded within more general policy measures related to the efficiency and equity aspects of the tax. In particular, progressivity and redistributive consequences of such reforms have been relevant policy issues to be considered. The evaluation of the impact of such reforms on income distribution and individual welfare is basically a matter of empirical assessment.

If policy makers' primary concern is to reduce (or increase) tax liability so that characteristics of the tax such as progressivity, redistribution, elasticity are preserved after reform, Pfähler's analysis of linear tax reforms applied to a unidimensional tax would be very useful for its simplicity and normative results[1].

Pfähler (1984) considers neutral-revenue tax reforms that reduce (or increa-

se) tax liability. According to his results, tax cuts maintaining residual progression unchanged are the most redistributive reforms. Additionally, whenever the income distribution is positively skewed and the tax function is progressive such cuts are also the most welfare-improving reforms. Furthermore, this option would be chosen in a majority vote process for income distributions with more 'poor' than 'rich' tax-payers, in a way that will be shown later. For tax increases, initial tax liability progression should be the relevant measure to be considered.

Pfähler's analysis deals with the case of only one tax function (in fact, he uses Income Tax as a case in his study). The following question is consequently of interest. Are the above

[1] A tax reform is named 'linear' if the post-reform average tax rate is obtained as a linear transformation of the initial average tax rate.

results still valid for dual taxes, or more generally, for tax cuts (or increases) which are shared by two or more different taxes?

In this paper, we extend the analysis of linear tax reforms to the dual tax case, both theoretically and empirically. Specifically, we analyze if normative assessments derived from Pfähler are accomplished in the case of dual taxes.

To do this, dual progression measures are defined for dual taxes and their relationship to Lorenz dominance is discussed. Regarding Lorenz dominance criterion, it is proved that a partial order among linear dual tax reforms can be established if certain condition -that will be stated later- on income distributions is fulfilled, provided also that tax cuts are neutral-revenue in the two different income bases. This result can be considered a benchmark to guide reforms.

In the empirical part of the paper, we use a micro-simulation model to illustrate the differential incidence analysis of linear tax reforms which comes from the theoretical results.

The rest of the paper is organized as follows. Section 2 formalizes linear tax reforms in the case of dual taxes. Progressivity measures applied to dual tax schedules are discussed. Section 3 analyzes the effect of linear tax reforms on income inequality. Reforms are also compared in terms of Lorenz dominance. Section 4 is dedicated to the analysis of tax elasticity. Section 5 analyzes the effects of different tax cuts applied to a dual tax by means of a micro-simulation model which uses a large sample of Spanish income tax returns from 2005. We also study the effect of the Spanish Personal Income Tax (Act '35/2006') on the income redistribution regarding the tax before reform. Finally, Section 6 concludes.

2 Dual taxes, tax cuts and progressivity measures

The current structure of the Spanish Personal Income Tax is a dual one. There are two different tax schedules applied separately onto two different income bases, henceforth called labor and capital income bases. Regarding the analysis of tax reforms, there are two matters of significance: 1) the effect on the income inequality and 2) the *degree* of progressivity of the post-reform tax. Pfähler studied these issues in the context of three types of linear tax reforms for unidimensional taxes. We shall substantiate Pfähler's work in the context of dual taxes. This extension could be useful in other scenarios, for instance, the one in which a government intends to carry out a reform on (unidimensional) personal income tax and on value-added tax simultaneously.

Pfähler assumed that a tax function consisted only of a tax schedule which, applied to the pre-tax income, provided the post-tax income. Given any progressive tax schedule, he proposed three different (unidimensional) tax reforms to be defined respectively either as fraction a of the tax liability $T(x)$, as a fraction b of the post-tax income $V(x) = x - T(x)$ or as a fraction c of the pre-tax income x. Each one of these reforms is neutral with respect to local measure of tax progressivity: *liability progression*, *residual progression* and *average rate progression* (see Musgrave and Thin, 1948). If $t(x) = T(x)/x$ is the average tax rate of the current tax schedule $T(x)$, let $T_1(x)$, $T_2(x)$ and $T_3(x)$ be the three reforms:

$$T_1(x) = T(x) - aT(x) \iff t_1(x) = (1-a)t(x)$$

$$T_2(x) = T(x) - bV(x) \iff t_2(x) = (1+b)t(x) - b$$

$$T_3(x) = T(x) - cz \iff t_3(x) = t(x) - c.$$

Generally, the derivative of the tax schedule is a sum of step functions defined by marginal tax rates and by income thresholds. In this particular case, it is easy to prove that applying the above linear cuts on the whole tax schedule is equivalent to transforming the marginal tax rates according to the linear function that defines the tax cut[2]. The three above transformations of the original tax are neutral-revenue if and only if $b = ag/(1-g)$ and $c = ag$, where $g = \frac{\overline{T}}{\overline{z}}$ is the quotient between the average tax liability $\overline{T}$ and the average pre-tax income $\overline{z}$. Observe that g is the proportion of the tax burden in terms of pre-tax incomes, whereas $g/(1-g) = \overline{T}/\overline{V}$ is the proportion of the tax burden in terms of post-tax incomes. Hence, if we assume the yield-equivalent condition holds, there is only one design parameter,

[2]See Remark 1 in the Appendix

let us say a, whereas g is an ex-ante parameter and c and b are obtained multiplying a by g and $g/(1-g)$ respectively.

A tax function maps tax-payers' pre-tax income to net post-tax income. In the case of the Spanish PIT, the tax function is organized as follows. First of all, the pre-tax income of each tax-payer is divided into labor and capital pre-tax income respectively. Then, three similar sequential steps are applied separately to both labor and capital incomes. In the first one, using tax-payer's monetary and non-monetary information, allowances are deducted from gross income to obtain the taxable labor and the taxable capital incomes respectively. In the second step, using only tax-payer's monetary information, two different tax rate schedules are applied to each of the taxable incomes to obtain the gross tax liability. The difference between the taxable income and the gross tax burden is called gross post-tax income. Finally, the net post-tax income is obtained from the gross post-tax income after applying a series of tax credits.

Because of the difficulty of introducing non-monetary information into an analytical model in the theoretical part of this paper and for the sake of simplicity, we do not consider the role of allowances and tax credits on the tax. This approach is the same that Pfähler (1984) implicitly followed. When it comes to real tax functions, allowances can be roughly approximated by linear transformations of the pre-tax income (see for instance Fries et al, 1982). If we assume that allowances and tax credits steps are essentially a homothety, then taxable incomes can be seen as re-scaled pre-tax incomes, whereas net post-tax incomes can be seen as re-scaled gross post-tax incomes and none of our theoretical results has to be abandoned when we consider the real input and output of the tax function. Simulation results in Section 5 are obtained according to our theoretical predictions.

In what follows let $x \geq 0$ be taxable labor income before tax and $y \geq 0$ taxable capital income before tax. Then, a *dual tax schedule* is defined by

$$T(x,y) = L(x) + K(y) \leq x + y \tag{1}$$

where $L(\cdot), K(\cdot)$ are unidimensional tax schedules. When both $L(\cdot), K(\cdot)$ are *progressive*[3], we say that $T(x, y)$ is *quasi-progressive.*

Consider a (finite) set of tax-payers with pre-tax labor incomes and pre-tax capital incomes given by distributions $\widetilde{x} = (x_1, ..., x_n)$ and $\widetilde{y} = (y_1, ..., y_n)$ respectively, where (x_k, y_k) are the incomes of the tax-payer k. Then, let $\overline{x}$ be the *average pre-tax labor income*, $\overline{y}$ the *average pre-tax capital income*, $\overline{L}$ be the *average labor liability* and $\overline{K}$ the *average capital liability.* We also introduce the following initial rates, $g_L = \frac{\overline{L}}{\overline{x}}$ and $g_K = \frac{\overline{K}}{\overline{y}}$.

We represent the dual tax schedule obtained from a dual tax schedule $T(x, y)$ applying a reform of type i on the labor tax schedule and a reform of type j on the capital tax schedule by

$$T_{i,j}(x, y) = \overbrace{\rho_i^L L(x) + \sigma_i^L x}^{L_i(x)} + \overbrace{\rho_j^K K(y) + \sigma_j^K y}^{K_j(y)}, \tag{2}$$

for $i = 1, 2, 3$ and $j = 1, 2, 3$. As an example, in the case $T_{1,2}$ the parameters are $\rho_1^L = 1 - a_L$, $\sigma_i^L = 0$, $\rho_1^K = 1 + b_K$ and $\sigma_i^K = -b_K$. By means of simple algebra, it is proven that the positive (resp. negative) change $\Delta R_{i,j}$ on the aggregate *total*[4] post-tax income when a tax cut (resp. a tax increase) of type $T_{i,j}(x, y)$ is applied to $T(x, y)$ is given by

$$\Delta R_{i,j} = \left(\left(1 - \rho_i^L\right) \overline{L} - \sigma_i^L \overline{x} + \left(1 - \rho_j^K\right) \overline{K} - \sigma_j^K \overline{y}\right) \cdot n. \tag{3}$$

Dividing equation (3) by $(\overline{x} + \overline{y}) \cdot n$, the yield-equivalent condition for the above tax cuts is

$$\overline{\Delta R} = \delta \cdot \left(\overbrace{\left(1 - \rho_i^L\right) g_L - \sigma_i^L}^{\overline{\Delta L_i}} \right) + (1 - \delta) \cdot \left(\overbrace{\left(1 - \rho_j^K\right) g_K - \sigma_j^K}^{\overline{\Delta K_j}} \right) \tag{4}$$

where $\delta = \overline{x} / (\overline{x} + \overline{y})$ and the relative change $\overline{\Delta R}$ on the aggregate total tax liability with respect to the aggregate total pre-tax income does not depend either on i or on j. The above expression generalizes the yield-equivalent condition introduced by Pfähler to the dual tax case.

[3] That is, $\frac{d}{dx}(\frac{L(x)}{x}) \geq 0$, $\frac{d}{dy}(\frac{K(y)}{y}) \geq 0$.

[4] We use total to refer to both labor and capital.

