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Deciding the sale of a life policy in the viatical market: Implications on individual welfare

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Abstract

In this paper, we present an economic model that allows a terminally ill policy-holder to decide whether or not to sell (part of) the policy in the viatical settlement market. The viatical settlement market emerged in the late 1980s in response to the AIDS epidemic. Nowadays it is part of the large US market in life settlements. The policies traded in the viatical market are those of terminally ill policyholders expected to die within the next two years.

The model is discrete and considers only the next two periods (years), since this is the maximum remaining lifetime of the policyholder. The decisor has an initial wealth and has to share it between his own consumption and the bequests left to his heirs. We first introduce the expected utility function of our decisor and then use dynamic programming to deduce the strategy that gives higher utility (not selling/selling (part of) the policy at time zero/selling (part of) the policy at time one). The optima depends on the value of the viaticated policy and on some personal parameters of the individual. We find an analitical expression for the optimal strategy and perform a sensitivity analysis.

Abstract

En aquest article, es presenta un model econòmic que permet determinar la venta o no d'una pòlissa de vida (total o en part) per part d'un assegurat malalt terminal en el mercat dels viatical settlements. Aquest mercat va aparèixer a finals de la dècada dels 80 a consequència de l'epidèmia del SIDA. Actualment, representa una part del mercat dels life settlements. Les pòlisses que es comercialitzen en el mercat dels viaticals són aquelles on l'assegurat és malalt terminal amb una esperana de vida de dos anys o menys.

El model és discret i considera només dos períodes (anys), ja que aquesta és la vida residual màxima que contempla el mercat. L'agent posseix una riquesa inicial que ha de repartir entre consum i herència. S'introdueix en primer lloc la funció d'utilitat esperada del decisor i, utilitzant programació dinàmica, es dedueix l'estratègia que reporta una utilitat més gran (no vendre/vendre (en part) la pòlissa en el moment zero/vendre (en part) la pòlissa en el moment un). L'òptim depèn del preu de la pòlissa venuda i de paràmetres personals de l'individu. Es troba una expressió analítica per l'estratègia òptima i es realitza un anàlisi de sensibilitat.

Keywords: Viatical settlement, expected utility, dynamic programming.

JEL codes: G22, C61.

1 Introduction

Secondary markets for life insurance policies started in the United States with the emergence of a new product, the viatical settlement. This product allows terminall ill policyholders with financial needs and without enough liquidity to sell their life insurance policy to a third party, a viatical firm, before the policy matures. The viatical firm pays an amount, the viatical settlement value (henceforth VSV), that provides the seller with an immediate cash amount. In exchange for the VSV, the original policyholder transfers all the rights of his life insurance policy, in the sense that the viatical firm shall pay all the remaining premiums to the life insurance company, shall becoming the new beneficiary of the policy and shall therefore collect the death benefits when the original policyholder dies.

Viatical settlements were initially created in the late 1980s for those AIDs patients who had to face high medical and living expenses but had no income or liquid assets. Individuals infected by AIDs were usually gay men and not particularly old, as they normally had no spouse or children, they had no a compelling reason to keep the policy in effect. Viatical settlements provided a way to extract value from the policy while the policyholder was still alive. However, improvements in the treatment of AIDS prolonged and improved the life expectancies of the affected individuals, and the viatical settlement market began to lose popularity as investors started to amass considerable losses. Thereafter, the viatical industry considered that those potential policyholders for taking out a viatical were not only AIDs patients but also all terminally/chronically ill individuals (for example, those suffering from Cancer, cardiovascular disease, Allzheimer's disease, etc) expected to die within the next two years. See Conning (1999) for more on the origins and evolution of the market.

In the early 2000s as the viatical market matured the industry expanded into a new product: life settlement. This also refers to the transfer of an existing life insurance policy, but it is characterized by the fact that the insured person is expected to die between the next two and fifteen years (see Bhuyan (2009) or Aspinwall et al. (2009) for more information about life settlement contracts). Nowadays, the vast majority of the transactions in the secondary market for life insurance policies are made through the recruitment of life settlements, and hence the viatical market is only one part of the large US market in life settlements. In fact, Giacalone (2001) indicates that reliable data on the viatical industry is not available, and that the size of the market is not well known, despite the foundation of the National Viatical Association (NVA) in 1993 and the Viatical and Life Settlement Association of America (VLSAA) in 1994; two trade associations whose objective is to regulate the secondary market for life insurance policies. Sood et al. (2005) were the first to document information about the structure and performance of the viatical settlement industry. Using the Freedom of Information Act (FOIA), they assembled a unique dataset of viatical transactions from firms licensed in states regulating viatical settlement markets (California, Connecticut, Kentucky, North Carolina, New York, Oregon and Texas) between 1995 and 2001. The study shows a strong decline in the number of viatical transactions and the total value of settlements over the time period (2,623 transactions in 1995 and 235 in 2001); a reduction in the number of firms in each state (for example, in California there were 13 firms in 1995 and only 5 firms in 2001); an increase in market concentration that may be associated with rising profit margins and declining prices, and a larger reduction in prices payed by firms, especially for those contracts with longer life expectancies due to changes in the market's perception of risk (i.e., unexpected improvements in treatments for chronic deseases).

Remark that life contracts traded on secondary markets are those of policyholders with impaired health (fifteen years maximum life expectancy) and with a positive cash surrender value. According to Fang and Kung (2010) the opportunity for the secondary life insurance market arises from two main features of these life insurance contracts. First, the insurance premium stays fixed over the course of the policy. This is what is known as *front-loading*, and means that at the beginning the policyholder pays higher premiums than the actuarial fair value and lower premiums as he gets older. However, if the policyholder presents impaired health, then the *front-loading* becomes much more favorable and the actuarial value of the life insurance policy becomes much higher. This fact may encourage the policyholder to sell his life policy. Secondary, the cash surrender value for life insurance policies does not depend on the health status of the policyholder. Thus, both viatical firms and life settlement firms when considering higher death probabilities on the calculation, respectively, of the VSV and LSV, which adequately reflect the impairment of the insured party, offer more cash than the insurers with the cash surrender value (CSV). Moreover, the lower the policy-holder's life expectancy, the greater the VSV or the LSV, and vice versa. In this sense, impaired policyholders have an opportunity to recover more money from their policies.

However, both viaticals and life settlements are not necessarily a good alternative for everyone. Deloitte's Report,Vadiveloo et al. (2005), confirms that selling a life insurance contract in the secondary market always provides more liquidity than surrendering the contract, and hence the VSV and the LSV always exceed the CSV. However, the policyholder should consider other alternatives such as preserving life insurance until the death of the insured. In that sense, the authors introduce a new concept in the literature, called intrinsec economic value (IEV), which quantifies the value of retaining the policy until death. They show that IEV exceeds VSV and LSV for most situations. This new measure and the paper itself have received some criticism in the literature, see e.g. Singer and Stallard (2005).

