UNIVERSITAT DE BARCELONA

Further Investigations into the Anomalies of Rational Intertemporal Choice

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Mistakes are the portals of discovery.

James Joyce (1882 - 1941)

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Presentation & Acknowledgements

It is not only that every choice we make is in essence an intertemporal choice, this is, a choice among objects located in different moments of time. It is much more. I believe the most precious objective in life, happiness, to be so closely linked to the way in which we make intertemporal decisions, that I cannot think of a better way to understand the individual welfare problem than to study the theory of intertemporal choice. And economic science deals mostly with this problem, as we economists shall not forget. I therefore have felt passionate about focusing my dissertation on this field...so passionate, that, in fact, I expect to continue this research for many years, somehow even through my entire life.

This dissertation is therefore the product of a true vocation. But this vocation grew in me thanks to many people. Looking back, I realize that the very beginning of it happened in the early undergraduate years, when I was first exposed to the influence of so many interesting fellow students and professors around “La Espiral”. This was an informal and interdisciplinary group we created to meet and discuss mainly philosophical questions aside from university; and I owe to all these people many ideas, interests and reading suggestions. Also, of course, the friendship, a true and long lasting friendship.

From those days I also remember how I was fascinated by the problems of ethics, and its implications for economic theory. I started attending several ‘ethics and economics’ courses, and reading both classical works (Stuart Mill, Hume, Smith, etc) and modern texts (Elster, Hirschman, Sen, etc). But the original and main source of inspiration of these interests was no doubt the course ‘Ética y Economía’ that Professor Antoni Domenech teached at Universitat de Barcelona. I feel specially in debt with him for so many insights, authors, ideas and reading suggestions...among others, and perhaps most importantly, the reading of the masterwork by Aristotle, Nicomachean Ethics. Ultimately, he and his work have helped me understand why intertemporal choice is so important for economic and political thinking.

All these interests culminated in me entering the PhD program in Mathematical Economics at the University of Barcelona. My choice of a mathematics department appeared surprising to many people around me, but I had a quite clear vision that ethics and economics and, specifically, economic rationality was, if taken seriously, going to demand more and more mathematical modelling. In that, and in many other things, I was influenced by the works and methodologies of Sen and Harsanyi. In fact, the problem of individual rationality emerged in me as the fundamental interest in my readings and in my first research ideas, since I realized then that most of the problems we could label as 'ethics and economics' problems, dealt ultimately with our utilitarian theory of the 'individual good', whose comprehension clearly demands some mathematical knowledge.

Once I had finished the doctoral courses, I discovered the work of Ken Binmore on justice. I am grateful to Félix Ovejero for this and many other advices. This episode was very important for me because Binmore was an economist speaking ethics in the language of game theory and mathematics. Also, his approach appeared to me in so many ways correct...Fortunately, I was then invited to visit precisely the centre directed by Professor Binmore, the Economic Learning and Social Evolution (ELSE) centre at Universtity College London. There I benefited from an atmosphere full of economic knowledge and scientific spirit, and, of course, from the strong influence that reading Professor Binmore's texts had on me. I am very grateful to Professor Binmore for giving me this possibility and must apologize for not being able at that moment to give back any scientific product. I am also grateful to the many good friends I made in the department during my visit.

If I where to choose one single finding of all I made while visiting London, I would pick the very simple, but important idea that our model for individual decision making was simultaneously interesting and incorrect. In those days I for the first time understood the fundamentals of decision under uncertainty, and, in general, the problem of measuring utility. Also, in those days I remember being captured by Kahneman and Tversky's 1979 paper. It was then when I decided to focus my research on individual decision making.

Once back in Barcelona, I entered the *Departament de Matemàtica Econòmica, Financera i Actuarial* at *Universitat de Barcelona*, where I have teached several courses in Mathematical Economics for the last six years. While working in this department I have enjoyed a very friendly atmosphere, made many good friends and learned a lot from my colleagues. I am truly grateful to all members in the department. Specifically, I want to mention Mª Gracia Casado, Pilar Báguena, Marina Nuñez, Pere Calleja, Josep María Izquierdo, Marcial Pérez, Jordi Sales, Javier Martínez Albéniz, Merce Boncompte, Jordi Esteve, Xavier Varea, Javier Sarrasí, Mari Angels Pons, Hortensia Fontanals, Merche Galisteo, Teresa Preixens, Teresa Costa, Fernando Espinosa, Jesús Montes, Jesús Marin, Jorge Navas, Carme Ribas, Oriol Roch, Anna Castañer, David Ceballos, Manu Bosch, Lluis Bermúdez, Didac Ramírez, Antoni Alegre and Carles Rafels.

During these years, I also started Netquest, a company dedicated to online fieldwork technologies and services, aimed at helping social and market researchers to conduct studies via the Internet. I must especially thank Mauricio Edo, Oriol Llaurado and Santiago Llobet in this company for their programming help in the experiments presented here, and also thank the company Ya.com for letting me access such a large sample of Internet Spanish population for these experiments. In general, I must thank everybody in Netquest for their support and ideas during these years.

Several ideas in this dissertation have been presented in seminars or conferences. An earlier version of chapter 3 was presented in Bari, Italy, within the Experimental Economics and Dynamic Choice workshop in January 2005, and I am grateful to the many valuable comments I received there. Also, part of this chapter was presented at the Barcelona Economics Decision Group seminar at Universitat Pompeu Fabra in February 2005, were I benefitted from important objections and insights. Finally, part of this thesis was also presented in the VIII Seminari en Finances at Universitat de Barcelona, from where I also obtained valuable comments. Previous versions of chapter 1 were presented at the Seminario de Teoría Social Analítica organized by José Noguera, and published in *Papers. Revista de Sociología*. I am very grateful to all members of this seminar for their support and for the stimulant intellectual discussions.

I want also to express my gratitude to everytbody in the Barcelona Economics Decision Group at *Universitat Pompeu Fabra*, especially to Robin Hogarth, Natalia Karelaia, Irina Cojuharenco, and to Marc Lemenestrel, whose generosity and help were truly helpful for this project. The existence of this group and its seminar here in Barcelona constitutes a great benefit for all of us who want to study individual decision making.

Chapters 2 and 3 owe very much to Professor Daniel Read. Both chapters are based on experiments designed and conducted jointly with him (and, for the experiment in chapter 2, also together with Mara Airoldi). Working with him and learning from his profound knowledge of intertemporal choice theory has been very stimulant for me.

Writing this dissertation had not been possible without my supervisor, Carles Rafels. He guided my initial enthusiasm and ideas into an academic project, and has always been ready to help me. Carles has been a true master for me, from whom I have learned not only mathematical economics. I am truly grateful to him also for showing such a confidence in me from the very begining. I plan to work hard to make this bet pay off.

My family and friends have also had a strong impact on this project. I am grateful to my parents and my sister for their support in so many ways along these years. Also I want to thank David Casassas, Alfonso Buil and Ramon Souto for their direct help, continuous encouragement and intellectual inspiration along these years. I believe ours to be 'perfect' friendships in aristotelian terms.

Finally, this dissertation has overcome several difficulties, the main one being the lack of time. It was conceived, developed and written while working full time in a private company, and working part time as a Lecturer at University. This means that writing it has disturbed virtually all vacations, weekends and leisure time of the last years. It has been, as I have said, the product of a true vocation. But also the product of true love, since the whole project had not been possible without the complicity of my wife Judith. I dedicate this dissertation to her and to our love.

Barcelona, 28 de octubre de 2008

General Introduction

Man seems to be deficient in nothing so much as he is in time.
Zeno

After the first ring, many of us press the 'snooze' button in the alarm clock. And this decision reflects aproximately the following reasoning: "I can't really wake up right now, but I will in 5 minutes". When this lapse of time is over, the typical next move is to press again. Usually this struggle goes on for two or three more rounds, until you reach an 'ultimatum wake up time', one that you know is the latest you can afford. Now we can ask ourselves, is something wrong about this? Is this behavior *anomalous*?

The concept of anomaly necessarily relates to that of proper function. If during a flight the captain claims there is an anomaly in the landing gear, we all interpret we are in trouble, something is not working properly. In economics, proper function is usually defined as rational action, so to claim there is an anomaly in someone's behavior, is in fact to claim that he is doing something wrong. Now, how can we say another person is wrong in his behavior when this behavior only affects him? Well, this is admittedly a very difficult task. But not an impossible one, since a person can –and usually does- disagree with himself. Take our previous example. The day before, he thinks he should wake up at 7:30 am, but when 7:30 am arrives, this person disagrees with his own previous plan and considers 7:35 am a better choice. And then at 7:35 am he considers 7:40 am a better one; and so on. Suppose he then, once asked, admits later on, retrospectively, that it had been much better for him to follow the initial plan (because he would have had more time to have breakfast and read the paper, had arrived on time at work, had had less risk of having an accident, etc). Could we then claim that pressing the 'snooze' button was wrong?

Well, let me postpone my own view on this question for the general discussion that closes this dissertation, and focus now on what the standard economic theory of intertemporal choice would have to say about the problem above. Now, put in simple words, intertemporal choice theory would say yes, this is wrong. This behavior is irrational. What makes it irrational is the dynamic inconsistency of it, the fact that it generates an internal struggle within the individual. And to make this behavior rational, economic theory would demand that either the original (previous-night) preference was changed into a more realistic wake up time, or that the new (first-ring) preference is changed into waking up[1]. But one thing must hold: the individual should not deviate from a previous plan. Such thing would be *objectively* wrong.

Whether this approach is correct or not is an unsolved question, as I shall defend later in this text. But what is without doubt is the fact that the ethical intuition underlying this position (i.e., inconsistency is bad for you) has normatively grounded what we call the theory of rational intertemporal choice for a long time. In fact, exponential discounting (often referred to simply as *discounted utility*) is considered the standard model for intertemporal decisions *because* it is a model that guarantees dynamic consistency and no other model based on additive utility functions does that. This explains why this model has the priveleged status of being considered *the* model for rational intertemporal choice.[2]

Although imperfect, the microeconomic theory of choice has thus a specific model of rational, proper action in intertemporal choices. Consequently, any empirically observed departure from this model can be labelled as an anomaly from the standpoint of economics. In other words: despite it being obvious that, until economic science solves all normative problems of the theory we should not call such deviations 'anomalies', it has been accepted in the discipline to label them as 'anomalies', much in the same spirit as the Allais Paradox is called an anomaly of decision under uncertainty. It is a technical meaning, and that is also the only meaning that this word will have in this dissertation.

Now there are several anomalies other than the snooze anomaly (dynamic inconsistency) that have been reported since Thaler (1981), which can be considered the starting point of this literature. All these 'mistakes' in the evaluation of delayed consumption in fact have served to cast light into the problem of finding a better model of human intertemporal decision making. A further investigation into these anomalies is therefore aimed at contributing to a more profound understanding of temporal decisions, and thus, ultimately, to a better understanding of the determinants of individual well being. I believe the findings presented next to be such a contribution, even if they

[1] See Strotz (1956) for a discussion about these two rational ways to cope with dynamic inconsistency.
[2] Note, however, that this privilege rests not only on the referred ethical intuition (1), but also on the assumption that intertemporal choice should be modelled via a utility function (2) that is additive (3).

tipically cast more new questions than probably solve old ones. But I truly believe all effort put into this direction to be extremely worthwhile, since I consider the problem of properly defining an individual's well being in the context of time to be one of the most urgent problems to solve in social sciences. "The good of man must be the end of the science of politics", as Aristotle put it. And we still don't know what the good of man is.

This dissertation is organized as follows. In the first chapter I present an in depth review of discounted utility as the standard for rational intertemporal choice, and of all anomalies reported in the literature. When reviewing the mathematical foundations of discounted utility, I have decided to also study the axiom system underlying the theory to a certain extent, in order to give the reader a better intuition of the value of this theory, and of the main assumptions on which it is built. And when reviewing the anomalies, I have tried to comment the main empirical findings, explain the theoretical models that have been proposed to capture such behaviors, and provide the relevant literature in each case. The main contribution of this chapter is thus to unify in a single text for the first time an axiomatic account of the theory together with a more state-of-the-art kind of text that presents simultaneously the history of the theory, and the detailed description of all important contributions. Also, there are some arguments, examples and proofs that are new or correct other texts. Finally, at the end of the chapter I have included a discussion on the normative inability of the theory, something too often disregarded as a problem in the literature.

The second chapter explores a very important question for the first time. It is known and accepted that there are several important anomalies in intertemporal choice. Possibly we could even say that hyperbolic discounting -one possible theoretical solution to dynamic inconsistency- together with excessive discounting –discounting more than financial market rates would recommend-, both are seen as two major anomalies. Now the question I put in this second chapter is whether these anomalies depend on the methodology used for experimentation or, on the contrary, are truly robust. During the past twenty five years, virtually all experiments in intertemporal choice have tried to elicit 'discount functions' from the individuals by asking them questions in a very specific way. But changing the way these questions are put turns out to have a strong impact, as I show. More generally, chapter 2 explores how much of a framing effect there is in several well established findings through a big experiment –the largest in size ever for intertemporal choice, to my knowledge- where a sample of the spanish (Internet) population is asked to make intertemporal decisions on a web questionnaire. This experiment is an important contribution since it shows that two of the most well known anomalies in intertemporal choice are dependent on method variance.

Chapter 3 deals with something different. Here I further explore possible connections between the three most important anomalies known, namely hyperbolic discounting, the magnitude effect and the sequence effects. The starting point is an effect virtually unattended in the literature, that I label the constant sequence effect. Individuals tend to be more patient (reveal lower discount rates) when choosing among constant sequences of outcomes than when choosing among single outcomes. Before the work I present here, the literature had considered this to be a mere side effect of another anomaly, hyperbolic discounting. It therefore had not atracted too much attention, since an explanation for it already existed, it was thought. But I will show in chapter three that this explanation is wrong, and provide a new and better one relating it to the magnitude effect, not to hyperbolic discounting. Although this is the main thing investigated in experiment 1, I will also show that there are signs of other, new underlying anomalous behavior behind the constant sequence effect. In a second experiment, I then further explore how individuals discount constant outcomes embedded in a sequence as compared to the discounting of single outcomes adding up to the same total amount, and find new anomalous behavior. It is struggling that such simple objects as constant sequences produce anomalous preferences, since we face constant sequences of outcomes very often in our everyday life, as, for example, whenever we decide on a purchase that consists of paying in several monthly installments. Chapter 3 thus opens what I believe to be an important new line of research, namely the evaluation of constant sequences as a good testing device of our theories. While a sequence of outcomes is simultaneously a set of multiple outcomes, *and* a set of subsequent outcomes, the literature has focused its attention until now in the second feature. And there it has found several anomalies because people are not indifferent among different *shapes* of a sequence; but my results indicate that the first feature is equally important, in fact, possibly even more fundamental, because it addresses more basic questions to the standard models in intertemporal decision making.

The dissertation will end with a summary and conclusion section, in which I comment on the main findings presented, and discuss future direction of research. At the end of the text I also provide tables and graphics for total and partial results of the experiments, together with the description of materials and an evaluation of the quality and representativity of participation.

In sum, the dissertation that starts in the next page is thus a combination of two chapters further exploring some of the main anomalies in intertemporal choice, together with one chapter presenting the historical development of the field, the mathematical foundations underlying its main findings, and also unsolved methodological questions surrounding the standard theory. So the fundamental objective of this dissertation is to contribute to the deepening of our understanding of how the presence of time affects decision making, hence, how time affects individual well being, individual happiness. As I hope the dissertation will show, we are still too far from a sufficient understanding of this problem so as to provide a radically better modelization of it; so to deepen our understanding of intertemporal choice is basically to contribute, step by step, to a better description of new anomalous phenomena occurring in intertemporal decision behavior, accumulating empirical work and discussing unfortunately mostly partial effects. We probably need now more data (experiments) than models, so that, if one day a Newton shows up, we make sure he has sufficient data available. Playing Newton with no previous systematic data collection, seems too difficult. Playing Tycho Brahe seems to come first.

Therefore, if this dissertation contributes to the available empirical evidence of effects in intertemporal choice, then it will have accomplished a first significant mission. Then, if it also served to open a passage through the anomalies of human decision making, so that someone can look through it and see the way out to a different, better solution for modelling intertemporal choice, it would have fulfilled also my highest expectations.

Chapter 1
Rational Intertemporal Choice Reviewed

> "(…) In conclusion, any connection between utility as discussed here and any welfare concept is disavowed. The idea that the results of such a statistical investigation could have any influence upon ethical judgements of policy is one which deserves the impatience of modern economists."
>
> *Paul Samuelson*
>
> *"A Note on the Measurement of Utility" (1937)*

1.1 Introduction

This final, cautious remark by the influential economist Paul Samuelson in 1937 was to be in fact the *birth* of what has ever since been the standard economic model for rational intertemporal choice: the theory of *discounted utility*. This theory has been so widely used by economists as to cover from an agent's investment behaviour to –of course- policy issues such as the proper allocations of wealth among different generations. It is claimed, in fact, that discounted utility is to inter-temporal decision-making what expected utility is to decision-making under uncertainty (Loewenstein & Prelec 1992). And there is no doubt Samuelson's 5 pages paper established a central approach to time preference in modern economics.

Now we may ask ourselves whether such a phenomenal success has been due, as Samuelson himself feared, to the impatience of economists, too ready to disregard the limitations of discounted utility. The answer, as we shall see in Chapter 1, is "not only". While we review its main features, we will see how, despite its simplicity, Samuelson's approach achieved to capture many previous insights on time preference into one single formula, solving several technical problems of the theories preceding his own. Discounted utility incorporated, for the good and for the bad, several assumptions that the intellectual legacy of many economic thinkers laid down to Samuelson, starting from Adam Smith and the Classical School, and going all along through neoclassical economics to reach the fundamental contribution of Stanley Fisher. No wonder, then, that discounted utility was instantly established as the orthodoxy.

Why then was Samuelson that much worried? To the just mentioned inherited assumptions, he had added new ones for mathematical convenience. The result was a very parsimonious theory of intertemporal choice that had significant limitations. Samuelson himself was perfectly aware of them, and that shall be no surprise. He had both the sensitivity to the need of the empirical grounding of economic theories[3] and a deep knowledge of mathematical economics. In other words, Samuelson could perfectly realize the mathematical attractiveness of his newborn theory, but knew also its rather limited scientific reach; and that is why he was concerned with an abuse of his model by modern economists.

This abuse occurred, and has actually lasted until the early 80s, when the first systematic experiments were done. Then there came the beginning of a vast literature studying the so-called *anomalies* of discounted utility, anomalies that have cast serious doubts on the validity of discounted utility as a proper theory of rational dynamic choice. It is the aim of this first chapter to review both the foundations of discounted utility and the major findings regarding the above mentioned anomalies, and describe the main alternative intertemporal choice models appeared in the second half of the 20th century, all of them challenging Samuelson's formula in one way or another.

Chapter 1 in this dissertation will, hence, consist of an up-to-date insight into the economic theory of rational intertemporal choice, starting at the early neoclassical authors and ending with the most recent findings in the field. My purpose is to develop a critical review that provides the reader of the remaining chapters with the theoretical background needed to evaluate the relevance of the results presented in this dissertation. Throughout the next pages I will therefore describe the mathematical and psychological foundations of discounted utility and its theoretical alternatives into some detail, and try to extract, in the conclusion to this first chapter, an evaluation of the normative and

3 In fact, Samuelson is the father of 'revealed preference', a theory aimed at founding microeconomics empirically (see Samuelson (1938)).

positive validity of discounted utility as the standard theory for rational intertemporal choice.

1.2 The Legacy of Classical Economic Thought

The idea that the value of a good depends on the timing of its consumption was already present in economic thinking in the 18th century[4]. But an in-depth study of the economic and psychological motivations underlying time preference had to wait until the publication in 1834 of *The Sociological Theory of Capital*, written by who is in consequence considered the father of intertemporal choice modelling: John Rae[5]. According to Rae, there are four determinants of what he labelled "the effective desire of accumulation":

I. The Bequest Motive
II. Self-restrain
III. Uncertainty of Human Life
IV. Excitement of Immediate Consumption

Factors I and II are considered to promote the desire of accumulation, while factors III and IV are supposed to limit it. All four factors jointly determine a person's time preference. Thus, for example, the more uncertainty an individual has over his life (factor III), the less he will care about the future, and, consequently, the lower will be his desire of accumulation. On the other hand, the higher his affections towards his heir (factor I), the more value he will give to the future, and, consequently, the higher will his desire of accumulation be.[6]

The marginalist William Stanley Jevons took years later the following more technical view (Jevons 1888 [1871]): to maximize total utility over time, a person ought to distribute consumption of a good over 'n' days so as to equal each days' marginal utility v_i, that depends upon consumption increasingly, times the probability p_i of the good remaining consumable:

[4] Even a formula for present value existed already in that times, thanks to contributions like that of Halley (1761), more famous for the comet that bears his name -as reported by Mark Rubinstein (2003)-.

[5] The relevance of Rae's work as a pioneering one in this topic is made clear by Irwin Fisher's dedication of his famous *Theory of Interest*: "To The Memory of John Rae and of Eugen von Böhm-Bawerk Who Laid the Foundations Upon Which I Have Endeavoured to Build"

[6] And, hence, according to Rae, the higher his nation's wealth will be. Interestingly, Rae's own theory on the Wealth of Nations was precisely based on the different desire of accumulation of their inhabitants.

(1) $v_1 p_1 = v_2 p_2 = \ldots = v_n p_n$

Since $(p_i)_{i=1,\ldots,n}$ is to be assumed a decreasing sequence for obvious reasons, and under the assumption of decreasing marginal utility, this equation means that future consumption of the commodity should be less than present consumption, since lower consumption means *greater* marginal utility[7]. Jevons assumed that this kind of time preference, owing to the fact that there is an intrinsic uncertainty on whether future consumption is ever going to happen, is completely rational. But he also acknowledged an *irrational* time preference, due to men not being perfectly foresighted and to take this fact into account, he defined q_1, q_2, q_3, *etc.,* to be *"the undetermined fractions which express the ratios of the present pleasures or pains to those future ones from whose anticipation they arise*" (Jevons, 1888, III.62). Such discounting factors completed his previous equation to make it as follows:

(2) $v_1 p_1 q_1 = v_2 p_2 q_2 = \ldots = v_n p_n q_n$

We can immediately see how these new 'irrational' factors become a further explanation of why people allocate less consumption to the future than to the present, for it is natural to assume that instant pleasures will be more valued than the anticipation of the same pleasures occurring sometime in the future. If, in consequence, $(q_i)_{i=1,\ldots,n}$ is also considered a decreasing sequence, then equation (2) means that individuals allocate lower consumption to the future than to the present for two distinct motives, of which one is deemed rational –intrinsic uncertainty because of the passage of time- while the other is considered irrational –undervaluation of future pleasures-.

In 1884, Eugen von Böhm-Bawerk (1890) claimed that this systematic tendency to underestimate future pleasures may be due to humans lacking the capacity to make a complete picture of their future wants, especially when it comes to remotely distant

[7] That lower consumption means higher marginal utility is true only under the assumption of diminishing marginal utility of consumption, a natural one to be done by Jevons. Under his view, there are thus two competing forces: diminishing marginal utility invites us to postpone consumption, while uncertainty recommends us not to do it.

ones[8]. Böhm-Bawerk's voluminous work *Capital and Interest* was mostly devoted to the study of time preferences. Together with other very insightful psychological considerations, this work interests our review because it further developed the methodology, already present in Jevons[9], based upon considering the allocation to different consumption periods as a mere *technical* question; in essence, as a matter of efficiency, given some preferences.

This feature is of crucial importance to the understanding of how and why contemporary intertemporal choice theory gradually moved away his focus from the psychological determinants of time preference, and hence abandoned, in favour of a robust mathematical theory, any rationality consideration (other than internal consistency, as we will see).

The 'technical' approach originated by Böhm-Bawerk was further perfected by the American economist Irwin Fisher, who achieved a formalization of time preference in terms of economic trade-offs among consumption in different periods (Fisher 1930). According to Fisher, every person has his own rate of 'impatience', one that depends upon objective (size and risk of future income) and subjective factors (foresight, strength of will, habit, uncertainty, selfishness, influence of fashion). But in a monetary market, people will freely lend and borrow until their personal rate of time preference equals the interest rate available in the market. Fisher's idea is that an individual whose degree of impatience is, for example, higher than the interest rate, will be willing to borrow money from another individual whose impatience is lower than the market interest rate. But once he has borrowed, his current income will be higher, making him automatically less impatient[10]. This process will last until his or her rate of impatience equals the market interest rate. An analogous reasoning is valid for someone whose impatience is lower than the interest rate. In equilibrium, thus, people's degree of impatience is expected to equal the market interest rate.

Fisher's theory was very close to discounted utility. In fact, it was a theory of discounted utility, although Fisher did not propose a single formula to evaluate

[8] This line of research has been lately further developed in Loewenstein, O'Donogue & Rabin (2003), who show that people systematically tend to mis-predict their future utility by considering it too similar to their current utility.
[9] I depart here of the opinion of Fredrick, Loewenstein and O'Donoghue (2002) who consider Böhm-Bawerk to be the first to favour this methodology. My position is based on the formulae just presented, where Jevons clearly argues for an allocation that maximizes utility over different periods as if they were different alternative 'uses' of the good.
[10] Here underlies the crucial assumption of the degree of impatience being inversely proportional to income, "Impatience Principle A" in Fisher's theory.

consumption paths, among other things, because he was too aware of the variability of all influencing factors. What Fisher proposed is that we view the individual in the market as allocating money to satisfy the following equilibrium-formula in *The Theory of Interest*:

$$x_0 + \frac{x_1}{1+i_1} + \frac{x_2}{(1+i_1)(1+i_2)} + ... + \frac{x_n}{(1+i_1)(1+i_2)\times ...\times(1+i_n)} = 0$$

This equation states that the *present value* of all 'additions' $x_0, x_1, x_2, ..., x_n$ equals zero, where additions for every period mean positive or negative amounts added to a period's income (because of borrowing or lending). Thus, if an individual wants $100 from next year's income allocated to his current income ($x_0 = +100$), then he will have to remove (subtract) *more* than $100 from next year's income ($x_1 = -110$, for example), so that the sum of present values of both additions equals zero. What interests us from this approach is both the fact that Fisher already thought in terms of compound interest, and also, as we can see in the formula, the fact that he considered a different interest rate for every period, arguing that interest rates certainly could differ among periods even for the same person.

The use of different interest rates per period together with his assumptions about the many psychological factors influencing intertemporal decision-making, all are signs of how economists before Samuelson treated intertemporal choice. Their fundamental legacy was double: on one hand, the multiplicity of psychological factors influencing time preference, some of them considered irrational; on the other hand, the possibility of achieving a technical-mathematical description of intertemporal efficient *allocations* based on interest rates. Discounted utility continued developing the latter, while leaving aside the former. From a descriptive point of view, intertemporal choice theory lost then its concern for the multiple phenomena affecting time preference. From a normative point of view, the theory lost all distinction among rational and irrational[11] causes of time discounting. It gained, nevertheless, the usefulness of a simple and robust mathematical formulation.

[11] Among which neoclassical economists counted lack of education and poverty (Peart 2000), conferring institutions the task of correcting these sources of irrationality.

1.3 The Formulation of Discounted Utility

In 1937, Paul Samuelson wrote a very influential paper: "A Note on the Measurement of Utility" (Samuelson 1937). In his paper, Samuelson showed that, under certain assumptions, it was possible to infer (to measure) the utility function underlying a series of choices made by an individual over a certain period of time. The basic idea was, thus, to find what *mathematical structure* makes it possible to unambiguously determine the form of an individual's utility function for consumption over a specified time period. The spirit of Samuelson's paper resembled in fact very much his idea of revealed preference, first published only one year later.