An example of the usefulness of the above expression is provided next. When carrying out a tax reform, the policymaker usually decides which extra aggregate amount of money the tax-payers can keep (or the Govern-

ment not get), in comparison to the current tax. For instance, the policyma-

ker wants to carry out a cut $T_{1,2}$ that supposes that each tax-payer keeps on average 10% more of the pre-tax income compared to the pre-reform tax, i.e. $\overline{\Delta R} = 0.1$. Notice that g_L and g_K depend only on the current dual tax, whereas the proportion of the average pre-tax labor income with respect to the pre-tax average income, δ, depend only on the income distribution, i.e. neither g_L, g_K nor δ depend on the reforms. For instance, $g_L = 0.3$, $g_K = 0.2$ and $\delta = 0.8$. In such case, (4) results on $24 \cdot a_L + 16 \cdot b_K = 10$. That is, there is only one degree of freedom: either a_L or b_K. If for some reason b_K has to be equal to 0.05, then $a_L = 0.383$.

Observe that a particular and more restrictive case is obtained where $\overline{\Delta L_i}$ and $\overline{\Delta K_j}$ are both equal across reforms. In such case, (4) reduces to the yield-equivalent condition introduced by Pfähler, separately applied on the labor and on the capital parts of the dual tax.

Regarding the measuring of the degree of progressivity of a dual tax, the first approach is to consider a unidimensional measure. Given a dual tax schedule $T(x, y)$, we could 1) apply the definition of liability progression directly to $T(x, y)$ seen as a unidimensional tax on the total income $x + y$ and 2) consider a weighted mean of $\alpha_L(x)$ and $\alpha_K(y)$, the liability progressions associated to labor and capital tax schedules respectively.

However, the two above approaches are not without their flaws. The first option is not valid as $T(x, y)$ need not be a function on $x + y$. Indeed, it is not difficult to find two tax schedules $L(x)$ and $K(y)$ and two pairs of incomes (x, y) y (x', y') such as $x + y = x' + y'$ and $T(x, y) \neq T(x', y')$. On the other hand, in the second option a more progressive labor tax could be shadowed by a regressive -or less progressive- capital tax. The drawbacks of both unidimensional approximations are the same as those arising when we try to extend the complete order structure of $\mathbb{R}$ to $\mathbb{R}^2$.

On the basis of what have been stated, the next step would be to establish bidimensional definitions of the progressivity of dual taxes[5]. Such definitions should be used to compare, whenever possible, the degree of progressivity of any pair of dual tax schedules.

Next we define the *labor post-tax income* $V_L(x) = x - L(x) \geq 0$, the *capital post-tax income* $V_K(y) = y - K(y) \geq 0$, the *total post-tax income* $V(x, y) = x + y - T(x, y) = V_L(x) + V_K(y) \geq 0$, the *average labor type* $t_L(x) = \frac{L(x)}{x}$, the *average capital type* $t_K(y) = \frac{K(y)}{y}$ and the *average total type* $t(x, y) = \frac{T(x,y)}{x+y}$. Let also $>$ be the ordinary partial order on $\mathbb{R}^2$: $(x_1, y_1) > (x_2, y_2)$ if $x_1 > x_2$ and $y_1 > y_2$. Then, the following three measures are proposed:

- **Dual Liability Progression:** the elasticities of labor liability and capital liability with respect to the pre-tax labor income and the pre-tax capital income respectively are defined as

$$(5) \qquad \overrightarrow{\alpha_T}(x, y) = (\alpha^L(x), \alpha^K(y)) = \left(\frac{dL(x)}{dx} \frac{x}{L(x)}, \frac{dK(y)}{dy} \frac{y}{K(y)} \right).$$

- **Dual Residual Progression:** the elasticities of both the post-tax labor income and the post-tax capital income with respect to the pre-tax labor income and the pre-tax capital income respectively are defined as

$$(6) \qquad \overrightarrow{\psi_T}(x, y) = (\psi^L(x), \psi^K(y)) = \left(\frac{dV_L(x)}{dx} \frac{x}{V_L(x)}, \frac{dV_K(y)}{dy} \frac{y}{V_K(y)} \right).$$

- **Dual Average Rate Progression:** the changes of both the labor average type and the capital average type with respect to the pre-tax labor income and the pre-tax capital income respectively are defined as

$$(7) \qquad \overrightarrow{\beta_T}(x, y) = (\beta^L(x), \beta^K(y)) = \left(\frac{d\frac{L(x)}{x}}{dx}, \frac{d\frac{K(y)}{y}}{dy} \right).$$

It is important to point out that the definition of Liability Progression for a unidimensional tax schedule $T(z)$ was introduced by Musgrave and Thin (1948) as the limit of the elasticity

[5] For a further justification see Example 1 in the Appendix.

of the tax liability with respect to pre-tax incomes $x_0 < x_1$

$$\alpha_T(x_0, x_1) = \frac{T(x_1) - T(x_0)}{T(x_0)} \cdot \frac{x_0}{x_1 - x_0}, \tag{8}$$

when x_1 approximates x_0. We introduce this definition because discrete definitions will be needed for some proofs (see Proposition 4 in the Appendix).

Finally, the larger $\alpha_T(x)$ is, the more progressive $T(x)$ is in x according to the liability progression criterion. Therefore, we say that $T(x,y)$ is more progressive than $\widehat{T}(x,y)$ according to the dual liability progression criterion if $\overrightarrow{\alpha_T}(x,y) \geq \overrightarrow{\alpha_{\widehat{T}}}(x,y)$ for all (x,y). Analogous comments can be made about the other two dual measures.

3 Linear reforms and their effect on the inequality of the post-tax income distribution

This section is devoted to the study of the relative effect that each of the reforms proposed has on the inequality of the post-tax income distribution. This effect can be studied 1) globally, i.e. understanding what happens to the whole post-tax income distribution for the different cuts proposed, according to some global criteria and 2) locally, i.e. comparing relative net gain (or loss) across reforms for each tax-payer.

3.1 Global effects

First we focus on the global effects of linear dual tax reforms. We specifically prove that, provided some constraints to be introduced below hold, Pfähler-based cuts $T_{i,j}$ can be compared according to the (restrictive) criterion of Lorenz domination[6]. A symmetrical study

[6] Given an income distribution $\widetilde{x} = (x_1 \leq ... \leq x_n)$, the Lorenz curve $L_{\widetilde{x}}$ is defined by the following ordered pairs $\left(\frac{p}{n}, L_{\widetilde{x}}(p)\right)$, for $p = 1, ..., n$, where

$$L_{\widetilde{x}}(p) = \frac{\sum_{i=1}^{p} x_i}{\sum_{i=1}^{n} x_i}.$$

Then, given two income distributions $\widetilde{x} = (x_1 \leq ... \leq x_n)$ and $\widetilde{y} = (y_1 \leq ... \leq y_n)$, we say that $\widetilde{x}$ is Lorenz dominated (LD) by $\widetilde{y}$ if and only if $L_{\widetilde{x}}(p) \leq L_{\widetilde{y}}(p)$, for $p = 1, ..., n$.

can be carried out for tax hikes.

Taking into account the hypothesis of neutral revenue, Pfähler proved that his three unidimensional tax cuts can be compared using the LD criterion on post-tax income distributions. According to his results, T_2 is the most redistributive reform whereas T_1 is the least. On the other hand, Jakobbson (1976) showed that it is the same to compare the residual progression between two different tax schedules and compared, under the Lorenz Dominance, the post-tax income distributions obtained applying the two different tax schedules to an arbitrary distribution[7].

Next we prove that the two above results can be extended to include dual taxes if the two following conditions hold,

- **(Condition 1)** The aggregate labor tax liability and the aggregate capital tax liability are equal across reforms.

- **(Condition 2)** The relative order of incomes of both labor and capital income distributions coincides.

Observe that Condition 1 applies to the dual tax cuts and tells us that they must be simultaneously labor yield-equivalent and capital yield-equivalent. On the other hand, Condition 2 is applied to income distributions. This last condition does not hold if we consider real sets of tax-payers. However, it enables us to obtain benchmark results. Moreover, this condition makes more sense if we consider groups of tax-payers instead of single tax-payers. In Section 5 we show that assuming Condition 2 is feasible in light of real income distributions.

Given a dual tax $T(x,y)$ and labor and capital income distributions $\widetilde{x}$ and $\widetilde{y}$ we denote by $V_2(\widetilde{x},\widetilde{y}) = \{x_i + y_i - T(x_i,y_i)\}_{i=1}^n$ the post-tax income distribution and by $T_2(\widetilde{x},\widetilde{y}) = \{T_2(x_i,y_i)\}_{i=1}^n$ the tax liability distribution. The connection between the Lorenz dominance

[7] In the Appendix the results needed from both Jakobbson (1976) and Pfähler (1984) are rewritten in the adequate way to be used in this paper.

criterion and the dual progressivity measures proposed in this paper is shown in the next Theorem.

Theorem 1 *Given a (finite) set of tax-payers with labor incomes* $\widetilde{x} = (x_1 \leq ... \leq x_n)$ *and capital incomes* $\widetilde{y} = (y_1 \leq ... \leq y_n)$ *and two arbitrary quasi-progressive dual tax schedules* $T_1(\cdot, \cdot)$ *and* $T_2(\cdot, \cdot)$ *that are labor and capital yield-equivalent, then*

1. $\overrightarrow{\psi_1}(x, y) < \overrightarrow{\psi_2}(x, y)$ *for all* (x, y) *implies that* $V_2(\widetilde{x}, \widetilde{y})$ *is LD by* $V_1(\widetilde{x}, \widetilde{y})$*. However, the reciprocal does not hold.*

2. $\overrightarrow{\alpha_1}(x, y) > \overrightarrow{\alpha_2}(x, y)$ *for all* (x, y) *implies that* $T_1(\widetilde{x}, \widetilde{y})$ *is LD by* $T_2(\widetilde{x}, \widetilde{y})$*. However, the reciprocal does not hold.*

Proof.