This paper focuses on viatical settlements. We present an economic model where a terminally ill policyholder has to decide whether or not to sell his life insurance policy and hire a viatical. The decision depends on where he maximizes his expected utility. Our model is based on Bhattacharya et al. (2004), but we focus on a different problem and allow a wider range of possible decisions. In Bhattacharya et al. (2004), the main objective is to compare two different contexts: The regulated secondary market, where the consumer faces a binding price floor and hence finds no possible buyers for his life policy, and the non-regulated secondary markets where the policyholder is free to sell the policy.

The paper is structured as follows. In Section 2 we present our model and describe all the possible strategies for the impaired policyholder, that is, not selling or selling (a part) of the policy. In Section 3, by using dynamic programming, we find an optimal solution for the general problem. In Section 4 we give a numerical illustration of our model. Considering a baseline data, we give the expected utilities derived from each decision, extracting for the consumer the optimal strategy that coincides with the one with higher expected utility. Moreover, as this optimal strategy depends on the personal characteristics of each individual, we make a sensitive analysis that shows how the optimal decision of the consumer changes as some of his personal parameters changes. Finally, Section 5 summarizes our findings and introduces items for further research.

2 General problem description

In this section, we present an economic model that allows terminally ill policyholders to decide between selling or not selling their life insurance policy by hiring a Viatical. Our results are obtained in the framework of the expected utility theory. Hence, our objective is to maximize the expected utility of an individual derived from the sale or not of his life insurance policy.

The model is discrete and considers only two periods (years), since this is the maximum remaining lifetime for the policies traded in the viaticals market (see e.g. Bhuyan (2009)). Hence, we consider a policyholder with a very high death probability for the first period q_x (very small survival probability $p_x = 1 - q_x$). Should he survive the first period, then we know for certainly that he will die in the second period, i.e., $q_{x+1} = 1$.

At the beginning of the first period, $t = 0$, the decisor has an initial wealth W which includes, among other properties, a life insurance policy with a death benefit A and annual premiums P. He consumes an amount $C_0 > 0$ during the first period and an amount $C_1 > 0$ during the second one. If he dies in the first period, he leaves to his heirs an amount $H_1 > 0$ at $t = 1$, and if he survives the first period, then he dies for sure in the second period and leaves to his heirs an amount $H_2 > 0$, at $t = 2$.

The expected utility of the policyholder depends on the utilities of the consumption and the bequests for both periods. At the beginning of the first period, the expected utility is given by

$$EU_0 = U(C_0) + \beta \cdot q_x \cdot V(H_1) + \beta \cdot p_x \cdot U(C_1) + \beta^2 \cdot p_x \cdot q_{x+1} \cdot V(H_2) \tag{1}$$

where $\beta \in [0, 1]$ is the yearly intertemporal discount factor or rate of time preference, which includes the interest rate for discounting, r, and some personal time preferences. A large value of β indicates greater concern of the economic agent with respect to his future, in our case, the next period.

Expression (1) can be rewritten as

$$EU_0 = U(C_0) + \beta \cdot q_x \cdot V(H_1) + \beta \cdot p_x \cdot [U(C_1) + \beta \cdot V(H_2)] \tag{2}$$

where $U(C_1) + \beta \cdot V(H_2)$ is the expected utility of the decisor computed at $t = 1$, EU_1.

At the beginning of both periods the agent has to decide whether or not to sell his policy. We consider the possibility of selling only a part of the policy. The policy is sold on the Viaticals market and the seller receives the Viatical Settlement Value (VSV). This amount is smaller than the death benefit but greater than the cash surrender value, since the insurance company will not take into account the illness of the insured, and the life expectation used in the actuarial computations will therefore be higher than two years (see e.g. Vadiveloo et al (2005)). The VSV could be significantly different depending on the US state where the policy is sold, and it could also differ from one viatical company to another. Without loss of generality, we can consider that the VSV is equal to the actuarial present value of the policy multiplied by a certain $\gamma \in [0, 1]$.

We next introduce all the possibilities for our economic agent. From now on, the term viaticate refers to the sale of (part of) the policy in a viatical market.

1. Viaticate a percentage δ, $0 \leq \delta \leq 1$, at $t = 0$ and viaticate $(1 - \delta)$ at $t = 1$.

   The economic situation is the one stated in the following graph:

	H_1	H_2
W	$(W + VSV_0^{\delta} - P' - C_0) \cdot (1 + r)$	
$+VSV_0^{\delta}$	$+ VSV_1^{1-\delta}$	
$-P'$	$- C_1$	

$-C_0$

where

$$VSV_0^{\delta} = \gamma \left[\delta \left(A \sum_{t=0}^{1} {}_{t/}q_x \cdot (1+r)^{-(t+1)} - P \sum_{t=0}^{1} {}_t p_x \cdot (1+r)^{-t} \right) \right],$$
$$VSV_1^{1-\delta} = \gamma \left[(1-\delta) \left(A \, (1+r)^{-1} - P \right) \right],$$
$$H_1 = (W + VSV_0^{\delta} - P^{'} - C_0)(1+r) + A^{'},$$
$$H_2 = [(W + VSV_0^{\delta} - P^{'} - C_0)(1+r) + VSV_1^{1-\delta} - C_1](1+r),$$

with $A' \, (A' < A)$, $P' \; (P' < P)$ denoting, repectively, the death benefit and the premium after a δ part of the policy has been sold.

2. Viaticate a percentage δ, $0 < \delta < 1$, at $t = 0$ and viaticate a percentage ρ of the remaining policy, $0 < \rho < 1$, at $t = 1$.

   In this case, the individual is keeping in effect a part of his policy equal to $(1-\rho)(1-\delta) > 0$. The economic situation is the one stated in the following graph:

	H_1	H_2
W	$(W + VSV_0^{\delta} - P^{'} - C_0) \cdot (1+r)$	
$+VSV_0^{\delta}$	$+VSV_1^{\rho}$	
$-P^{'}$	$-P^{''}$	
$-C_0$	$-C_1$	

where

$$VSV_0^{\delta} = \gamma \left[\delta \left(A \sum_{t=0}^{1} {}_{t/}q_x \cdot (1+r)^{-(t+1)} - P \sum_{t=0}^{1} {}_t p_x \cdot (1+r)^{-t} \right) \right],$$
$$VSV_1^{\rho(1-\delta)} = \gamma \left[\rho \left(A^{'} \cdot (1+r)^{-1} - P^{'} \right) \right],$$
$$H_1 = (W + VSV_0^{\delta} - P^{'} - C_0)(1+r) + A^{'},$$
$$H_2 = [(W + VSV_0^{\delta} - P^{'} - C_0)(1+r) + VSV_1^{\rho} - P^{''} - C_1](1+r) + A^{''},$$

with $A' \, (A' < A)$, $P' \; (P' < P)$ denoting, repectively, the death benefit and the premium after a δ part of the policy has been sold and $A'' \; (A'' < A')$, $P'' \, (P'' > P')$ denoting the death benefit and the premium once the $\rho \, (1-\delta)$ part has been sold.