To achieve such a 'measurement of utility', Samuelson stated the need of several assumptions. A most fundamental one is the one that assumes that "during any specified period of time, the individual behaves so as to maximize the *sum* of all future utilities". Mathematically, the individual is thus supposed by Samuelson to maximize the following integral:

$$J = \int_0^T \mathrm{V(x,t)} \cdot \mathrm{dt} \qquad (1)$$

Where $V(x,t)$ is the utility value of income x in time t. But then, future utility is not directly comparable to present utility. Future utilities need to be "reduced to comparable magnitudes by suitable time discounting", in Samuelson's words. Now the natural question to ask is *how* should such a discounting occur. And here came Samuelson's most decisive assumption: discounting was considered independent (separate) from utility. An individual's future utilities were disentangled into a regular utility function for money[12] $\mathrm{U(x)}$ -also called 'instantaneous utility'-, and a discount function $D(t)$ that assigns a weight to utility at every time period. The original utility function is substituted by the following one:

$$\mathrm{V(x,t)=U(x)} \cdot D(t)$$

The individual is assumed to have an 'instantaneous utility' for money that does not depend on the timing of its consumption. Any time preference is thus confined to the

[12] Samuelson's theory was intended to explain only preferences over money income.

discount function $D(t)$. And in consequence, the individual is now assumed to maximize the following integral:

$$J = \int_0^T \mathrm{U(x)} \cdot D(t)\mathrm{dt} \qquad (2)$$

The disentanglement of value into instantaneous utility and discount function is an arbitrary assumption that imposes a particular *structure* on time preference. Samuelson did, thus, impose two fundamental assumptions: behaviour as maximization of the *sum* of future utilities[13], and discounting of future utilities computed as a *separate* function.

In fact, Samuelson went further and proposed a specific discount function to arrive at a solution to his original measurement problem: the exponential discount function. The choice of exponential discounting was inspired by the following reasoning: in Samuelson's setting, an individual is supposed to efficiently allocate a stream of money income over time. If for a certain period the individual chooses not to consume, the unconsumed money yields an interest in the next periods. And it was reasonable for Samuelson to compute this interest as a 'compounded interest', since compound-interest was at that time already widely used in financial economics. For the sake of simplicity, Samuelson chose continuously compounded *constant* interest for his model. Under all these assumptions, the integral assumed to be maximizing by the individual became the following:

$$J = \int_0^T \mathrm{U(x)} \cdot e^{-r \cdot t}\mathrm{dt} \qquad (3)$$

where $r = \ln(1+i)$ is a constant rate, and i is the *per-period* equivalent interest rate to *continuously* compounding at interest r.

Samuelson's model is also called 'constant discounting' or 'exponential discounting'. And Samuelson's choice of this model was not as arbitrary as one may be tempted to think. As we will next see, exponential discounting has a rationale as a model for intertemporal choice.

[13] Which implies 'consumption independence' (see axiom 3 in section 1.4.2) and therefore excludes certain common preference patterns.

1.4 The Axiomatic Foundations of Discounted Utility[14]

1.4.1. The Axiom System Underlying Discounted Utility

I will now present and discuss the complete axiom system underlying Discounted Utility. In fact, different axiom systems were proposed in the second half of the 20th century[15]. Here I have chosen mainly to combine the classical work by Krantz et al (1971), with two contributions by Wakker (1989) and Ahlbrecht & Weber (1995). The reason for this choice is that, first, Krantz et al (1971) is the most general approach to additive representations (as I will show later); second, Wakker (1989) is the most intuitive approach to additive representations; and third, Ahlbrecht & Weber (1995) is a clear discussion on the implications of every axiom for the specific theory of intertemporal choice, with emphasis in normative questions regarding the rationality of the different discounting models I will consider.

The following development is, thus, a true combination of all three pieces of work with my own elaborations, to give the reader of this dissertation the best possible unified description of the implications of the axioms for rational intertemporal choice. The *spirit* of this reconstruction resembles that of Ahlbrecht & Weber (1995); nevertheless, the difference the reader will find with that paper is that (a) I present the algebraic (more general) approach to additive representations, while they refer to the topological approach; (b) I use the more standard Cartesian product structure as set of alternatives, while they use a more sophisticated set; and (c) I enter into many mathematical details that their development does not[16]. In sum, the development that follows tries to describe the axiom system for intertemporal choice in the spirit of Ahlbrecht & Weber (1995), but with the rigor of the classical reference Krantz et al. (1971) and enhanced by examples, illustrations and some proofs on myself.

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[14] By the term 'discounted utility' I will from now on refer to the model implied by Samuelson's integral (3), i.e. exponential discounting.

[15] Axiom systems for discounted utility have been proposed by Koopmans (1960), Lancaster (1963), Fishburn & Rubinstein (1982). See also Ok & Masatlioglu (2003) for a more recent axiomatic theory of time preference that takes in several departures of the standard discounting model.

[16] I describe the axioms and theorems together with examples, and give, where possible, sketches of the main proofs (in this section, 1.4.1). Also, I have tried to correct Ahlbrecht & Weber (1995) regarding certain imperfections: the authors do not mention in their theorem 2 that they need at least three periods for the additive representation to result, while I do not only mention it, but also justify it and give examples of why this is so. Also, they use a set of alternatives for which, to my knowledge, no additive representation theorem exists, while I, as mentioned before, use the more standard Cartesian product -Wakker (1989) shows that the Cartesian product is crucial for additive representation (see remark III.4.1, page 73)-.

The starting point is the following setting. The objects of choice will be streams of consequences (consumption paths). Let $I=\{0,1,...,n\}$, with $n\in\mathbb{N}_0$, be a finite index set. Possible consequences in period $i\in I$ are real numbers belonging to an arbitrary[17] non empty set $X_i\subset\mathbb{R}$. I will also use $\mathbb{R}^+$ to refer to the set of positive real numbers including zero, and $\mathbb{R}^{++}$ to refer to the positive real numbers excluding zero. For a consequence stream I write $x=(x_0,x_1,...,x_n)$, where $x_i\in X_i$ indicates consumption in period i. Thus, I define the set of alternatives as a Cartesian product structure as follows:

$$\Omega=\prod_{i\in I}X_i$$

Now consider a binary relation $\succeq$ defined over the set Ω to describe a decision-maker's preferences, so that if he weakly prefers stream x over y then we write $x\succeq y$. Now let us establish the following standard axioms:

AXIOM 1 (transitivity)
$\forall x,y,z\in\Omega$, if $x\succeq y$ and $y\succeq z$, then $x\succeq z$
AXIOM 2 (completeness)
$\forall x,y\in\Omega$, either $x\succeq y$ or $y\succeq x$

Also, two associated binary relations $\sim$ and $\succ$ are defined from $\succeq$ as usual:
$x\sim y\Leftrightarrow x\succeq y$ and $y\succeq x$
$x\succ y\Leftrightarrow x\succeq y$ and not $x\sim y$

Axioms 1 and 2 guarantee that the individual's preferences $\succeq$ over Ω are a *weak order*[18], and are common to those of choice theory without the time dimension. I will from now on refer to $(\Omega,\succeq)$ as an *intertemporal choice problem* in which $\succeq$ is a weak order

[17] I will follow here the so-called algebraic approach that dispenses with the topological assumptions on X_i by using instead two axioms, restricted solvability and the Archimedean axiom. I will introduce them next. For a discussion on the pros and cons of the algebraic or the topological approach to additive representations, see Wakker (1988).
[18] A weak order differs from a total order in that it need not be *antisymmetric*, this is, in a weak order (or preference) it is not necessarily the case that if $x\succeq y$ and $y\succeq x$ then $x=y$. In contrast, a total order is a weak order that is antisymmetric. A natural example of simple order is $\langle\mathbb{R},\geq\rangle$: it is a weak order (complete and transitive), and it is also antisymmetric.

or preference relation defined over the set of alternatives Ω, given an index set of periods I.

But more mathematical structure than $(\Omega, \succeq)$ is needed to arrive at a tractable model of intertemporal choice. In particular, we want to disentangle two sources of utility, on one hand the utility of consumption, and on the other hand the utility/disutility of how long one has to wait for that consumption. For such disentanglement, intertemporal choice theory first establishes the existence of an *additive* representation, meaning utility of a consequence stream equals *the sum of utility of consumption in each separate period*, as we saw in Samuelson's formulation. To obtain such a representation we will need to introduce more assumptions.

Our first step concerns the independence of the weak order with respect to equal *substreams*. Let me introduce this concept:

Substreams: Consider $\varnothing \neq A \subset I$ and let x_A denote the element of $\prod_{i \in A} X_i$ with *i-th* coordinate x_i for all $i \in A$. I will call x_A a *substream*. Given x, x_A can be considered the restriction of x to A. Of course, $x_{\{i\}} = x_i$ and $x_I = x$. In general, the length of x_A is $|A|$. Similarly, let x_{-A} denote the element of $\prod_{i \notin A} X_i$ with *i-th* coordinate x_i for all $i \notin A$. Finally, for $x, y \in \prod_{i \in I} X_i$, $x_{-A} y_A$ denotes the stream with i-th coordinate x_i for all $i \notin A$, and with i-th coordinate y_i for all $i \in A$.

I now introduce the crucial axiom for the existence of an additive utility function representing the weak order $\succeq$. This axiom is called 'independence of equal substreams', and was already discussed informally by Fisher (1927).

AXIOM 3 (independence of equal substreams (IES))

For any non-empty $A \subset I$, take arbitrary $x, x', y, y' \in \Omega$; then

$$x_{-A} y_A \succeq x'_{-A} y_A \Leftrightarrow x_{-A} y'_A \succeq x'_{-A} y'_A$$

This axiom imposes independence of common consequences[19]. It means that preferences over streams of consumption are not affected by changes in substreams that are common to those streams. For example, imagine the set $X_i \subset \mathbb{R}^+$ represents possible salaries to be earned in year i from the company you work at. Consider a five-year salary stream $x = (x_0, x_1, x_2, x_3, x_4)$, where x_i represents the annual salary to be received in year i (expressed in thousands of euro), with $i \in I = \{0,1,2,3,4\}$. Now let an individual choose among the following two options:

$x = (10,15,10,20,10)$

$x' = (10,10,10,20,16)$

These two options share the amounts in periods 0, 2 and 3. So let us define $A = \{0,2,3\}$, and use the language of substreams to say that both options have the *common* substream $y_A = (10,10,20) \in X_0 \times X_2 \times X_3$. We in fact can refer to the above mentioned streams as:

Stream A: $x_{-A} y_A = (10,15,10,20,10)$

Stream B: $x'_{-A}\, y_A = (10,10,10,20,16)$

Now say he prefers $x_{-A} y_A$ over $x'_{-A}\, y_A$ (stream A over stream B), because he is willing to sacrifice six thousand euro in period 4 in order to get five thousand in period 1. Then IES implies that this preference should not be affected by changes in the amounts that are common to both alternatives. In particular, consider a choice among the following new options in which the common substream $y_A = (10,10,20)$ has been replaced by $y_A' = (8,12,14)$:

Stream C: $x_{-A} y_A' = (8,15,12,14,10)$

[19] For the case of A being a singleton, IES is equal to what is called coordinate-independence (CI), so that IES implies CI; but it is also important to observe that, for finite Cartesian products like the one we are considering (Ω), CI implies IES: if the preference among two alternatives x and x' (with common substreams, say $x_{-A} y_A \succeq x'_{-A}\, y_A$) is unaffected by the replacement of a common coordinate (because of CI), we may replace one by one all identical coordinates y_i for $i \in A$ without affecting the preference by any of these replacements, and thus arriving at IES.
Also, when the complementary set, $I \setminus A$, is a singleton, IES becomes *weak separability*.

Stream D: $x'_{-A}y_A' = (8,10,12,14,16)$

If the individual chose A over B before, then IES demands that he now chooses C over D. But in general, this means that our intertemporal choice model will not be able to capture many preferences for specific 'patterns' of consumption. For many people the fact that stream D has an increasing pattern makes this option automatically more valuable. So many people would violate IES by choosing A over B and D over C. Probably, for such people, the utility of the salary in one year depends on other year's salaries, a preference that is incompatible with the assumption of independence among consumption in different periods[20]. (There is plenty of empirical evidence for such preferences; see, for example, the pioneer work of Loewenstein & Sicherman (1991). I refer here to section 1.7.3 for a more detailed comment on this anomaly)[21].

Let us now continue our reconstruction of the axiom system for intertemporal choice. Under certain topological assumptions on the sets of alternatives -X_i to be a connected topological space, and Ω endowed with the product topology-, which I here have not imposed, axioms 1, 2 and 3 would already be sufficient for an additive representation for any continuous weak order, provided there were at least 3 essential coordinates[22]. Note that an essential subset of coordinates is one for which the weak order yields not indifference for all of its elements.

DEFINITION 1

Suppose $(\Omega, \succeq)$ is an intertemporal choice problem. For a subset $A \subset I$ and a subset $\mathrm{P} \subset \Omega$ we define that A is essential on P if and only if

$x_{-A}v_A \succ x_{-A}w_A$

for some $x_{-A}v_A$ and $x_{-A}w_A$ in P

---

20 A simple model, were utility in one year only depends on utility in the year before has, unfortunately, proven not sufficient to fully explain observed violations of independence; see Loewenstein & Prelec (1993).

21 Note that a preference for an increasing stream *per se* can be captured by additive utility, provided that one assigns negative time preference to the individual (i.e., that he prefers late consumption over soon consumption); in the previous example, nevertheless, the individual revealed positive time preference with the choice of B over A, which means that his preference for an increasing sequence are not due to negative time preference.

22 This result is to be found in Debreu (1960) and Wakker (1989, pp.49). I will comment later on the rather surprising case of only two essential coordinates, which needs one more axiom to guarantee the existence of an additive representation.

In other words, it means that the periods in A are relevant for the ordering of elements in P. I will refer to an essential *coordinate* whenever A is a singleton.

From now on I will write $(\Omega, \succeq, IES)$ to refer to an intertemporal choice problem in which the preference relation $\succeq$ satisfies the independence of equal subalternatives (axiom 3). The classical result by Debreu (1960) says that given such an $(\Omega, \succeq, IES)$ in which $\succeq$ is continuous, there exists an additive representation provided that there are at least 3 essential periods, and that we assume certain topological properties on Ω.

For an approach without topological properties on Ω, I present next the axioms restricted solvability and the Archimedean axiom, following Krantz *et. al* (1971), pp.301-303. A solvability condition was already present in Fisher (1927), but the first use in an algebraic version of additive representation is in Luce (1966).

AXIOM 4 (*restricted solvability*)

A binary relation $\succeq$ on Ω satisfies *restricted solvability* if, for all $i \in I = \{0,1,...,n\}$, whenever there exist $x_i, \tilde{y}_i, \underset{\sim}{y}_i \in X_i$ and $v_{-i}, w_{-i} \in \prod_{j \in I \setminus \{i\}} X_j$ for which

$\tilde{y}_i w_{-i} \succeq x_i v_{-i} \succeq \underset{\sim}{y}_i w_{-i}$

then there also exists $x_i' \in X_i$ such that $x_i' w_{-i} \sim x_i v_{-i}$.

More visually, if you think of $n+1$-dimensional streams, this property states that for every $i \in I$, whenever

$(w_0, ..., \tilde{y}_i, ..., w_n) \succeq (v_0, ..., x_i, ..., v_n) \succeq (w_0, ..., \underset{\sim}{y}_i, ..., w_n)$

then there exists $x_i' \in X_i$ such that

$(w_0, ..., x_i', ..., w_n) \sim (v_0, ..., x_i, ..., v_n)$

For the case of two-period streams, $I = \{0,1\}$, we can interpret restricted solvability geometrically as follows (see Figure 1.1). The indifference curves represent the sets of indifferent streams.

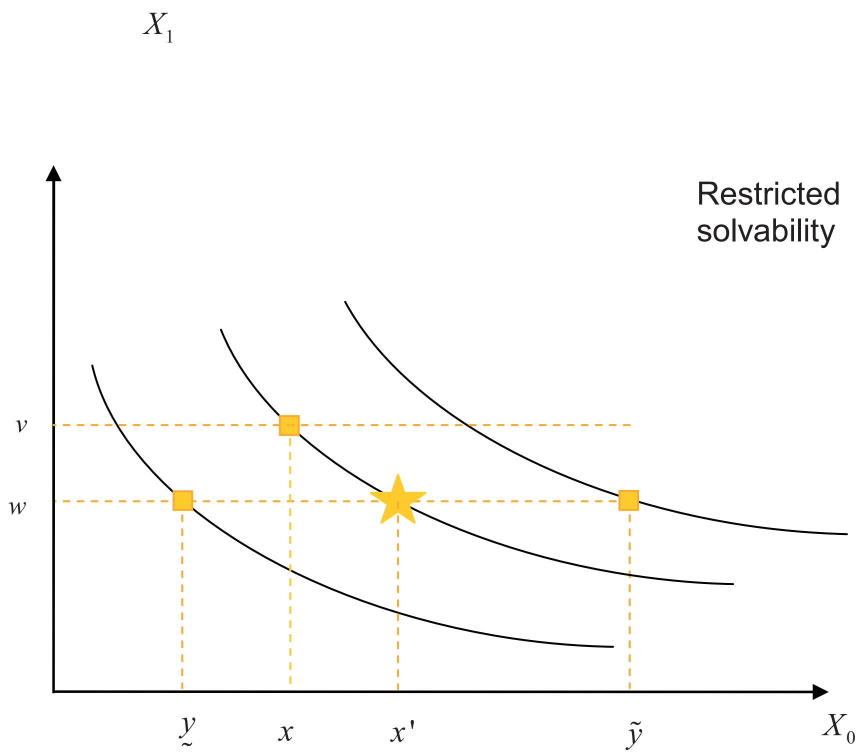

Figure 1.1

One way to interpret this is that, given the inequalities such as, for example, $\tilde{y}w \succeq xv$ and $xv \succeq \underset{\sim}{y}w$, we can always solve them by finding the appropriate element x' in X_0 that yields $x'w \sim xv$, this is, that solves the inequalities. Or, put in other words –and for the (n+1)-periods case-, it excludes the possibility of measuring holes in the sets X_i of our Cartesian product structure in the following sense: whenever you find a weak preference like $\tilde{y}_i w_{-i} \succeq x_i v_{-i} \succeq \underset{\sim}{y}_i w_{-i}$, then you have to be able to find a $x_i' \in X_i$ so that you can solve the equation into $x_i' w_{-i} \sim x_i v_{-i}$. The absence of such a x_i' would mean there are elements missing in X_i that prevent us from measuring the value of the difference expressed in the above weak preference in terms of the coordinate i.[23]

[23] More generally, restricted solvability is a structural axiom that asserts that solutions exist to certain classes of equations or inequalities. For example, imagine you want to measure the lengths of two different shoes. It is clear that, in order to measure this difference, we need a sufficiently enough dense measuring rod so that we can measure their difference precisely. Put in other words, imagine shoe a is longer than shoe b, and we write $a \succ b$; Imagine also I have a measuring rod made of equally spaced points. Then,

How restricted solvability is related to continuity

The topological assumption assumed by the approaches to additive representation in Debreu (1960) or Wakker (1989) imposes that every X_i is a connected topological space[24]. Now, put together, the continuity of a weak order and the connectedness[25] of a topological space imply restricted solvability (see Lemma III.3.3 in Wakker 1989, p.44). And this is one crucial connection between the so-called topological approach to additive representations and the algebraic approach. I have chosen to follow the algebraic approach because restricted solvability is a more general assumption. To see why, I present next an example of a continuous weak order defined over a two-period space $X_0 \times X_1$, where both X_0 and X_1 are not connected, and still $\succeq$ satisfies restricted solvability.

Define possible consumption in each period as a subset of the natural numbers as follows:

$X_0 = X_1 = \mathbb{N} \setminus \{0\}$

Note that these sets are not connected, since for both sets it is possible to find two disjoint subsets $\mathrm{O}_1 = \{1, 2, 3\}$ and $\mathrm{O}_2 = \{4, 5, ...\}$ that are closed, and such that $X_0 = \mathrm{O}_1 \cup \mathrm{O}_2$ and $X_1 = \mathrm{O}_1 \cup \mathrm{O}_2$.

Now define $\succeq$ on $X_0 \times X_1$ by:

$$(x, y) \succeq (x', y') \Leftrightarrow x + y \geq x' + y'$$

restricted solvability means that I will always find a c belonging to this measuring rod such that $a \sim b \circ c$. Restricted solvability says that I can make the measuring rod as dense as I *need* to solve my equations.

24 The topological assumption in fact assumes also *separability* -to deal with the trivial case of only one essential coordinate; see Wakker 1989, p.43- and the Cartesian product $\prod_{i \in I} X_i$ to be endowed with the product topology.

25 A binary relation $\succeq$ on Ω is continuous if for any $x \in \Omega$ the sets $\{y \in \Omega / x \succeq y\}$ and $\{y' \in \Omega / y' \succeq x\}$ are closed sets. On the other hand, a topological space X is connected if it cannot be partitioned into two disjoint closed sets O_1 and O_2 such that $X = O_1 \cup O_2$. An example of such a set is an interval of the real numbers.

It is clear that $\succeq$ is a continuous weak order: completeness and transitivity hold, and continuity also holds, since for all $(x', y') \in X_0 \times X_1$, the sets

$\{(x'', y'') \in X_0 \times X_1 : (x'', y'') \succeq (x', y')\}$ and $\{(x, y) \in X_0 \times X_1 : (x', y') \succeq (x, y)\}$

are closed sets.

And see that restricted solvability holds also. Suppose you have:

$$(n_1, m) \preceq (n', m') \preceq (n_2, m)$$

For restricted solvability to hold it is needed that we can guarantee there exists $x \in X_0$ such that

$(x, m) \sim (n', m')$ or, equivalently, $x + m = n' + m'$

The question reduces now to see whether there will always exist $x = n' + m' - m$ that is positive. But from the first inequality we know that $n' + m' \geq n_1 + m$, which implies $x > 0$.

As this example shows, $\succeq$ is a continuous weak order that satisfies restricted solvability, while being defined over a Cartesian product whose X_i are *not* connected.

It is also possible to find an example of a weak order that is *not* continuous but does satisfy restricted solvability. Define possible consumption in period i as the set $X_i = \mathbb{R}^+$, and consider the space of two-period streams, $X_0 \times X_1$. Define the following function:

$$\tilde{u}(x, y) = \begin{cases} -1 & \text{if } (x, y) = (0, 0) \\ xy & \text{if } (x, y) \neq (0, 0) \end{cases}$$

Now define a binary relation $\succeq$ as follows:

$$(x, y) \succeq (x', y') \Leftrightarrow \tilde{u}(x, y) \geq \tilde{u}(x', y')$$

The idea of such a preference relation is to modify a well-known preference relation defined by the product of the components, by making all streams in the axes indifferent among themselves, except stream $(0,0)$. This weak order is complete and transitive, but not continuous. To see why, define the set W of all streams weakly preferred to $(0,1)$. This set will consist of all streams but not $(0,0)$, which belongs to the boundary of W. This means that W is not closed, and thus, that $\succeq$ is not continuous.

This preference relation nevertheless satisfies restricted solvability. Due to the symmetry of $\succeq$ we only need to prove it for one period. Take $x, \tilde{y}, \underset{\sim}{y} \in X_0$ and $v, w \in X_1$ for which $\tilde{y}w \succeq xv \succeq \underset{\sim}{y}w$. I will show there exists $x' \in X_0$ such that $xv \sim x'w$ for each of three possible situations:

(a) Either $x = 0$ or $v = 0$ (but not both)

Then, $\tilde{u}(x,v) = 0$ by definition. In consequence,

1. choose $x' = 0$ in case $w \neq 0$ and you get $\tilde{u}(x',w) = 0$
2. choose any $x' \in X_0$ in case $w = 0$ and you get $\tilde{u}(x',w) = 0$

(b) Both $x = 0$ and $v = 0$

Then, $\tilde{u}(x,v) = -1$ and because $(x,v) \succeq (\underset{\sim}{y}, w)$, we know also $(\underset{\sim}{y}, w) = (0,0)$. In consequence, choose $x' = 0$ and you get $\tilde{u}(x',w) = \tilde{u}(x,v) = -1$.

(c) Both $x \neq 0$ and $v \neq 0$

Then, because $xv \succeq \underset{\sim}{y}w$, necessarily $w \neq 0$. Thus, choose $x' = \frac{xv}{w}$ and you get $\tilde{u}(x',w) = \tilde{u}(x,v) = xv$.

□

Restricted solvability is, nevertheless, a sufficient but not a necessary axiom of additive representations (Krantz *et al* 1971, p.23). In contrast, the last axiom we need to ensure an additive representation of $\succeq$ seems to be a necessary axiom: the Archimedean axiom[26]. It is called Archimedean because it corresponds to the Archimedean property of the real numbers that states that, for any positive number x (no matter how small), and for any number y (no matter how large), there exists an integer n such that $nx \geq y$. The meaning of the Archimedean property is that any two positive numbers are comparable, this is, that their ratio is not infinite. In the context of our measurement problem, the Archimedean property makes sure that it is always possible to compare differences in amounts of different periods, i.e., that consumption in one period is never infinitely better than consumption in another one. In order to present more formally the Archimedean axiom in our intertemporal choice problem, I need the following two definitions.

DEFINITION 2 (*induced relations* $\succeq_i$)

Given $(\Omega, \succeq, IES)$, let $A \subset I$ and $\mathrm{P} = \prod_{i \in A} X_i \subset \Omega$. I will call $\succeq_A$ an induced order and define it by:

for $x_A', x_A \in \mathrm{P}$,

$x_A' \succeq_A x_A$ if and only if for some $y_{-A} \in \prod_{j \notin A} X_j$, $x_A' y_{-A} \succeq x_A y_{-A}$.

LEMMA 1 ($\succeq_A$ are weak orders)

For all $A \subset I$, the binary relation $\succeq_A$ of Definition 2 is a weak order.

Proof: first suppose $\succeq_A$ is not complete; then, there exist $x_A, x_A' \in \mathrm{P}$ such that both $x_A' \not\succeq_A x_A$ and $x_A \not\succeq_A x_A'$. This means, consequently, that there is no y_{-A} such that $x_A' y_{-A} \succeq x_A y_{-A}$ or $x_A y_{-A} \succeq x_A' y_{-A}$, which contradicts the completeness of $\succeq$.

Second, suppose there exist $x_A, x_A', x_A'' \in \mathrm{P}$ such that $x_A'' \succeq_A x_A'$ and $x_A' \succeq_A x_A$. This would mean there exist $y_{-A}, y_{-A}' \in \prod_{j \in I \setminus A} X_j$ such that:

$x_A'' y_{-A} \succeq x_A' y_{-A}$

$x_A' y_{-A}' \succeq x_A y_{-A}'$,which by IES also implies $x_A' y_{-A} \succeq x_A y_{-A}$.

26 For a discussion on necessary, sufficient and independent axioms of additive representations, see Krantz *et. al* (1971), p. 21-25.

Then, by the transitivity of $\succeq$,

$x_A\,"y_{-A} \succeq x_A\,'y_{-A}$ and $x_A\,'y_{-A} \succeq x_A y_{-A}$ implies $x_A\,"y_{-A} \succeq x_A y_{-A}$, which implies that $x_A\," \succeq_A x_A$.

As we can see, then, the independent weak order $\succeq$ over the whole Cartesian structure of consumption streams induces weak orders $\succeq_A$ for consumption in a subset of periods A.

□

The second definition I need before I can present the Archimedean axiom is that of standard sequence. The idea underlying standard sequences was also already used by Fisher (1927), but the formal use for an algebraic approach to additive representations I make here is due to Krantz (1971).