1. By definition, if $\overrightarrow{\psi_1}(x, y) < \overrightarrow{\psi_2}(x, y)$ for all (x, y) we have that for all x

   $$\psi_1^L(x) < \psi_2^L(x) \tag{9}$$

   and for all y

   $$\psi_1^K(y) < \psi_2^K(y). \tag{10}$$

   From Proposition 1 in Jakobbson (1976) -see Proposition 4 in the Appendix-, we have that (9) and (10) are equivalent to $V_2^L(\widetilde{x}) = \{x_i - L_2(x_i)\}_{i=1}^n$ LD by $V_1^L(\widetilde{x}) = \{x_i - L_1(x_i)\}_{i=1}^n$ and $V_2^K(\widetilde{y}) = \{y_i - K_2(y_i)\}_{i=1}^n$ LD by $V_1^K(\widetilde{y}) = \{y_i - K_2(y_i)\}_{i=1}^n$. On the other hand, from hypothesis[8] and Lemma 4 in the Appendix we have that $V_2(\widetilde{x}, \widetilde{y}) = \{x_n - L_2(x_n) + y_n - K_2(y_n)\}_{i=1}^n$ is LD by $V_1(\widetilde{x}, \widetilde{y}) = \{x_i - L_1(x_n) + y_i - K_1(y_i)\}_{i=1}^n$.

[8] It is important to take into account that both $L(x)$ and $K(y)$ as well as both $L_i(x)$ y $K_j(y)$ for all $i = 1, 2, 3$ and $j = 1, 2, 3$ are non-decreasing functions and therefore do not alter the relative order of the income distributions. The same can be said about both $V^L(x)$ and $V^K(y)$ as well as $V_i^L(x)$ and $V_j^K(y)$ for all $i = 1, 2, 3$ and $j = 1, 2, 3$.

To check that the reciprocal does not hold it is sufficient to take any two income distributions $\widetilde{r} = (r_1 \leq ... \leq r_n)$ and $\widetilde{s} = (s_1 \leq ... \leq s_n)$ so that $\sum_{i=1}^n r_i = \sum_{i=1}^n s_i$, $V_1^L(\widetilde{x}) = \widetilde{r}$, $V_1^K(\widetilde{y}) = \widetilde{s}$, $V_2^L(\widetilde{x}) = \widetilde{s}$, $V_2^K(\widetilde{y}) = \widetilde{r}$ and $\widetilde{s}$ LD by $\widetilde{r}$. Then $V_2(\widetilde{x}, \widetilde{y}) = V_1(\widetilde{x}, \widetilde{y})$ holds but neither $\overrightarrow{\psi_1}(x, y) < \overrightarrow{\psi_2}(x, y)$ nor $\overrightarrow{\psi_1}(x, y) > \overrightarrow{\psi_2}(x, y)$ do hold.

2. Part 2 can be proved analogously to Part 1.

■

We are now in a position to prove that the nine linear reforms proposed can be compared in a specific way: they form a lattice[9]. However, before doing so we need to introduce some definitions.

Let $\Gamma(\gamma, \varphi, T)$ be the set of nine linear dual tax reforms of a quasi-progressive dual tax $T(x, y)$ with aggregate labor tax liability γ and aggregate capital tax liability φ. We also denote by $\Gamma^V(\gamma, \varphi, T, \widetilde{x}, \widetilde{y})$ (resp. $\Gamma^T(\gamma, \varphi, T, \widetilde{x}, \widetilde{y})$) the set of nine curves $LV_{i,j} = LV_{i,j}(\widetilde{x}, \widetilde{y})$ (resp. $LT_{i,j} = LT_{i,j}(\widetilde{x}, \widetilde{y})$) where $T_{i,j}(\cdot, \cdot) \in \Gamma(\gamma, \varphi, T)$ and $\widetilde{x}$ and $\widetilde{y}$ are respectively the labor and capital pre-tax income distributions.

For any $(\gamma, \varphi) \in \mathbb{R}_+^2$, given two reforms $T_{i,j}(\cdot, \cdot), T_{k,l}(\cdot, \cdot) \in \Gamma(\gamma, \varphi)$, we define the following operations $\vee, \wedge : \Gamma(\gamma, \varphi, T) \times \Gamma(\gamma, \varphi, T) \to \Gamma(\gamma, \varphi, T)$:

$$T_{i,j} \vee T_{k,l} = T_{i \oplus k, j \oplus l}, \quad T_{i,j} \wedge T_{k,l} = T_{i \ominus k, j \ominus l}, \tag{11}$$

where the commutative operators $\oplus, \ominus : \{1, 2, 3\} \times \{1, 2, 3\} \to \{1, 2, 3\}$ are defined by $1 \oplus 1 = 1$, $1 \oplus 2 = 2$, $1 \oplus 3 = 3$, $2 \oplus 2 = 2$, $2 \oplus 3 = 2$, $3 \oplus 3 = 3$ and $1 \ominus 1 = 1$, $1 \ominus 2 = 1$, $1 \ominus 3 = 1$, $2 \ominus 2 = 2$, $2 \ominus 3 = 3$, $3 \ominus 3 = 3$. We define as well $\vee^V, \wedge^V : \Gamma^V(\gamma, \varphi, T, \widetilde{x}, \widetilde{y}) \times \Gamma^V(\gamma, \varphi, T, \widetilde{x}, \widetilde{y}) \to$

[9] A given set S is a *partially ordered set (poset)* if there is a reflexive, antisymmetric and transitive binary relation $\preceq$ that orders some pair of elements in S. We say a poset S is a *lattice* if any element in S has supremum and infimum, i.e, there are operations $\vee, \wedge : S \times S \to S$ so that for any $x, y \in S$ there is $x \vee y \in S$ (resp. $x \wedge y \in S$) so that $x, y \preceq x \vee y$ (resp. $x, y \succeq x \wedge y$) and for all $z \in S \setminus (x \vee y)$ (resp. $z \in S \setminus (x \wedge y)$) so that $x, y \preceq z$ (resp. $z \preceq x, y$), $x \vee y \preceq z$ (resp. $z \preceq x \vee y$).

$\Gamma^V(\gamma,\varphi,T,\widetilde{x},\widetilde{y})$ and $\vee^T,\wedge^T : \Gamma^T(\gamma,\varphi,T,\widetilde{x},\widetilde{y}) \times \Gamma^T(\gamma,\varphi,T,\widetilde{x},\widetilde{y}) \to \Gamma^T(\gamma,\varphi,T,\widetilde{x},\widetilde{y})$ by:

$$LV_{i,j} \vee^V LV_{k,l} = LV_{i\oplus k, j\oplus l}, \quad LV_{i,j} \wedge^V LV_{k,l} = LV_{i\ominus k, j\ominus l} \tag{12}$$

$$LT_{i,j} \vee^T LT_{k,l} = LT_{i\ominus k, j\ominus l}, \quad LT_{i,j} \wedge^T LT_{k,l} = LT_{i\oplus k, j\oplus l} \ . \tag{13}$$

Next we prove that linear dual tax cuts form a lattice.

Proposition 1 *Let $T(\cdot,\cdot)$ be a quasi-progressive dual tax schedule, $(\gamma,\varphi) \in \mathbb{R}_+^2$ and a (finite) set of tax-payers with income distributions $\widetilde{x} = (x_1 \leq ... \leq x_n)$ and $\widetilde{y} = (y_1 \leq ... \leq y_n)$. Then*

1. *$\Gamma^V(\gamma,\varphi,T,\widetilde{x},\widetilde{y})$ is a lattice endowed with the partial order $\preceq$ defined by the Lorenz Dominance and the supremum and infimum operators $\vee^V,\wedge^V$ defined in (12). Moreover, the Lorenz curve of $\widetilde{x+y} = \{x_n + y_n\}_{n\geq 1}$ is LD by the infimum of the lattice.*

2. *$\Gamma^T(\gamma,\varphi,T,\widetilde{x},\widetilde{y})$ is a lattice endowed with the partial order $\preceq$ defined by the Lorenz Dominance and the supremum and infimum operators $\vee^T,\wedge^T$ defined in (13). Moreover, the supremum of the lattice is LD by the Lorenz curve of $\widetilde{x+y} = \{x_n + y_n\}_{n\geq 1}$.*

Proof.

1. Part 1 can be easily proved from Pfähler (1984), Lemma 4 in the Appendix and Theorem 1.

2. Part 2 can be proved analogously to Part 1.

■

Corollary 1 *Given $(\gamma,\varphi) \in \mathbb{R}_+^2$, $\widetilde{x}$ and $\widetilde{y}$ labor and capital pre-tax income distributions respectively and $T(\cdot,\cdot)$ a quasi-progressive dual tax schedule, $\Gamma(\gamma,\varphi,T)$ is a lattice endowed with the partial order $\preceq$ defined by the Lorenz Dominance of the post-tax income distributions and the supremum and infimum operators $\vee,\wedge$ defined in (11). Moreover, $T_{2,2}(\cdot,\cdot)$ is the most progressive reform and $T_{1,1}(\cdot,\cdot)$ the least progressive.*

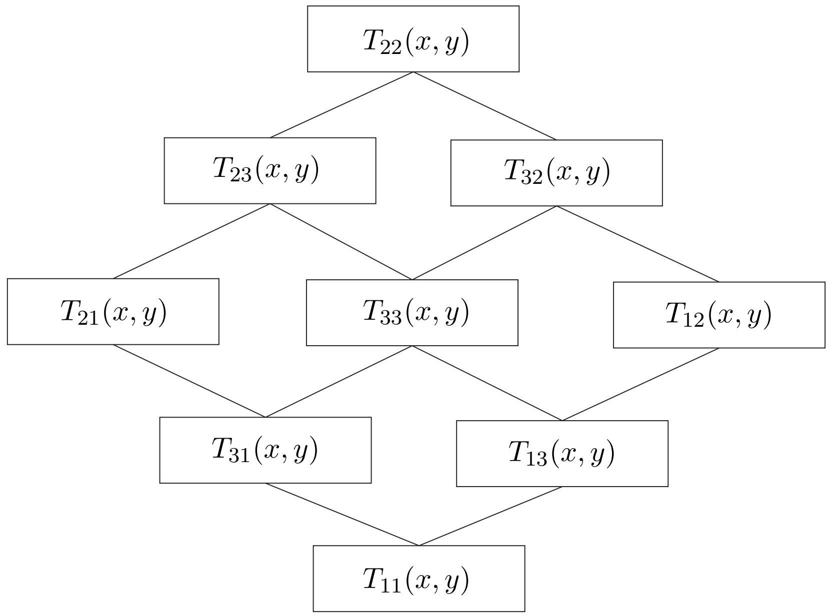

Figure 1: **Lattice structure of the 9 linear dual tax reforms.**

The lattice structure is shown in Figure 1. In a lattice it is only possible to compare elements bearing a vertical relationship.