3. Viaticate a percentage δ, $0 < \delta < 1$ at $t = 0$ and not at $t = 1$.

   The economic situation is the one stated in the following graph:

	H_1	H_2
W	$(W + VSV_0^{\delta} - P^{'} - C_0) \cdot (1+r)$	

$+VSV_0^\delta$	$-P^{'}$
$-P^{'}$	$-C_1$
$-C_0$	

where

$$VSV_0^\delta = \gamma\left[\delta\left(A\sum_{t=0}^{1} {}_{t/}q_x \cdot (1+r)^{-(t+1)} - P\sum_{t=0}^{1} {}_{t}p_x \cdot (1+r)^{-t}\right)\right],$$
$$H_1 = (W + VSV_0^\delta - P^{'} - C_0)(1+r) + A^{'}$$
$$H_2 = [(W + VSV_0^\delta - P^{'} - C_0)(1+r) - P^{'} - C_1](1+r) + A^{'},$$

with $A'\,(A' < A)$, P' $(P' < P)$ denoting, repectively, the death benefit and the premium after a δ part of the policy has been sold.

4. Do not viaticate at $t = 0$ and viaticate a percentage δ, $0 < \delta < 1$ at $t = 1$.

   The economic situation is the one stated in the following graph:

	H_1	H_2
W	$(W - P - C_0) \cdot (1+r)$	
$-P$	$+VSV_1^\delta$	
$-C_0$	$-P$	
	$-C_1$	

   where

$$VSV_1^\delta = \gamma\left[\delta\left(A \cdot (1+r)^{-1} - P\right)\right],$$
$$H_1 = (W - P - C_0)(1+r) + A,$$
$$H_2 = [(W - P - C_0)(1+r) + VSV_1^\delta - P^{'} - C_1](1+r) + A^{'},$$

   with $A'\,(A' < A)$, P' $(P' < P)$ denoting, repectively, the death benefit and the premium after a δ part of the policy has been sold.

5. Do not viaticate at $t = 0$ and do not viaticate at $t = 1$.

   The economic situation is the one stated in the following graph:

	H_1	H_2
W	$(W - P - C_0) \cdot (1+r)$	
$-P$	$-P$	
$-C_0$	$-C_1$	

   where

$$H_1 = (W - P - C_0)(1+r) + A$$
$$H_2 = [(W - P - C_0)(1+r) - P - C_1](1+r) + A.$$

Observe that in case where the individual survives the first period, he will sell all his policy with the strategy stated in 1, only a part of the policy with 2, 3, 4 and no part of the policy in 5. Hence, only in situation 1 will the heirs not receive any part of the death benefit at the end of the second period.

Next, we determine the optimal strategy for our decisor. It will be the one that maximizes the expected utiliity in expression (2).

3 An optimal solution for the general problem

In this section, we consider the optimization problem of a decision maker who determines the optimal strategy with respect to the sale of his life insurance policy in the viaticals market as the solution to the following maximization problem:

$$\begin{aligned} \max EU_0 &= U(C_0) + \beta \cdot q_x \cdot V(H_1) + \beta \cdot p_x \cdot [U(C_1) + \beta \cdot V(H_2)] \\ &= U(C_0) + \beta \cdot q_x \cdot V(H_1) + \beta \cdot p_x \cdot EU_1. \end{aligned} \tag{3}$$

For $i = 0, 1$, let the utility functions of consumptions and bequests be given by:

$$\begin{aligned} U(C_i) &= \ln C_i, \\ V(H_{i+1}) &= \alpha \ln H_{i+1}, \ \alpha > 0, \end{aligned}$$

where α indicates how the consumer values bequests in relation to consumption. For $0 < \alpha < 1$ consumptions are valued more highly than bequests, while values of $\alpha > 1$ indicate higher valuation of bequests (equal valuation is obtained for $\alpha = 1$).

With respect to the form of the utility functions, remark that they are a particular case of potential utility functions. Logarithmic utility functions are also considered in Bhattacharya et al (2004). The economic interpretation of this family of utility functions can be found e.g. in Karatzas and Shreve (1998).

As we have seen, there are five strategies for the decisor. In the sequel they will be referred to as case 1 to case 5. The optimal strategy will be the one that maximizes the utility in (3). In order to avoid repetition of similar computations, we group the fives cases into two: the first group considers that the decisor sells a part of the policy at $t = 0$ (cases 2, 3), and the second group considers that no part of the policy is sold at $t = 0$ (cases 4, 5). The special case where the decisor has sold all his policy at the beginning of the second period (case 1) is included in group 1. The solution to the problem is found by using the dynamic programming technique (see e.g. Bertsekas (2000)). For each group, we first compute the optima at $t = 1$ and then update this optima to $t = 0$. The optimal strategy (selling or not selling a part of the policy at $t = 0$) will be the one that gives rise to a higher optimal value for EU_0.

3.1 Viaticate a percentage δ at $t = 0$

At $t = 0$, when the economic agent decides to sell a part δ of his life policy, he receives the amount VSV_0^{δ}, and then his initial wealth, W, increases by that quantity. Moreover, as a $(1 - \delta)$ part of the policy will be active during the first period, covering a death benefit A', the policyholder pays the premium $P^{'}$.

At $t = 1$, if the consumer survives, he has three possibilities with respect to the sale of his policy:

- Selling the rest of it, i.e., the $(1-\delta)$ percent (**case 1**).
- Selling a percentage $\rho\,(1-\delta)$ such that a certain part of the policy remains in effect (**case 2**).
- Not selling (**case 3**).