DEFINITION 3 (*standard sequence*)
Given $(\Omega, \succeq, IES)$. For any set S of consecutive integers (positive or negative, finite or infinite), a set $\{x_i^s \,/\, x_i^s \in X_i, s \in S\}$ is a standard sequence on period i if and only if there exist $y_{-i}, z_{-i} \in \prod_{j \in I \setminus \{i\}} X_j$ such that not $y_{-i} \sim_{-i} z_{-i}$, and for all $s, s+1 \in S$, $x_i^s y_{-i} \sim x_i^{s+1} z_{-i}$.

The notion of standard sequence is of crucial importance for the understanding of additive representations, and thus, also for the understanding of an additive intertemporal choice model as Discounted Utility. Let me therefore present the construction of a standard sequence in a two-period setting (see Figure 1.2).

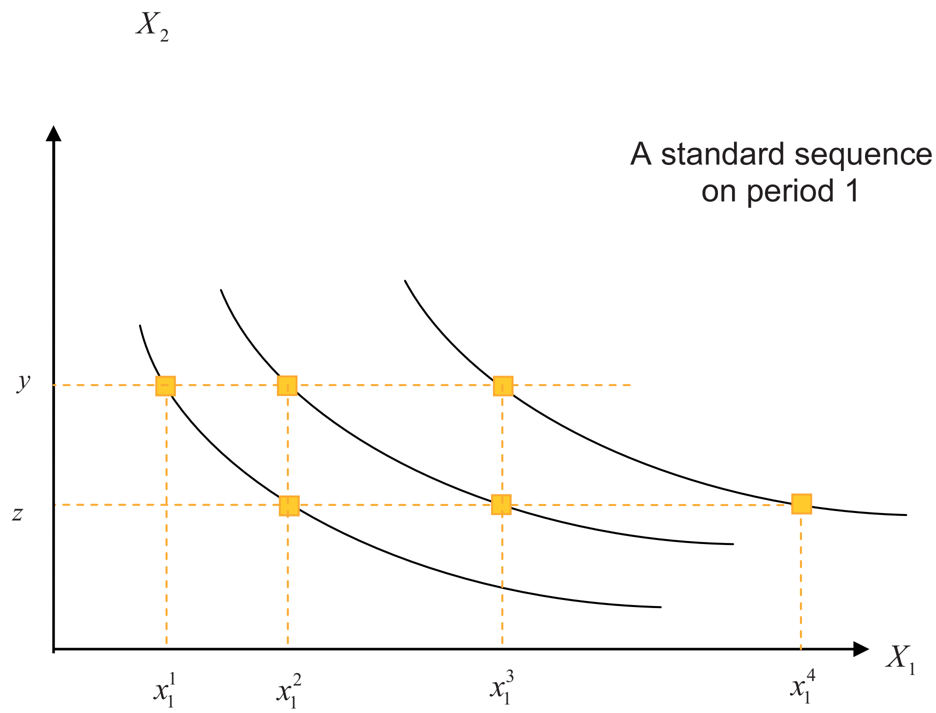

Figure 1.2

Given a weak order $\succeq$ represented by the indifference curves in the example of Figure 1.2, elements x_1^1, x_1^2, x_1^3 and x_1^4 form a standard sequence on period one. The way to construct such a standard sequence is thus clear: start at an arbitrary point $x_1^1 y$ and descend along the indifference curve that goes through this point until an arbitrary $x_1^2 z$. Then, go up vertically until you reach the height y and meet the indifference curve that goes through $x_1^2 z$. Descend along this indifference curve until you reach back the height z at point $x_1^3 z$. You may continue this process to construct a standard sequence as long as you want and can (depending on the nature of the set of consequences in period 1 and on the nature of the preference $\succeq$).

The notion of a standard sequence is therefore crucial to additive measurement: Suppose we had an additive representation of the preference relation $\succeq$, so there existed functions u_1, u_2 into the real numbers such that

$$(x', y') \succeq (x, y) \Leftrightarrow u_1(x') + u_2(y') \geq u_1(x) + u_2(y).$$

Then, what we are doing here is establishing that the change in value of moving from consuming x_1^1 to consuming x_1^2 is equivalent both to that of moving from x_1^2 to x_1^3 and to that of moving from x_1^3 to x_1^4, since all three 'intervals' value the same in terms of elements in period 2, namely the value we give to moving from z to y. We thus say that x_1^1, x_1^2, x_1^3 and x_1^4 are equally spaced in period one. The term equally spaced is therefore sometimes used in the literature instead of that of standard sequence.

The difference in value of moving from z to y acts thus as a 'measuring rod' establishing the value of the difference among consuming any two subsequent elements in the standard sequence. For example, we can establish that

$$u_1(x_1^1)+u_2(y)=u_1(x_1^2)+u_2(z)$$
$$u_1(x_1^2)+u_2(y)=u_1(x_1^3)+u_2(z)$$
$$u_1(x_1^3)+u_2(y)=u_1(x_1^4)+u_2(z)$$

So that in fact, for example, we could say that $u_1(x_1^4)-u_1(x_1^1)=3(u_2(y)-u_2(z))$, which means that changing consumption from x_1^1 to x_1^4 equals three times the change in consumption from z to y.

DEFINITION 4 (*strictly bounded standard sequence*)

Given $(\Omega,\succeq,IES)$. A set $B=\{x_i^1,x_i^2,...,x_i^s,...\}$ is a strictly bounded standard sequence on period i if it is a standard sequence and there exists $\overline{x}_i \in X_i$ such that $\overline{x}_i > x_i^s$ for all $x_i^s \in B$.

Now, if an additive representation exists, then if a standard sequence is infinite, it cannot be strictly bounded. The reason is that if it could be strictly bounded, then the value of moving consumption from any of its elements to the above bound would not exist (since it would be infinite times the measuring rod).

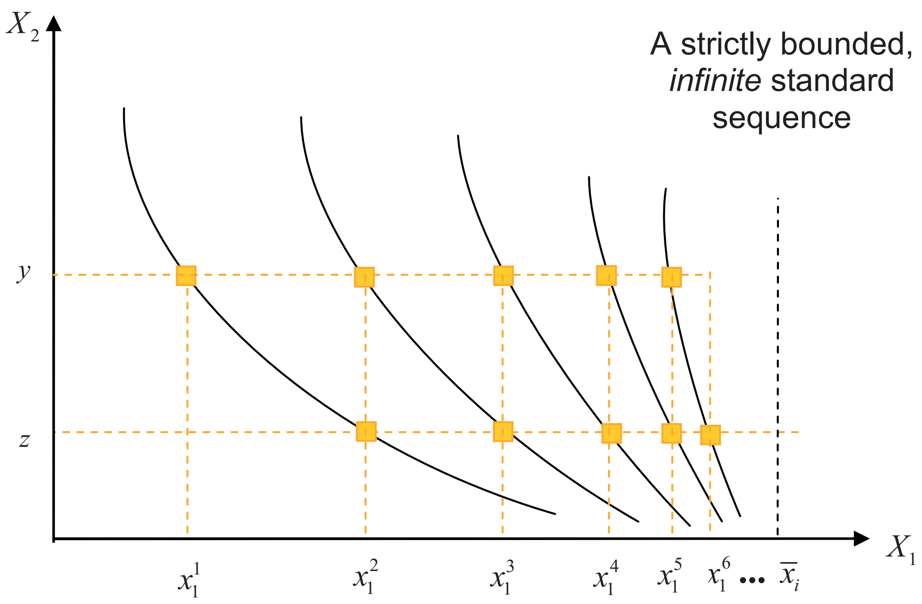

Figure 1.3

This principle states that consumption in one period cannot become infinitely more relevant than consumption in another period (see Figure 1.3). Consumption in one period shall always be 'comparable' to consumption in a different period.

The condition I just discussed corresponds to the so-called Archimedean property of the real numbers: for any positive number x, no matter how small, and for any number y, no matter how large, there exists an integer n such that $nx \geq y$. This is equivalent to saying that any two positive numbers are comparable, i.e. their ratio is not infinite. Another way to express this property is the following: the set of integers n for which $y > nx$, is finite.

Now, since this property is true for the real numbers, it has to be also true in our preference relation, since we are seeking a representation of preferences into the real numbers. The following axiom is thus a necessary axiom for additive representations.

AXIOM 5 (*Archimedean axiom*)

Every strictly bounded standard sequence is finite.

We are now ready to formulate an additive representation theorem. Note that, since $I = \{0,1,...,n\}$, then $n \geq 2$ means three or more periods.

THEOREM 1 (Additive Representation Theorem; Krantz, Luce, Suppes & *Tversky, 1971)*
Suppose there is $(\Omega, \succeq, IES)$ with $n \geq 2$, where $\succeq$ satisfies restricted solvability and the Archimedean axiom, and suppose also at least three periods are essential. Then there exist real-valued functions u_i on X_i, $i \in I$, such that for all $x = (x_0, x_1, ..., x_n)$ and $y = (y_0, y_1, ..., y_n)$ belonging to Ω,

$$(x_0, x_1, ..., x_n) \succeq (y_0, y_1, ..., y_n) \Leftrightarrow \sum_{i \in I} u_i(x_i) \geq \sum_{i \in I} u_i(y_i)$$

If $\{u_i'\}$ is another such family of functions, then there exist numbers $\alpha > 0$ and β_i, with $i = 0,1,...,n$, such that

$$u_i' = \alpha u_i + \beta_i$$

The first version of a theorem of additive representation was proven by Debreu (1960), although, as I have mentioned before, Debreu's approach uses topological assumptions on the sets X_i. I have preferred the more general algebraic approach by Krantz, et. al (1971) (see pp. 307-309 for their proof), which replaces those assumptions with restricted solvability and the Archimedean axiom. Another (topological) version of this theorem is proposed by Wakker (1989), who presents a more intuitive proof (Wakker, 1989, pp. 49).

<u>The case of only two essential periods</u>

The additive representation theorem refers to the necessity of having at least three essential periods, something it is easy to assume in the context of intertemporal choice problems. Nevertheless, it is interesting to note that, surprisingly, additive representations need one more axiom in the case of only two essential coordinates[27]: this axiom is the so-

[27] Note that the case of only one essential period is of no interest, since it would have a trivial additive representation by assigning zero utility to any consumption in the non-essential periods.

called Thomsen condition. Let me briefly explain what the Thomsen condition is, and why it is needed in the case of only two essential periods.
Suppose you had an additive representation over $\succeq$ on $X_0 \times X_1$. Then, it would be true that

$$(x,v) \succeq (y,w) \Leftrightarrow u_0(x)+u_1(v) \geq u_0(y)+u_1(w)$$
$$(y,s) \succeq (z,v) \Leftrightarrow u_0(y)+u_1(s) \geq u_0(z)+u_1(v)$$

Adding the two inequalities, we get

$$u_0(x)+u_1(v)+u_0(y)+u_1(s) \geq u_0(y)+u_1(w)+u_0(z)+u_1(v)$$

Substracting $u_0(y)$ and $u_1(v)$ to both sides, this yields

$u_0(x)+u_1(s) \geq u_1(w)+u_0(z)$, which by definition is equivalent to $(x,s) \succeq (z,w)$.

As we have seen, thus, a *necessary* condition for additivity is that, whenever $(x,v) \succeq (y,w)$ and $(y,s) \succeq (z,v)$, then $(x,s) \succeq (z,w)$. This property is called double cancellation (also called the Thomsen condition[28] when you replace $\succeq$ with $\sim$).[29] Now when $n \geq 2$ (three or more periods), then IES implies the Thomsen condition, and the representation theorem can therefore dispense with it. But for the case of only two periods, this is not the case. Next I develop an example where a weak order $\succeq$ over $X_0 \times X_1$ satisfies IES but not the Thomsen condition:

Define $\succeq$ over $X_0 \times X_1$, where $X_0 = X_1 = \mathbb{R}^+$, by:
$$(x,v) \succeq (y,w) \Leftrightarrow x+v+\min\{x,v\} \geq y+w+\min\{y,w\}$$

This preference relation does not satisfy the Thomsen condition, and can therefore not be represented by an additive utility function. An example that contradicts the Thomsen condition follows:

[28] See, for example, Wakker (1989), page 67; or Krantz et al. (1971), page 251.
[29] Note that another necessary axiom for additivity is independence (IES), which, in fact, is sometimes called single cancellation.

$$(6,6) \sim (18,0)$$
$$(18,18) \sim (42,6)$$

but

$$(6,18) \nsim (42,0)$$

Graphically, this example looks as follows (see Figure 1.4):

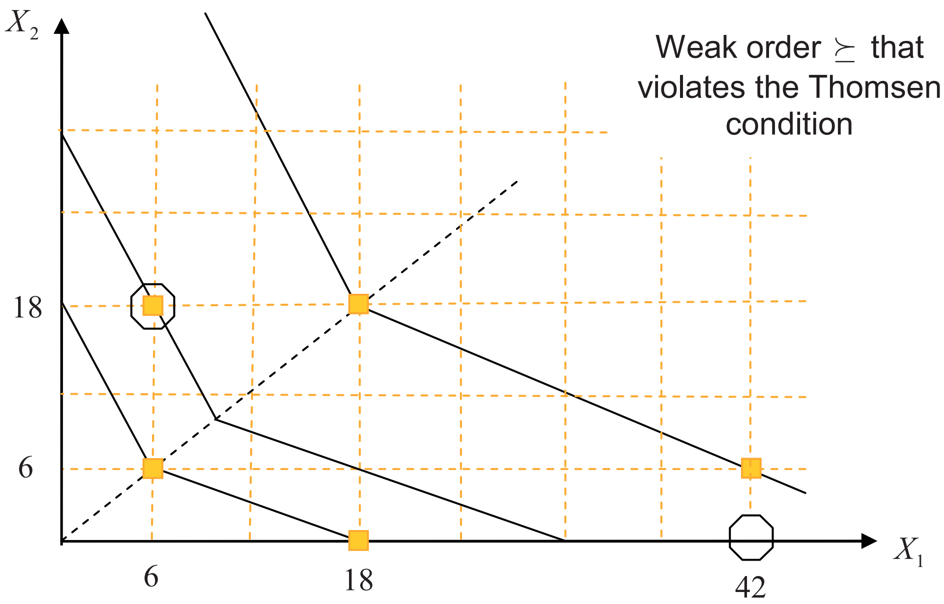

Figure 1.4

The intuition behind this condition is that if you traded off 12 units of the second period against 24 units of the first period (by declaring indifference among (18,18) and (42,6)); and you traded off 6 units of the second period against 12 units of the first (by declaring indifference among (6,6) and (18,0)), then these trade offs should also 'sum', i.e., you should also be ready to trade off 12+6 units of period two against 24+12 of period one and declare indifference among (6,18) and (42,0). If this is not the case (as in the example, as incicated by the circled points) then, clearly, an additive representation will fail to exist.

Note that this preference relation is a weak order (easy to see), and satisfies the independence of equal subalternatives (IES). To see why, suppose it would not satisfy

IES. This would mean there exist $x, y \in X_0$ and $v, w \in X_1$ such that $(x,v) \succeq (y,v)$ but $(y,w) \succ (x,w)$, which is impossible because, following the definition of $\succeq$, $(x,v) \succeq (y,v)$ would imply $x \geq y$, while $(y,w) \succ (x,w)$ would imply $y > x$.

IES is a stronger condition for three or more periods than it is for two, which is why IES is sufficient for a weak order (under restricted solvability and the Archimedean axiom) to guarantee an additive representation whenever there are three or more essential periods: the Thomsen condition is, in the case of three or more essential periods, already implied by axioms 1-5. In fact, it is interesting to note that if we translated the previous example into an analogous one with three periods, where preferences were defined by

$(x,v,s) \succeq '(y,w,t) \Leftrightarrow x+v+s+\min\{x,v,s\} \geq y+w+t+\min\{y,w,t\}$,

then it would not anymore satisfy IES, as can be seen by the following example:

$(5,5,3) \sim '(6,4,3)$

$(5,5,5) \nsim '(6,4,5)$

This may give an intuition of the fact that IES is a much stronger condition for three or more periods than it is for two.

**

Let me now take up again the reconstruction of the axiom system. Axioms 1 to 5 imply the existence of an additive representation of the intertemporal preferences $\succeq$ over Ω. A natural question to ask now is whether, if periods i and j share the same set of possible consequences ($X_i = X_j$), then their utility functions are equal ($u_i = u_j$); or, more generally, whether they are proportional to each other ($u_i = \delta_i \cdot u$). The conditions for equal utility functions are, obviously, strong: it is necessary that a property called 'permutability' holds; for example, with streams of consequences over only two periods, this would mean that, whenever $xv \succeq yw$, then also $vx \succeq wy$ (since this is obviously a necessary condition for $u_1 = u_2$). Permutability can also be easily shown to be sufficient for equal utility functions.

On the other hand, the necessary and sufficient condition for all utility functions u_i to be *proportional* to u is especially interesting for us, since Discounted Utility is based crucially on it. This condition is that *standard sequences be invariant across periods*. This is, if the set $\{x_i^1, x_i^2, ..., x_i^n\}$ of elements in X_i is a standard sequence in period i, then it has to be even so a standard sequence in period j. In fact, it suffices to state this property with three-term standard sequences as follows: if a_i, b_i, c_i is a standard sequence on period i, and $a_j = a_i, b_j = b_i, c_j = c_i$, then a_j, b_j, c_j is also a standard sequence in period j.[30]

AXIOM 6 (*invariance of standard sequences*)
Given $(\Omega, \succeq, IES)$, with $I = \{0, ..., n\}$ and $X_0 = X_1 = ... = X_n = X$. The preference relation $\succeq$ satisfies the invariance of standard sequences if, whenever $\{a_i, b_i, c_i\}$ is a standard sequence in period i, with $a_i, b_i, c_i \in X$, then, for any $j \in I$, $\{a_j, b_j, c_j\}$ whith $a_j = a_i, b_j = b_i, c_j = c_i$ is also a standard sequence in period j.

Graphically we can get a better intuition of what this property means. The following Figure (Figure 1.5) shows a standard sequence $\{a_i, b_i, c_i\}$ in period i. Note that intervals between consecutive elements of this standard sequence are all equivalent to the interval $[s, t]$ in period i'.

[30] The reason why stating this property only for three-term standard sequences is sufficient is that we can build any arbitrarily longer standard sequence by an overlapping concatenation of three-term standard sequences as follows: if $\{x_i^1, x_i^2, x_i^3\}, \{x_i^2, x_i^3, x_i^4\}, ..., \{x_i^{n-2}, x_i^{n-1}, x_i^n\}$ are standard sequences in period i, then there exist by definition $n-2$ measuring rods $v, w; v', w'; v'', w''; ... \in \prod_{j \neq i} X_j$. Thus, if an additive representation exists, then by definition this implies $u_{-i}(v) - u_{-i}(w) = u_{-i}(v') - u_{-i}(w') = u_{-i}(v'') - u_{-i}(w'')...$, which means there exists a common measuring rod for all elements, and thus, $\{x_i^1, x_i^2, ..., x_i^n\}$ is also a standard sequence in period i.

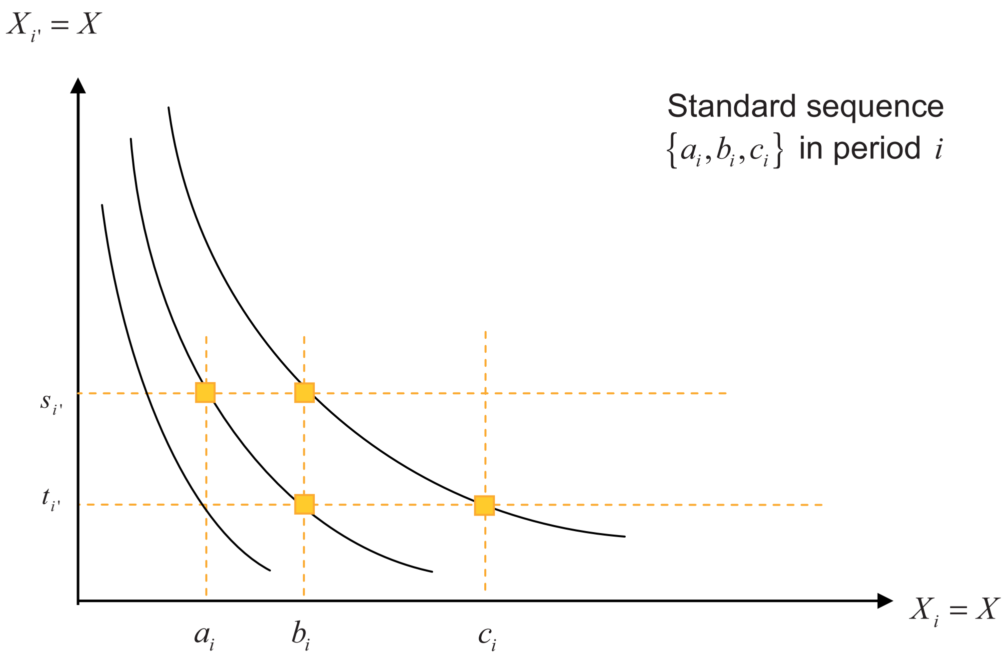

Figure 1.5

Now imagine you take this standard sequence and put it in a different period j by making $a_j = a_i, b_j = b_i, c_j = c_i$. The question is whether there will exist p_k and q_k in another period $k \neq j$ such that $a_j p_k \sim b_j q_k \Rightarrow b_j p_k \sim c_j q_k$. If the answer is yes then this would mean that $\{a_i, b_i, c_i\}$ is also a standard sequence in period j. Of course, p_k and q_k need not be the same as $s_{i'}$ and $t_{i'}$. The important thing is that $\{a_i, b_i, c_i\}$ keeps being a standard sequence, for whatever 'measuring rod' in period k. The idea behind this property is that preferences over *differences* among elements in a certain period *are maintained* across periods: in period i the individual values equally going down from c_i to b_i than going down from b_i to a_i; so in period j he should even so (see Figure 1.6).

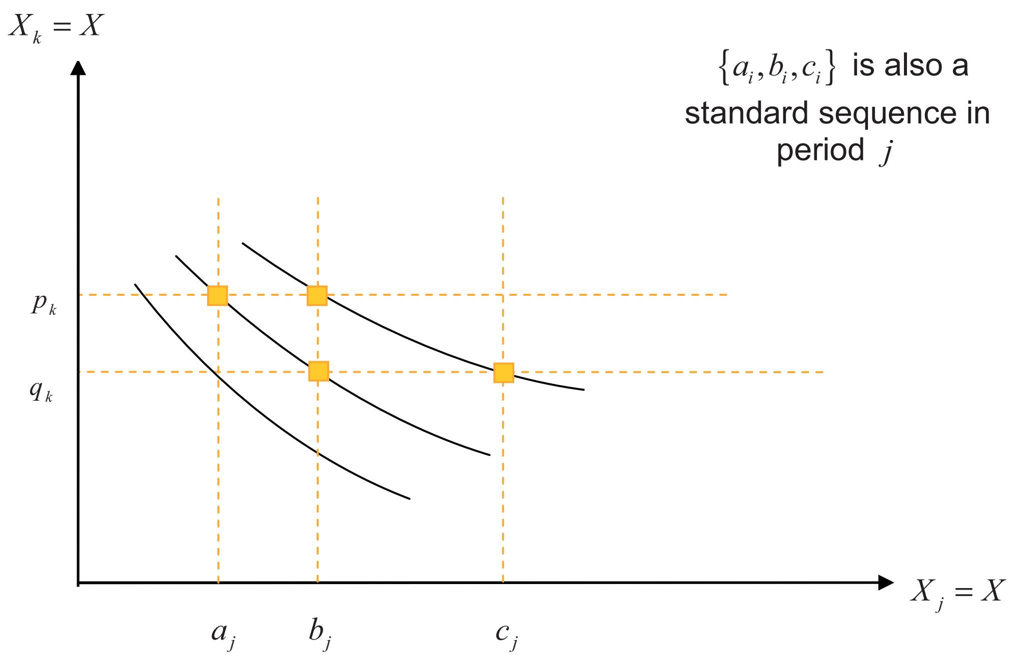

Figure 1.6

In general, differences in consumption need to be proportional across periods. An example may illustrate this principle. Suppose someone, in period 1, values going down from €100 to €90 double than going down from €30 to €20 (for example, suppose $u_1(100)-u_1(90)=100-90=10$ and $u_1(30)-u_1(25)=30-25=5$). Then, the invariance-of-standard-sequence property demands that in another period, say period 2, going down from €100 to €90 is also seen as double to going down from €30 to €20, even if possibly, in this new period utility for money is different, say, $u_2(100)-u_2(90)=80-72=8$ and $u_2(30)-u_2(25)=24-20=4$. Of course, when u_1 and u_2 are proportional, then $u_2(x)=\delta u_1(x)$, and $u_2(x)-u_2(y)=\delta(u_1(x)-u_1(y))$, which implies that standard sequences are invariant across the two periods.

We can now establish the next important result:

THEOREM 2 (Additive Representation with Weights)

Given $(\Omega, \succeq, IES)$, where $X_0 = X_1 = ... = X_n = X$ and $n \geq 2$ (three or more periods), suppose that $\succeq$ satisfies restricted solvability and the Archimedean axiom, and suppose also at least three periods are essential. There exist a real-value function u on X and nonzero numbers $\delta_0, \delta_1, ..., \delta_n$, such that for all $x = (x_0, x_1, ..., x_n)$ and $y = (y_0, y_1, ..., y_n)$ in $\Omega = X^{n+1}$,

$$(x_0, x_1, ..., x_n) \succeq (y_0, y_1, ..., y_n) \Leftrightarrow \sum_{i \in I} \delta_i \cdot u(x_i) \geq \sum_{i \in I} \delta_i \cdot u(y_i)$$

<u>if and only if</u>,
the preference relation $\succeq$ satisfies the invariance of standard sequences axiom.

Moreover, the scalars δ_i, $i = 0, ..., n$ are unique up to a multiplication by a positive constant.

The proof of this result is to be found, for example, in Krantz et al. (1971), page 310. Let me sketch the 'sufficiency' case, which states that, under axioms 1-5, the invariance-of-standard-sequences property implies the existence of an additive utility function with weights:
By hypothesis, there exists a representation of the form

$$\sum_{i=1}^{n} u_i(x_i)$$

Now consider a standard sequence in period zero made of $\{b^0, b^1, ...\}$, with measuring rod in another period i such that $p_i b^r \sim q_i b^{r+1}$, and let, without loss of generality, $u_j(b^0) = 0$ for all $j \in I$. Let also $u_0(b^1) = 1$ and for all $j \neq 1$, let $u_j(b^1) = \delta_j$. Now, for every $b \succ_0 b^1$, extend the previous standard sequence $\{b^0, b^1, ...\}$ until b (so that the distance of the last term in the sequence from b is less than the mesh of the sequence; also, let $\tilde{n}$ designate the number of steps through the standard sequence that you have needed to reach this point, and $\tilde{m}$ designate the number of steps you would need to reach b^1). We can then, with the help of this standard sequence, find the following approximations:

$u_0(b) \approx \tilde{n}\left[u_i(p_i) - u_i(q_i)\right]$, and also

$$1 = u_0\left(b^1\right) \approx \tilde{m}\left[u_i\left(p_i\right) - u_i\left(q_i\right)\right]$$

Thus,

$$u_0\left(b\right) \approx \tilde{n} \cdot \frac{u_0\left(b^1\right)}{\tilde{m}} = \frac{\tilde{n}}{\tilde{m}}$$

But now –and this is the key point-, since the previous standard sequence is also a standard sequence in any other period, we can find the *same* approximations in period j. It is interesting to note that, obviously, to reach element b in this new period requires the same number of 'steps' in the standard sequence as before, because we are in fact using the same standard sequence:

$u_j\left(b\right) \approx \tilde{n}\left[u_k\left(r_k\right) - u_k\left(s_k\right)\right]$, and also

$$\delta_j = u_j\left(b^1\right) \approx \tilde{m}\left[u_k\left(r_k\right) - u_k\left(s_k\right)\right]$$

Now,

$$u_j\left(b\right) \approx \tilde{n} \cdot \frac{u_j\left(b^1\right)}{\tilde{m}} = \delta_j \frac{\tilde{n}}{\tilde{m}}$$

and thus, we arrive at

$$u_j\left(b\right) \approx \delta_j \cdot u_0\left(b\right)$$

which, in the limit, and stating $u_0 = u$, becomes $u_j\left(b\right) = \delta_j \cdot u\left(b\right)$

**

The next step in our axiomatic derivation of Discounted Utility is to study under what circumstances $\delta_i = \delta^i$ for all periods $i \in I$, this is, under what circumstances discounting is exponential as it is in Samuelson's original model. The crucial property is that of *stationarity*. The intuition behind this principle is that only time distance *among* the objects is relevant for the preference order, but not distance *to* the objects of choice. For example, if receiving 10€ today is preferred to receiving 11€ tomorrow, then stationarity implies that receiving 10€ in 364 days has to be preferred to receiving 11€ in 365 days.