Finally, it is important to point out that the lattice structure referred to holds for any set of nine linear tax cuts satisfying Condition 1, given any labor and capital income distributions satisfying Condition 2. Moreover, if either Condition 1 or Condition 2 do not hold, we can find quasi-progressive tax schedules and income distributions where the lattice structure does not apply (see Example 1 in the Appendix). Hence, Condition 1 and Condition 2 are necessary and sufficient conditions.

3.2 Local effects

We have so far focussed on the effect that linear dual tax reforms have on income distribution. Let us now focus on the effect any of these reforms on a single tax-payer, according to Pfähler (1984).

Let $\Delta V_{i,j}(x,y) = \Delta V_i^L(x) + \Delta V_j^K(y)$ be the total post-tax income of a tax-payer with respect to the $T_{i,j}$ reform, where $\Delta V_i^L(x)$ and $\Delta V_j^K(y)$ are, respectively, the post-tax labor and capital incomes. A sufficient condition for $V_{i,j}(x,y) \geq V_{k,l}(x,y)$ is that $V_i^L(x) \geq V_k^L(x)$ and $V_j^K(y) \geq V_l^K(y)$. By Pfhäler (1984) and Theorem 1 we obtain the following result.

Proposition 2 *Let $\widetilde{x} = (x_1, ..., x_n)$ and $\widetilde{y} = (y_1, ..., y_n)$ be the labor and capital income distributions of a (finite) set of tax-payers and let $(x_g, y_g) = (l^{-1}(g_L), k^{-1}(g_K))$, where $T(x,y) = L(x) + K(y)$ is the current quasi-progressive dual tax schedule. Then,*

1. *If $(x,y) \leq (x_g, y_g)$, the chosen tax reform by a tax-payer with incomes (x,y) is $T_{2,2}$.*

2. *If $(x,y) \geq (x_g, y_g)$, the chosen tax reform by a tax-payer with incomes (x,y) is $T_{1,1}$.*

Notice that there is no restriction on the relative order on the capital and labor income distributions. That is, unlike in Theorem 1, we only require Condition 1 and not Condition 2. Also observe that a (finite) set of tax-payers can be divided into three subsets. Indeed, given a tax-payer with a pair of incomes (x,y) we will say he or she is *rich* if $(x,y) \geq (x_g, y_g)$, *poor* if $(x,y) \leq (x_g, y_g)$ and *middle-class* otherwise. Then, if poor tax-payers account for more than half the population, the $T_{2,2}$ reform would be chosen in an election to decide which reform out of the nine proposed should be carried out, provided that all tax-payers voted rationally. In such a case, these interests would be aligned with those of a Government concerned about the redistribution of post-tax incomes.

4 Revenue elasticity

Progressivity and redistribution of taxes are relevant equity issues to be considered when adopting tax reforms. Related to this analysis, there is also an empirical issue which is important not only from an equity perspective but from a government one when preparing a Budget, which is the elasticity of tax revenue with respect to income. With progressive taxation, revenue is elastic with respect to a proportional growth of all incomes, and the amount of this elasticity is also crucial for macroeconomic projections.

Hutton and Lambert (1979) show that increasing the average rate progression $\beta(x)$ of a tax at every point of the income distribution raises the elasticity of the revenue function. Considering this result, Pfähler-based $T_{i,j}$ tax cuts for dual taxes can also be partially ordered by the responsiveness of the aggregate tax liability with respect to income. Indeed, let $Z = \sum_{i=1}^{n} (x_i + y_i)$ be the pre-tax aggregate total income and $R = \sum_{i=1}^{n} (L(x_i) + K(y_i))$ the aggregate total tax liability. If Z changes only due to equiproportionate changes in $\widetilde{x}$ and $\widetilde{y}$[10], R can be viewed as function of Z.[11] Suppose that, at some initial point, $Z = Z_0$. Then, $R = R(kZ)$ with $R(Z) = R_0 = R(Z_0)$.

The *average-rate responsiveness* and the *elasticity* of the aggregate total tax liability with respect to the aggregate total income are defined respectively as

$$A(Z) = \frac{R'(Z)Z - R(Z)}{Z^2} = \left(\frac{R(Z)}{Z}\right)', \tag{14}$$

$$E(Z) = \frac{R'(Z)Z}{R(Z)}. \tag{15}$$

Next, we obtain the following result, that is proved in the appendix[12].

Lemma 1 *Let $\widetilde{x} = (x_1, ..., x_n)$ and $\widetilde{y} = (y_1, ..., y_n)$ be the labor and capital income distributions respectively of a (finite) set of tax-payers and $T(x, y) = L(x) + K(y)$ is the current quasi-progressive dual tax schedule. Then,*

$$A(Z) = \frac{\sum_{i=1}^{n} \left(\beta^L(x_i) \cdot x_i^2 + \beta^K(y_i) \cdot y_i^2\right)}{Z^2},$$

$$E(Z) = 1 + \frac{\sum_{i=1}^{n} \left(\beta^L(x_i) \cdot x_i^2 + \beta^K(y_i) \cdot y_i^2\right)}{\sum_{i=1}^{n} T(x_i, y_i)}.$$

Hence, we are in a position to extend Pfähler's result (1984).

Proposition 3 *Let $\widetilde{x} = (x_1, ..., x_n)$ and $\widetilde{y} = (y_1, ..., y_n)$ be the labor and capital income distributions respectively of a (finite) set of tax-payers. Let also $T(\cdot, \cdot)$ be a quasi-progressive*

[10] $\widetilde{r} = (r_1, ..., r_n)$ is obtained from a equiproportionate change of $\widetilde{s} = (s_1, ..., s_n)$ if $r_i = ks_i$ for all $i = 1, ..., n$.

[11] Hutton and Lambert (1979) make the same assumption.

[12] The proof, although similar to Hutton and Lambert (1979), is made for discrete distributions.

dual tax schedule and $(\gamma,\varphi)\in\mathbb{R}^2_+$*. Then,* $\Gamma(\gamma,\varphi,T,\widetilde{x},\widetilde{y})$ *is a lattice endowed with a partial order* $\preceq$ *defined by the elasticity* $E(Z)$ *of the aggregate total tax liability and the supremum and infimum operators* $\vee,\wedge$ *are defined in (11).*

Proof. According to Pfähler (1984), $\beta_1(x)<\beta(x)$, $\beta_2(x)>\beta(x)$ and $\beta_3(x)=\beta(x)$, where $\beta(x),\beta_1(x),\beta_2(x)$ and $\beta_3(x)$ are the average rate progression of the unidimensional taxes $T(x),T_1(x),T_2(x)$ and $T_3(x)$, respectively. Then, the proof is immediate from Lemma 1 and the definition of dual average rate progression. ■

Again, notice that there is no restriction over the relative order on the capital and labor income distributions. That is, Condition 2 is not needed. Moreover, Figure 1 also represents Proposition 3. Observe that, as in the unidimensional tax case, there is no trade-off between the preferences of the Government concerning the aggregate total tax liability and the redistribution of the post-tax income.

5 An empirical application: the case of the Spanish Income dual Tax

This section describes an empirical application related to the dual income tax introduced in Spain since 2007. By means of a static micro-simulation model SIMESP (Arcarons and Calonge, 2008) and a very large data set of taxpayers drawn from 2004 Spanish Income Tax Returns population we assess the redistributive and progressivity effects of linear tax reforms. We consider two empirical applications.

First, yield-equivalent tax cuts applied on the dual income tax are compared. Specifically, $T_{1,1}$, $T_{2,2}$ and $T_{3,3}$ tax cuts defined in Section 2 are simulated to analyze their progressivity and redistributive effects on income distribution. Elasticity coefficients of the simulated tax functions are also estimated.

In our second empirical exercise, the current dual tax applied since 2007 is compared to the income tax which had been applied before that date. As we shall comment later on, 2007 dual tax reform introduced significant changes in the fiscal law. According to Spanish Government budget forecasts, the impact of this reform was estimated as a 6% reduction on aggregate tax revenue. Considering this, we analyze how much income redistribution the dual tax introduced regarding the previous tax and, consequently, who the gainers and the losers from this reform are, if any. To do so we apply a residual-progressive linear tax cut on the previous tax to reach the same revenue of the dual tax reform. Then a differential redistribution analysis is carried out by comparing the two tax burden distributions.

For both applications results are computed for the whole range of the income distribution and locally for pre-tax income deciles. Before showing the results of these two applications we shall briefly describe the main features of the Spanish Income Tax and the SIMESP micro-simulation model we use.

5.1 Description of the current Spanish Income Tax (2007) Act '35/2006'

The most important feature of the current Spanish Personal Income Tax (PIT) is its dual structure. On one hand, 'labor' income base includes salary, entrepreneur and professional income, pension plan and rental income. This income base is rated according to a four-bracket tax schedule, ranging from 24% to 43% as the highest marginal tax rate. On the other hand, capital income and realized capital gains constitute what is called the 'savings' base -or 'capital' income base- which is levied at 18% fixed rate, with a 1500 € deduction for dividends.

Standard income deductions to be applied on general base are salary-related expenses, social security contributions and the amount invested in pension plans. Furthermore, there is a progressive allowance on earned income. In order to get taxable income, losses on earned and capital incomes can only be offset within their own base. Other allowances are related to the demography of the family: personal allowance for each tax payer (plus an addition allowance according to age), dependent children and ascendants. Further, these allowances are augmented in the case of disabled members. Labor income-tax schedule is applied to the total allowances, which deduces the gross-tax liability of PIT.