Hence at $t = 1$, in order to cover all three possibilities, we can consider that he receives the amount VSV_1^* defined by

$$VSV_1^* = \begin{cases} VSV_1^{1-\delta} & \text{if viatical of } (1-\delta) \text{ percent,} \\ VSV_1^{\rho(1-\delta)} & \text{if viatical of } \rho(1-\delta) \text{ percent,} \\ 0 & \text{if not viatical.} \end{cases}$$

Since a part of the policy could be active during the second period, covering a death benefit A^* which is equal to

$$A^* = \begin{cases} 0 & \text{if viatical of } (1-\delta) \text{ percent,} \\ A^{''} & \text{if viatical of } \rho(1-\delta) \text{ percent,} \\ A^{'} & \text{if not viatical,} \end{cases}$$

the policyholder pays the premium

$$P^* = \begin{cases} 0 & \text{if viatical of } (1-\delta) \text{ percent,} \\ P^{''} & \text{if viatical of } \rho(1-\delta) \text{ percent,} \\ P^{'} & \text{if not viatical.} \end{cases}$$

3.1.1 Optima at $t = 1$

At $t = 1$, the only unknown variables to work with are C_1 and H_2. Indeed, we consider that the first period has finished, so C_0 is a known variable and H_1 has not ocurred because the individual is still alive. The consumer's problem is to maximize the expected utility from consumption and bequests subject to the corresponding constrains for each case:

$$\max_{C_1, H_2} EU_1 = U(C_1) + \beta V(H_2)$$

subject to:

$$(W + VSV_0^{\delta} - P^{'} - C_0)(1+r) + VSV_1^* - P^* - C_1 \begin{cases} > 0 & \text{if viatical of } (1-\delta) \text{ percent at } t = 1, \\ \geq 0 & \text{otherwise,} \end{cases}$$

$$\{C_1, H_2\} > 0.$$

C_1 and H_2 are required to be strictly positive. Then, should the consumer decids to sell all his policy (case 1), there will no longer be a death benefit and the consumption for the second period has to be strictly smaller than the consumer's current wealth at $t = 1$.

From optimality conditions, after some calculations, we obtain a solution that depends on a bound of the remaining death benefit, A^*. Let

$$\overline{A} = \alpha\beta[(W + VSV_0^{\delta} - P' - C_0)(1+r)^2 + (VSV_1^* - P^*)(1+r)], \tag{4}$$

then:

(a) If $A^* < \overline{A}$, the optimal solution is

$$C_1^* = \frac{(W + VSV_0^\delta - P' - C_0)(1+r) + VSV_1^* - P^* + A^*(1+r)^{-1}}{(1+\alpha\beta)}, \qquad (5)$$

$$H_2^* = \frac{\alpha\beta}{1+\alpha\beta}[(W + VSV_0^\delta - P' - C_0)(1+r)^2 + (VSV_1^* - P^*)(1+r) + A^*]. \qquad (6)$$

(b) If $A^* \geq \overline{A}$, the optimal solution is

$$C_1^* = (W + VSV_0^\delta - P' - C_0)(1+r) + VSV_1^* - P^*, \qquad (7)$$

$$H_2^* = A^*. \qquad (8)$$

There is an economic interpretation for the dual solution obtained. After selling a part of the policy, if the remaining death benefit A^* is considered by the individual to be a small amount, insufficient for the heirs, then he will decide not to consume everything and save some part of his current wealth. On the other hand, if he considers that A^* is big enough, then he only leaves this capital to his heirs. Note that for case 1, viaticate all the policy, since $A^* = 0$ the optima is always given by the first solution.

3.1.2 Optima update at $t = 0$

At $t = 0$, the optimization problem that enables us to obtain the complete solution for all the variables involved in the problem is as follows:

$$\max_{C_0, H_1} EU_0 = U(C_0) + \beta \cdot q_x \cdot V(H_1) + \beta \cdot p_x \cdot EU_1^*$$

with

$$EU_1^* = \max[EU_1^*(\text{case 1}); EU_1^*(\text{case 2}); EU_1^*(\text{case 3})]$$

subject to

$$W + VSV_0^\delta - P' - P^*(1+r)^{-1} - C_0 \geq 0$$

$$\{C_0, H_1\} > 0$$

C_0 and H_1 are now required to be strictly positive, and it will be fulfilled in the three cases for values of C_0 smaller or equal to the consumer's current wealth at $t = 0$, since there is always a certain death benefit at $t = 1$ (i.e. $H_1 > 0$). Observe that the consumer's current wealth is equal to the initial wealth increased by VSV_0^δ and decreased by the premium payments (P' at $t = 0$ and P^* at $t = 1$).

From optimality conditions, by considering results in the previous section, we obtain the complete solution to the optimization problem. Consider $\overline{A}$ as defined in expression (4), then:

(a) If $A^* < \overline{A}$, the optimal solution for the consumption of the first period is

$$C_0^* = \min\left\{C_0^{*(1)}, \overline{C}_0^*\right\}$$

where

$$C_0^{*(1)} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

with

$a = (1+r)^2[1+\alpha\beta q_x + \beta \cdot p_x \cdot (1+\alpha\beta)]$,

$b = (-1)(1+r)[(W+VSV_0^{\delta} - P^{'})(1+r)(2+\alpha\beta q_x + \beta \cdot p_x \cdot (1+\alpha\beta)) + (VSV_1^* - P^*)(1+\alpha\beta q_x) + A^{'}(1+\beta \cdot p_x(1+\alpha\beta)) + A^*(1+r)^{-1}(1+\alpha\beta q_x)]$,

$c = (1+r)^*[(W+VSV_0^{\delta} - P^{'}) + A^{'}(1+r)^{-1}] \cdot [(W+VSV_0^{\delta} - P^{'})(1+r) + VSV_1^* - P^* + A^*(1+r)^{-1}]$,

and

$$\overline{C}_0^* = W + VSV_0^{\delta} - P^{'} - P^*(1+r)^{-1}.$$

(b) If $A^* > \overline{A}$, the optimal solution for the consumption of the first period is

$$C_0^* = \min\left\{C_0^{*(2)}, \overline{C}_0^*\right\}$$

where

$$C_0^{*(2)} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

with

$a = (1+r)^2(1+\alpha\beta q_x + \beta \cdot p_x)$,

$b = (-1)(1+r)[(W+VSV_0^{\delta} - P^{'})(1+r)(2+\alpha\beta q_x + \beta \cdot p_x) + (VSV_1^* - P^*)(1+\alpha\beta \cdot q_x) + A^{'}(1+\beta \cdot p_x)]$,

$c = [(W+VSV_0^{\delta} - P^{'})(1+r) + A^{'}] \cdot [(W+VSV_0^{\delta} - P^{'})(1+r) + VSV_1^* - P^*]$,

and

$$\overline{C}_0^* = W + VSV_0^{\delta} - P^{'} - P^*(1+r)^{-1}.$$

By substituting $C_0 = C_0^*$ in

$$H_1 = (W + VSV_0^{\delta} - P^{'} - C_0)(1+r) + A^{'}$$

and expressions (5), (6) or (7), (8); we obtain all the optimal values for consumption and bequests for any of the three possibilities considered. The best strategy (case 1, 2 or 3) will be the one that gives a higher optimal value for EU_0 :

$$EU_0^{*(1)} = \max[EU_0^*(\text{case 1}); EU_0^*(\text{case 2}); EU_0^*(\text{case 3})].$$

3.2 Not viaticate at $t = 0$

Alternatively, at $t = 0$, the economic agent can decide not to sell his policy. He then pays the premium P, and if he dies during the first period, at the end of this period the hears will receive the savings made until the moment of death, plus the entire death benefit of the policy, A.

At $t = 1$, if the consumer survives, he has now two possibilities with respect to the sale of his policy:

- Selling a percentage δ of the policy, $0 < \delta < 1$ (**case 4**).
- Not selling (**case 5**).