Let me state this principle formally:

AXIOM 7 (stationarity)

Given $(\Omega, \succeq, IES)$, where $n \geq 3$, the preference relation $\succeq$ is stationary if and only if there exists $x \in X$ such that, for all $a^0, a^1, ..., a^{n-1}, b^0, b^1, ..., b^{n-1} \in X$,

$$\left(a^0, a^1, ..., a^{n-1}, x\right) \succeq \left(b^0, b^1, ..., b^{n-1}, x\right)$$

if and only if

$$\left(x, a^0, a^1, ..., a^{n-1}\right) \succeq \left(x, b^0, b^1, ..., b^{n-1}\right)$$

Note that this condition is much weaker than the above mentioned 'permutability', since only certain equivalences for special permutations are preserved. But it suffices to imply the invariance-of-standard-sequences property, which means that it ensures the existence of an additive representation with weights (Theorem 2). To see why, imagine a, b, c is a standard sequence (Definition 3) in the first period, with mesh equal to the interval $[q, p]$ in the second period. Then, by definition of standard sequence, we have that $ap \sim bq$ and $bp \sim cq$[31]. Now, for any $a^3 \cdots a^{n-1}x$,

$apa^3 \cdots a^{n-1}x \sim bqa^3 \cdots a^{n-1}x$, and

$bpa^3 \cdots a^{n-1}x \sim cqa^3 \cdots a^{n-1}x$

Then, by stationarity, both

$xapa^3 \cdots a^{n-1} \sim xbqa^3 \cdots a^{n-1}$ and

$xbpa^3 \cdots a^{n-1} \sim xcqa^3 \cdots a^{n-1}$, which means that a, b, c is a standard sequence in the second factor, with mesh equal to $[q, p]$ in the third.

But stationary *also* implies exponential discounting, i.e. $\delta_i = \delta^i$.

THEOREM 3 (Additive Representation with Exponential Discounting)

Suppose that $\succeq$ on X^{n+1}, with $n \geq 2$ (three or more periods), is a binary relation that satisfies axioms 1, 2, 3, 4 and 5, and suppose also at least three periods are essential in the sense of Definition 1. There exist a real-value function u on X and a unique number $\delta > 0$, such that for all $x = (x_0, x_1, ..., x_n)$ and $y = (y_0, y_1, ..., y_n)$ in X^{n+1},

[31] I am using here $\sim$ to mean $\sim_{\{0,1\}}$.

$$\left(x_0, x_1, ..., x_n\right) \succeq \left(y_0, y_1, ..., y_n\right) \Leftrightarrow \sum_{i \in I} \delta^i \cdot u\left(x_i\right) \geq \sum_{i \in I} \delta^i \cdot u\left(y_i\right)$$

<u>if and only if</u>,

the preference relation $\succeq$ satisfies the axiom of stationarity.

I already showed that stationarity implies the invariance-of-standard-sequences property. Now let me sketch the proof of stationarity implying exponential discounting for a three periods case (for a complete proof of this Theorem see Koopmans 1960 (with countably infinite many periods); Fishburn & Rubinstein 1982 (based on single outcomes, not streams); or Strotz 1956 (whose approach is based on dynamic consistency, as I will show in the next section):

Suppose

$a^1 a^2 x \sim b^1 b^2 x$. Then, by definition,

$\delta_1 u\left(a^1\right) + \delta_2 u\left(a^2\right) + \delta_3 u\left(x\right) = \delta_1 u\left(b^1\right) + \delta_2 u\left(b^2\right) + \delta_3 u\left(x\right)$, and thus,

$$\delta_1 u\left(a^1\right) + \delta_2 u\left(a^2\right) = \delta_1 u\left(b^1\right) + \delta_2 u\left(b^2\right)$$

$$u\left(a^1\right) - u\left(b^1\right) = \frac{\delta_2}{\delta_1}\left(u\left(b^2\right) - u\left(a^2\right)\right)$$

$$\frac{u\left(a^1\right) - u\left(b^1\right)}{u\left(b^2\right) - u\left(a^2\right)} = \frac{\delta_2}{\delta_1}$$

But also, because of stationarity,

$x a^1 a^2 \sim x b^1 b^2$ and

$\delta_1 u\left(x\right) + \delta_2 u\left(a^1\right) + \delta_3 u\left(a^2\right) = \delta_1 u\left(x\right) + \delta_2 u\left(b^1\right) + \delta_3 u\left(b^2\right)$, thus,

$\delta_2 u\left(a^1\right) + \delta_3 u\left(a^2\right) = \delta_2 u\left(b^1\right) + \delta_3 u\left(b^2\right)$ and

$$u\left(a^1\right) - u\left(b^1\right) = \frac{\delta_3}{\delta_2}\left(u\left(b^2\right) - u\left(a^2\right)\right)$$

$$\frac{u\left(a^1\right) - u\left(b^1\right)}{u\left(b^2\right) - u\left(a^2\right)} = \frac{\delta_3}{\delta_2},$$

which means that

$$\frac{\delta_2}{\delta_1} = \frac{\delta_3}{\delta_2} = \delta$$

Stationarity thus yields the so-called constant (exponential) discounting model for intertemporal choice, after which people 'discount' the value of an outcome with a constant per-period discount factor. To get a better intuition of why this happens, note the following: just before theorem 3, we stated that stationarity implies the invariance-of-standard-sequences property. Under stationarity, if a standard sequence in period i has mesh $[q, p]$ in period $i+1$, then, as we saw, it is even so a standard sequence in period $i+1$, and it has *the same mesh* $[q, p]$ in period $i+2$ (see the argument above). In other words, if 10€ in period i are equivalent to 11€ in $i+1$, then 10€ in $i+1$ are also equivalent to 11€ in $i+2$.

The property of stationarity has been regarded as a *dynamic consistency* property in much literature, because, if people are assumed to have stationary preferences, and provided these preferences are maintained across periods[32], then stationarity guarantees that any consumption plan the individual chooses, he will stick to it.

For example, suppose that $(0,0,0,0,11) \succeq (0,0,0,10,0)$ for a binary relation $\succeq$ on X^5, meaning the individual prefers receiving 11€ in 5 days rather than receiving 10€ in 4 days. Imagine that, contradicting stationarity, he also has the preference $(10,0,0,0,0) \succ (0,11,0,0,0)$, possibly because of impulsiveness. Clearly, this will prevent the individual from behaving consistently *if he maintains this preference structure over the next 4 days*. Only in that case it could be the case that he today *planned* to wait until day 5 to get 11€, but once in day 4, he would *reconsider* this plan and choose to get 10€ immediately. Stationarity is thus said to be a dynamic consistency condition only under the assumption of invariant preferences over time.

Theorem 3 completes our reconstruction of the axiom system underlying Discounted Utility. Nevertheless, and because it was such an important result –and had such an

32 If the individual changes his preferences from one period to another, of course nothing prevents him from abandoning previous plans, even if the mathematical structure of these preferences is of the exponential kind. For an interesting discussion on the important role of this invariance-of-preferences (hidden) assumption in considering stationarity as dynamic consistency, see Ahlbrecht & Weber (1995).

impact on economic theory-, let me next present a discrete-time version of the mathematical argument by the pioneer work founding Samuelson's exponential discounting model on the condition of dynamic consistency.

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1.4.2 Dynamic Consistency and Exponential Discounting

In effect, the very first to show that dynamic consistency forces $\delta_i = \delta^i$, for all $i \in I$, was Robert Strotz in the seminal paper "Myopia and Inconsistency in Dynamic Utility Maximization" (Strotz 1956). As can be guessed, his main objective was to study time preference from a dynamic perspective; this is, to analyse the effect of individuals continuously maximizing Samuelson's integral (see section 1.3) at *every point in time*. But the dynamic re-examination of the optimization problem, Strotz proved, entails the risk of dynamic inconsistency: an individual, for example, may now choose to save money for a whole year, but in six months re-evaluate his decision, and choose to spend it, abandoning his initial plan. This problem, to which Strotz devoted much of his paper, casts, as we shall see, ultimate doubts on the fundamental concept of consumer sovereignty. Strotz therefore studied what is mathematically needed in order to rule out the possibility of dynamic inconsistency. And there was, as we have just seen in the previous section, an answer to this question: 'exponential discounting'[33].

I will present the essence of Strotz' mathematical argument in a very simple setting, in which I consider an individual who wants to distribute his leisure time for Saturday and Sunday from the perspective of Friday, but is then allowed to re-evaluate his decision again from the perspective of Saturday. I thus will consider only three time periods, and will use a discrete-time, three-period approach (see Strotz for the general, continuous case). Basically all interesting considerations on dynamic consistency as founding exponential discounting will appear already in such a simple formulation. For the continuous and more general approach I nevertheless refer the reader to the original paper, Strotz (1956).

[33] It is important to note that Samuelson himself already realized that exponential discounting had the property of dynamic consistency. From Samuelson's paper one can even interpret that this was a further reason for Samuelson to choose this particular structure. But the first in depth axiomatic study of dynamic inconsistency was Strotz (1956).

Suppose it is Friday and an individual –call him David- realizes this weekend he only can afford k hours of his preferred activity -watching sports on TV- since he needs to get some work done over the weekend. He thus decides to allow himself a total of k hours TV during the weekend. Suppose also his preferences only depend on TV consumption, and not on what he does the rest of the time, and that they are such that the pleasure he derives from watching TV marginally decreases with consumption[34]. Now take David's preferences at time τ over the set of all possible TV consumption paths to be represented by the following utility function:

$$u_\tau(x_0,...,x_n) = \sum_{i=0}^{n} \delta_{i-\tau} \cdot \sqrt{x_i}$$

where i is the period at which the object of choice is located, and τ is the moment at which the individual makes the choice (both $i,\tau \in \mathbb{N}$); $(x_0,...,x_n) \in \mathbb{R}_+^{n+1}$ represents a consumption plan, and $\sqrt{x_i}$ is an 'instantaneous utility function' assigning a marginally decreasing value to consumption x at period i, while $\delta_{i-\tau} : \mathbb{Z} \to (0,1]$ is a discount function that weights the utility of consumptions depending on the time-distance between a future (or past) period i and the present period τ. In other words, suppose David behaves as if he maximized the discrete-time equivalent to Samuelson's integral (2), to which we have incorporated a specific instantaneous utility function. We can now ask what maximization problem does in fact face David on *Friday*. David's utility as seen on Friday (at $\tau = 0$) is the following:

$$u_0(x_0, x_1, x_2) = \delta_0 \cdot \sqrt{x_0} + \delta_1 \cdot \sqrt{x_1} + \delta_2 \cdot \sqrt{x_2} \quad ,$$

where x_0, x_1, x_2 are TV hours watched, respectively, on Friday, Saturday and Sunday. We want to find a maximum for this function in the subset

[34] The problem becomes trivial if that is not the case: under the assumption of positive time preference, in the absence of marginally decreasing utility David would allocate all budgeted leisure into the nearest possible period. It is the assumption of diminishing marginal utility that makes intertemporal choice problems interesting, since it constitutes a counterbalance to positive time preference.

$$S_k = \left\{ (x_0, x_1, x_2) \in \mathbb{R}_+^3 : x_0 + x_1 + x_2 = k \right\}$$

Now, because S_k is a compact set and u_τ continuous in S_k we know there exists a maximum. Also, since u_τ is strictly concave in S_k (u_τ is the sum of strictly concave functions), any maximum we find is global in this domain. So let us first consider any possible interior solution ($x_0, x_1, x_2 > 0$). The maximization problem in this case is as follows:

$$\begin{aligned} &\max \quad u_0(x_0, x_1, x_2) \\ &s.t. \\ &x_0 + x_1 + x_2 = k \end{aligned}$$

Using Lagrange multipliers we find the following first-order conditions for a maximum:

$$x_0^* = \frac{\delta_0^2 \cdot k}{\delta_0^2 + \delta_1^2 + \delta_2^2}$$

$$x_1^* = \frac{\delta_1^2 \cdot k}{\delta_0^2 + \delta_1^2 + \delta_2^2}$$

$$x_2^* = \frac{\delta_2^2 \cdot k}{\delta_0^2 + \delta_1^2 + \delta_2^2}$$

Thus, we find an optimal consumption plan $\left(x_0^*, x_1^*, x_2^*\right)$ (there is no need of second-order conditions due to the concavity of u_τ), with which the individual achieves maximal utility. Now, in order to formulate a dynamic consistency problem, the question to ask is what decision-problem this individual faces one day after, on Saturday (before consumption). He has already consumed x_0^* on Friday, and at $\tau = 1$ his utility function now looks as follows:

$$u_1(x_0^*, x_1, x_2) = \delta_{-1} \cdot \sqrt{x_0^*} + \delta_0 \cdot \sqrt{x_1} + \delta_1 \cdot \sqrt{x_2}$$

And his new maximization problem is

$$\max \quad u_1(x_0^*, x_1, x_2)$$
$$s.t.$$
$$x_1 + x_2 = k - x_0^*$$

First-order conditions for a maximum are now the following:

$$x_1^{**} = \frac{k \cdot (\delta_1^2 + \delta_2^2)}{(\delta_0^2 + \delta_1^2 + \delta_2^2) \cdot (1 + \frac{\delta_1^2}{\delta_2^2})}$$

$$x_2^{**} = \frac{k \cdot (\delta_1^2 + \delta_2^2)}{(\delta_0^2 + \delta_1^2 + \delta_2^2) \cdot (1 + \frac{\delta_0^2}{\delta_1^2})}$$

Thanks to the concavity of $u_1(x_0^*, x_1, x_2)$, we find again that the solution (x_1^{**}, x_2^{**}) is the unique maximum of this program, and that the individual will thus follow this new consumption plan. The natural question to ask now is under what conditions will this plan be consistent with the previous one; in other words, when will optimal consumption for Saturday and Sunday as seen from the standpoint of Friday (x_1^*, x_2^*) equal optimal consumption for Saturday and Sunday as seen from the standpoint of Saturday (x_1^{**}, x_2^{**}) (i.e., when will David behave dynamically consistent). Note that, $x_1^* = x_1^{**} \Rightarrow x_2^* = x_2^{**}$, since x_0^* is fixed when choosing x_1^{**} and $k = x_0 + x_1 + x_2$. To impose dynamic consistency we therefore only need that optimal consumption for Saturday as seen from Friday equals optimal consumption for Saturday as seen from the standpoint of Saturday:

$$x_1^* = x_1^{**}$$

$$\frac{\delta_1^2 \cdot k}{\delta_0^2+\delta_1^2+\delta_2^2}=\frac{(\delta_1^2+\delta_2^2)\cdot k}{(\delta_0^2+\delta_1^2+\delta_2^2)(1+\frac{\delta_1^2}{\delta_0^2})}$$

$$\delta_1^2=\frac{\delta_1^2+\delta_2^2}{1+\frac{\delta_1^2}{\delta_0^2}}$$

$$\delta_1^2\cdot(1+\frac{\delta_1^2}{\delta_0^2})=\delta_1^2+\delta_2^2$$

$$\frac{\delta_1^4}{\delta_0^2}=\delta_2^2$$

$$\frac{\delta_1^2}{\delta_0}=\delta_2$$

$$\frac{\delta_1}{\delta_0}=\frac{\delta_2}{\delta_1}$$

As we can see, a necessary and sufficient condition for the consistency of David's behavior is that the relative importance of Saturday and Sunday is the same both from the point of view of Friday and Saturday. An inconsistency may thus only arise if David does not discount according to this particular structure.

Also, note that the previous argument easily extends to any number of periods: from the standpoint of $\tau=0$, the individual would choose

$$x_0^*=\frac{\delta_0^2\cdot k}{\delta_0^2+\ldots+\delta_n^2}$$

$$x_1^*=\frac{\delta_1^2\cdot k}{\delta_0^2+\ldots+\delta_n^2}$$

...

$$x_n^*=\frac{\delta_n^2\cdot k}{\delta_0^2+\ldots+\delta_n^2}$$

while from the standpoint of $\tau=1$ he would choose

$$x_0^{**} = \frac{\delta_{-1}^2 \cdot k}{\delta_{-1}^2 + \ldots + \delta_{n-1}^2}$$
$$x_1^{**} = \frac{\delta_0^2 \cdot k}{\delta_{-1}^2 + \ldots + \delta_{n-1}^2}$$
...
$$x_n^{**} = \frac{\delta_{n-1}^2 \cdot k}{\delta_{-1}^2 + \ldots + \delta_{n-1}^2}$$

Now if we impose that every x_i^* planned at $\tau = 0$ should coincide with every x_i^{**} planned at $\tau = 1$, we get that

$$\frac{\delta_0^2 \cdot k}{\delta_0^2 + \ldots + \delta_n^2} = \frac{\delta_{-1}^2 \cdot k}{\delta_{-1}^2 + \ldots + \delta_{n-1}^2} \Rightarrow \frac{\delta_0^2}{\delta_{-1}^2} = \frac{\delta_0^2 + \ldots + \delta_n^2}{\delta_{-1}^2 + \ldots + \delta_{n-1}^2}$$
$$\frac{\delta_1^2 \cdot k}{\delta_0^2 + \ldots + \delta_n^2} = \frac{\delta_0^2 \cdot k}{\delta_{-1}^2 + \ldots + \delta_{n-1}^2} \Rightarrow \frac{\delta_1^2}{\delta_0^2} = \frac{\delta_0^2 + \ldots + \delta_n^2}{\delta_{-1}^2 + \ldots + \delta_{n-1}^2}$$
...
$$\frac{\delta_n^2 \cdot k}{\delta_0^2 + \ldots + \delta_n^2} = \frac{\delta_{n-1}^2 \cdot k}{\delta_{-1}^2 + \ldots + \delta_{n-1}^2} \Rightarrow \frac{\delta_n^2}{\delta_{n-1}^2} = \frac{\delta_0^2 + \ldots + \delta_n^2}{\delta_{-1}^2 + \ldots + \delta_{n-1}^2}$$

which implies

$$\frac{\delta_1}{\delta_0} = \frac{\delta_2}{\delta_1} = \ldots = \frac{\delta_n}{\delta_{n-1}}$$

And this condition means automatically that, for every $i \neq \tau$,

$$\frac{\delta_i}{\delta_\tau} = \frac{\delta_{i+1}}{\delta_{\tau+1}} = \ldots = \frac{\delta_n}{\delta_{n-i+\tau}} = \delta^{i-\tau}$$

which guarantees consistency among any two arbitrary moments[35].

The previous results yield the following specification of the discount function for outcomes at distance $i - \tau$:

[35] The argument is the same I just followed for the special case $\tau = 0$ and $\tau' = 1$.

$$D(i-\tau)=\delta^{i-\tau}$$

This is known as exponential discounting, and in fact, if we make

$$\delta=\frac{1}{(1+r)}$$

we can express the constant discount factor δ in terms of a per-period interest rate r. According to exponential discounting, thus, an outcome x_i at distance $i-\tau$ has the following present value:

$$u_\tau(x_i)=\frac{1}{(1+r)^{i-\tau}}\cdot u_i(x_i)$$

This way of discounting can therefore be seen also as the inverse of compounded interest. Now consider the continuously compounded case: we need just to evaluate the limit of the exponential discount function when the period has been divided in infinitely small parts:

$$\lim_{n\to\infty}\frac{1}{\left(1+\frac{r}{n}\right)^{n\cdot(i-\tau)}}=\lim_{n\to\infty}\left(1+\frac{1}{n/r}\right)^{-n\cdot(i-\tau)}=\lim_{m\to\infty}\left(1+\frac{1}{m}\right)^{-m\cdot r\cdot(i-\tau)}=e^{-r\cdot(i-\tau)}$$

We thus get to $D(i-\tau)=e^{-r(i-\tau)}$, which is exactly Samuelson's proposed discount function (Samuelson, as we saw before, chose this discount function because it was natural for him to think of the individual as receiving continuously compounded interest for the postponed money). What we have proved here, nevertheless, is that this structure is not only convenient, but necessary, if we want to guarantee the individual's dynamic consistency.

In sum, we have seen that in order to rule out dynamic inconsistency from the individual's behavior, we need to assume *exponential* discounting. We have shown this result only for the discrete time case, and also just considering a specific instantaneous utility function; I refer to Strotz (1956) to find a more general proof where time is continuous and instantaneous utility can be any concave utility function.

Exponential discounting appears thus as the necessary mathematical structure to *guarantee* dynamic consistency if an additive utility framework is adopted; and hence, since dynamic consistency is *per se* considered a rationality requirement, exponential discounting is regarded also as *the* rational intertemporal choice model. I will discuss what this exactly means -and whether it is true- later in the conclusions to chapter 1 (section 1.8). What needs to be stated at this point is that Strotz (1956) linked the mathematical structure of Samuelson's intertemporal choice model to the intuitively rational principle of dynamic consistency. And also, that despite the importance of dynamic consistency, other strong assumptions underlie the theory of Discounted Utility. Let me now translate all these mathematical assumptions into their psychological implications.

1.5 Psychological Assumptions Inherent in Discounted Utility

All axioms we just saw imply restrictions to intertemporal preferences. 'Measuring' utility with the Discounted Utility model relies thus upon many psychological assumptions whose empirical validity will be discussed later in this chapter. Let us now summarize these assumptions.

(a) Constant Time Preference

As we just saw, stationarity as a dynamic consistency principle only makes sense under the assumption that the mere passage of time has no influence upon a person's time preferences. In other words, constant time preference assumes that humans keep their time perception constant across time. This assumption goes against the common intuition that a child's time perception may well differ from his own once he has grown up. For a child, to wait for a month is an enormous effort; for an adult, that is a short period of time. Constant time preference is, nevertheless, a necessary assumption in order to defend exponential discounting as the rational intertemporal choice model. Without it, exponential discounting would not anymore be grounded on the basis of dynamic consistency.

(b) Dynamic consistency

Dynamic consistency, the rationale for stationarity, is one of the strongest psychological assumption in Discounted Utility. It demands from the individual that his preferences for any two objects of choice do not depend on the particular moment of decision, but only on the relative distance among these objects. If waking up at 7am in the morning is seen as the best option from the standpoint of the previous night, then it has to be preferred also when the alarm starts ringing at 7am. This particular form of invariance is in fact violated by virtually everybody when the decision problem is conveniently chosen (as we will see later in this chapter).

(c) Positive Time Preference

Although not a necessary assumption, positive time preference (impatience) is usually accepted as part of the discounted utility model[36]. Psychologically, it means that decision makers prefer to have goods sooner rather than later, and 'bads' later rather than sooner, which seems a reasonable hypothesis to stick to, and is observed empirically almost invariantly. Surprisingly, though, some violations have been found for this apparently weak assumption: people sometimes have preferences for 'happy endings', for example, when they prefer to hear the best song at the end of a concert; or, also, many people prefer 'bads' sooner rather than later, for example: people who know they necessarily have to pass a painful experience (operation), often prefer to go through it as soon as possible. (I will present and comment the literature in section 1.7.3)

(d) Utility Independence

A further assumption in Discounted Utility is that a person's well-being in one period is independent of his or her consumption in any other period. According to this, one's preferences over, let us say, having pizza for dinner today, should be unaffected by the fact that we had pizza yesterday. As Koopmans (1960) put it, 'we cannot claim a high degree of realism for such a postulate, because there is no clear reason why complementarity of goods could not extend over more than one time period'. This assumption is, nevertheless, what makes it possible to represent intertemporal preferences by an additive utility function as we saw in Theorem 1. As we saw in the last section, under the discounted utility model all value of a sequence of outcomes is obtained by adding the discounted values in each period. The independence of equal substreams excludes preferences for specific distributions of utility across time as, for example, an increasing sequence of payments. This assumption has also proved empirically wrong, as I will show in section 1.7.3.

[36] In fact, Koopmans (1960) derived impatience as a necessary trait in time preferences provided the number of periods is countably infinite.

(e) Stationary Instantaneous Utility

Another underlying assumption in discounted utility is that the instantaneous utility function $u(x)$ remains the same across time, or, put in different words, that the individual's tastes do not change over time (see Theorem 2 in the previous section). This is obviously not true in humans, for reasons including maturation, satiation, social influence, or even the physiological effects of aging (see Loewenstein & Angner 2003 for an up to date review on preference change).

(f) Domain Independence

If one uses the same discounted utility model to describe intertemporal preferences in different domains, one is then assuming a unitary time preference, invariant across all forms of consumption. But the truth is that people discount utility from different sources at very different rates. Chocolate bars can be expected to be discounted at a much higher discount rate than money, for example (a chocolate bar now or two in a week?). While this is not a hypothesis within the model –remember that Samuelson restricted the validity of discounted utility to preference over money income- it is still worth bearing this assumption in mind, since many of the multiple applications of Discounted Utility have been in domains other than money.

Discounted Utility relies thus upon several strong assumptions, possibly the most salient one being dynamic consistency. Moreover, since discounted utility is the only intertemporal choice model that guarantees dynamic consistency, and dynamic consistency is *per se* considered a rationality requirement, Samuelson's model has been regarded as *the* rational intertemporal choice model. This strong normative support for exponential discounting had two effects in the further development of discounted utility. First, it gave fresh impetus to the descriptive validity of the theory: given that it was deemed implausible that regular people could survive without being able to stick to their own plans, exponential discounting was hard to deny also from a positive perspective[37]. And second, it had a blinding effect: the rest of the assumptions underlying Discounted Utility received little attention. As a result, discounted utility established not only as the

[37] Of course, many of these arguments do not pass severe scrutiny. Here I just want to describe how such arguments have helped the establishment of discounted utility; in the conclusions to Chapter 1 I will try to show why they may be wrong, and what implications this has had for the development of intertemporal choice theory.

standard normative model, but also as the best attempt for a positive theory; it established, in fact, as the standard theory of rational dynamic choice.

1.6 Hyperbolic Discounting

Over the last 25 years the view of discounted utility as the standard, rational theory for dynamic choice has changed dramatically. After the widening of experiments in intertemporal choice, virtually all assumptions in discounted utility have proven invalid as general principles of behavior. And interestingly, the first principle of discounted utility to be contradicted was constant discounting: instead of remaining constant over time, observed discount rates appear to decline with time (or, equivalently, discount factors δ_i are increasing in time). A common interpretation of this phenomenon is that people consider postponing consumption one period a bigger sacrifice when the period is near than when it is far in the future, contrary to Samuelson's assumption. Discount rates that decline with time-distance reveal 'decreasing impatience', or, as it is often referred to in the literature, *hyperbolic discounting*.

The finding of hyperbolic discounting has not only directly challenged discounted utility, but also opened the Pandora's Box of empirical testing, which has produced a large series of experiments revealing many so-called *anomalies of intertemporal choice*[38]. Let us now revise all these anomalies in some detail.