Main tax credits of PIT are related to housing: 15% tax credit on the interest mortgage and capital (with a maximum of 9,000 €). Other tax credits are applied to business activities. From January 2008, 10.05% of the rent paid on the main home can be deducted, provided that the taxable income is less than 24,020€ per year. Tax liabilities and tax credits are shared by the central government and the regional-level administration at percentages of 67% and 33% respectively. Finally, although PIT has an individual nature, joint-returns are also permitted by aggregating the incomes of each member in the family.

5.2 Data and the micro-simulation model

The empirical analysis is based on a very large data set of a million of tax payers containing fiscal information from the 2004 Spanish Income Tax File-Return. The database is supplied by the I.E.F. 'Instituto de Estudios Fiscales' to researchers. The sample contains almost 200 variables related to fiscal information of tax-payers and their relatives.

Stratitification of the sample (1176 stratas) has been carried out by province, income base and the type of tax-return (individual and joint tax returns). Richest recipients have been oversampled in order to get a better description of the highest part of income distribution. Grossing-up factors are derived from the stratification scheme and they are used to estimate population totals and other statistics from the sample (see Picos et al., 2007).

SIMESP is a static micro-simulation model for modeling reforms on PIT. As a static model, it does not simulate behavior responses to changes in tax policies. Its output is interpreted as short-term of first-round effect of the policy changes, but by using the large data set mentioned before on administrative records, SIMESP takes advantage of the huge heterogeneity regarding personal and demographic characteristics of the tax-payer population. The model is able to providee some dynamic analysis projecting the monetary variable of the models. Computation of variables, tax and income distributions and statistical indices are grossed-up by means of the sampling weights in order to obtain population aggregates.

To interpret the empirical section we should first point out the following issues. Our theoretical predictions about the effect of linear dual tax cuts on income redistribution are derived by using taxable income as the income variable of the analysis. Our simulations are also based on the taxable-income variable, but their effects on income distribution are measured as well considering the recipient pre-tax income. Pre-tax income is a more accurate measure of the individual welfare than taxable income. The use of pre-tax income takes into account the impact of allowances and deductions on PIT.

5.3 Comparing linear dual-tax cuts

In this exercise we compare three different linear tax cuts, $T_{1,1}$, $T_{2,2}$ and $T_{3,3}$, according to the neutral revenue hypothesis. Table 1 describes 'labor' and 'capital' tax schedules for the current Spanish Income Tax –which is the baseline of the simulation – and the proposed linear tax cuts after applying a 10% reduction on both labor and capital gross-tax liabilities.

Tax rates of simulated tax cuts are obtained according to equation (3). For both income bases, we use parameters a, b, c instead of ρ_i^L, σ_i^L, ρ_j^K, σ_j^K for simplicity. As explained in Section 2, parameters a and g represent the tax cut as a percentage of the initial fiscal revenue (10%) and average tax rates calculated as the ratio between gross-tax liabilities and taxable income, respectively. Parameters c and b are obtained multiplying a by g and $g/(1-g)$ respectively. The average tax rate on capital income g_K is calculated before a $1,500$€ allowance for dividends is applied.

The distribution impact of neutral-revenue tax cuts is measured either globally or locally at a particular point of the income distribution. Table 2 summarizes the global effect of the different tax cuts on total revenue, liability progression and income redistribution.

Tax schedule				
	Baseline Tax	$T_{1,1}$	$T_{2,2}$	$T_{3,3}$
'Labor' base				
Tax Schedule				
0-17, 360€	24%	21, 60%	21, 25%	21, 34%
17, 360 − 32, 360€	28%	25, 20%	25, 40%	25, 34%
32, 360 − 53, 360€	37%	33, 30%	34, 72%	34, 34%
> 53, 360€	43%	38, 70%	40, 94%	40, 34%
Parameter	$g_L = 26.56\%$	$a_L = 10\%$	$b_L = 3.61\%$	$c_L = 2.65\%$
'Savings' base Tax Schedule	18%	16, 20%	16, 29%	16, 27%
Parameter	$g_K = 17.28\%$	$a_K = 10\%$	$b_K = 2.08\%$	$c_K = 1.72\%$

Table 1. Baseline tax and simulated tax reforms.

	Gross Tax Liability (10^6€)	K	S	RE	η	η_2	ΔR
Ref.	72765	0.136	0.148	0.033			
$T_{1,1}$	65489	0.136	0.148	0.029	1.454	1.066	952
$T_{2,2}$	65509	0.142	0.157	0.031	1.470	1.078	963
$T_{3,3}$	65495	0.140	0.154	0.030	1.466	1.074	960

Table 2. Main aggregate results[13].

The first column of the Table 2 includes the initial revenue obtained by the microsimu-

[13]Parameter η represents the tax-elasticity coefficient obtained by simulating an 1% increase on the pre-tax income components of each tax-payer. Taxable-income elasticity is estimated as 1.33643 * Simulated Gross Tax Liability after 1% pre-tax income increased.

lation model and the revenue figures simulated for the three tax cuts. Loss-revenue under the three tax cuts accounts for approximately 10.50% of the gross-tax liability, instead of the simulated 10% tax cut. The half point percent of discrepancy between these figures is mainly due to the initial tax rate applied to the 'saving base', 17.27%, which is lower than 18%, the statutory tax rate (the capital average tax rate is computed before the dividend allowance is applied). The proposed tax cuts are compared by using aggregate indices of liability progression and redistributive effect on income distribution. Tax-progressivity indices as Kakwani (1976), K, and Suits (1977), S, are measures of liability progression. Values of the progressivity indices for each tax cut reveal $T_{2,2}$ as the most progressive one followed by $T_{3,3}$. Similar results can be inferred considering a measure of residual progression. The redistributive effect of PIT on income distribution is measured by the Redistributive Effect or Reynolds-Smolensky index, $RE = G_x - G_{x-T}$, where G_x and G_{x-T} are the Gini indices of pre-tax and post-tax income respectively. Results reveals, again, that $T_{2,2}$ increases income redistribution the most, whereas $T_{1,1}$ is the least redistributive tax cut.

Next, the differential distributional effects of linear tax cuts are also compa-

red in terms of Lorenz dominance by representing concentration curves of tax-payments (or post-tax income) against the cumulated shares of pre-tax income recipients. To test whether $T_{2,2}$ is more liability progressive than $T_{1,1}$ or $T_{3,3}$ we only need to prove that the concentration curves $CT_{1,1}$ or $CT_{3,3}$ dominate $CT_{2,2}$, which implies that the distance between curves is positive for each decile of the income distribution. In a similar way, $T_{2,2}$ has a greater redistribution on income with respect to $T_{1,1}$ or $T_{3,3}$ if the difference, $LV_{2,2} - LV_{1,1}$ or $LV_{3,3} - LV_{1,1}$, is always positive through income distribution, where $LV_{1,1}$, $LV_{2,2}$ and $LV_{3,3}$ are the income post-tax distributions. The redistributive profile of PIT reforms $T_{2,2}$ and $T_{3,3}$ with respect to $T_{1,1}$ is represented in Figure 2.

These results bear out the theoretical ones described in Section 3 indicating a clear Lorenz dominance order $T_{2,2} \succ T_{3,3} \succ T_{1,1}$ for residual progression. Similar results on liability

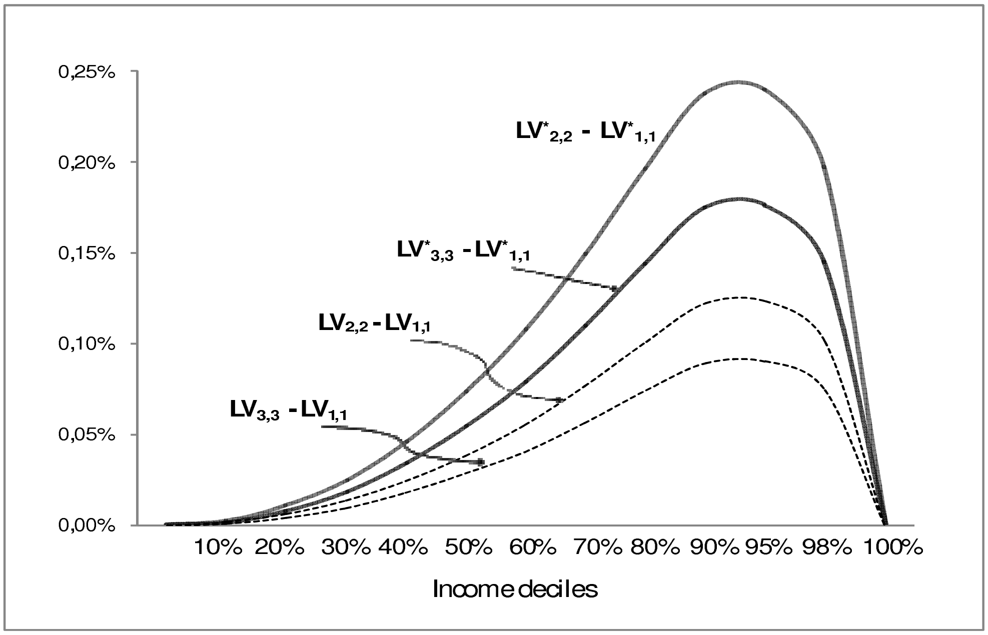

Figure 2: **Difference between post-tax taxable income Lorenz curves (continuous curves refer to taxable income whereas dashed curves refer to pre-tax income).**

progression, not reproduced here, are obtained for tax concentration curves.

The analysis of local differential effects on the income distribution can also be based on the absolute increase of post-tax income that a particular tax-payer obtains from each reform. To analyze how different taxes affect local inequality, the average gain is computed for a set of income deciles. Figures 3 and 4 compare respectively $T_{2,2}$ tax cuts against $T_{1,1}$ and $T_{3,3}$ tax cuts in terms of the percentage of gainers and the per capita amount of gaining for a set of deciles of the pre-tax income distribution. $T_{2,2}$ provides greater per capita gains till the 95% percentile of the income distribution.