Hence at $t = 1$, in order to cover the two possibilities, we can consider that he receives the amount VSV_1^* defined by

$$VSV_1^* = \begin{cases} VSV_1^\delta & \text{if viatical of } \delta \text{ percent,} \\ 0 & \text{if no viatical.} \end{cases}$$

Since a part of the policy will be always active during the second period, covering a death benefit A^* which is equal to

$$A^* = \begin{cases} A' & \text{if viatical of } \delta \text{ percent,} \\ A & \text{if no viatical,} \end{cases}$$

the policyholder pays the premium

$$P^* = \begin{cases} P' & \text{if viatical of } \delta \text{ percent,} \\ P & \text{if no viatical.} \end{cases}$$

We proceed as before for obtaining the optima. The proceedings and economic interpretations are similar and can be ignored.

3.2.1 Optima at $t = 1$

The consumer's problem to solve is

$$\max_{C_1, H_2} EU_1 = U(C_1) + \beta V(H_2)$$

subject to

$$(W - P - C_0)(1 + r) + VSV_1^* - P^* - C_1 \geq 0,$$

$$\{C_1, H_2\} > 0.$$

The solution we obtain again depends on a bound of the remaining death benefit, A^*. Let

$$\overline{A} = \alpha\beta[(W - P - C_0)(1 + r)^2 + (VSV_1^* - P^*)(1 + r)], \tag{9}$$

then:

(a) If $A^* < \overline{A}$, the optimal solution is

$$C_1^* = \frac{(W - P - C_0)(1 + r) + VSV_1^* - P^* + A^*(1 + r)^{-1}}{(1 + \alpha\beta)}, \tag{10}$$

$$H_2^* = \frac{\alpha\beta}{1 + \alpha\beta}[(W - P - C_0)(1 + r)^2 + (VSV_1^* - P^*)(1 + r) + A^*]. \tag{11}$$

(b) If $A^* \geq \overline{A}$, the optimal solution is

$$C_1^* = (W - P - C_0)(1 + r) + VSV_1^* - P^*, \tag{12}$$

$$H_2^* = A^*. \tag{13}$$

3.2.2 Optima update at $t = 0$

At $t = 0$, the optimization problem is as follows:

$$\max_{C_0, H_1} EU_0 = U(C_0) + \beta \cdot q_x \cdot V(H_1) + \beta \cdot p_x \cdot EU_1^*$$

with

$$EU_1^* = \max[EU_1^*(\text{case } 4); EU_1^*(\text{case } 5)]$$

subject to

$$W - P - P^*(1+r)^{-1} - C_0 \geq 0$$

$$\{C_0, H_1\} > 0$$

From optimality conditions, by considering the results in previous section, we obtain the complete solution to the optimization problem. Consider $\overline{A}$ as defined in expression (9), then:

(a) If $A^* < \overline{A}$, the optimal solution for the consumption of the first period is

$$C_0^* = \min\left\{C_0^{*(1)}, \overline{C}_0^*\right\}$$

where

$$C_0^{*(1)} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

with

$a = (1+r)^2[1 + \alpha\beta q_x + \beta \cdot p_x \cdot (1+\alpha\beta)]$,

$b = (-1)(1+r)[(W-P)(1+r)(2 + \alpha\beta q_x + \beta \cdot p_x \cdot (1+\alpha\beta)) + (VSV_1^* - P^*)(1+r)(1+\alpha\beta \cdot q_x) + A(1 + \beta \cdot p_x(1+\alpha\beta)) + A^*(1+\alpha\beta \cdot q_x)(1+r)^{-1}]$,

$c = [(W-P) + A(1+r)^{-1}] \cdot [(W-P)(1+r) + VSV_1^* - P^* + A^*(1+r)^{-1}]$

and

$$\overline{C}_0^* = W - P - P^*(1+r)^{-1}.$$

(b) If $A^* > \overline{A}$, the optimal solution for the consumption of the first period is

$$C_0^* = \min\left\{C_0^{*(2)}, \overline{C}_0^*\right\}$$

where

$$C_0^{*(2)} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

with

$a = (1+r)^2(1 + \alpha\beta q_x + \beta \cdot p_x)$,

$b = (-1)(1+r)[(W-P)(1+r)(2 + \alpha\beta q_x + \beta \cdot p_x) + (VSV_1^* - P^*)(1+r)(1+\alpha\beta \cdot q_x) + A(1+\beta \cdot p_x)$,

$c = [(W-P)(1+r) + A] \cdot [(W-P)(1+r) + VSV_1^* - P^*]$, and

$$\overline{C}_0^* = W - P - P^*(1+r)^{-1}.$$

By substituting $C_0 = C_0^*$ in

$$H_1 = (W - P - C_0^*)(1+r) + A$$

and expressions (10), (11) or (12), (13); we obtain all the optimal values for consumption and bequests for any of the two possibilities considered. The best strategy (case 4 or 5) will be the one that gives a higher optimal value for EU_0 :

$$EU_0^{*(2)} = \max[EU_0^*(\text{case } 4); EU_0^*(\text{case } 5)].$$

At this point, we have found the solution for the optimization problem stated in (10) since

$$\max EU_0 = \max\left\{EU_0^{*(1)}, EU_0^{*(2)}\right\}.$$

4 Numerical illustration

4.1 Optimal solution

In this section, we illustrate previous results by a numerical example. We take as the starting point an economic agent of age x with the following parameters (all monetary units are expressed in euros):

$W = 100,000$	$A = 50,000$	$P = 1,500$	$r = 0.04$
$\gamma = 0.8$	$\beta = 0.6$	$\alpha = 0.5$	$q_x = 0.7$

(14)

He has an initial wealth of $100,000$ and a life insurance policy with death benefit $50,000$, and constant annual premiums equal to $1,500$. We consider a risk free interest rate of 0.04. The intertemporal discount factor or rate of time preference, β, is equal to 0.6. Observe that a 0.6 intertemporal discount factor means that the individual is neutral between 60 euros today or 100 euros the next time unit, in our case, the next year. The marginal rate of substitution, α, between consumption and bequest is 0.5, i.e., heritage is valued as a half of the consumpion. Finally, since the consumer is a terminally ill policyholder who may have a very high death probability for the first year, we assume that $q_x = 0.7$. The death probability for the second period is equal to 1.

We consider the following possibilities for our consumer:

1. Viaticate a percentage $\delta = 0.6$ at $t = 0$ and viaticate the rest of the policy, i.e. $(1-\delta) = 0.4$, at $t = 1$.

   The economic situation will be as follows:

   At $t = 0$, the initial wealth increases by

   $$VSV_0^{0.6} = 21,882.96,$$

   which corresponds to 60% of the actuarial present value of the part of the policy sold. Since 40% of this policy remains active during the first period, covering a death benefit $A' = 0.4 \cdot A = 20,000$, the policyholder pays a premium $P' = 0.4 \cdot P = 600$.