Thaler (1981) was the first study to test the declining discount rate hypothesis. He asked subjects to specify the amounts in one month, one year and ten years they considered equivalent to receiving \$15 now. The median responses were, respectively, \$20, \$50 and \$100. If we compute the annual equivalent discount rate underlying these choices, we find that subjects revealed an annual discount rate of 345% for the one-month period, 120% for the one-year period and 19% for the ten years horizon, a pattern that clearly supports the hyperbolic discounting hypothesis. Other studies have found

[38] As explained previously (see the general introduction), the term 'anomaly' is used in the literature to mean a departure from the behavior that a normative model (here, discounted utility) would prescribe. The term was adopted by the researchers specially after the influential article Loewenstein & Prelec (1992) was entitled "Anomalies in Intertemporal Choice" to establish a parallelism between these anomalies and the widely known anomalies in the field of choice under uncertainty.

similar results (Benzion, Rapoport and Yagil 1989; Chapman 1996; Chapman & Elstein 1995; Pender 1996; Redelmeier and Heller 1993) [39].

A second type of empirical support for hyperbolic discounting comes from experiments on dynamic inconsistency. Several studies report systematic preference reversals between two rewards as the time-distance to these rewards diminishes (Green, Fristoe & Myerson 1994; Kirby & Herrnstein 1995; Millar & Navarick 1984; Solnick et al. 1980)[40]. For example, many people do in fact prefer €101 in thirty-one days over €100 in thirty days but at the same time €100 now to €101 tomorrow (see Figure 1.7). Kirby & Herrnstein (1995) looks at such reversals and finds an astonishing 34 out of 36 subjects who behave inconsistently. Such overwhelming results are to be explained by the fact that their questionnaires adapted to individual preferences: the authors first asked subjects what is the shortest delay for €101 at which they would still prefer €100 today. Now imagine someone said two-days, meaning for shorter delays he would prefer the larger-later amount; then they would move both amounts forward (keeping constant the two-days distance between them) and ask subjects to choose again until either their preference reversed in favour of €101, or a certain number of questions passed. This methodology has the advantage of yielding the maximal amount of preference reversals, although, on the other hand, it may be questioned methodologically by the fact that subjects may perceive they are *expected* to reverse their preference. The authors did nevertheless post-experimental interviews, and report that subjects were in fact expressing their true preferences. And, in general, the finding of dynamic inconsistency is today considered robust in the literature.

[39] See, however, the findings I present in Chapter 2 showing that this effect disappears if subjects are told what interest rates underlies each choice, making the validity of the standard decreasing-discount-rates finding rely upon a certain experimental methodology, namely asking subjects to choose among money quantities withour indicating the underlying interest rates.

[40] These results have been also replicated in pigeons (Ainslie & Herrnstein 1981; Green et al. 1981).

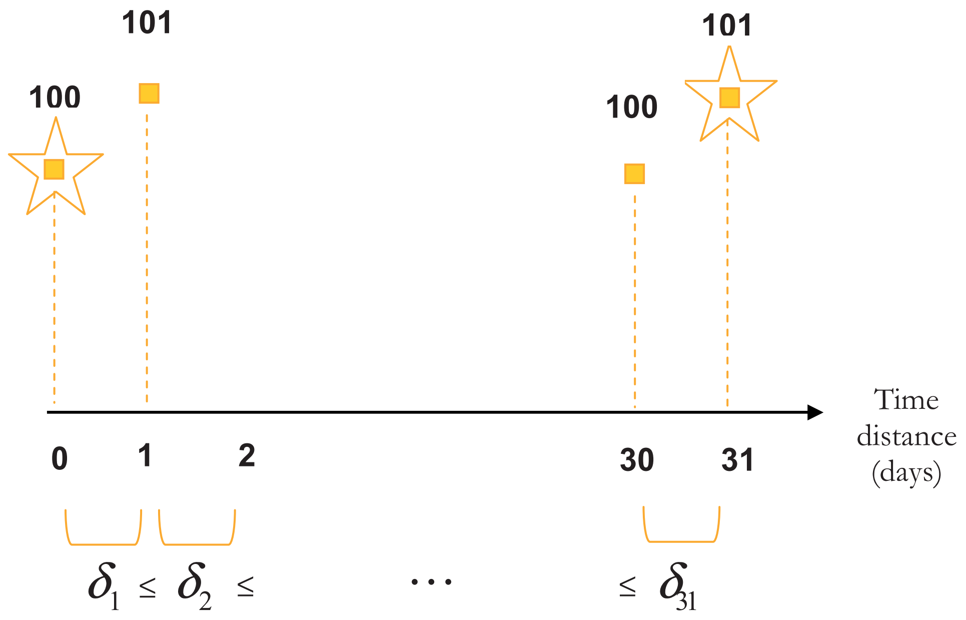

Figure 1.7

1.6.1. Mazur Discounting

Let me now present the so-called Mazur discounting through a numerical example in which an individual, call him David, has a non-exponential discount function, and incurs into dynamically inconsistent behavior. Imagine David has to choose among two possible actions/consumption-paths x and y :

- x : watching TV 2 hours on Saturday and 3 hours on Sunday; or
- y : watching TV 4 hours on Saturday and only 1 hour on Sunday.

Suppose David's instantaneous utility function is the following:

$$u : \mathbb{R}^{++} \to \mathbb{R}$$
$$x \to u(x) = \ln(x)$$

and take a non-exponential discount function known as Mazur-discounting by experimental psychologists:

$$\delta(i-\tau)=\frac{1}{1+0,7\cdot(i-\tau)} \qquad (41)$$

	Saturday	Sunday	
	$i=1$	$i=2$	*Leisure Budget*
X	2	3	= 5
Y	4	1	= 5
Instantaneous Utility			
$u(X)=\ln(X)$	0,69	1,10	
$u(Y)=\ln(Y)$	1,39	0	
Standpoint of Friday ($\tau=0$)			
$i=$	1	2	
$\delta(i-\tau)$	0,59	0,42	*Friday's choice:*
$u^0(X)$	0,4071	0,462	= 0,8691
$u^0(Y)$	0,8201	0	= 0,8201
			Thus, $X \succ Y$

Table 1.1

On Friday David chooses consumption path x. He prefers to work a little bit harder on Saturday, and be able to watch TV on Sunday also (see Table 1.1). But then Saturday comes. It would be a mistake to conclude that David will necessarily stick to Friday's considerations. Rather, if he is to maximize utility as seen from the point of view of Saturday, he must reconsider his choice. Suppose his discount function remains the same, but both the present moment τ and the values of the discount function for each

[41] Of course, any person with a minimal financial expertise will find a daily 'interest rate' of 0.7 as completely foolish to have. But such rates have been observed, specially for non-monetary objects of choice (see, for example, the empirical estimates on discount rates reported in Fredrick, Loewenstein & O'Donoghue 2002).

period have changed.

	Saturday	Sunday	
	$i=1$	$i=2$	*Leisure Budget*
X	2	3	= 5
Y	4	1	= 5
Instantaneous Utility			
$u(X)=\ln(X)$	0,69	1,10	
$u(Y)=\ln(Y)$	1,39	0	
Standpoint of Saturday ($\tau=1$)			
$\delta(i-\tau)$	1	0,59	*Saturday's choice:*
$u^1(X)$	0,69	0,649	= 1,339
$u^1(Y)$	1,39	0	= 1,39
			Thus, $Y \succ X$

Table 1.2

This leads David to behave inconsistently. On Friday he chose consumption path x, but the next day he chooses consumption path y (see Table 1.2). This shift in David's preferences is due only to the fact that his discounting is not constant. In fact,

$$\frac{0,59}{1} \neq \frac{0,42}{0,59} \left(\frac{\delta_1}{\delta_0} \neq \frac{\delta_2}{\delta_1} \right),$$

which means David does not discount time so as to guarantee dynamic consistency.

The discount function we have used in this example is the simplest hyperbolic discounting model, first explored by the psychologist Mazur[42] (Mazur 1984).

[42] Although the first to propose this discount function was Herrnstein (1981).

Mazur's Discount function[43]:

$$D(i)=\left(\frac{1}{1+r\cdot i}\right)$$

And its correspondent per-period discount factors are:

$$\delta_i=\left(\frac{1+r\cdot(i-1)}{1+r\cdot i}\right) \qquad (i\geq 1)$$
$$\delta_0=1 \qquad (i=0)$$

Note that the discount factor is not constant, but an increasing function of time. When objects of choice are distant in the future, deferring consumption one period is not too relevant (example: $i=50$ would mean $\delta_{50}=0,9818$ for a per-period $r=0,2$, meaning a difference in value of aprox. 2% among objects in periods 49 and 50). In contrast, deferring consumption one period starting from the present is much more significant ($i=1$ would mean $\delta_1=0,8333$ for the same per-period $i=0,2$, meaning a difference in value of aprox. 17% among outcomes in periods 0 and 1). This preference pattern may therefore produce dynamic inconsistent choices.

1.6.2. Finding a General Hyperbolic Discounting Model

A more general hyperbolic discounting function was proposed in Loewenstein & Prelec (1992). Let us present here their main result. Their starting point considers an

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43 Mazur's discount function resembles very much the model of 'simple interest' in mathematical finance. The only (but important) difference is to find in the different definition of the discount factor. The 'simple interest' model defines the discount *factor* as follows:

$$\delta_i=\left(\frac{1}{1+r\cdot i}\right)$$

Such a function yields intransitive choices, while Mazur's discount factor yields transitive choices. The reason for this is that Mazur discounting always evaluates objects from the perspective of the present moment, while the simple interest model acts more dynamically, i.e. it compares the relative values of objects from the perspective of the timing of those objects (not from the present). Interestingly, the reason for this subtle different formulation is the fact that mathematical finance was created to explain *exchanges* of financial prospects and thus to describe how people will act in the future, while decision theory is devoted to describe preferences an individual has over future objects of choice from his standpoint. When an individual evaluates two distant objects, he does so from the perspective of the present. But if you think in terms of exchanges in a financial market, the financial value of two distant objects is to be compared *at the time when the eventual exchange occurs*.

additive and separable utility function describing preferences over streams of consequences[44]:

$$U(x_0,...,x_n) = \sum_{i=0}^{n} v(x_0) \cdot D(i)$$

where $v(x_i)$ is a value function assigning values to departures from a reference point (or status quo)[45], and $D(i)$, the discount function, is an arbitrary real function assigning weights to consumption in period i. Periods belong to an index set $I = \{0,1,...n\}$. What interests us here is their derivation of a specific structure for the discount function.

The authors' idea is to capture the empirical finding that "people are more sensitive to a given time delay if it occurs earlier rather than later". This principle can be formulated as follows: if a person is indifferent between receiving $x > 0$ immediately and $y > x$ at some later period j, and we suppose, as standard, that $D(0) = 1$, then he/she will strictly prefer the better outcome when both outcomes are delayed by a common lapse i:

$$v(x) = v(y) \cdot D(j) \Rightarrow v(x) \cdot D(i) < v(y) \cdot D(i + j)$$

Both $i, j \in I$; now in order to re-establish equality, the larger outcome would need to be delayed by a greater delay. If we postulate that this delay is a linear function of the delay to the smaller, earlier outcome (i), then

$$v(x) = v(y) \cdot D(j) \Rightarrow v(x) \cdot D(i) = v(y) \cdot D(ki + j)$$

for some positive constant k (Note that in the case of $k = 1$ we would get the standard stationarity assumption). The same is true, obviously, for a different common delay i':

$$v(x) = v(y) \cdot D(j) \Rightarrow v(x) \cdot D(i') = v(y) \cdot D(ki' + j)$$

[44] It is worth noting that Loewenstein & Prelec's model was conceived in fact as a reference-dependent model in the spirit of Kahneman & Tversky (1979), but their result has ended up widely used in the context of regular utility functions as the one I will present.

[45] Loewenstein & Prelec (1992) develops an intertemporal choice model that accounts for several anomalies; the use of a value function aims at being maximally general, capturing the different preferences for gains and losses in the spirit of Kahneman & Tversky (1979).

We then can compute the discounted value of any point laying in between as follows:

$$\begin{aligned} v(x)\cdot D(\lambda i+(1-\lambda)i') &= v(y)\cdot D(k(\lambda i+(1-\lambda)i')+j) \\ &= v(y)\cdot D(\lambda(ki+j)+(1-\lambda)(ki'+j)) \end{aligned}$$

where $\lambda \in [0,1]$. Now from

$v(x)\cdot D(i) = v(y)\cdot D(ki+j)$, and under the natural assumptions that $D(.)$ is a monotonic function and $v(y)\neq 0$, we can state both that

$$ki+j = D^{-1}\left(\frac{v(x)\cdot D(i)}{v(y)}\right)$$

and

$$v(x)\cdot D(i') = v(y)\cdot D(ki'+j)$$

$$ki'+j = D^{-1}\left(\frac{v(x)\cdot D(i')}{v(y)}\right)$$

from where we can obtain

$$v(x)\cdot D(\lambda i+(1-\lambda)i') = v(y)\cdot D\left(\lambda\cdot D^{-1}\left(\frac{v(x)\cdot D(i)}{v(y)}\right)+(1-\lambda)\cdot D^{-1}\left(\frac{v(x)\cdot D(i')}{v(y)}\right)\right).$$

Let now

$$r=\frac{v(x)}{v(y)};w=D(i);z=D(i') \text{ and } u=D^{-1}$$

and we arrive at the equation

$$ru^{-1}(\lambda u(w)+(1-\lambda)u(z)) = u^{-1}(\lambda u(rw)+(1-\lambda)u(rz)),$$

whose only solutions are the logarithmic and power functions[46]:

[46] See Aczel (1966), p152 equation 18.

$u(i) = c\ln(i) + d$
or
$u(i) = ci^{\tau} + d$

Now since $D(i) = u^{-1}(i)$, then we find that the discount function must be a generalized hyperbola:

$$D(i) = (1+\alpha i)^{-\frac{\beta}{\alpha}}, \quad \alpha, \beta > 0$$

And the corresponding (increasing) per-period discount factors are

$$\delta_i = \left(\frac{1+\alpha(i-1)}{1+\alpha i} \right)^{\frac{\beta}{\alpha}}$$

Loewenstein and Prelec's general hyperbolic discount function is extremely flexible: α captures how much the function departures from exponential discounting. In the limit, when α goes to zero, we obtain the exponential discounting model:

$$\lim_{\alpha \to 0} (1+\alpha i)^{-\frac{\beta}{\alpha}} = e^{-\beta i}$$

And whenever $\alpha = \beta$, then we get Mazur discounting:

$$D(i) = (1+\alpha i)^{-\frac{\beta}{\alpha}} = \frac{1}{(1+\beta i)}$$

1.6.3. Quasi-hyperbolic discounting

Another simple functional form that captures a preference for immediacy has received recently a lot of attention. Phelps and Pollak (1968) first proposed the following discount function to study intergenerational altruism (people in one generation caring for people in subsequent generations):

$$D(i) = \begin{cases} 1 & \text{if } i = 0 \\ \beta\delta^{i} & \text{if } i > 0 \end{cases}$$

Note that in the first period, utility is discounted by $\beta\delta$, while subsequent periods are discounted by δ only. This very parsimonious model, afterwards applied by Elster (1982) and specially Laibson (1997) to individual decision-making, assumes constant discounting in all but the first period, in which the individual is supposed to be more impatient ($\beta < 1$). Recently this model has received new important support thanks to McClure, Laibson, Loewenstein & Cohen (2004). In his paper the authors show that two *separate* neural systems are in fact involved in the evaluation of immediate and delayed rewards. Parts of the limbic system (the β-system, they call it) are preferentially activated by decisions involving immediate rewards, while parts of the lateral prefrontal cortex and posterior parietal cortex (the more 'deliberative' or rational δ-system) are uniformly activated independently of delay, thus making it possible to associate the relative participation of each system with the choice of the immediate or delayed reward.

The importance of the prefrontal cortex to intertemporal choice is widely accepted, among other things thanks to the incredible story of Phineas Gage, a member of a railway construction gang in Vermont, in the year 1848. When he was preparing an explosion, he started tamping directly onto the explosive powder with his iron rod. The sparks immediately struck fire, producing a big blast that shot up the iron rod towards his head. The iron penetrated his left cheek bone and went out through the top of his head, crossing the whole frontal part of his brain and landing 300 feet away. He survived the accident without any apparent damage to his mental capacities. He could even return to work in a few weeks, and no difference was to be observed in his behavior. But, according to everyone who had contact with him thereafter, there was in fact one difference: Phineas Gage had become impulsive, capricious and completely unable to plan ahead, and spent the rest of his life drifting in the moment, from one abandoned job to another, until his early death at age thirty-eight (Damasio 1994; Macmillan 2000).

For a review of recent developments in Neuroeconomics, see Camerer, Loewenstein & Prelec (2005).

The also called (β, δ) discounting, together with other hyperbolic discounting models, all have helped recently explain many phenomena: they have been used to study paradoxes in the consumption-saving behaviour (Laibson 1997; Laibson, Repetto & Tobacman 1998; Angeletos et al. 2001), or even procrastination, since (β, δ) preferences lead a person to put off an onerous activity more than he would like to from a prior perspective (O'Donoghue & Rabin 1999b, 2001; Fischer 1999). (β,δ)-preferences also have been used to analyse addiction (O'Donoghue & Rabin 1999a, 2000a; Gruber & Koszegi 2000; Carrillo 1999), since it predicts over-consumption of highly addictive products. In sum, the hyperbolic discounting literature has been very influential and celebrated as a first big success of the interaction between psychology and economics.

1.6.4. Subadditive Intertemporal Choice and the Experimental Challenge to Hyperbolic Discounting.

More recently, however, much of this enthusiasm with hyperbolic discounting has come up against important criticism. Read (2001) has questioned the usual inference made from existing experimental evidence by researchers in intertemporal choice. Typically, subjects are confronted with the choice of a smaller-sooner (SS) outcome and a larger-later (LL) outcome, occurring at delays t_1 and t_2, respectively. But then, Read points out, the standard procedure is to set $t_1 = 0$, i.e., to present the choice among an *immediate* SS and a delayed LL, thus confounding two factors possibly affecting choice: the *delay* (to LL) and the *interval* (among SS and LL). The typical finding in experiments is that, the farther away you set LL, the greater the implicit discount factor. Example: imagine someone declares he is indifferent between €400 now and €450 in six months, but also between €400 now and €475 in 12 months. The usual interpretation is that he discounts the first six months with

$$\delta_{0\to 6} = \frac{400}{450} = 0,889\,,$$

and the second six-month-period with

$$\delta_{6\to 12} = \frac{450}{475} = 0,947$$

meaning an increasing discount factor (increasing patience or decreasing impatience) that supports the hyperbolic discounting hypothesis. Of course this interpretation builds on one fundamental assumption: the idea that discounting over a certain period is independent of whether this period is embedded in a longer interval or not. Put in other words, we need to make sure that, for example, when discounting over 12 months, the individual 'uses' his six-month-factor to discount the first half, and a new factor for the second half, rather then a completely new discount factor for the whole period.

Read has done several experiments to test these hypothesis (Read 2001; Read & Roelofsma 2002; Read, Airoldi & Loewe 2005) and has found there is in fact no evidence for an increasing discount factor. In one of his experiments he found the smaller-sooner and the larger-later amounts among which a participant was indifferent, and computed the resulting yearly discount factor for several intervals. A clear result in Read's experiments is that annual-equivalent discount factors are lower for shorter intervals, and higher for longer intervals, something he labels 'subadditive discounting'. When individuals discount over a longer interval they do behave more patiently; but, interestingly, there is no evidence of discount factors increasing with *delay* to that interval. When Read expands delay keeping the interval constant, he finds no evidence of an increasing discount factor, a finding that contradicts hyperbolic discounting[47].

Read's argument, thus, is that the hyperbolic discounting literature has been mixing interval discounting with delay discounting: since virtually all experiments set the SS at delay equal zero, they thus confound interval discounting with delay discounting, and conclude there is an increasing discount factor. What Read has shown, on the contrary, is that individuals discount over intervals, and do not change their discount factors with *delay to interval*.

The results obtained in Read's experiments cast important doubts on the existence of hyperbolic discounting, and starkly show how far we still are from successfully capturing behaviour into a mathematical model. In fact, while Read does not find any evidence of hyperbolic discounting in experiments based on choice tasks, he does find some evidence of it in experiments based on so-called matching tasks (Read 2003), where subjects are directly asked to state equivalent amounts, instead of having to choose among options. As was already conjectured by Ahlbrecht & Weber (1997), true hyperbolic discounting (increasing discount factors) is only observed in matching tasks, something that reminds us of how determinant frames can be in intertemporal choice, too[48].

An objection that someone could be tempted to make against Read's challenge to hyperbolic discounting is that dynamic inconsistency apparently is a robust empirical finding, and it's only explanation seems to be precisely hyperbolic discounting. But, as

[47] But not 'quasi-hyperbolic' discounting, a model compatible with the absence of increasing patience.
[48] See chapter 2 within this dissertation for a further framing effect in intertemporal choice.

Read shows (Read 2003), dynamic inconsistency can be explained in other ways[49]: first, it can be explained very parsimoniously by another, well established anomaly called 'magnitude effect', according to which individuals discount more the lower an amount is (see section 1.7.2). If, for example, an individual prefers one apple today over two apples tomorrow, but at the same time he prefers two apples in 365 days over one apple in 364 days –as in the classic example by Richard Thaler-, we may understand this dynamic inconsistency as one provoked by a different per-period discount factor for one apple than for two apples (magnitude effect). Imagine one apple having utility 10, and two apples having total utility 14. Now imagine that, due to the magnitude effect, one apple is discounted with a constant factor of 0.5 while the two apples are discounted with 0.7. Then, one apple today would be preferred to two apples tomorrow because $10 \cdot 0.5^0 > 14 \cdot 0.7^1$; while two apples in 365 days would be preferred to one apple in 364 days due to $14 \cdot 0.7^{365} > 10 \cdot 0.5^{364}$.

Another possible account of the dynamic inconsistency phenomenon is impulsivity. It is in fact very difficult to find a dynamic inconsistent behaviour in the choice of things such as gasoline or paper, as Hoch & Loewenstein (1991) pointed out. Dynamic inconsistency appears related to either impulsive desires (chocolate) or myopia (deciding to go early to bed, then watching movie until late at night), but many decisions do not produce neither one nor the other, so modelling individuals' time preferences in general with hyperbolic functions may not be justified at all. Rather, we could think of explaining dynamic inconsistency *ad hoc* as something produced by visceral influences on behaviour (Loewenstein 1996)[50].

Read has not been the only one casting doubts on the validity and convenience of hyperbolic discounting. Rubinstein, A. (2003) showed that hyperbolic discounting could be challenged empirically in a similar way as exponential discounting has been challenged before (see Chapter 2 for a detailed description of his experiment 2). Rubinstein

[49] Read himself claims that, in fact, the hyperbolic discounting explanation of dynamic inconsistency is particularly bad since it leads to absurd predictions. If one is to explain the typically observed dynamic inconsistencies with hyperbolic discounting, he needs to assume very low discount factors; and such low discount factors would predict the same person prefers, for example, 27.000€ now over 1 million in one year, which, of course, is a wrong prediction (Read 2003).

[50] Read (2003) even points out a third alternative explanation of dynamic inconsistency, one based on the theory of 'temporal construal', developed in Liberman & Trope (2003). Following these authors, time distance to consumption alters the relative weights of the central and peripheral features of objects of choice: when buying a car, if the car is delivered right away, the individual may prefer a sports car, underweighting the central feature of a car (family transport) in favour of a peripheral feature (having fun driving). On the other hand, if the car is delivered in six months, the same buyer may choose a family van.

concludes from several experiments that finding a good description of behaviour requires to open up the black box of human decision-making, rather than to simply try out alternative mathematical structures within the same framework.

Despite all this criticism, hyperbolic discounting has established as an important alternative to exponential discounting, one representing the benefits of experimental economics and psychology. In fact, the question we may ask ourselves is why has not any hyperbolic discounting model -say quasi-hyperbolic discounting, for example- achieved to *displace* exponential discounting, as one could reasonably expect (it explains dynamic consistency, it fits data statistically better – see Fredrick, Loewenstein & O'Donoghue (2002)-, it is parsimonious, and it fits intuition). I will try to answer this question in the conclusions to this chapter.

1.7 Other Discounted Utility Anomalies

As I have shown earlier, discounted utility not only relies upon the hypothesis of constant discount rates. Many other discounted utility assumptions exist and have been tested, and a collection of other well known anomalies has established in the literature in a similar way as there is a collection of expected utility anomalies (Loewenstein & Prelec 1992; Fredrick, Loewenstein & O'Donoghue 2002). Hence, as we will next see, there are many common preference patterns that Samuelson's proposed mathematical structure is unable to capture.

1.7.1. Excessive Discounting

Although the discounted utility formula does not limit the discount rate to any specific amount, we may consider discounting at, say 100% a year an 'anomaly'. In contrast with other anomalies reviewed in this section, the 'irrationality' underlying excessive discounting does not consist in any violation of the previously reviewed axioms of intertemporal choice; rather, it consists in the fact that one can always borrow money in the market at an interest rate that is most of the time between 3% and 8%[51]. Someone choosing €100 now over €200 in one year seems therefore not to be a rational economic

[51] This argument was first developed by Fisher (1930).

agent: taking instead €200 in one year, he or she could still borrow today €100 in the market at some market interest rate (obtaining thus immediate utility from €100), pay back the loan in one year using the €200 chosen before, and end up being better off. Empirical results nevertheless show dramatic deviations from this normative behavior. Thaler (1981), Benzion, Rapoport & Yagil (1989) or Fredrick, Loewenstein & O'Donoghue (2002) find discount rates ranging from 25% to 3000% per year. Excessive discounting occurs both when choosing among non-monetary objects and when choosing among monetary objects, although at very different degrees depending on the characteristics of these objects. For example, small amounts are usually discounted at a much higher rate than big amounts, as we will next see.

1.7.2. The 'Magnitude Effect'

Larger outcomes are discounted at a lower rate than smaller outcomes. In virtually all studies that vary outcome size, a clear 'magnitude effect' has been revealed by the choices of subjects, an effect that is now one of the most robust findings in intertemporal choice (Ainslie & Haendel 1983; Benzion, Rapoport & Yagil 1989; Chapman & Winquist 1998; Green, Fristoe & Myerson 1994; Green, Fry & Myerson 1994; Holcomb & Nelson 1992; Kirby 1997; Kirby & Marakovic 1996; Kirby, Petry & Bickel 1999; Loewenstein 1987; Raineri & Rachlin 1993; Shelley 1993; Thaler 1981). See, for example, 1 month discount rates in Table 1.3 found in Thaler (1981)[52] :

Amount	1-month equivalent
$15	$20 (345%)
$250	$300 (219%)
$3000	$3100 (39%)

Table 1.3

The 'magnitude effect' strongly affects intertemporal choices, and has been shown to explain several phenomena (see, for instance, chapter 3).

[52] 1-month equivalents are median values. Discount rates (in parenthesis) correspond to continuously compounded rates.

1.7.3 Sequence Effects

Discounted utility evaluates a sequence of outcomes by adding each outcome's discounted values, independently of whether the sequence has any particular shape (increasing or decreasing, for example). The literature has nevertheless found that people systematically prefer increasing sequences of consumption over decreasing ones that add up to the same total amount (Ariely & Carmon 2003; Fredrick & Loewenstein 2002; Loewenstein & Prelec 1993; Loewenstein & Sicherman 1991). For example, Loewenstein & Sicherman (1991) found that people prefer an increasing wage profile over a declining or flat one, even after being reminded that a decreasing sequence has a higher total present value due to higher interests of the larger-sooner outcomes. Analogously, when sequences were framed as streams of pain (headache pain, for example), respondents showed a clear preference for decreasing over increasing sequences, indicating pain was preferred sooner rather than later (Chapman 2000). Another version of this effect, the preference for 'happy endings', was found by Ross & Simonson (1991) and Loewenstein & Prelec (1993): if a good outcome is embedded in a (small) sequence together with not-so-good outcomes, people prefer the sequence where the good outcome occurs later rather than sooner, contradicting positive time preference and potentially causing violations of IES.