What we observe from the distribution of gainers and losers in both figures is that the $T_{2,2}$ tax cut is favorable to the 90% tax payer population with less income. Furthermore, a significant number of tax payers located between the 90% and 95% percentiles still remain gainers under $T_{2,2}$ tax cuts. This result indicates that the 'break-even' income (pre-tax income value for which tax units would be indifferent to any of the proposed tax cuts) is located in this income group. At the top of the income distribution, the situation would be the reverse. Tax payers above 'break-even' income become losers for $T_{2,2}$ (or gainers for $T_{1,1}$ and $T_{3,3}$). According to Pfähler, the 'break-even' income defines a threshold which determines tax payers' voting preferences between tax cuts[14]. Results reveal that $T_{2,2}$ would be a preferred policy for the majority of tax payer population.

Finally, we focus on the elasticity of the tax functions corresponding to the reforms proposed, according to the theoretical results obtained in Section 4. Let η measure the elasticity of gross-tax liability with respect to income. This parameter has been computed after comparing the fiscal revenues obtained from the corresponding tax $T_{1,1}$, $T_{2,2}$ and $T_{3,3}$ before and after multiplying all pre-tax income sources by 1.01. The elasticity coefficients obtained by micro-simulation are important in magnitude $(1.46, 1.47)$. However, they do not strictly reflect the elasticity due to the 'shape' of the tax function, the coefficient we are interested in. Since taxable income is obtained from pre-tax income after application of allowances and income-related deductions, we may expect substantial increase in elasticity for taxable income if those fiscal credits are not upgraded in line with income. There is also another effect which refers to those recipients with zero taxable income, which is upgraded into liability as a consequence of the income increase.

We formulate the tax elasticity respect to pre-tax income as the product of the taxable-income η_1 and tax rate η_2 elasticities, $\eta = \eta_1\eta_2$, where η_1 depends basically on allowances and deductions. Coefficient η_1 is directly estimated by using the micro-simulation model and it takes the same value for each tax cut, which is equal to 1.36. Elasticity coefficients associated with the rate structure of each tax function are then computed from the ratio between η and η_1, which are shown in Table 2. Results reveals that $T_{2,2}$ tax cut produces the most elastic revenue source of the three types of progressivity-cuts, and obviously, $T_{2,2}$ generates more revenue than the other tax cuts. By contrast, $T_{1,1}$ generates the lowest revenue of the three taxes, confirming the hypothesis outlined in Section 3.

[14]To be consistent with the theoretical part, the 'break-even' income should be defined for both income bases. For simplicity, we only focus on the aggregate (unidimensional) 'break-even', which approximates the sum of the capital and labor break-even incomes.

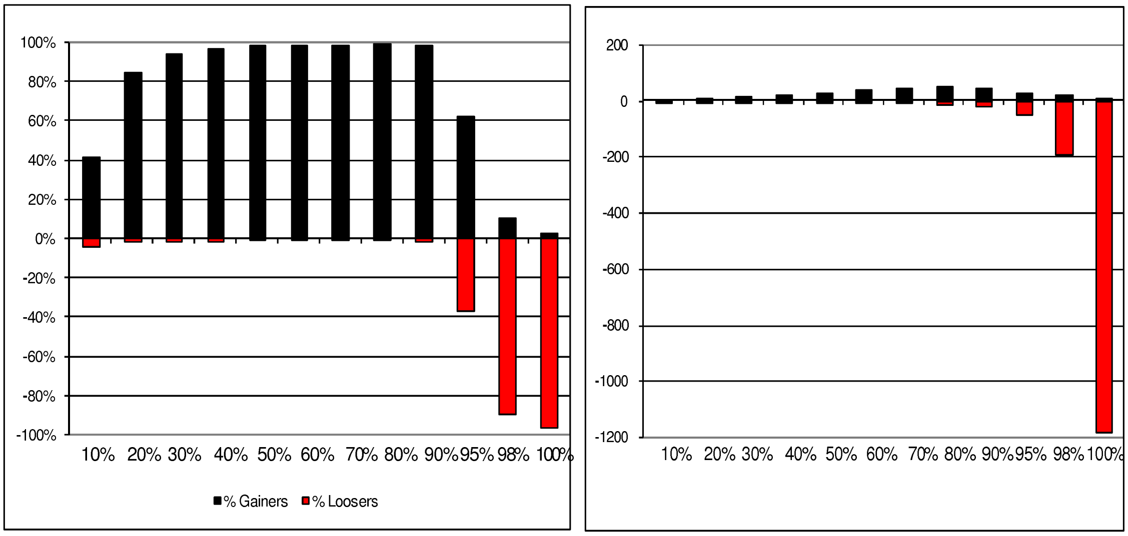

Figure 3: **Comparing $T_{2,2}$ against $T_{1,1}$: percentage of gainers and losers (left) and average gaining by income groups (right).**

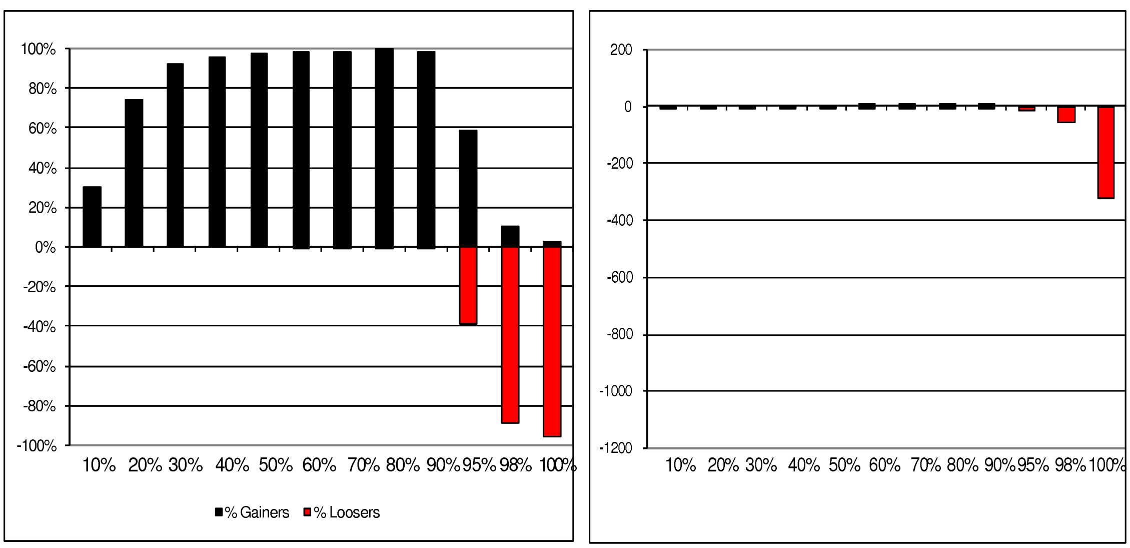

Figure 4: **Comparing $T_{2,2}$ against $T_{3,3}$: percentage of gainers and losers (left) and average gaining by income groups (right).**

The consequences for the distribution of tax burdens among tax units will be depend on the tax elasticity at all values of observed pre-tax incomes. We compute 'average' elasticity coefficients η and η_2 for each income group (see Table A.3 in the Appendix). As we observe from this table, the elasticity values of the tax schedule bear out the theoretical results, in the sense that $\eta_2(T_{2,2}) \geq \eta_2(T_{3,3}) \geq \eta_2(T_{1,1})$.

Another empirical aspect to consider from the estimated elasticity coefficients is related to the potential distortion originated by no-inflation adjustments. No indexation of the tax affects all income groups, but their redistributional consequences could be different among them. As an example, suppose we focus on the tax elasticity estimated for $T_{2,2}$ tax cuts. We can distinguish two different mechanisms. First, tax burden would be altered if allowances and other deductions applied before obtaining taxable income not updated with inflation. This fiscal drag is predominant for low income groups, where the tax function is quite flat, according to values of η_2 near to 1.01. Second, the 'bracket creep' effect through the tax schedule essentially affects the highest income groups. See Immerwooll (2005) for a detailed discussion on how a partial adjustment or simply, the absence of adjustment by inflation, can affect the distributional properties of the income tax.

5.4 Effects of the Spanish dual tax reform

Spanish Income Tax (2007) Act '35/2006' was announced as a tax reform by the government in 2005 to be applied from the fiscal year 2007. Regarding the previous tax, a number of fiscal changes introduced by the reform can be viewed as a simple updating of certain parameters of the fiscal law (specifically, those related to minimum threshold, income limits, allowances related to the labor income, private pension contributions, etc).[15] Other changes in income tax are structural elements related to the new tax design.

Next we focus on the elements which make the dual income-tax different with respect to

[15] For example, income allowance deducted from salaries was adjusted up to 13%, although this percentage is still below the accumulate inflation generated since the previous revision.

what it was before the reform. These features can be summarized as follows:

Tax schedule, marginal tax rates and demographic allowances.

As we see in Table 3, income brackets are wider in the dual tax. Basically, changes introduced were 1) elimination of the first income bracket 2) updating the income limits and 3) a lowering the highest marginal tax rate from 45% to 43%. Changes on the income bracket limits are not exclusively an inflation update but they are also related to the fact that allowances linked to the sociodemographic characteristics of the family are applied after gross-tax liability is obtained. Specifically, this allowance is obtained after applying the general tax schedule to the total amount of family deductions. The main goal of the above changes is to introduce greater progressivity on the tax burden distribution.

New taxation on capital income.

In the case of the dual tax, capital income is levied at an 18% fixed rate with a 1,500€ allowance for dividends. In contrast, capital income - including capital gains realized within the year - before the reform was taxed according to the individual's marginal tax rate, t' (except long term realized capital gains which were also taxed at 15% fixed rate). This is indeed the most notable aspect of the reform. If we consider income from interest, dividends, etc as a surplus added to the earned income in the case of most tax payers, the new tax clearly favors those taxpayers with a previous marginal tax rate on regular income higher than 18%. However, the previous tax allowed for 40% deduction on taxable capital income for long-run savings, so an extra euro on this type of income was marginally taxed at $0,6t'$. Furthermore, there was a 40% tax credit for dividends.

Other features.

Another important difference between the two taxes, in our opinion, is that the way of compensating different types of incomes has been canceled out by the reform. Any negative yields on capital income were integrated into the income base before the reform, whereas currently these should be offset exclusively on the capital income base belonging to further fiscal years.