   At $t = 1$, all the policy is sold so there is no death benefit or premium. In return, the economic agent receives the amount

   $$VSV_1^{0.4} = 14,904.62.$$

2. Viaticate a percentage $\delta = 0.6$ at $t = 0$ and viaticate a percentage $\rho = 0.5$ of the remaining policy at $t = 1$.

   The economic situation at $t = 0$ is exactly the same as in case 1.

   At $t = 1$, 50% of the remaining policy is sold and the agent receives the amount

   $$VSV_1^{0.5(0.4)} = 7,452.31$$

   Since 20% of the policy remains active covering a death benefit $A'' = 0.2 \cdot A = 10,000$, the policyholder pays a premium $P'' = 0.2 \cdot P = 300$.

3. Viaticate a percentage $\delta = 0.6$ at $t = 0$ and do not viaticate at $t = 1$.

   Again the economic situation at $t = 0$ is the one stated in case 1.

   At $t = 1$, there is no additional sales so 40% of the policy remains active covering a death benefit $A' = 0.4 \cdot A = 20,000$. The corresponding premium is $P' = 0.4 \cdot P = 600$.

4. Do not viaticate at $t = 0$ and viaticate a percentage $\delta = 0.6$ at $t = 1$.

   At $t = 0$, since no part of the policy is sold, the death benefit and the premium are the initial ones, that is $A = 50,000$ and $P = 1,500$.

   At $t = 1$, 60% of the policy is sold and the agent receives the amount

   $$VSV_1^{0.6} = 22,356.92.$$

   Since 40% of the policy remains active covering a death benefit $A' = 0.4 \cdot A = 20,000$, the policyholder pays a premium $P' = 0.4 \cdot P = 600$.

5. Do not viaticate at $t = 0$ and do not viaticate at $t = 1$.

   In this case, the parameters for both periodes are the initial ones.

By solving all the possible paths of the policyholder, we obtain the optimal consumptions and bequests for each case and the associated expected utilities. The results are presented in the following table.

Case	C_0^*	H_1^*	C_1^*	H_2^*	EU_0^*
Case 1	$95,423.63$	$46,893.71$	$32,152.55$	$10,031.60$	$16,09038$
Case 2	$96,135.47$	$46,153.39$	$33,016.22$	$10,301.06$	$16,10067$
Case 3	$96,821.21$	$45,440.22$	$24,840.22$	$20,000.00$	$16,08912$
Case 4	$91,523.50$	$57,255.56$	$29,012.49$	$20,000.00$	$16,10933$
Case 5	$78,437.72$	$70,864.77$	$19,364.77$	$50,000.00$	$15,97654$

Table 1: Optima and maximum expected utility

Hence, for an individual with the above characteristics, the optimal solution corresponds to case 4, that is, the utility is maximized when only a part of the policy, $\delta = 0.6$, is sold at $t = 1$.

Observe that the solution found so far is a particular solution valid for an individual with the above personal characteristics. It can be easily improved if the viatical market allows the agent to sell any percentage of the policy at each time. In that case, we just have to check when the policy should be sold and what percentage must be sold in order to maximize the expected utility. This analysis is done in next section, where we also consider the effect on the optimal solution of some other relevant individual parameters.

4.2 Sensitivity analysis

In this section, we analize the influence of the individual parameters on the optimal solution. By considering the initial wealth, for instance, intuitively it seems clear that when W increases, the necessity of selling the life insurance policy must decrease. This fact is next proven for our particular example.

The parameters included in the analysis are: W -the initial wealth-, α -the marginal rate of substitution between consumption and bequest-, β -the intertemporal discount factor-, γ -the percentage of the actuarial present value of the policy that the viatical company pays- and δ, ρ -the part of the policy sold by the policyholder-. Remark the relevance of the analysis of these last parameters, δ, ρ, since they will complete the solution of the problem. Indeed, observe that the optimal solution found so far could be a particular one since we have assumed a particular value for δ, ρ. The optimal strategy for our agent is to sell the part of the policy that gives rise to a higher value of his current expected utility.

4.2.1 Modifying W

The data is that given in (14), except for the value of W. For some increasing values of W, Table 2 gives the optimal decision with respect to the sale of the policy, while Figure 1 reflects the evolution of the optimal values $C_0^*,\ H_1^*,\ C_1^*,\ H_2^*$.

W	Decision
$10,000$ $20,000$ $30,000$	Viaticate δ at $t = 0$ and viaticate $(1 - \delta)$ at $t = 1$
$40,000$ $50,000$ $60,000$ $70,000$ $80,000$	Viaticate δ at $t = 0$ and viaticate ρ at $t = 1$
$90,000$ $100,000$ $200,000$	Not viaticate at $t = 0$ and viaticate δ at $t = 1$
$400,000$	Not viaticate at $t = 0$ and not viaticate at $t = 1$

Table 2: Consumer's decision when W is modified

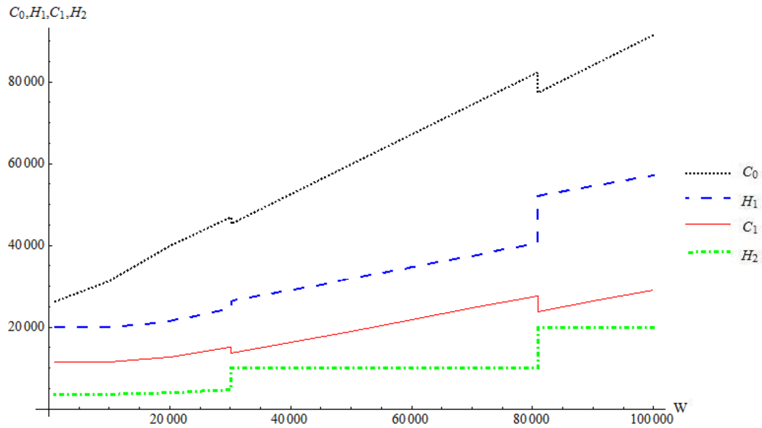

Figure 1: Distribution between consumptions and bequests when W varies

Observe that a consumer with a small W sells all his policy if he survives the first period. At $t = 0$, the individual sells part of the policy and consumes almost all his current wealth ($W + VSV_0^{0.6}$). If he dies, the heritage at the end of the first period will be the remaining insured capital, A', plus the few savings of the period, which are almost zero for values of $W < 20,000$. If he survives, he sells at $t = 1$ the rest of the policy. The current wealth at that time is $VSV_1^{0.4}$ plus the few savings made during the first period. Most of this amount is consumed and the the heritage at the end of the second period contains only the few savings left for the heirs (in all cases $H_2 < 5,000$).