In addition to this preference-for-improvement finding, other sequence effects are reported in the literature. Loewenstein & Prelec (1993) found a preference for uniformly spreading of outcomes in a sequence. People tend to prefer (0,1,0,1,0,1,0) rather than (0,0,1,1,1,0,0) showing a tendency to like evenly distributed sequences. Also, research in retrospective evaluation of experiences started by Kahneman et al. (1993) has shown a preference for two particular 'moments' in a sequence, the *peak* and the *end*. Several studies have found that a weighted average of experiences at these two particular points in time suffices to explain individuals' retrospective overall evaluation of experiences. The reason for it is that memory stores only *highlights* of an experience, and this is the only information that is afterwards used when the time comes to make a retrospective evaluation (see also Ariely & Carmon 2003 for a review on the evidence).

In sum, preferences for sequences appear to be *essentially* different from preferences for single outcomes. Once people perceive they are choosing among objects embedded in a sequence, they act according to a collection of new reasons regarding the specific shape of the sequence (see Read & Powell 2002 for a qualitative study on these reasons). Put in other words: we can conclude that 'gestalt' properties matter; and discounted utility does not account for them (see section 1.4.1).

1.7.4. The Sign Effect

In many studies gains are discounted at a higher rate than losses. Imagine someone receives a traffic ticket and is asked how much he is willing to pay to delay the payment by three months. Now consider the same problem framed with gains: someone has won a prize and is asked how much he would need to be paid to accept receiving the money three months later. Thaler (1981) showed that the underlying discount rates differ significantly for both framings. In fact, in many studies subjects have shown a preference to incur in a loss immediately rather than delay it (Benzion, Rapoport & Yagil 1989; Loewenstein 1987; MacKeigan et al. 1993; Mischel, Grusec & Masters 1969; Redelmeier & Heller 1993; Yates & Watts 1975).

1.7.5. The Delay-Speedup Asymmetry

Loewenstein (1988) found that respondents who expected receiving a VCR in one year would pay an average of $54 to receive it immediately, while those who expected receiving it immediately demanded an average of $126 to delay its receipt by a year. Other studies have confirmed these findings (Benzion, Rapoport & Yagil 1989; Shelley 1993), which suggest that an individual's reference point is relevant for his intertemporal choices.

1.7.6. The Date-Delay Effect

Future outcomes are discounted significantly more when time is expressed in delays (e.g., 'in six months') compared to when it is expressed in calendar dates (e.g., 'on June 14th.'). This anomaly has been found very recently by Read et. al (2005). In one of their experiments the authors find, for example, that while only 29% of the subjects chose 450 in 13 months over 370 in 4 months, 60% of the subjects did make that choice when time was expressed as calendar dates (450 on June 25, 2004 over 370 on September 26, 2003). Moreover, the authors found no evidence of increasing discount factors (i.e., hyperbolic discounting) when calendar dates were used, while they did find it when time was expressed as delays. We can therefore conclude that the way time is expressed strongly affects decision-making.

The reason for this effect remains still unanswered. The authors nevertheless suggest an explanation inspired by Rubinstein, A. (2003) and Leland (2002), who proposed a procedural decision-making theory according to which subjects look at the time dimension of the objects of choice and, if they find the two timings *similar*, then they decide upon the money dimension, which implies the choice of the larger-later amount, a choice that we then interpret as a more patient choice[53]. Now Read et.al propose that when timings are expressed as calendar dates, they are perceived as more similar than when they are presented as delays, possibly because dates usually are quite similar (e.g., 2003 vs 2004). Moreover, they propose that similarity between calendar dates separated by a common interval does *not* change the later they occur, and for that reason we do not observe increasing discount factors in the calendar dates framing; in contrast, when asked in terms of delays, the further away a given interval lays, the more similar the timings of SS and LL are perceived by the individual, and thus the higher the induced discount factor appears to be. I will have more to say on the possible explanations to this effect in the final conclusions chapter.

1.8 Alternative Models of Intertemporal Choice

We have seen a collection of anomalies that casts serious doubts on discounted utility as a theory of intertemporal choice. The reported findings are quite intuitive, too; in fact, most of us could well be represented by these preference patterns, despite which they cannot be accommodated into Samuelson's mathematical structure, as I have shown. Now the obvious question is the following: what alternative models have been proposed to account for the observed anomalies? Following Fredrick, Loewenstein & O'Donoghue (2002), I will next classify them into two distinct groups: on one hand, there are models that maintain the basic structure of DU but modify either the instantaneous utility function or the discount function (or both); on the other hand, there are models that depart more radically from DU by adopting completely different views of intertemporal decision making phenomena.

[53] See chapter 3 for a more detailed description of Rubinstein's similarity theory.

1.8.1. Models Modifying or Enriching Discounted Utility

The main alternative model within this category is obviously Hyperbolic Discounting, already reviewed in previous section 1.6., including several different hyperbolic discounting functions. Within this family of alternative models we can -as we have seen (section 1.6)- refer to (β, δ) discounting as possibly the most succesful one in explaining many relevant-to-economics phenomena (consumption-saving behavior, procrastination, addiction or information acquisition).

Other models departing partially from DU are those enriching in several ways the instantaneous utility function: first, there are *habit-formation* models, in which utility from current consumption can be affected by the level of past consumption (first proposed by Duesenberry (1952), and further developed by Pollak (1970) and Ryder & Heal (1973)), therefore affecting intertemporal choices. Second, there are *reference-dependent* models incorporating standard findings of prospect theory such as value derived from departures from a reference point, that might depend upon past consumption, expectations, social comparison, status quo, etc; or loss aversion, meaning negative departures from reference point yield more 'disutility' than equivalent positive departures; and diminishing sensitivity for gains and losses, meaning that the value function is concave over gains and convex over losses. A good example of such a model is Loewenstein & Prelec (1992), where the authors apply a reference-dependent model in order to simultaneously explain hyperbolic discounting, the magnitude effect, the sign effect and the delay-speedup effect. Other contributions of reference-dependent models applied to intertemporal choice are Bowman, Minehart & Rabin (1999) and Shea (1995a, 1995b), both studying how loss aversion in consumption affects consumption growth over the years.

Third, there are models incorporating *utility from anticipation*. The idea underlying these models, as in Loewenstein (1987), is that individuals derive utility from two sources, one of them being current consumption and the other one being the anticipation of future consumption. Such a model can be used to explain several of the previously presented anomalies: for example, it can explain why people may have a preference for improving sequences, since the further away an outcome lays, the more anticipatory utility it yields, which means that higher outcomes produce more overall utility if located later rather than sooner; also, and for similar reasons, it can explain the sign effect, and other effects like inconsistency or the fact that people discount different goods at different rates. Fourth, there exist models accounting for *visceral influences* on behavior such as hunger, sexual desire, physical pain, etc. These models incorporate the impact of such visceral states into behavior, and provide an alternative explanation for the typical preference reversal finding usually attributed to hyperbolic discounting, the explanation being that after having planned to, say, diet, then, once in front of a chocolate cake, the visceral state suddenly determines behavior in the opposite direction as the one initially planned.

Finally, there are also models that combine several of the previous modifications to explain behavior, as for example Loewenstein & Prelec (1993), combining a preference for improvement together with hyperbolic discounting to explain a preference for U-shaped sequences. A very recent and powerful example is the model developed in Baucells & Heukamp (2007), that accounts simultaneously for expected utility, discounted utility, prospect theory and hyperbolic discounting, together with the more and more fundamental magnitude effect.

1.8.2. Models Explaining Other Phenomena Relevant to Intertemporal Choice

In the last 25 years several totally new perspectives have been developed to overcome the problems of DU. First, there appeared models based on a *multiple-self* perspective. The basic idea is to postulate a myopic self who is in conflict with more farsighted 'selves', and then solve this conflict with tools from strategic interaction economics as game theory, principal-agent, etc. Examples of such models are Winston (1980), Thaler & Shefrin (1981), Schelling (1984), Elster (1985), Ainslie & Haslam (1992). Second, different models have been developed around the idea of *mental accounting*. According to this perspective, people do not treat all money as fungible, but assign different expenditure types to their different 'mental accounts'. Elster (1985) suggest that such behavior may explain why people tend to spend small amounts of money loosely but big amounts very carefully: small amounts are assigned to the category 'spending money', while big amounts are assigned to the category 'saving money'. Other models using the mental-accounts approach are Benartzi & Thaler (1995) and Prelec & Loewenstein (1998).

Third, there are models based on *choice bracketing*. Read, Loewenstein & Rabin (1999) showed how people's ability to 'broad-bracket' when facing several simultaneous decisions usually helps them to make better overall decisions. Unfortunately, in reality people tend to 'narrow-bracket' when facing several decisions at a time. When this effect is applied to intertemporal decision making, we can use the idea of narrow 'temporal-bracketing' to understand or predict several anomalies related to multiple-outcomes effects, as the ones discussed in Loewenstein & Prelec (1993). Fourth, Gul & Pesendorfer (2001) developed a model considering the influence of what they labelled 'temptation utility' to intertemporal choice. In their model, individuals have an incentive to eliminate desired options, something that resembles a preference for commitment as the one suggested already by Strotz (1956) to overcome dynamic inconsistency.

Fifth, recently Loewenstein, O'Donoghue & Rabin (2003) have developed a model to explain how well decision makers predict their future changes in tastes. Their main finding is that people systematically exaggerate the degree to which their future tastes will resemble their current tastes, thus incurring into what the authors label '*projection bias*'. This important effect has many implications for economic behavior, such as inconsistent planning or misguided purchases of durable goods.

All these alternative intertemporal choice models are today being used by economists to enhance their modelization of human behavior in intertemporal decision making. But, unfortunately, there is no established unified theory for intertemporal choice yet. To find a diagnostic of why this has not yet happened, I will now complete my review with a discussion on the role of the rationality requirement in the development of intertemporal choice theory.

1.9 Concluding Remarks: The Role of Rationality in Intertemporal Choice Theory

As it is widely known, choice theory is based upon the idea of preference relations describing people's behaviour. Preference relations' particular structure – completeness and transitivity- ensures we can represent such preferences by a (utility) function assigning a value to each object of choice, which makes it possible to interpret an individual's choices as if he aimed at maximizing the value of this function. And rational choice theory consists basically in assuming people do in fact have (stable[54]) preference relations. But why should they?

[54] Stability of preference relations is very important, since the opposite -changing preferences- would be completely non-informative; any possible behaviour would fit in the preference relations' theory (Rosenberg 1992).

Many economists like to answer with the money-pump argument. According to this idea, an individual with intransitive preferences would be easy to exploit by an arbitrageur. Suppose an individual's preferences over three objects were as follows:

$$A \succ B \text{ and } B \succ C; \text{ but } C \succ A$$

This individual will thus be ready to exchange C plus a certain quantity (say, one cent) to obtain B, which he values more. But then, he will also accept exchanging B plus one cent to get A; and finally, also A plus one cent to get C. Such a cycle would bring him back to his original situation possessing C, but having spent 3 cents! Thus, this subject can easily be exploited by an arbitrageur, who could keep indefinitely pumping money out of him until complete ruin.

The money-pump argument is far from being uncontroversial from a technical-economical point of view (see, among others, Cubitt & Sudgen 2001; Machina 1989; McClennen 1990; Sudgen 1991; Anand 1993; Kelsey & Milne 1997; Yaari 1998). But its fundamental idea is that economic theory has reasons to believe that most of the people behave most of the time transitively, or otherwise they would not 'survive'. Thus, a basic normative principle –it is bad to be ruined- becomes a positive theory by arguing that exposure to a money-pump would immediately be exploited by arbitrageurs.

Let us now go back to intertemporal choice. In a previous section we saw that dynamic consistency is considered a rationality principle for intertemporal choice. The rationality of dynamic consistency can be founded ultimately on the same argument we just saw. A dynamic inconsistent individual could be brought to ruin by an arbitrageur. To illustrate this, imagine that Alex has a lasting preference for €11 in 24 hours over €10 in 22 hours, but prefers €10 immediately to €11 in two hours. Helene offers to sell him €11 in 24 hours for €10 in 22 hours. Alex agrees. 22 hours later, Alex gives Helene €10. But now his preferences have changed and he would prefer keeping the €10 rather than getting €11 in two hours. Helene thus offers to give him back the €10, if he agrees to pay her €11 plus, say, one cent in two hours. Alex agrees and in two hours Helene is one cent better off. Helene then offers to sell Alex €11 in 24 hours…and so on. Now a hyperbolic discounter, the argument continues, has precisely such a preference structure, and thus could easily be exploited to ruin. On the contrary, exponential discounters are invulnerable to such money-pumps, and are therefore expected to proliferate in society.

There are problems with such simplifications. In effect, imagine most of the people were sophisticated hyperbolic discounters as follows: if they detect an arbitrageur, they do not make any deal with him (they give up a certain opportunity to be better off in order to morally punish arbitrageurs)[55]; in any other situation, they behave hyperbolically. This means there would be no incentives at all for arbitrageurs to operate. We could then perfectly observe a vast majority of hyperbolic discounters behaving dynamically inconsistent in their every day life, without causing their selves any fatal damage by doing so. The money-pump-based justification of dynamic consistency as a rationality principle thus relies upon viewing people as naive decision-makers, whose preferences are invariant even in the presence of someone who is trying to bring them to complete ruin.

Moreover; suppose now there were no sophisticated behaviours, and that hyperbolic discounters were naively exposed to exploitation. We still need certain preconditions in order to support the money-pump defence of exponential discounting: for example, it is needed that the specific arbitrage opportunities are enough to make a living out of it. How much work would be needed by the arbitrageurs to earn how much money? Would this be more worth doing than a regular job? Would the moral discomfort outweigh the monetary incentives? If these circumstances are not met, we simply cannot infer dynamically inconsistent individuals would not survive. Hence, the money-pump argument also relies upon the worthiness of arbitrage for arbitrageurs.

Finally, I want to point at a third objection to the money-pump argument for dynamic consistency, due to Ahlbrecht & Weber (1995). Many decisions are completely binding, not subject to future reconsideration. For those kinds of decisions there is no possible argument in favour of exponential discounting based on dynamic consistency. When decisions are irrevocable (buying a house, for example), nothing prevents the individual from discounting future values hyperbolically since such behaviour would not produce any dynamic inconsistency to be exploited by arbitrageurs. Thus, even if the money-pump argument had any validity, this validity would still not be independent of the decision context.

As we can see, then, dynamic consistency and money-pump arguments are not definitive principles to sustain exponential discounting as a descriptive theory of intertemporal choice. But, are they valid *normative* principles? Is it true that being dynamically consistent is necessarily 'good'? The answer is also 'no'. There is no ultimate

[55] The existence of punishing behaviour that goes against one's interests is now empirically well founded in the literature (see, for example, Thaler (1989) on the so-called ultimatum game).

reason why someone should obey his previous decisions, for we cannot systematically grant a past 'self' a higher moral authority than a present 'self'. As Strotz put it, we may ask ourselves "at which date should sovereignty inhere in the [decision] maker" (Strotz 1956, p179). The concept of consumer sovereignty has no meaning in the context of dynamic decision-making (Strotz 1956) and, for that reason, Samuelson himself regarded discounted utility as lacking normative legitimacy as a rational intertemporal choice model (recall the quote at the beginning of this chapter).

Despite all these considerations, economic thought has put the 'straitjacket' of dynamic consistency to intertemporal choice theories. First it has used the principle stating dynamic inconsistency is bad *per se* to automatically assimilate dynamic consistency to rationality. Then it has shown that only exponential discounting is completely free of inconsistencies, and thus the only possible rational intertemporal choice theory. Finally, it has used this normative support to argue that exponential discounting ought to be also the standard descriptive intertemporal choice model. Experimental work has uncovered the flaws of such reasoning. But the strong normative case for exponential discounting has restricted much of the attention to the exponential- vs-hyperbolic discounting debate, and blinded the many other fundamental problems in discounted utility we have reviewed in the previous section; anomalies that actually cast doubts on the many other assumptions underlying discounted utility theory.

Principles like dynamic consistency have of course helped to specify intertemporal choice theory; but they also have hindered the ability of the theory to evolve between the 50's and the 90's, and, to some extent, it continues obstructing its further development. The dynamic consistency requirement has proven particularly hard to withdraw from intertemporal choice, since its normativity relies upon a basic intuition identifying inconsistency with the bad[56]. In my view, this intuition is wrong. There is no ultimate reason why someone behaving only sometimes inconsistently is producing harm to himself, for there is no ultimate reason why the 'planner' makes always good plans for the 'doer'. There may indeed be reasons to consider that someone who is continuously abandoning his plans is producing himself harm; but the fact that someone's preferences are such that they may result in dynamic inconsistency *once* does not imply he will be condemned to repeated dynamic inconsistency, as if an iron rod had totally damaged his

[56] We could see the strong power of this moral intuition in the US presidential campaign 2004, where huge amounts of money were devoted to prove electors that one candidate was a "flip-flap", changing continuously his mind on important issues.

brain permanently. The identification of dynamic inconsistent preference structures with the bad relies thus also on an invariance assumption saying that preferences remain the same across all periods and circumstances, including the situation in which someone perceives he is stuck in a money-pump. Of course, any mathematical representation of choice needs to make such simplifications, or the theory would become completely impossible to handle; *but the choice of the normative principles that ought to define validity of theoretical models should not drag these simplifications.* There is thus no solid grounding to demand dynamic consistency to our intertemporal choice models.

Fortunately, today intertemporal choice seems to be starting to undo this straitjacket and rapidly generating valuable alternative theories that are free from its complex about dynamic inconsistency. Experimental work in intertemporal choice has been crucial in this process, since it has made the invisible straitjacket visible. Thanks to it we have developed an important body of knowledge regarding intertemporal decision-making. Nevertheless, the main conclusion we should extract from the first chapter in this dissertation is that we still lack *both* a normative *and* a positive theory of intertemporal choice. Normatively, discounted utility is *not* a definitive theory, although it is indeed much more solid as a normative than as a positive theory. Descriptively, as we have seen, discounted utility and, in general, all alternative models based on the idea of adding up discounted outcomes together, look *not* able to capture the fundamentals of behaviour. Chapters two and three in this dissertation aim thus at contributing to the task of finding a better intertemporal choice model. And in order to do this, it seems natural to investigate more deeply into the anomalies of the current theory.

1.10 Summary

Let me at this point summarize the main ideas contained in chapter 1:

1. The neoclassical economists already systematically studied intertemporal choice problems (section 1.2), but the first general formula describing intertemporal preferences was Samuelson's in 1937 (section 1.3), known as *discounted utility*, and based upon adding (section 1.4.1) up *exponentially* discounted values (section 1.4.2).
2. Disregarding Samuelson's advice, discounted utility established both as the normative and positive orthodoxy thank to (a) its ability to capture many insights into one single and parsimonious formula; and, more decisively, (b) the fact that it is the only model in the context of additive utility whose structure is completely immune to dynamic inconsistent behaviour, and thus the only apparently rational model (sections 1.4.1 and 1.4.2).
3. The hegemony of discounted utility lasted undisturbed until 1981, when Thaler first showed that behaviour was *not* dynamically consistent and therefore discounting also not exponential. A long series of other challenging experiments followed Thaler's, and hyperbolic discounting models appeared as a valid explanation of many anomalies (section 1.6). Possibly the most promising descriptive model that this experimental revolution produced is (β, δ)-discounting, also called quasi-hyperbolic discounting (section 1.6.3).
4. After hyperbolic discounting, many other anomalies in discounted utility have emerged. Possibly the most significant ones are the preferences observed over sequences of outcomes with particular shapes, and the magnitude effect. The first of these phenomena casts significant doubts on the positivity of time preference and on the validity of the additivity of discounted utility (section 1.7.3), while the second challenges the fundamental hypothesis of a time preference that is independent of the magnitudes at stake (section 1.7.2). Different alternative models have emerged to account for the many anomalies of DU observed, but no unified theory for rational intertemporal choice has yet established (section 1.8).

5. Also, although research in intertemporal choice has deepened our understanding of human temporal decision-making along the past years, it still has achieved neither a completely satisfactory normative theory, nor a positive one (section 1.9). New and better intertemporal choice models may come from the interaction between economics and neurobiology.

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Chapter 2
Excessive Discounting and Hyperbolic Discounting Explored under Experimental Method Variance

Summary

In intertemporal choice experiments people usually choose between smaller-sooner and larger-later amounts of money. That is, they make tradeoffs in terms of nominal amounts. Yet the currency of intertemporal tradeoffs in the outside world is usually the interest rate. In this study I tested whether two major phenomena that occur when trading off nominal amounts, *excessive discounting* and the *hyperbolic-interval effect*, would also occur when trade-offs are made in terms of interest rates. The answer is they are not. We conducted a large-scale (N=1960) internet study and found that when the tradeoffs were described in terms of nominal amounts, the discount rates were high and there was a sizable hyperbolic-interval effect (replicating earlier studies). When they were described as both amounts and interest rates, discount rates were much lower, and there was no interval effect. When the tradeoffs were described as interest rates only, discount rates were even lower, and the hyperbolic-interval effect was *reversed*. It seems that some of the most-cited results in intertemporal choice research are unique to a specific way of eliciting discount rates.

2.1. Introduction

The economic model of intertemporal choice unambiguously predicts how rational people will trade off money over time. Given a choice between a smaller amount to be received sooner and a larger amount to be received later, agents will choose based on their current financial status and their opportunities on the capital market. Those who are already investing money (i.e., saving) will take the smaller-sooner amount if and only if the rate of return, once transaction costs have been accounted for, is lower than the rate they would get from their best alternative investment having the same degree of risk. Their discount rate will then be equivalent to their best investment rate. Imagine someone who currently has money invested at 5% who is offered a choice between €100 now and €104 in a year. She will take the early €100, because over the year it can be transformed into €105. But she would take a later payment of €106, because this is more than she could earn otherwise. Those who have an immediate need for cash, on the other hand, will take the smaller-sooner amount if and only if its rate of return is no greater than their best alternative borrowing opportunity. Imagine someone who is hard up for cash but can borrow €100 at 10% interest. He will take €100 now over €109 in a year, but take €111 in a year over €100 now. Given that people vary in their circumstances, their discount rates for money will vary, but only within the range dictated by the capital market.

In 1981, Thaler tested this economic model with a series of experiments (see section 1.6), and in so doing introduced a method that has been used, with modest variation, in most subsequent studies (e.g., Benzion, Rapaport & Yagil, Chapman &

Elstein, 1995; Green, Myerson & McFadden, 1997; Kirby, 1997; Madden, Bickel & Jacobs, 1999; Read, 2001; Shelley, 1993). This method involves asking decision makers to choose between a smaller-sooner (SS) and larger-later (LL) outcome ("Would you prefer €200 in one month or €400 in ten months?"), or else to equate two delayed outcomes, such as stating what LL is equivalent to a specified SS ("How much would you demand in ten months to forgo receiving €200 in one month?").

Thaler's study revealed several striking deviations from the normative model, including two that are the focus of the present paper: excessive discounting (Ainslie & Haendel, 1983, called this 'monumental impatience') and the hyperbolic-interval effect.[57] Both these findings have been widely replicated (see reviews in Frederick, Loewenstein & O'Donoghue, 2002; Read, 2004). On one hand, excessive discounting means that people discount future outcomes more than they should based on the capital market they face. Because few people in the developed world will earn today much more than 3% on investments, or have to pay more than 20% to borrow money, their personal discount rate should be somewhere between these values.[58] Yet reported discount rates rarely get as low as 20%, and frequently reach levels of 100% or more (Frederick et al., 2002; Thaler, 1981). For very short delays, such as a few days or a week, the discount rate can go through the roof, reaching values of several thousand or even millions of percent.[59]
On the other hand, the hyperbolic-interval effect is that the discount rate decreases as the interval separating SS and LL increases. Thaler found that the median amount demanded to forego $250 immediately was $300 for a wait of 3 months (107% discount rate), and $500 for a wait of 3 years (26% discount rate).

Because most demonstrations of excessive discounting and the hyperbolic-interval effect are based on modest variations of Thaler's original method, the large number of replications may overstate the strength of the evidence. They may reflect people's true discount functions for money, but they may also be just reflecting how

57 As discussed in chapter 1, other anomalies discovered by Thaler (1981) are the sign effect (losses are discounted at a lower rate than gains) and the magnitude effect (larger amounts are discounted less than smaller ones).

58 This dissertation was written between 2005 and 2007. Within these dates, the best rate in countries like Spain or the UK for borrowing €2500 was around 15 %. For smaller amounts the rates would be higher, and one would normally use a credit card, which will mean a rate of approximately 19%. The best possible savings rate is around 5% -- but these are 'introductory offers' that depend on using an internet account – and investment in bonds or stocks can yield slightly more.

59 To take one example, the median respondent in Kirby and Marakovic's Experiment 1 (Kirby & Marakovic 1996), revealed an annual discount rate of well over 8 million percent over a three day delay. Viewed from the perspective of the actual preferences observed, however, this does not seem so outrageous – the respondent was indifferent between $25 now and $28.50 in three days.

people are asked. It is well known that different ways of asking people formally identical questions can yield significantly different stated preferences and even beliefs. Consequently, before interpreting a specific pattern of response, we need to establish how robust it is in the face of methodological variation. The present paper addresses this issue by eliciting intertemporal preferences using an apparently modest variation on Thaler's method: In most studies, such as Thaler's original ones, respondents make tradeoffs among nominal amounts[60], as in "€50 today versus €100 in a year." Yet in the 'real' world of everyday decision making it is more common to decide based on rates of return, as in "€50 today versus investing it for a year at 5%." Credit cards and mortgages are chosen because they offer the lowest rate, bonds and savings accounts are chosen because they offer the highest rate.[61]

We investigate whether people make the same intertemporal decisions regardless of the tradeoff currency, and learn they do not. Excessive discounting and the hyperbolic-interval effect characterize tradeoffs expressed in nominal amounts, but not those expressed in interest rates.

2.1.1. Interest rates and excessive discounting

The only previous study that compared intertemporal tradeoffs given different descriptions was the one by Coller and Williams (1999).[62] They compared intertemporal choices for nominal amounts (the 'money-only' description, in our terminology), with those for amounts combined with interest rates ('interest+money' description). The 'interest+money' description reduced the median discount rate by 7%.[63] Their results already suggest that providing information about interest rates can reduce the discount rate.

There are many possible reasons for this. One is that interest rates remind people they are dealing with money, which has a market price. Another is that they eliminate misconceptions about what different interest rates earn. To illustrate, imagine a consumer

[60] Nominal amounts do not take inflation into account. For instance, if prices go up by 5% a nominal payment of €110 in one year is a real payment of slightly less than €105.
[61] Coller and Williams (1999) put it thus another way, observing that in experiments outcomes are typically priced in a different *currency* (i.e., nominal amounts) than they are in the real world (interest rates).
[62] Harrison, Lau & Williams (2002) studied a sample of Danish consumers using a version of Coller and Williams' Interest+Money questions, but did not compare this description to any other. They did suggest, however, that this was the correct way to ask about discount rates.
[63] Coller and Williams did not conduct a single experiment with random assignment to groups, but rather a series of experiments with differing conditions. Therefore, we cannot rule out the possibility that irrelevant differences between experiments (e.g., non-equivalent samples, maturation, history, etc.) influenced their results.

who knows his best annual equivalent rate (AER)[64] is 6%, yet who also believes this yields €1.40 per annum for each €1.00 invested. Given this mistaken belief, he would rationally choose €100 now over €139 in one year. Asking him what future amount he would demand, therefore, confounds his desired interest rate with his false belief about what it will earn.