To compare both taxes under the equal-revenue hypothesis and focusing on the main changes discussed above, some elements of the tax law before and after reform have been restricted to be equal. The following parameters have been set as the same ones for both taxes: realized capital gains taxed at 18% fixed rate, labor allowances calculated according to the formula introduced by the reform and a 15% tax credit on housing investment (capital and mortgage interest rates). Other limits related to private pension contribution are also fixed to be the same ones. Once these changes had been introduced in the tax before reform, the aggregate revenue and average tax rate were computed to simulate the tax schedule which produced the same revenue as the dual tax.

Pre-reform tax schedule			**Dual tax schedule**	
	(1)	(2)		(3)
0-4,000€	15%	12.50%	0-17360€	24%
4,000-13,800€	25%	21.75%	17360-32360€	28%
13,800-25,800€	30%	25.85%	32,360-52,360€	37%
25,800-45,000€	37%	35.15%	>52360€	43%
>45.000€	45%	43.35%		
Capital gains	15%	18%	**Savings base**	18%

Table 3. Tax rates structure of simulated taxes

In this exercise we focus on comparing taxes (2) and (3), that are the previous tax schedule that produces the same revenue than the dual tax and the dual tax itself, respectively. Tax burden comparisons are based on net tax liabilities, since, in both taxes, tax credit are approximately equal in both taxes. In the context of this two yield-equivalent tax comparison, we can say that differences, if any, in post-tax income distribution between the pre-tax and post-tax reform are mainly related to the three points outlined above. Obviously, the distribution of gainers and losers we should expect from this analysis is also related to those fiscal changes.

As a first result of this simulation, we stress that the reform produces a large number of losers: a 45.60% tax payers pay more than they would have to pay before reform, against a 35% percentage of gainers. Approximately 25% of the recipients are indifferent between both reforms. Tax payers who benefit or (do not) from the tax reform are not uniformly distributed throughout the income scale. The tax reform clearly benefits the two lowest income groups. There are scarcely not losers at the bottom of the income distribution. Percentages of 6.69% and 28% of tax payers from the first and second income deciles benefit from the tax reform. Percentages of losers are significant for the remaining income deciles. Regarding the population located between the 40% and 98% percentiles, losers percentages are higher than gainers ones (see Table 4).

Income deciles	10%	20%	30%	40%	50%	60%	70%	80%	90%	95%	98%	100%
% Gainers	6.69	28.51	42.31	28.01	32.35	31.08	27.81	24.00	18.97	24.35	30.26	55.64
Per capita gains (e)	157	151	168	201	220	248	332	405	513	740	977	4,214
% Losers	0.00	0.53	8.95	45.17	51.84	59.54	67.20	76.64	80.58	75.60	69.53	44.27
Per capita losses (e)	1.127	104	64	103	125	148	184	265	375	416	584	818

Table 4. Distribution of share and cumulated shares of pre-tax income, taxable-income and gross-tax liability T by income deciles.

The distribution of losers comes mainly from the dual tax schedule designed. The change of sociodemographic deductions from the 'income base system' to 'tax credits' system favors low-income tax-payers. The rest of the population experiments an increase in the marginal tax rate as a consequence of that change.

6 Conclusions

The distributive effects of a class of linear tax cuts (or tax increases) are established by Pfáhler (1984). His main results show that yield-equivalent tax cuts can be ranked according to a social welfare criterion (Lorenz dominance). His analysis identifies the most redistributive policy as the one which the initial residual progression of the tax. However when dual taxes are concerned (where two tax functions are applied to different income bases), Pfähler's results need further development.

In this paper we define a bi-dimensional progression measure which is needed to establish the Lorenz dominance criterion among post-tax income distribution. To do this, the dual tax cuts must be simultaneously labor and capital yield-equivalent. Moreover, the relative order of both labor and capital income distribution must coincide. We set up a lattice whereby Pfähler-type linear tax refoms are compared.

Finally, by means of a microsimulation model and a large dataset of income tax payers we empirically illustrate the effect of linear tax cuts on dual taxes. Our analysis is carried out to focus on redistribution and progressivity effects as well as the elasticity of the tax functions.

7 Appendix

7.1 Theoretical part

Remark 1

Consider a unidimensional tax schedule defined by

$$T(x) = \sum_{i=0}^{m+1} T_i(x)$$

where $0 < t_1 < ... < t_m < t_{m+1} < 1$ are the marginal types, $0 = x_0 < x_1 < ... < x_m < x_{m+1} = \infty$ are the marginal incomes and

$$T_i(x) = \max\{0, \min\{x_i - x_{i-1}, x - x_{i-1}\}\} \cdot t_i$$

for all $i, 1 \leq i \leq m+1$. We say that $T(x)$ is a stepwise tax schedule.

It can be easily checked that $T(x)$ is progressive and convex. Further, given a linear tax reform $t'(x) = d \cdot t(x) + h$ defined on the average type $t(x) = T(x)/x$ we next prove that the new tax schedule is stepwise where $0 < d \cdot t_1 + h < ... < d \cdot t_m + h < d \cdot t_{m+1} + h < 1$ are the marginal types and $0 = x_0 < x_1 < ... < x_m < x_{m+1} = \infty$ are the marginal incomes.

As $t(x) = \left(\sum_{i=0}^{m+1} T_i(x)\right) \backslash x$ we have

$$\begin{aligned}
t'(x) &= d \cdot t(x) + h \\
&= \frac{\sum_{i=0}^{m+1} d \cdot T_i(x) + h \cdot x}{x} \\
&= \frac{\sum_{i=0}^{m+1} d \cdot \overbrace{\max\{0, \min\{x_i - x_{i-1}, x - x_{i-1}\}\}}^{T_i(x)} \cdot t_i}{x} \\
&+ \frac{h \cdot \overbrace{\sum_{i=0}^{m+1} \max\{0, \min\{x_i - x_{i-1}, x - x_{i-1}\}\}}^{x}}{x} \\
&= \frac{\sum_{i=0}^{m+1} \max\{0, \min\{x_i - x_{i-1}, x - x_{i-1}\}\} \cdot \overbrace{(dt_i + h)}^{t_i'}}{x}.
\end{aligned}$$

Lemma 2 (Pfähler) *Let $T(x)$ a unidimensional tax schedule and $T_i(x), i = 1, 2, 3$ the Pfähler-based linear cuts with the same aggregate total tax liability. Then $\alpha(x) = \alpha_1(x) < \alpha_3(x) < \alpha_2(x)$ and $\psi(x) = \psi_2(x) < \psi_3(x) < \psi_1(x)$.*

Proposition 4 (Jakobsson) *Let $T_1(x)$ and $T_2(x)$be two dual tax schedules. For any pre-tax income distribution $\widetilde{x} = (x_1, ..., x_n)$ of a (finite) set of tax-payers,*

1. *$\widetilde{T_1(x)} = (T_1(x_1), ..., T_1(x_n))$ is LD by $\widetilde{T_2(x)} = (T_2(x_1), ..., T_2(x_n))$ if and only if $\alpha_1(x) > \alpha_2(x)$ for all $x \geq 0$.*

2. $\widetilde{V_1(x)} = (x_1 - T_1(x_1), ..., x_n - T_1(x_n))$*is LD by* $\widetilde{V_2(x)} = (x_1 - T_2(x_1), ..., x_n - T_2(x_n))$ *if and only if* $\psi_1(x) > \psi_2(x)$ *for all* $x \geq 0$.

It is important to point out that Jakobsson's proof (1976) assumes implicitly that a more restrictive constraint than $\alpha_2(x_0) < \alpha_2(x_0)$, for all x_0, holds. In fact, he assumes that $\alpha_2(x_0, x_1) < \alpha_2(x_0, x_1)$ for all $x_0 < x_1$, given an income distribution $\widetilde{x} = \{x_0 \leq x_1 \leq x_2 \leq ...\}$. However, when $x_0 \approx x_1$ both conditions almost coincide. Indeed, $\alpha(x_0, x_1) = \frac{T(x_1) - T(x_0)}{x_1 - x_0} \cdot \frac{x_0}{T(x_0)} = T'(x_0) \cdot \frac{x_0}{T(x_0)} + o(x_1 - x_0)$. Therefore, if a income distribution $\widetilde{x}$ is *dense*, i.e. any x_i is close enough to x_{i+1} then Jakobsson assumptions are correct. Finally notice that this final assumption would not hold for the highest tail of the income distribution. However, in such cases the tax schedule is almost proportional and both measures coincide.

Lemma 3 (Pfähler) *Given a unidimensional tax schedule* $T(x)$ *and given the three Pfähler-based unidimensional tax cuts*

$$x \lesseqqgtr t^{-1}(g) = x_g \Rightarrow V_2(x) \gtreqqless V_3(x) \gtreqqless V_1(x)$$

and

$$x \lesseqqgtr t^{-1}(g) = x_g \Rightarrow \frac{dV_2(x)}{dx} \gtreqqless \frac{dV_3(x)}{dx} \gtreqqless \frac{dV_1(x)}{dx}$$

where $V_i(x)$ *is the post-tax income of a tax-payer with pre-tax income* x *when reform* i *is applied to* $T(x)$ *for all* $i = 1, 2, 3$.[16]

Pfähler (1984) shows that the magnitude $t^{-1}(g) = x_g$ *-break-even pre-tax income-* is important for two reasons:

- Consider the proportional tax schedule proportional $T_p(x) = gx$. Then,

$$T(x) \geq T_p(x) = gx \Leftrightarrow \frac{T(x)}{x} \geq g \Leftrightarrow t(x) \geq g \Leftrightarrow x \geq x_g$$

[16]We rewrite and extend what Pfähler (1984) says in his paper.

- x_g is the same for the three reforms as it only depends on $g = \frac{\bar{T}}{\bar{x}}$.