When W increases (in our example, for $W > 30,159$), the individual can increase both consumption and savings by selling part of the policy at $t = 0$. Then, if he survives the first period he does not need to sell all the remaining policy in order to keep the consumption level for the second period.

Observe that, without selling his policy at $t = 0$, an individual with a relatively high initial wealth (in our example, with $W > 80,834$) can in most cases consume more than another individual in the previous stage of wealth who sells his policy. At $t = 1$, if he survives, the "richest" individual has to sell part of the policy in order to keep the consumption level for the second period.

Finally, observe that an individual with a very high initial wealth ($W > 351,000$) can achieve the optimal consumption levels for both periods without selling the policy and therefore leave the entire insured amount to the heirs. This last resut is shown in the figure below:

4.2.2 Modifying α

The data is that given in (14), except for the value of α. The parameter α reflects how the consumer values bequest with respect to consumption. For some increasing values of α (i.e., increasing valuation of bequests), Table 3 gives the optimal decision with respect to the sale of the policy.

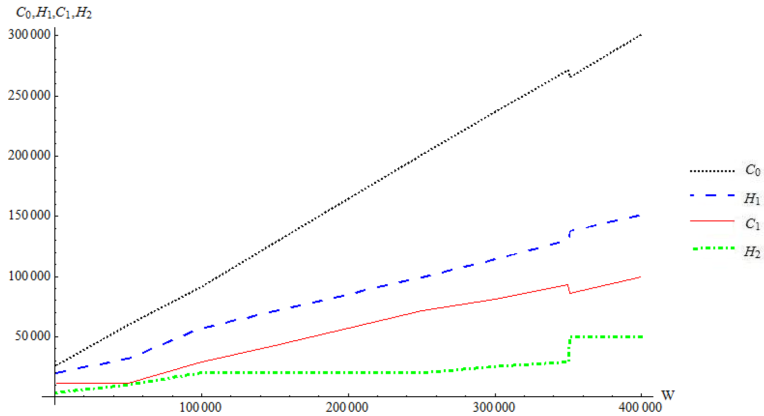

Figure 2: Distribution between consumptions and bequests for $W > 351,000$

α	Decision
0.01 0.1 0.2	Viaticate δ at $t = 0$ and viaticate $(1-\delta)$ at $t = 1$
0.3 0.4	Viaticate δ at $t = 0$ and viaticate ρ at $t = 1$
0.5 0.6 0.7 0.8 0.9 1	Not viaticate at $t = 0$ and viaticate δ at $t = 1$
2	Not viaticate at $t = 0$ and not viaticate at $t = 1$

Table 3. Consumer's decision when α is modified

As expected, when α increases there is less tendency to sell the policy. For small values of α the consumer sells all his policy, and as it increases, since bequests are more highly valued, the consumer keeps at least a part of his policy. For values of $\alpha > 1.30$, all the policy is kept and heritages reach their maximum values. The analysis will be more accurate if the evolution of the optimal values C_0^*, H_1^*, C_1^*, H_2^* are observed, as can be seen in Figure 3.

For small values of α ($\alpha \to 0$), consumption in the first period reaches almost all the current wealth; the consumer keeps a part of the policy, and, if he dies the heritage at the end of the first period will be the remaining insured capital, A'. If he survives, he sells the remaining policy and in the second period consumes almost everything, the heritage at the end tends to zero. As α increases, the consumption level for the first period clearly decreases. There are decreasing jumps that correspond to the change in the decision to sell (less of the policy is sold). However, for each of the resulting intervals, the tendency to consume decreases (bequest left to the heirs increases). Observe that the consumption level for the second period is quite low. Hence, even for an increasing α, the remaining death benefit corresponding to the part of the policy that has not been sold gives the desired level of heritage for this second period, and the consumption level does not always decrease.

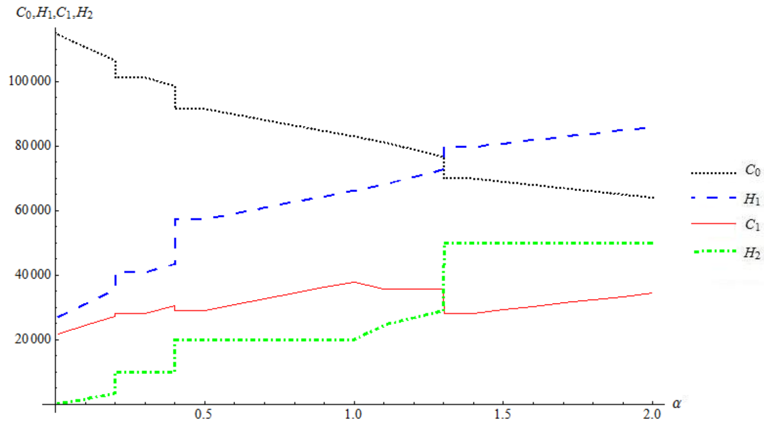

Figure 3: Distribution between consumptions and bequests when α varies

4.2.3 Modifying β

The data is that given in (14), except for the value of β. The parameter β corresponds to the yearly intertemporal discount factor. Since we are only considering two periods, a small value of β indicates that the individual does not care about what might happen the next year. Higher values of β indicate greater concern with respect to this second period.

Table 4 shows how the decision to sell changes when β increases. Observe that for our particular data, the optimal decision is always to sell or at least part of the policy. As we might expect, as β increases, less of the policy is sold and this happens later.

β	Decision
0.01	
0.1	Viaticate δ at $t = 0$
0.2	and viaticate $(1 - \delta)$ at $t = 1$
0.3	
0.4	Viaticate δ at $t = 0$
0.5	and viaticate ρ at $t = 1$
0.6	
0.7	Not viaticate at $t = 0$
0.8	and viaticate δ at $t = 1$
0.9	
1	

Table 4. Consumer's decision when β is modified

For small values of β, the individual tends to consume almost all his current wealth today, rather than saving to be able to consume more in the next period. As β increases, the individual is more concerned about the next period. Therefore, the consumption level for the first period decreases with β, while the corresponding to the second period increases with β. These results can be observed in Figure 4, where the optimal values C_0^*, H_1^*, C_1^*, H_2^* are represented.