On the other hand, those who usually think of tradeoffs in terms of nominal amounts, and who are unfamiliar or misinformed about interest rates, might choose the wrong rate because they are mistaken about what it will earn. Consider, for example, an investor who is the mirror-image of the one just described. She wants €140 in a year for each €100 invested, and believes she will get this by investing at 6%. She will agree to a 6% rate as long as she doesn't know how little it will earn.

In our experiment we therefore tested discounting given three ways of describing the tradeoff currency: as nominal amounts, as interest rates, or as both. We predicted that the discount rate would be reduced whenever interest rates were given. Morever, because the usual way of thinking about these tradeoffs is in terms of rates, we expected that when both descriptions were given, the discount rates would be intermediate but closer to the interest rate only description.

2.1.2. Interest rates and the hyperbolic-interval effect

I earlier described the hyperbolic-interval effect, a term I used because it is frequently attributed to hyperbolic discounting[65] (e.g., Ainslie, 1975; Kirby, 1997; Mazur, 1987; Rachlin, 1989), according to which the discount rate is marginally decreasing in delay. Much evidence for hyperbolic discounting has recently been challenged because the method used to test it does not distinguish between the delay effect and the (true) interval effect.[66] I illustrate this with the aid of the following figure:

---

[64] The AER is the interest rate earned in a year taking compounding into effect. The formula is $\text{AER} = \left(1 + \frac{i}{n}\right)^n - 1$,
where *i* is the nominal (annual) interest rate, and *n* is the number of compounding intervals in a year. For example, a 5% annual interest rate compounded daily would correspond to an AER of approximately 5.12%.

[65] This should not be confused with *quasi-hyperbolic* discounting or *present-biased preferences*. According to quasi-hyperbolic discounting (Laibson, 1997) and present-biased preference (O'Donoghue & Rabin, 2000a), the discount rate is stationary after a 'jolt' of excess discounting applied to any delayed outcome. (see section 1.6.3).

[66] The argument that follows receives a fuller treatment in Read and Roelofsma (2003).

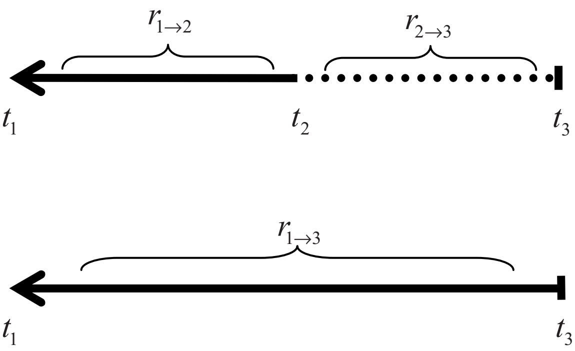


Figure 2.1

In a typical experiment, discounting is measured over intervals that start at the same time, but differ in length, such as t1→t2 versus t1→t3. If we use $r_{i\to j}$ to denote the annual-equivalent discount rate over an interval that starts at t_i and ends at t_j (i.e., with length $t_j - t_i$), the usual result is that r is greater over the shorter interval than over the longer one: $r_{1\to 2} > r_{1\to 3}$ for $0 \le t_1 < t_2 < t_3$. But when interpreting these results, researchers tipically assume that discounting over the long interval is the product of discounting over its parts,

$$\left(1+r_{1\to 3}\right)^{t_1-t_3} = \left(1+r_{1\to 2}\right)^{t_1-t_2}\left(1+r_{2\to 3}\right)^{t_2-t_3}$$

Obviously, this means

$$\left(1+r_{2\to 3}\right)^{t_2-t_3} = \frac{\left(1+r_{1\to 3}\right)^{t_1-t_3}}{\left(1+r_{1\to 2}\right)^{t_1-t_2}}$$

which, for $r_{1\to 2} > r_{1\to 3}$ (as usually observed in experiments), yields

$$\frac{\left(1+r_{1\to 3}\right)^{t_1-t_3}}{\left(1+r_{1\to 2}\right)^{t_1-t_2}} < \left(1+r_{1\to 2}\right)^{t_2-t_3}$$

and, consequently,

$r_{1\to2} > r_{2\to3}$

An analogous argument yields also $r_{1\to3} > r_{2\to3}$.

Therefore, assuming that discounting over the long interval is the product of discounting over its parts, together with the empirical observation of $r_{1\to2} > r_{1\to3}$, implies two testable predictions:

(1) $r_{1\to2} > r_{2\to3}$

(2) $r_{1\to3} > r_{2\to3}$

That is, that the discount rate for the second segment of the interval from t1→t3 is lower than that for either the first segment or for the interval taken as a whole.

Relationships (1) and (2) have rarely been tested directly, and have received little support. Relationship (2) has never been reported, and (1) only rarely. The more typical result is that discount rates are equal for the first and second part of the interval, and higher for the second part than for the undivided interval: (1´) $r_{2\to3} = r_{1\to2}$; (2´) $r_{2\to3} > r_{1\to3}$ (e.g., Baron, 2000; Gigliotti & Sopher, 2004; Holcomb & Nelson, 1992; Read, 2001; Read & Roelofsma, 2003). If we know all three values of $r_{i\to j}$, therefore, we can distinguish between the two theoretically important effects mentioned above:

A) the *delay effect* when the discount rate is lower for intervals that start later ($r_{1\to2} > r_{2\to3}$), which would be true hyperbolic discounting; and

B) the *interval effect* when the discount rate is lower for longer intervals ($r_{1\to2}, r_{2\to3} > r_{1\to3}$).

In this study, I obtained discount rates for three consecutive 6-month intervals, and for the corresponding 18-month interval that spanned them. I expected an interaction between the interval effect and tradeoff description: When the tradeoff was asked in nominal amounts, the prediction is the standard interval effect consisting of more discounting for longer intervals. But when it was asked in interest rates, I predicted either no interval effect or even a reverse-interval effect of less discounting for longer intervals.

This reverse-interval effect is, in fact, the norm in the financial marketplace. That is, longer investment periods yield higher returns. We can see this in bank accounts, which have to offer higher rates in exchange for a longer period of notice before savings can be accessed, because consumers value liquidity. In general, the longer the investment period, the higher the liquidity premium.[67] This leads us to expect that when pricing tradeoffs in terms of interest rates, people might demand higher rates for longer intervals. Indeed, we conducted a pilot study that suggests just this. We asked 112 respondents to an internet survey the following question:

> *Imagine that you have won €10,000. As a condition of the prize you can either (a) take all the money immediately, or (b) invest it for three years and earn interest. If you choose (b) you will not receive any of the money until the three years are up. We want to know what is the minimum yearly interest rate you would ask to compensate you for waiting three years to get the prize.*

After giving this three-year rate, the same respondents were asked to give a one year rate. On average, they demanded a greater rate for the longer interval (18%) than for the shorter one (14%), ($t(111)=3.1$, $p=.002$). In other words, these people showed the opposite of the traditional hyperbolic effect, and discounted more the longer the interval.

2.2. Experimental overview and hypotheses

I conducted an experiment with 16 conditions, corresponding to four descriptions (Interest-only, Interest+Money, Money-only, and No-investment) and four discounting intervals per description (1→7 months, 7→13 months, 13→18 months, and 1→19 months – three short intervals and one long one). The first three descriptions framed the tradeoff in terms of an investment decision, while the fourth was like the Money-only condition except that no mention was made of investment.

[67] The interest rate over the long term can be thought of as arising from two factors -- a liquidity premium, and expectations about future rates. Any constraint on liquidity pushes rates higher over longer periods, while expectations pushes these rates in the direction of those expectations. Since rates are, in the long run, equally likely to go up or down the norm is for long term rates to be greater than short term ones.

I tested five hypotheses. The first two concern the effect of describing options in terms of interest rates or nominal amounts. I predicted that providing interest-rate information would decrease discount rates:

H1. The Interest-only and Interest+Money conditions will yield lower discount rates than the Money-only condition.

I also predicted that providing both kinds of information would yield intermediate discount rates:

H2. Discount rates in the Interest+Money condition will fall between those in the Interest-only and Money-only condition.

Three further hypotheses concern the delay and interval effects. First, I expected no delay effect, and a standard interval effect only for nominal amounts:

H3. Discount rates for same-length intervals will be unaffected by variations in the delay to their onset.
H4. In the Money-only condition, the discount rate will be higher for shorter intervals than for longer ones.

On the other hand, however, I predicted a reverse-interval effect for the Interest-only condition:

H5. In the Interest-only condition, the discount rate will be higher for longer intervals.

A final question concerns the No-investment condition. I formed no specific hypotheses about it, although I anticipated it would mirror the Money-only condition (i.e., show the effects predicted in H2, H3 and H4). If there was any additional effect, I expected it to have higher rates than any of the investment-frame conditions.

2.3. Method

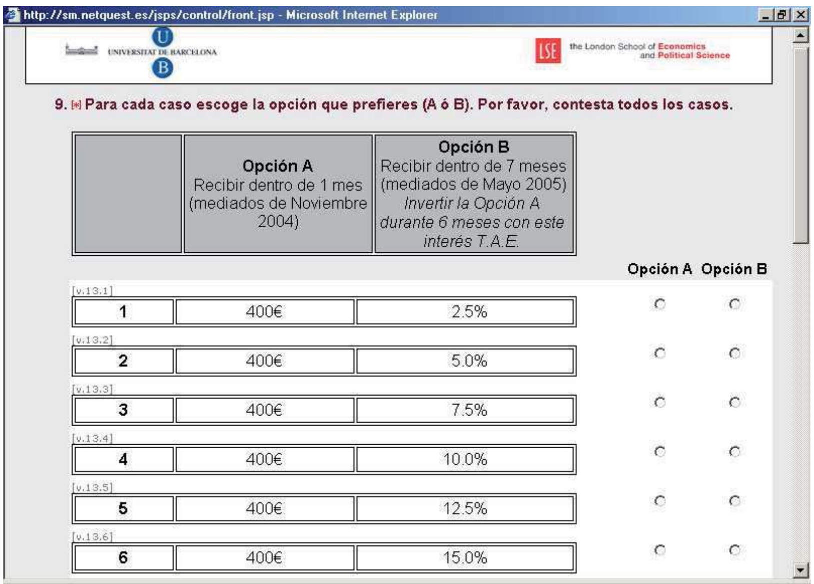
2.3.1. Sample

On October 14th, 2004 an invitation to participate in a study of "financial preferences" was sent to 3,936 members of *Metascore*, a representative panel of Spanish Internet users (see Annex 3 for more details on this representativity). The e-mail contained a link to one of the 16 questionnaires, programmed so that it could only be used once. The data-collection was successful: 64% of those invited opened the e-mail, 83% of these clicked on the link, and 94% of these finished the questionnaire, for a total of 1,960 completed.

The incentive to participate was a random lottery (e.g., Cubitt, Starmer & Sugden, 1998). Participants were (truthfully) informed that one of their choices would be paid for real, and that payment would be based on the choice made to one randomly drawn question.

2.3.2. Materials

Participants made 20 choices between Smaller-Sooner and Larger-Later options, presented in a tabular format (See Annex 2 for a description of the values used). The SS option was €400 and the LL option was the result from investing €400 for 6 or 18 months, presented either as money-only, as interest-only (see for example Figure 2.2) or as both.



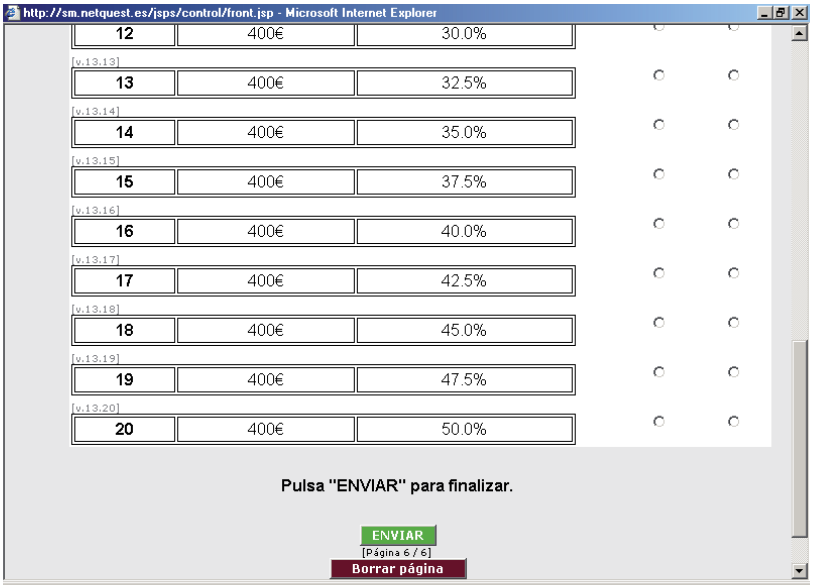


Figure 2.2

As can be seen, the timing of outcomes was described using both the month and year of receipt, and the time until receipt. Participants chose the preferred option by clicking on a radio button. Table 2.1 shows how the options were described in the four different conditions:

<table>
<tr><td colspan="3"><u>Description of choices for the four experimental conditions</u>
All conditions included all intervals for a complete 4 × 4 design. To avoid redundancy, this table shows each description assigned to one interval only.</td></tr>
<tr><th>Condition</th><th>SS</th><th>LL</th></tr>
<tr><td rowspan="2">Interest-only</td><td>OPTION A: received in 1 month (mid November 2004).</td><td>OPTION B: received in 7 months (mid May 2005).

Invest Option A for 6 months at the following AER.</td></tr>
<tr><td>€ 400</td><td>2.5%</td></tr>
</table>

Interest+Money	**OPTION A**: received in 7 months (mid May 2005).	**OPTION B**: received in 13 months (mid November 2005). Invest Option A for 6 months and receive the following at end of the investment period (AER in parentheses).
	€ 400	**€ 404 (2.5%)**
Money-only	**OPTION A**: received in 13 months (mid November 2005).	**OPTION B**: received in 19 months (mid May 2006). Invest Option A for 6 months and receive the following at end of the investment period.
	€ 400	**€ 404**
No-investment	**OPTION A**: received in 1 month (mid November 2004).	**OPTION B**: received in 19 months (mid May 2006).
	€ 400	**€ 404**

Table 2.1

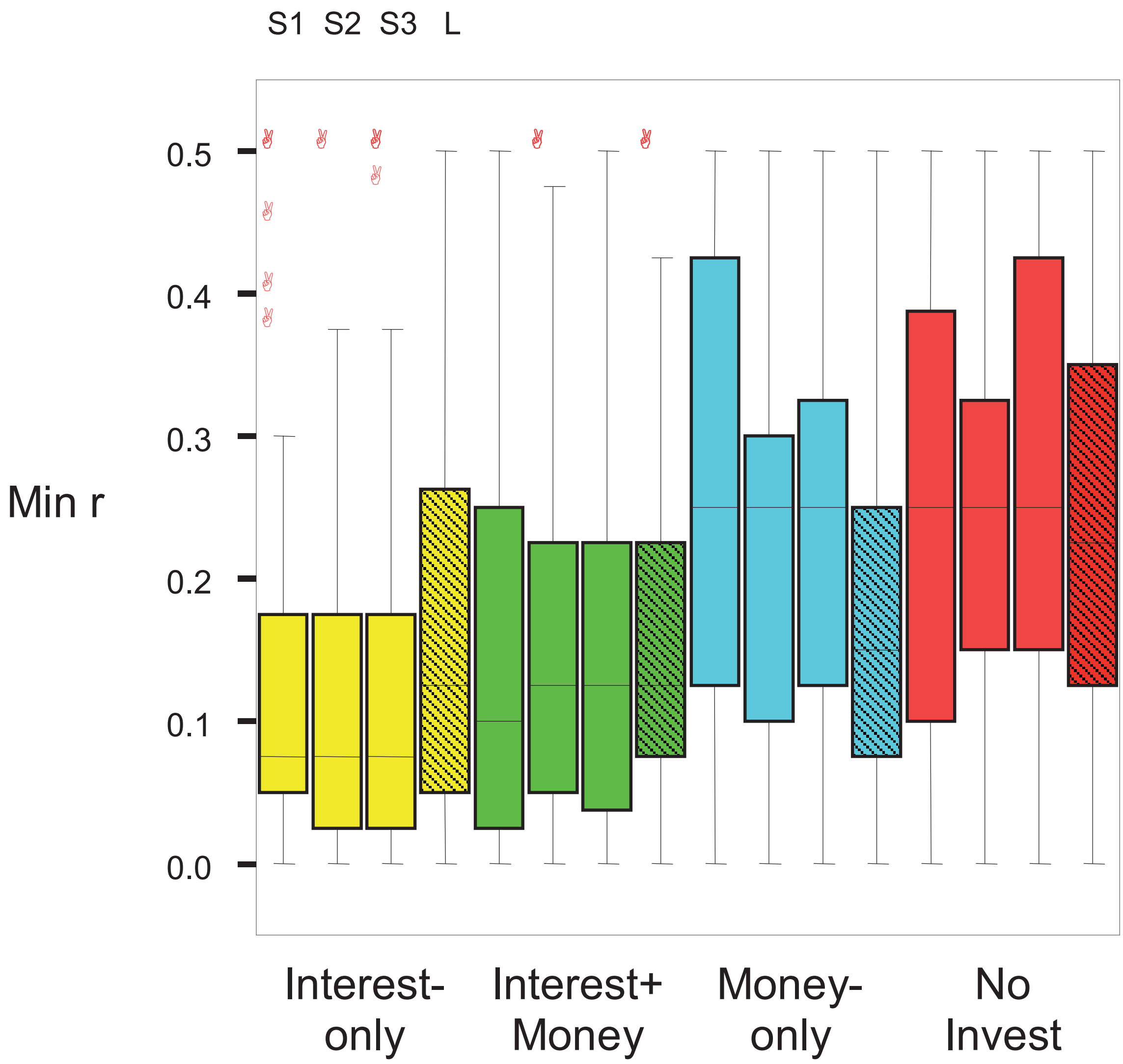
2.4. Results

The results are summarized in Figure 2.3 in the form of box plots, and in numerical form in Table 2.2. The dependent variable is *Min*, the lowest discount rate consistent with the respondent's choices. *Min* was computed as follows:

1. Min = X%, if the respondent preferred SS for every interest rate up to X% and then switched to LL for X+2.5%;
2. Min = 0%, if the respondent preferred LL for every interest rate;
3. Min = 50%, if the respondent preferred SS for every interest rate.

Case 1 describes the normal situation in which the respondent chooses SS for low interest rates, and then changes to a preference for LL. The true discount rate will then be between Min (the highest rate at which they chose SS) and Min+2.5% (the lowest rate at which they chose LL)[68]. The discount rate of respondents who always chose LL could take any value below 2.5%, so we coded this as a Min = 0%. The discount rate of respondents who always chose SS could take any value above 50%. Because there is some uncertainty about the range of possible discount rates when Min is 0% or 50%, the median Min is the most accurate measure of central tendency.

[68] It is not possible to interpret the responses of those who switched back and forth, or chose LL at lower interest rates and SS at higher ones. Data from these respondents was not analysed.



Box plot showing results from all conditions
S1, S2, S3: Short intervals, 1→7, 7→13, 13→19 months respectively
L: Long interval, 1→19 months

Figure 2.3

Descriptive statistics for all conditions. Means and medians are based on the minimum discount rate (*Min*). True discount rates fall between *Min* and *Min*+2.5%.

Condition	Interval	Mean	σ	Median	Max*	Included	Excluded**	N
Interest-only	**1→7**	11.5%	.12	7.5%	4%	109	16	125
	7→13	10.2%	.11	7.5%	2%	106	12	118
	13→19	12.0%	.13	7.5%	6%	105	17	122
	1→19	17.5%	.16	12.5%	10%	112	6	118
Interest+Money	**1→7**	16.1%	.16	10.0%	12%	99	12	111
	7→13	15.4%	.15	12.5%	7%	107	8	115
	13→19	14.5%	.13	12.5%	7%	123	9	132
	1→19	16.8%	.14	12.5%	8%	112	8	120
Money-only	**1→7**	26.4%	.17	25.0%	19%	105	2	107
	7→13	23.7%	.16	25.0%	13%	135	0	135
	13→19	23.8%	.16	25.0%	16%	122	5	127
	1→19	16.8%	.12	15.0%	2%	124	6	130
No-investment	**1→7**	24.9%	.17	25.0%	19%	111	3	114
	7→13	24.8%	.15	25.0%	13%	117	3	120
	13→19	26.6%	.16	25.0%	17%	133	5	138
	1→19	23.9%	.15	22.5%	8%	124	4	128

* Percent of respondents who always chose the SS option.

** Number of subjects who switched more than once. They were excluded from the analysis.

Table 2.2

My Hypothesis 1 was that the Interest-only and Interest+Money conditions would show the lowest discount rates. Table 2.2 shows clear evidence for this: the median *Min* was 25% in the Money-only condition and between 7.5% and 12.5% in the two interest-rate conditions. A median test comparing the combined Interest-only and Interest+Money conditions to the Money-only condition confirmed this, $\chi^2(1)=89.4$, $p<10^{-5}$. As can be seen in Figure 2.3, however, this relationship holds only for the 6-month intervals, an observation confirmed by separate median tests:

Interval (months)	$\chi^2(1)$	
1→7	33.1	$p<10^{-5}$
7→13	44.2	$p<10^{-5}$
13→19	34.8	$p<10^{-5}$
1→19	0.8	*p* ns

Table 2.3

This finding is examined below, when discussing Hypotheses 4 and 5.

Hypothesis 2, that discount rates in the Interest+Money condition would fall between those in the Interest-only and Money-only condition, was also supported. As already discussed, the Interest-only and Money-only conditions differed only when the interval was six months, and for all six month intervals there was intermediate discounting in the Interest+Money condition. Median tests comparing the combined Interest+Money description to the separate ones revealed they differed significantly:

	Interest+Money compared to:			
Interval (months)	Interest-only		Money-only	
1→7	2.7	$p=.10$	19.0	$p<10^{-4}$
7→13	6.5	$p=.01$	20.5	$p<10^{-4}$
13→19	5.7	$p=.02$	21.8	$p<10^{-4}$

Table 2.4

In line with our earlier discussion, this suggests the interest rate description makes people more patient, and the money description makes them less so. Moreover, the

Interest+Money rates were closer to the Interest-only than Money-only ones, suggesting that interest rate information predominates.

Hypothesis 3 is that discount rates for same-length intervals will be unaffected by the front end delay. This received strong support. Within each question frame, discount rates were virtually identical for all the short intervals. An overall median test ($\chi 2(2)=1.3$), as well as separate analyses for each frame ($\chi 2(2)=0.87$, 0.85, 3.69 and 0.49 respectively) indicated no hint of a significant effect.

Hypothesis 4 and 5 will be discussed together. I predicted that the discount rate would increase with interval length in the Money-only condition, and decrease with interval length in the Interest-only condition. Both predictions were supported – indeed, the 18-month rates were virtually identical in both conditions. Median tests comparing long with short intervals revealed a significant effect in both the Money-only ($\chi 2(1)=28.1$, $p<10-5$) and Interest-only ($\chi 2(1)=15.0$, $p<10-3$) conditions. Because the discount rates in the Interest-only and Money-only condition are virtually identical, I expected the Interest+Money condition (predicted to fall between them) to yield the same discount rate, and this is what happened. A median test comparing the 18 month interval in all three investment frame conditions revealed no difference, $\chi 2(2)=0.87$.

Comparisons with No-investment condition. I now consider the effect of the 'Investment' frame. I expected results from the No-investment condition to be like those from the Money-only condition, perhaps with a higher overall discount rate in the No-investment condition. The No-investment condition was as expected in that the discount rates were substantially higher than when interest-rates were provided. However, while I expected that the 18 month interval in the No-investment condition would yield a lower discount rate than the 6 month intervals, there was little evidence of such a difference. The discount rate was slightly lower, but not significantly so. Moreover, it was significantly higher than the corresponding 18 month interval in the Money-only condition, $\chi 2(1)=9.3$, $p=.003$.

While I cannot rule out the possibility that the difference between the No-investment and Money-only condition is real and meaningful, I believe that the No-investment condition reveals an unrepresentative effect. The interval effect, for outcomes described as nominal amounts, has been replicated many times. Moreover, even these data give some evidence that discounting is lower in the long interval conditions.

2.5. Comparison with Coller and Williams (1999)

Coller and Williams compared discounting over 2-month intervals, in a Money-only versus an Interest+Money frame (their Experiments 1 and 2). They found, as I did for 6-month intervals, that discount rates were lower in the Interest+Money frame. As can be seen in the table below, which shows the median discount rates in their study and mine, their discount rates were slightly higher than mine. But the differences are small, and can be readily accounted for by the fact that they studied students, whereas we studied members of the general population: Harrison et. al. (2002) reported that the discount rate was 4% higher for students than other groups.

	My study		Coller & Williams'	
Interval (months)	Interest+ Money	Money-only	Interest+ Money	Money-only
1→3			17.7%	25.2%
1→7	11.25%	26.25%		
1→19	13.75%	16.25%		

Table 2.5

2.6. Discussion

This study is one of the largest ever experiments into intertemporal choice, and differs from most others in that (a) the respondents were members of the general public and not students, (b) incentives were provided by means of a random lottery, (c) discounting was measured over several intervals differing only in the delay to their onset, and, most importantly, (d) future payments were framed in terms of interest rates as well as (the usual) nominal amounts. The effect of this framing was substantial. For short intervals, discounting was much lower when payments were framed as interest rates. And when both kinds of information were given – in the Interest+Money frame – the pattern of discounting was consistent with two principles of economic rationality. First, the average discount rate was close to the market rate: the mean was between 15 and 17.5%, the median between 10 and

12.5%, somewhere between what it would cost to borrow such a small amount and what could be earned from investing it. Second, the discount rate was stationary, being affected by neither the delay nor the interval length. Since psychological accounts of intertemporal choice suggest people routinely violate both these principles, it is startling to discover that they do not violate them when they are provided with such a small amount of additional information (interest rates) on top of what they normally receive in experiments. Such a striking example of method variance has great implications for how we interpret experimental studies of intertemporal choice.

In general, studies of individual judgment and choice, including intertemporal choice, can be divided into two categories based on the kinds of conclusions sought. The first looks for generalizations about groups; the second looks for generalizations about circumstances. An example of the first category is found in studies of confidence judgments, which seek to test the generalization that "people are overconfident." Researchers have tested this generalization in many ways. Although some controversy remains, it appears that while overconfidence can be reduced, it does not go away. Because overconfidence has proved so robust when tested in so many ways, we can be justified in accepting the generalization.

The second category of research focuses on method variance, or the effects of circumstances on preferences. This research is exemplified by Kahneman and Tversky's (1984) study of framing effects. People's preferences over the same gamble differ depending on whether it is framed as a gain or loss – sometimes they are risk averse, other times risk-seeking. It would clearly be unwarranted to conclude that "people are risk-averse." Rather, we have to offer contingent conclusions, such as "risk attitude depends on the question frame in such-and-such way." Because very few generalizations survive the scrutiny of multiple-methods, most research programs in decision making are of the second type, investigations of how people want A when asked question X, and want B when asked question Y.

The conclusions from studies of intertemporal choice are usually presented as being of the first type. The generalizations tested are that 'people can be characterized by excessive discounting and the hyperbolic-interval effect.' But, unlike the study of overconfidence, very few research methods have been attempted. Indeed, as already discussed, virtually everyone has adopted minor variations on Thaler's method (1981).