Lemma 4 *Let* $\widetilde{x^1} = (x_1^1 \leq ... \leq x_n^1)$, $\widetilde{x^2} = (x_1^2 \leq ... \leq x_n^2)$, $\widetilde{y^1} = (y_1^1 \leq ... \leq y_n^1)$ *and* $\widetilde{y^2} = (y_1^2 \leq ... \leq y_n^2)$ *be income distributions such that* $\widetilde{x^2}$ *is LD by* $\widetilde{x^1}$ *and* $\widetilde{y^2}$ *is LD by* $\widetilde{y^1}$. *If* $\sum_{i=1}^n x_i^1 = \sum_{i=1}^n x_i^2$ *and* $\sum_{i=1}^n y_i^1 = \sum_{i=1}^n y_i^2$ *then* $\widetilde{x^2 + y^2}$ *is LD by* $\widetilde{x^1 + y^1}$.

Proof. The Lorenz curve of $\widetilde{x^1 + y^1} = (x_1^1 + y_1^1 \leq ... \leq x_n^1 + y_n^1)$ is defined by

$$L_{x^1+y^1}(p) = \frac{\sum_{i=1}^p \left(x_i^1 + y_i^1\right)}{\sum_{i=1}^n \left(x_i^1 + y_i^1\right)}$$

for $p = 1, ..., n$. Analogously, the Lorenz curve of $\widetilde{x^2 + y^2} = (x_1^2 + y_1^2 \leq ... \leq x_n^2 + y_n^2)$ is defined by

$$L_{x^2+y^2}(p) = \frac{\sum_{i=1}^p \left(x_i^2 + y_i^2\right)}{\sum_{i=1}^n \left(x_i^2 + y_i^2\right)}$$

for $p = 1, ..., n$. Then, from hypothesis, for $p \neq 0, n$,

$$L_{x^1+y^1}(p) > L_{x^2+y^2}(p) \Leftrightarrow \sum_{t=1}^p \left(x_i^1 + y_i^1\right) > \sum_{t=1}^p \left(x_i^2 + y_i^2\right).$$

Finally, given $0 < p < n$,

$$\sum_{t=1}^p \left(x_i^1 + y_i^1\right) = \sum_{t=1}^p x_i^1 + \sum_{t=1}^p y_i^1 > \sum_{t=1}^p x_i^2 + \sum_{t=1}^p y_i^2 = \sum_{t=1}^p \left(x_i^2 + y_i^2\right)$$

for $\widetilde{x^2}$ is LD by $\widetilde{x^1}$, $\widetilde{y^2}$ is LD by $\widetilde{y^1}$ and the hypothesis of the lemma. ■

Proof. [Proof of Lemma 1] We have

$$R(kZ) = \sum_{i=1}^n L(kx_i) + K(ky_i). \tag{16}$$

If we take the derivative of (16) with respect to k and evaluate it in $k = 1$ we obtain

$$R'(Z)Z = \sum_{i=1}^n L'(x_i)x_i + K'(y_i)y_i.$$

On the other hand, from (7),

$$\begin{aligned} Z^2A(Z) + R(Z) &= \sum_{i=1}^n L'(x_i)x_i + K'(y_i)y_i \Leftrightarrow \\ Z^2A(Z) &= \sum_{i=1}^n \left(L'(x_i)x_i + K'(y_i)y_i\right) - \sum_{i=1}^n \left(L(x_i) + K(y_i)\right) \Leftrightarrow \\ Z^2A(Z) &= \sum_{i=1}^n \left(\frac{L'(x_i)x_i - L(x_i)}{x_i^2}x_i^2 + \frac{K'(y_i)y_i - K(y_i)}{y_i^2}y_i^2\right). \end{aligned}$$

■

Example 1

Consider the quasi-progressive dual tax schedule $T(x,y) = L(x) + K(y)$where $L(x)$ and $K(y)$ are unidimensional tax schedules, on the labor income and on the capital income respectively, that are applied to the following income distribution $z = \{(2,1),(3,4),(6,5)\}$ in the way defined in the table below:

$\mathbf{x}$	$\mathbf{y}$	$\mathbf{x+y}$	$\mathbf{T(x,y)}$	$\mathbf{t(x,y)}$	$\mathbf{L(x)}$	$\mathbf{l(x)}$	$\mathbf{K(y)}$	$\mathbf{k(y)}$
2	1	3	1.05	0.35	0.80	0.40	0.25	0.25
3	4	7	3.25	0.46	1.25	0.42	2.00	0.50
6	5	11	5.60	0.51	2.60	0.43	3.00	0.60

Table A.1. Income distributions and quasi-progressive dual tax schedule.

Consider also the following two reforms: $T_{1,3}(x,y) = T(x,y) - 0.1 \cdot L(x) - 0.24 \cdot y$ and $T_{3,2}(x,y) = T(x,y) - 0.232 \cdot x - 0.6585 \cdot (y - K(y))$, which applied to the above tax and income distributions result in the table below:

$\mathbf{x}$	$\mathbf{y}$	$\mathbf{L_{1,3}(x)}$	$\mathbf{K_{1,3}(y)}$	$\mathbf{T_{1,3}(x,y)}$	$\mathbf{L_{3,2}(x)}$	$\mathbf{K_{3,2}(y)}$	$\mathbf{T_{3,2}(x,y)}$
2	1	0.72	0.01	0.73	0.34	0.20	0.54
3	4	1.13	1.04	2.17	0.55	1.87	2.42
6	5	2.34	1.80	4.14	1.21	2.87	4.08

Table A.2. Dual tax cuts.

Observe that the two reforms proposed are not separately labor and capital yield-equivalent, although they are globally yield-equivalent -the aggregate total tax liability is approximately equal to 7.04-. That is, Condition 1 does not hold.

On the other hand, if we take into account the discrete definition of the residual progression it can be obtained[17] that, given an income distribution $z = (z_1 \leq ... \leq z_n)$, and two

[17] $\frac{V_1(z_{i+1})-V_1(z_i)}{V_1(z_i)} \cdot \frac{z_i}{z_{i+1}-z_i} > \frac{V_2(z_{i+1})-V_2(z_i)}{V_2(z_i)} \cdot \frac{z_i}{z_{i+1}-z_i} \Leftrightarrow \frac{V_1(z_{i+1})-V_1(z_i)}{V_1(z_i)} > \frac{V_2(z_{i+1})-V_2(z_i)}{V_2(z_i)} \Leftrightarrow \frac{V_1(z_{i+1})}{V_1(z_i)} - 1 > \frac{V_2(z_{i+1})}{V_2(z_i)} - 1 \Leftrightarrow \frac{V_1(z_{i+1})}{V_1(z_i)} > \frac{V_2(z_{i+1})}{V_2(z_i)}$

reforms $T_1(z)$ **y** $T_2(z)$**, if** $\psi_{T_1}(z_i, z_{i+1}) > \psi_{T_2}(z_i, z_{i+1})$ for some z_i**,** $i \in \{1, ..., n-1\}$, then

$$\frac{V_1(z_{i+1})}{V_1(z_i)} > \frac{V_2(z_{i+1})}{V_2(z_i)}. \tag{17}$$

Hence, it can be checked that, by (17),

$$\frac{V^L_{1,3}(3{,}4)}{V^L_{1,3}(2{,}1)} < \frac{V^l_{3,2}(3{,}4)}{V^L_{3,2}(2{,}1)}, \quad \frac{V^K_{1,3}(3{,}4)}{V^K_{1,3}(2{,}1)} > \frac{V^K_{3,2}(3{,}4)}{V^K_{3,2}(2{,}1)}, \quad \frac{V_{1,3}(3{,}4)}{V_{1,3}(2{,}1)} > \frac{V_{3,2}(3{,}4)}{V_{3,2}(2{,}1)},$$

$$\frac{V^L_{1,3}(6{,}5)}{V^L_{1,3}(3{,}4)} < \frac{V^L_{3,2}(6{,}5)}{V^L_{3,2}(3{,}4)}, \quad \frac{V^K_{1,3}(6{,}5)}{V^K_{1,3}(3{,}4)} > \frac{V^K_{3,2}(6{,}5)}{V^K_{3,2}(3{,}4)}, \quad \frac{V_{1,3}(6{,}5)}{V_{1,3}(3{,}4)} < \frac{V_{3,2}(6{,}5)}{V_{3,2}(3{,}4)},$$

and that $V_{T_{1,3}}(\widetilde{x}, \widetilde{y}) = (2.27; 4.84; 6.86)$ and $V_{T_{3,2}}(\widetilde{x}, \widetilde{y}) = (2.46; 4.58; 6.92)$ are not LD-comparable, although labor and capital tax schedules are separately comparable by $\psi(x)$ and $\psi(y)$, and also using the Lorenz Domination criterium. Although income distribution of this example is not dense, we obtain exactly what Jakobbson (1976) predicts (see Calonge and Tejada, 2009, for a further argument on how density of can affect Jakobsson results).

In conclusion, if Condition 1 does not hold, dual tax cuts $T_{1,3}$ and $T_{3,2}$ cannot be compared, neither by $\psi(x+y)$ nor $\overrightarrow{\psi}$ (x,y) nor using the Lorenz Domination criterium on the total post-tax income distribution, as it is the case when such condition does hold. Therefore, Condition 1 is necessary. Similar examples show that Condition 2 also necessary.

7.2 Empirical part

deciles		10%	20%	30%	40%	50%	60%	70%	80%	90%	95%	98%	100%
$T_{1,1}$	η	2.945	2.483	2.507	2.149	1.568	1.374	1.324	1.359	1.304	1.437	1.452	1.236
	η_2	1.040	1.009	1.005	1.004	1.001	1.006	1.007	1.094	1.108	1.246	1.288	1.158
$T_{2,2}$	η	2.937	2.482	2.507	2.149	1.568	1.374	1.326	1.378	1.324	1.480	1.495	1.255
	η_2	1.038	1.008	1.005	1.004	1.001	1.005	1.008	1.109	1.125	1.283	1.327	1.175
$T_{3,3}$	η	2.939	2.482	2.507	2.149	1.568	1.374	1.325	1.373	1.319	1.469	1.484	1.250
	η_2	1.038	1.008	1.005	1.004	1.001	1.006	1.008	1.105	1.120	1.273	1.316	1.171
	η_1	2.831	2.461	2.495	2.141	1.566	1.366	1.315	1.242	1.177	1.154	1.127	1.068

Table A.3. Losers and gainers analysis: elasticities.

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