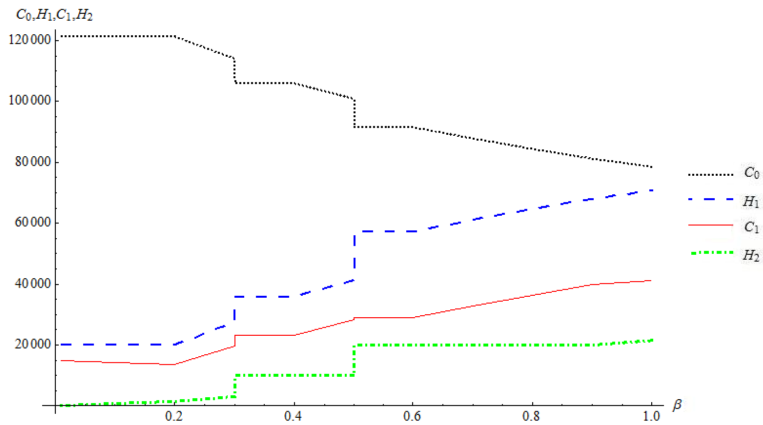

Figure 4: Distribution between consumption and bequests when β varies

4.2.4 Modifying γ

The parameter γ reflects the price payed by the viatical company to the policyholder. It is the percentage of the actuarial present value of the policy actually paid. Again, the data is that given in (14), except for the value of γ. Intuitively it seems clear that for small values of γ no part of the policy is sold while for values of $\gamma \to 1$, all the policy is sold. Intermediate values of γ necessarily imply sale of a part of the policy, which will be sold earlier as γ increases. These conclusions are shown in Table 5.

γ	Decision
0.01	Not viaticate at $t = 0$
0.1	and not viaticate at $t = 1$
0.2	
0.3	
0.4	Not viaticate at $t = 0$
0.5	and viaticate δ at $t = 1$
0.6	
0.7	
0.8	
0.9	Viaticate δ at $t = 0$ and viaticate ρ at $t = 1$
1	Viaticate δ at $t = 0$ and viaticate $(1 - \delta)$ at $t = 1$

Table 5. Consumer's decision when γ is modified

Figure 5 reflects the evolution of the optimal values C_0^*, H_1^*, C_1^*, H_2^*. Obviously, consumption associated to both periodes increases with the sale of part of the policy. For the widest range of possible values, $0.1 < \gamma < 0.8$, the optimal decision is selling part of the policy in the second period. The optimal level of consumption for the first period increases in this interval, but it must imply that the heritage left at the end of this first period decreases. Once part of the policy is sold, consumption associated to the second period increases and the heritage left is equal to the remaining death benefit.

Before concluding this section, it should be pointed out that, considering the four parameters analyzed so far, there is a possibility regarding the decison of selling that is never optimal. This corresponds to case 3, that is, viaticate a percentage δ at $t = 0$ and do not viaticate at $t = 1$.

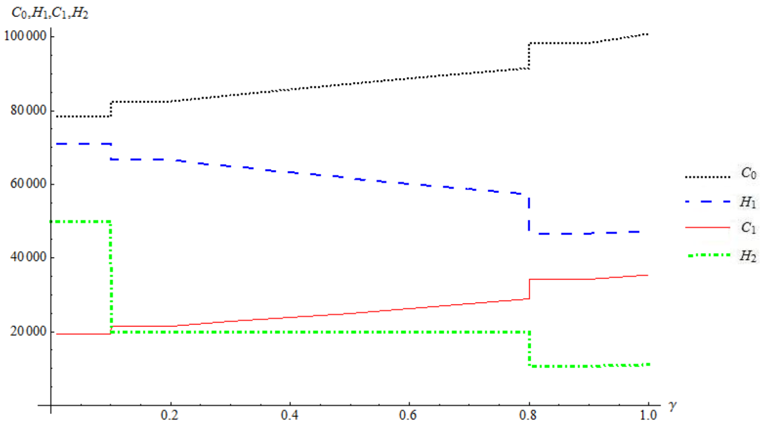

Figure 5: Distribution between consumptions and bequests when γ varies

Then, for our particular data, should the decisor sell at $t = 0$, he should alsosell at $t = 1$ in order to maximize his expected utily. Intuitively, this result seems logical; if the optimal is selling at $t = 0$, then the decision of selling stands at $t = 1$.

4.2.5 Modifying δ

Analysis of the parameter δ will always be necessary since it will give the optimal solution to our particular problem. Indeed, given the individual parameters, the optimal strategy will be to sell the part of the policy that yields to a higher value of the expected utility. As one may observe in the table, for our particular data, the optimal decision is always to sell, at least a part of the policy. By comparing the expected utilities for all possible values of δ, we can conclude that the optimal strategy for our agent is selling 80% of the policy at $t = 1$.

δ	Decision	Utilities
0	Viaticate δ at $t = 0$	$16,10944701$
0.1	and viaticate $(1-\delta)$ at $t = 1$	$16,10649701$
0.2		$16,10466377$
0.3	Viaticate δ at $t = 0$	$16,10688164$
0.4	and viaticate ρ at $t = 1$	$16,10726568$
0.5		$16,10541234$
0.6		$16,10933485$
0.7	Not viaticate at $t = 0$	$16,11829377$
0.8	and viaticate δ at $t = 1$	$16,11942477$
0.9		$16,11494859$
1	Viaticate δ at $t = 0$ and viaticate $(1-\delta)$ at $t = 1$	$16,07574794$

Table 6. Consumer's decision when δ is modified

The optimal strategy found corresponds to case 4; that is, not to viaticate at $t = 0$ and viaticate a percentage $\delta = 0.8$ at $t = 1$. Since no part of the policy is sold At $t = 0$, the death benefit and the premium are the initial ones; that is, $A = 50,000$ and $P = 1,500$. At $t = 1$, 80% of the policy is sold and the agent receives the amount

$$VSV_1^{0.8} = 29,809.23.$$

Since 20% of the policy remains active covering a death benefit $A' = 0.2 \cdot A = 10,000$, the policyholder pays a premium $P' = 0.2 \cdot P = 300$. The optimal values for consumptions and bequests are

$$C_0^* = 94,931.94 \, , H_1^* = 53,710.78 \, , C_1^* = 33,220.01 \, , H_2^* = 10,000.$$

Finally, just remark that there is a last step to be taken for those cases where the optimal solution is found by viaticating δ at $t = 0$ and ρ at $t = 1$. Indeed, the parameter ρ that gives rise to a higher expected utility should be found.

5 Final remarks

In this paper, we obtaine the analytical form of the solution of the optimization problem considered. It corresponds to a terminally ill policyholder with a maximum life expectancy of two years, who must decide between selling or not selling (part of) the policy at the beginning of each of his two remaining years of life. We work on the framework of the expected utility theory, and therefore the optimal solution corresponds to the decision that has a higher expected utility. We consider a particular case of our general model, since we also consider logarithmic utility functions. Remark that logarithmic utility functions are a particular case of potential utility functions. Therefore, our results could easily be extended to this wider range of utility functions.

The results in this paper are valid for policies traded in the viatical market. A natural extension of our results is to consider the entire life settlement market. Policies traded are those of policyholders expected to die in the next fifteen years. Considering a discrete setting with more than two periods is worthless, since analytical solutions are too cumbersome to work with, or simply cannot been reached for most situations. Hence, for the life settlement market, we are oblied to move to a continuous model, where further research must be done with respect to the survival function corresponding to an impaired insured.

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