When we explore the effects of adopting different methods of measurement, the results become quite different. In this chapter, I showed that the imputed discount rate and the form of the discount function depend on the currency in which intertemporal choices are made. Other studies have shown that the discount rate depends on how time is described (see Read, Frederick, Orsel & Rahman (2005)), on whether the tradeoff involves delaying or speeding up the receipt of an outcome (Loewenstein (1988)), and that the form of the discount function depends on whether questions are answered using choice or matching (Ahlbrecht & Weber (1997); Read & Roelofsma (2003)), and on whether people answer a series of questions by moving backwards or forwards in time (Malkoc & Zauberman (2005)).

To sum up, I suggest that research into intertemporal choice needs to develop in the way that other areas of judgment and decision making have, by caring about the decision circumstances. Any claims made about either the magnitude of the discount rate, or the form of the discount function, should be recognized as contingent claims that are not generalizations from multiple methods, but a description of what happens when a specific methodological choice is made.

Chapter 3
Magnitude Effect and Hyperbolic Discounting in the Choice of Constant Sequences

Summary

Existing literature in both Psychology and Economics shows that people are more patient when choosing among constant sequences of outcomes than when choosing among single isolated outcomes. This effect has been attributed to hyperbolic discounting (Ainslie and Monterrosso 2003, Kirby and Guastello 2001, Ainslie 1986) or similarity (Rubinstein, A. 2003). In experiment 1 (n=501) I show that both explanations are incorrect, and show that the magnitude effect is a better explanation of this effect, although not the only one. People show more patience over sequences because a higher amount is at stakes, and because more outcomes are compared to one-another. In a second experiment (n=1.482), I further explore how outcomes in a constant sequence are discounted, and find that only delay to first outcome seems to play a role in the choice of small constant sequences, while no discounting is observed for later subsequent outcomes, something I label *zero intra-sequence discounting*. These results imply new departures from standard assumptions in discounted utility.

3.1. Introduction

The parsimonious model proposed for inter-temporal choice by Samuelson (1937) describes an individual's preferences over sequences of future outcomes by the *sum* of the present value of each outcome in the sequence (see section 1.4.1). Under the *discounted utility* model, a sequence is a mere collection of outcomes to be evaluated separately with the help of a 'discount function' that depends upon a single parameter capturing an individual's rate of time preference (a unique discount rate that yields constant discount factors across periods). Moreover, the standard approach in economics has been for many years ever since to consider that such a discount rate can be obtained from a choice among single outcomes and then be used to predict behaviour in the choice of more complex decision objects like wages profiles, environmental consequences or health conditions during a certain period of time, which constitute in fact sequences of outcomes or consumption streams, as they have been also called.

As we saw in chapter 1, this approach has unfortunately proven problematic. Preferences over sequences of consumption are not always reducible to preferences over single outcomes in the way discounted utility does. During the last 15 years, several empirical studies have clearly shown how individuals have *specific* preference patterns concerning 'gestalt' (shape) properties of the sequences they evaluate, which sometimes imply a violation of independence of equal sub-streams (IES) (see section 1.4.1, axiom 3)[69]; in section 1.7.3 I commented typical examples as the preference for increasing sequences or the preference for uniformly spread outcomes. In fact, Loewenstein & Prelec (1993) build on these empirical findings to develop a model in which they explain preferences over sequences by discounted utility plus two additional factors: the degree of improvement of a sequence and its degree of spread. Their underlying hypothesis is that individuals have a preference for improvement – possibly due to deriving utility from departures from an increasing reference-point -, and also a preference for the uniformly spreading of outcomes in a sequence –possibly also due to the effect of reference dependent utility.

In sum, preferences for sequences have been found to be *essentially* different from preferences for single outcomes. Or, put in other words, empirical research in intertemporal choice has proven that, contrary to what discounted utility assumes, 'gestalt' matters.

Only 'gestalt' matters?

Now the question we may ask ourselves is whether all observed anomalies are *only* due to these 'gestalt' properties. Suppose there were no 'gestalt' differences among two sequences; can we *then* explain preferences with the parsimonious discounted utility model? Unfortunately, the answer is no. Several empirical results have shown that people are more patient when evaluating constant sequences than when evaluating single outcomes, even if, in such case, the compared sequences have equal, uniform shape, making it impossible to call upon any 'gestalt' effect to account for such extra patience. In the search for an alternative explanation, existing literature has attributed this effect –which I will call the constant sequence effect- to either hyperbolic discounting (Ainslie, 1986; Kirby & Guastello 2001; Ainslie & Monterosso 2003) or procedural decision making based on similarity assessments (Rubinstein, A. 2003).

But a third, more plausible explanation remains yet untested: the magnitude effect (see section 1.7.2). More patience when choosing among constant sequences may occur

[69] Not all 'gestalt' preferences necessarily violate this principle: a preference for increasing sequences can be modelled maintaining the separable discounted utility model and assuming *negative* time preference instead of the more natural positive time preference (goods preferred earlier rather than later).

simply because more money is at stakes. Thus, the first objective of this chapter is to test whether the higher patience observed in the choice of constant sequences is due to the magnitude effect and not to hyperbolic discounting or similarity-based decision-making (Experiment 1). A second objective of this chapter is to investigate how discounting occurs in the choice of constant sequences. In particular, I want to test whether people value a single, big amount of money differently than a sequence that starts at the same point in time and is made of small outcomes adding up to the same total amount (Experiment 2). My plan is as follows: in section 3.2 I will review the evidence for the constant sequence effect and formulate a new explanation for it. In sections 3.3 and 3.4 I will present experiments 1 and 2, and finally, in section 3.5 I will discuss the findings and conclusions.

3.2. The Constant Sequence Effect

Let me begin by defining what I will call a constant sequence. The framework will be the same as presented in section 1.4.1 (in the case of equal consumption sets for all periods): periods will be denoted by $i \in I = \{1, 2, ..., n\}$, possible consumption in each period will consist of elements of an arbitrary set X, and streams of consumption will consist of elements of the Cartesian product X^n. I will suppose preferences are captured by a weak order $\succeq$ defined over X^n that satisfies axioms 3, 4 and 5 (i.e., that admits an additive representation with weights as in Theorem 2, section 1.4.1).

DEFINITION (constant sequence)

Let periods be denoted by $I = \{1, 2, ..., n\}$. A stream of consumption $x = (x_1, x_2, ..., x_i, ...x_n) \in X^n$ is a constant sequence if and only if there exist $S \subseteq I$ and $a \in \mathbb{R}, a \neq 0$ such that:

1. $x = a \cdot 1_S$, where 1_S is the vector with all components whose index belongs to S equal to one, and all other components equal zero.
2. For $S = \{i_1, i_2, ..., i_s\}$ with $i_1 < i_2 < ... < i_s$, the following is satisfied:

$$i_2 - i_1 = i_3 - i_2 = ... = i_s - i_{s-1}$$

A constant sequence is thus any stream of consumption with equal amounts that are uniformly spread. For example, $(2,2,2,2)$, $(0,3,0,3,0,3)$ or $(0,0,4,4)$ all are constant sequences.

Now the constant sequence effect consists in the observation that people show more patience (less discounting) when choosing among constant sequences than when choosing among single outcomes. Current evidence includes two experimental psychology studies, one in humans (Kirby & Guastello (2001) and the other one in rats (Ainslie & Monterosso 2003); and one economic decision-making experiment in Rubinstein, A. (2003). I will consider first the evidence from experimental psychology, and present the evidence in Rubinstein, A. (2003) later, together with the author's explanation of this effect.

3.2.1. Empirical Evidence

Kirby & Guastello (2001) presented their subjects the following choice task: (example for one particular individual[70])

A. Receive $7,90 today
B. Receive $8,80 in 6 days
C. Receive $7,90 today, in 10 days, in 20 days, in 30 days and in 40 days.
D. Receive $8,80 in 6 days, in 16 days, in 26 days, in 36 days and in 46 days.

The authors found that 21 out of 22 subjects incurred in a preference reversal when making such choices, preferring A over B but D over C. These results are incompatible with the discounted utility model, since, as we saw in section 1.4.1, they constitute a violation of stationarity (axiom 7). Clearly, for any δ $(0<\delta\leq 1)$ to be the daily discount factor, if someone chooses A over B,

$$\delta^0 u(7,90) > \delta^6 \cdot u(8,80)$$

[70] In their study, Kirby and Guastello presented each subject with specific numbers and delays, according to previously revealed individual discount rates and delay to reversal, in the spirit of Kirby & Herrnstein (1995).

which implies, for any $k \geq 0$,

$$\delta^k \cdot u(7,90) > \delta^{k+6} \cdot u(8,80)$$

Now since this implies that every element in sequence C is preferred to his 'counterpart' in sequence D, this implies C preferred to D, contrary to what the authors observed. In their study, Kirby and Guastello analysed also whether this effect persisted when the "linking" of the outcomes in the sequences was loosened. In a condition they called free-linking condition, they asked subjects to choose among A and B, and indicated the subjects they would afterwards sequentially face five identical choices (to occur in the same delays as in C and D). Also, in a condition called suggested-linking condition, they asked subjects the same as in the free-linking condition, but mentioning subjects the fact that if they started choosing the smaller-sooner option, they probably would also do so in all subsequent choices. As we can see in Figure 3.1, the percentage of preference reversals appears to be dependent on the degree to which a collection of outcomes is presented as an imposed sequence or rather as a mere collection of repeated choices.

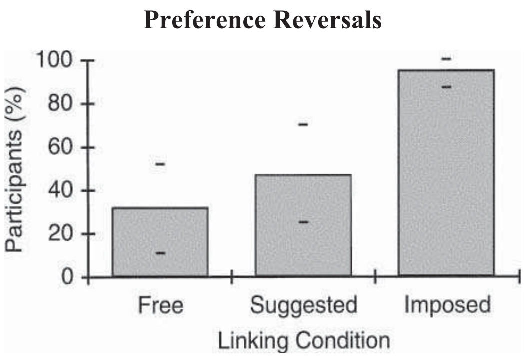


Figure 3.1

The authors found a strong effect for constant sequences (imposed-linking condition), and still some effect for both the free- and suggested-linking conditions. Clearly, constant sequences were perceived as something different than just a series of single outcomes. People appear to behave more patiently when facing sequences. Moreover, such a result was also found in rats. Ainslie & Monterosso (2003) found what they call a "bundled-rewards" effect, meaning rats chose a larger-later option over a smaller-sooner one more often when the choice was among constant sequences than when it was among single isolated rewards.

3.2.2. The Hyperbolic Discounting Explanation

Now, why would people behave more patiently when facing constant sequences of outcomes? According to Ainslie, the answer can be found in hyperbolic discounting. The intuition behind his argument is the following: the more elements we add to a sequence, the farther away they are, which means the higher become the period discount factors we use to evaluate them, implying the relatively more attractive become the larger-later outcomes. Hyperbolic discounting -Ainslie and others claim- is able to explain a preference reversal as the one observed by Kirby and Guastello[71]. And it is again something that exponential discounting cannot explain. Let me illustrate their idea with an example:

Suppose $I = \{0,1,...,7\}$ is the set of days in a week, beginning in day zero (today), and suppose also possible consumptions in each period are positive (or zero) amounts of money. Take the following constant sequences:

$$a = (8,0,0,0,0,0,0,0) \;\; b = (0,0,0,9,0,0,0,0)$$
$$c = (8,0,8,0,8,0,0,0) \;\; d = (0,0,0,9,0,9,0,9)$$

Imagine we observed the following inconsistent preferences:

$$a \succeq b$$
$$d \succeq c$$

[71] In fact, and despite the fragility of this argumentation –as we shall see-, Ainslie and others have even developed a whole theory of self-control based on it.

Under hyperbolic discounting, daily discount factors δ_i would be increasing, so that $\delta_{i+1} > \delta_i$ for all $i \in I$. Therefore, $a \succeq b$ and $d \succeq c$ could perfectly be explained by the fact that second and third (non zero) elements in the sequences are discounted less than first (non zero) elements. For example, the following hyperbolic[72] discount factors (increasing whenever $i \geq 1$):

$$\delta_i = \left(\frac{1 + 0,042 \cdot (i-1)}{1 + 0,042 \cdot i} \right) \qquad \text{if } \; i \geq 1$$
$$\delta_0 = 1 \qquad \text{if } \; i = 0$$

would explain the previous preferences taking, for example, $u(x_i) = x_i$, since

$$u(8) \geq \prod_{i=0}^{3} \delta_i u(9)$$
$$8 \geq (0,8881) \times 9$$
$$8 \geq 7,99$$

and, at the same time,

$$\prod_{i=0}^{3} \delta_i u(9) + \prod_{i=0}^{5} \delta_i u(9) + \prod_{i=0}^{7} \delta_i u(9) \geq \delta_0 u(8) + \prod_{i=0}^{2} \delta_i u(8) + \prod_{i=0}^{4} \delta_i u(8)$$
$$(0,8881) \times 9 + (0,8264) \times 9 + (0,7728) \times 9 \geq (1) \times 8 + (0,9225) \times 8 + (0,8562) \times 8$$
$$22,39 > 22,23$$

According to Ainslie, thus, increasing discount factors are able to explain the constant sequence phenomenon while, as we have seen before, exponential discounting –meaning constant discount factors- is not. Moreover, Ainslie and others have claimed the constant sequence effect constitutes a further confirmation of hyperbolic discounting.

3.2.3. Rubinstein's Alternative Explanation Based on Similarity

Unfortunately, there is a very simple and obvious objection to the previous argument by Ainslie. What if you constructed the constant sequences backwards (towards the present),

[72] This corresponds to the standard Mazur discounting model. (See section 1.6.1)

starting from a certain future moment, rather than forwards? In that case, every added outcome would be discounted *more* (due to a lower per-period discount factor), making the difference among both sequences move in favour of the shorter-sooner one. The prediction of hyperbolic discounting in that case would be anything but *not* a reversal from less patient to more patient. Individuals acting according to hyperbolic discounting should start being more patient for a short and *distant* sequence, and possibly end up being less patient when new outcomes were added towards the present. But, as I will show, this is not the kind of empirical evidence we find.

In a series of recent experiments, Rubinstein, A. (2003) has precisely shown that hyperbolic discounting can be refuted experimentally inducing in the same way exponential discounting has been refuted before[73]. In his paper, three experiments conducted with students show situations in which hyperbolic discounting is refuted, while a procedural approach based on similarity can, according to Rubinstein, explain observations better. I will comment here only the second of his experiments, the only one involving constant sequences.

Rubinstein presents the following four options:

A		Apr 1 $1000			Jul 1 $1000			Oct 1 $1000		Dec 1 $1000
B	Mar 1 $997			Jun 1 $997			Sept 1 $997		Nov 1 $997	
C										Dec 1 $1000
D									Nov 1 $997	

Figure 3.2

The hyperbolic discounting approach predicts that every subject choosing D over C will choose B over A. The reason is that if he is ready to sacrifice $3 to advance the payment

[73] The moral in Rubinstein's story, as he famously writes in this paper, is that economists should try more profound modifications of inter-temporal choice theory than hyperbolic discounting, and "open up the black box of decision-making", instead of trying out minor modifications in the standard model.

from December to November, then he must be even more ready to do it when periods are closer in time, since (δ_i) is weakly increasing (and thus, per-period discount factors get lower the closer periods are). But Rubinstein observes a strong preference reversal: 22% of the subjects chose D over C but A over B (and only 6% chose C over D and B over A). People apparently were more patient when choosing among the sequences than when choosing among the single outcomes and this could not be because of hyperbolic discounting, as Ainslie proposed.

Why then? Rubinstein presents a procedural decision-making approach as a possible explanation for this phenomenon, an explanation that departs clearly from the discounted utility theory. According to Rubinstein, when people evaluate two pairs (x,t) and (y,s), they may be going through a three-stage decision procedure using two similarity relations (one in the money dimension and one in the time dimension):

I. The decision maker looks for dominance: if $x > y$ and $t < s$ then he chooses (x,t)
II. The decision-maker looks for similarities between *x* and *y* and between *t* and *s*. If he finds similarity in one dimension only, he determines his preference using the dimension in which there is no similarity.
III. If the first two stages are not decisive, the choice is made using a different criterion.

Using this procedure, based on the notion of similarity (see Luce 1956, Tversky 1977, Rubinstein 1988), Rubinstein explains his results as follows: "…many subjects viewed the alternative as a pair, a sequence of dates and a sequence of $ amounts. The sequence of dates (April1, July1, Oct1, Dec1) was considered similar to the sequence (March1, June1, Sept1, Nov1) whereas the sequence of payments ($1000, $1000, $1000, $1000) was considered less similar to ($997, $997, $997, $997) than ($1000) was to ($997)." The idea is that when moving from single outcomes to sequences, the small difference among two very similar outcomes becomes significant, since this difference occurs several times in the sequence. And this triggers a different decision-stage. When choosing among sequences, people would be deciding according to stage II: since time dimension is similar, and money dimension favours A over B, then they choose A. While with single outcomes, both the time dimension and the money dimension are perceived similar, and people would thus jump to stage III, and decide according to something different.

While realizing the relevance of similarity to decision making, I do believe Rubinstein's explanation is wrong. As I will show in my Experiment 1, it is possible to replicate Rubinstein's results while completely eliminating the similarity of sequences A and B in the time dimension, which makes the three-stage procedural explanation impossible.

3.2.4. Magnitude Effect as an Alternative Explanation

Surprisingly, neither Rubinstein nor any of the experimental psychologists mentioned above do consider the most natural explanation[74] for the higher patience observed in the choice of sequences: the magnitude effect (see section 1.7.2). It is a robust finding in inter-temporal choice that smaller outcomes are discounted more than large ones. Thaler (1981), for example[75], found the following average annual discount rates depending on amounts:

Amount	Average Annual Discount Rate
$60	139%
$350	34%
$4000	29%

Table 3.1

Now, when people evaluate a sequence of outcomes, the total amount at stakes is in fact the sum of all amounts in the sequence. Consider for instance the following options:

A. Receive $60 in one year
B. Receive $30 now
C. Receive six annual payments of $60 starting in one year

[74] This explanation is not the only alternative to Rubinstein's. His results could also be explained by increasing discount rates, as was observed by Fredrick, Loewenstein & O'Donoghue (2002), footnote 16.
[75] See among others Kirby & Marakovic (1996) and Green, Myerson & McFadden (1997).

D. Receive six annual payments of $30 starting now

If people's time preferences are affected by the total amount at stakes in a degree as the one found in Thaler's study, we should expect many people choosing B over A but C over D. In that case, the constant sequence effect would just be a different manifestation of the well-known magnitude effect. And this effect would predict preference reversals both when considering onwards constructed sequences and when considering backwards constructed ones.

3.3. Experiment 1

Research Hypothesis

My Experiment 1 was designed to test whether the constant sequence effect persists when both the hyperbolic discounting and the similarity based explanations are ruled out. Thus, my research hypothesis number one (H1) is that neither one nor the other is a necessary condition for a constant sequence effect. My second hypothesis (H2) is that the constant sequence effect disappears when you control for the magnitude effect. In other words, I expect the constant sequence effect to disappear when single choices are among outcomes amounting to the same total value as the sum of outcomes in the sequences. Both hypothesis 1 and 2 will be tested in experiment 1.

3.3.1. Design

The questions were as follows[76]:

[76] Here I have omitted both the verbal statement and the intensity question scale. For an exact display of a questionnaire's page see Annex 2. Later in this section I describe the intensity of preference questions that followed every question (Q1-Q4).

		OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
Q 1	A				**€200**					
	B								**€210**	

		OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
Q 2	A		**€200**	**€200**	**€200**	**€200**				
	B						**€210**	**€210**	**€210**	**€210**

		OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
Q 3	A				**€800**					
	B								**€840**	

		OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
Q 4	A				**€200** **€200** **€200** **€200**					
	B								**€210** **€210** **€210** **€210**	

Figure 3.3

Consider only Q1 and Q2. This design was intended to rule out either a hyperbolic or a similarity-based explanation of an eventual constant sequence effect (=individuals choosing A in Q1 but B in Q2). The intuition of why it rules out hyperbolic discounting is as follows: if a hyperbolic discounter takes A over B in Q1, then he is willing to sacrifice €10 in order to advance consumption from period 8 (May) to period 4 (Jan). And this, because of increasing discount factors δ_i, means that he must also prefer (€200, DEC) to (€210, APR), and (€200, NOV) to (€210, MAR). Thus, this person would prefer the first three €200 outcomes in Q2 better than their counterparts in the €210 sequence. And this preference is *not* to be compensated by an eventual preference of (€210, JUN) over (€200, FEB); not in any standard hyperbolic discounting model[77]. Let me next give a formal proof:

Consider a hyperbolic discounter whose time preferences would be captured by a positive and monotonically increasing instantaneous utility function for money u, together with a general hyperbolic discounting function as in Loewenstein & Prelec (1992):

$$D(i) = \frac{1}{(1+\alpha i)^{\frac{\beta}{\alpha}}},$$

with $\alpha, \beta > 0$, where $i \in I = \{0,1,...,n\}$ represents the period of consumption.

Step 1

A preference for A over B in Q1 would mean that

$$\frac{u(200)}{(1+\alpha 4)^{\frac{\beta}{\alpha}}} \geq \frac{u(210)}{(1+\alpha 8)^{\frac{\beta}{\alpha}}}$$

Let

$$\frac{u(210)}{u(200)} = k, \quad \text{with } u(200), u(210) \neq 0$$

so that

[77] With the term 'standard' I mean any model within the general family of hyperbolic discounting models developed in Loewenstein & Prelec (1992) (see section 1.6.2). In fact, it is possible to capture the mentioned preferences if you depart from Loewenstein & Prelec's general model, and choose specific discount factors so that, for example, period 9 is not discounted at all ($\delta_9 = 1$), while all the other periods are discounted by the same discount factor $\delta_{i \neq 9} = 0{,}9873$. While this can be considered hyperbolic discounting in the general sense of (δ_i) being a weakly increasing sequence, it would be a completely ad hoc model, very easy to contradict in any further choice by the individual. Also, note that, in fact, such an odd case would be ruled out if single outcomes in Q1 were placed in months FEB and JUN, as was the case in Rubinstein's model.

$$\frac{(1+\alpha 8)^{\frac{\beta}{\alpha}}}{(1+\alpha 4)^{\frac{\beta}{\alpha}}} \geq k \qquad (1)$$

$$\left[\frac{1+\alpha 8}{1+\alpha 4}\right]^{\frac{\beta}{\alpha}} \geq k$$

$$\frac{\beta}{\alpha}\ln\left[\frac{1+\alpha 8}{1+\alpha 4}\right] \geq \ln k$$

from where we get to the following condition for β:

$$\beta \geq \frac{\alpha \cdot \ln k}{\ln \frac{1+\alpha 8}{1+\alpha 4}} \qquad (2)$$

Now suppose the same individual would choose B over A in Q2. This would mean that

$$k \cdot \left[\frac{1}{(1+\alpha 6)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 7)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 8)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 9)^{\frac{\beta}{\alpha}}}\right] \geq$$
$$\geq \left[\frac{1}{(1+\alpha 2)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 3)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 4)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 5)^{\frac{\beta}{\alpha}}}\right] \qquad (3)$$

In the coming steps I will show this is impossible.

Step 2

Consider again the inequality obtained from Q1:

$$\frac{(1+\alpha 8)^{\frac{\beta}{\alpha}}}{(1+\alpha 4)^{\frac{\beta}{\alpha}}} \geq k$$

Obviously, this inequality holds also if we multiply the left hand side by a higher number than the right hand side. To this end, for convenience we can use discount factors δ_8 and δ_4, because:

$$\delta_8 = \left(\frac{1+\alpha 7}{1+\alpha 8}\right)^{\frac{\beta}{\alpha}} > \delta_4 = \left(\frac{1+\alpha 3}{1+\alpha 4}\right)^{\frac{\beta}{\alpha}}$$

Thus,

$$\left(\frac{1+\alpha 7}{1+\alpha 8}\right)^{\frac{\beta}{\alpha}} \frac{1}{(1+\alpha 4)^{\frac{\beta}{\alpha}}} > k \left(\frac{1+\alpha 3}{1+\alpha 4}\right)^{\frac{\beta}{\alpha}} \frac{1}{(1+\alpha 8)^{\frac{\beta}{\alpha}}}$$

$$\frac{1}{(1+\alpha 3)^{\frac{\beta}{\alpha}}} > k \frac{1}{(1+\alpha 7)^{\frac{\beta}{\alpha}}}$$

And, analogously,

$$\frac{1}{(1+\alpha 2)^{\frac{\beta}{\alpha}}} > k \frac{1}{(1+\alpha 6)^{\frac{\beta}{\alpha}}} \qquad (4)$$

As a result, and because of increasing discount factors δ_i, the choice of A over B in Q1 means the individual prefers the first three €200 outcomes in Q2 better than their counterparts in the €210 sequence.

Now if the individual (by hypothesis) prefers the €210 sequence over the €200 sequence – chooses B in Q2-, this necessarily means that he has a strong enough preference for (€210, JUN) over (€200, FEB) so as to compensate the preference of the three €200 sooner outcomes over their €210 counterparts.

Step 3

We can reduce the inequality

$$k \cdot \left[\frac{1}{(1+\alpha 6)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 7)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 8)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 9)^{\frac{\beta}{\alpha}}} \right] \geq$$

$$\geq \left[\frac{1}{(1+\alpha 2)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 3)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 4)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 5)^{\frac{\beta}{\alpha}}} \right]$$

by using expressions (1) and (4), into this one:

$$k \cdot \left[\frac{1}{(1+\alpha 7)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 9)^{\frac{\beta}{\alpha}}} \right] \geq \frac{1}{(1+\alpha 3)^{\frac{\beta}{\alpha}}} + \frac{1}{(1+\alpha 5)^{\frac{\beta}{\alpha}}} \tag{5}$$

Now if (5) holds for a certain β, then it must also hold for a $\beta' < \beta$, so that we can in fact substitute β for its smallest possible value, determined by (2). Let me show this formally:

LEMMA 1

For all $a, b \in \mathbb{R}^{++}$, $a > b$, $\beta' < \beta$, and given $k \geq 1$,

$k \cdot \frac{1}{(1+\alpha a)^{\frac{\beta}{\alpha}}} \geq \frac{1}{(1+\alpha b)^{\frac{\beta}{\alpha}}}$ implies $k \cdot \frac{1}{(1+\alpha a)^{\frac{\beta'}{\alpha}}} \geq \frac{1}{(1+\alpha b)^{\frac{\beta'}{\alpha}}}$.

Proof:

Define $\Delta\beta = \beta - \beta'$. Suppose the antecedent inequality is satisfied. We can show that the consequent inequality also necessarily holds using the fact that

$$\left(\frac{1+\alpha a}{1+\alpha b} \right)^{\frac{-\Delta\beta}{\alpha}} < 1 \quad \text{for all } a > b,\ \Delta\beta \geq 0,\ \alpha > 0$$

as follows:

$$k \cdot \frac{1}{(1+\alpha a)^{\frac{\beta}{\alpha}}} \geq \frac{1}{(1+\alpha b)^{\frac{\beta}{\alpha}}} \left(\frac{1+\alpha a}{1+\alpha b} \right)^{\frac{-\Delta\beta}{\alpha}}$$

$$k \cdot \frac{1}{(1+\alpha a)^{\frac{\beta}{\alpha}} (1+\alpha a)^{\frac{-\Delta\beta}{\alpha}}} \geq \frac{1}{(1+\alpha b)^{\frac{\beta}{\alpha}} (1+\alpha b)^{\frac{-\Delta\beta}{\alpha}}}$$

$$k \cdot \frac{1}{(1+\alpha a)^{\frac{\beta-\Delta\beta}{\alpha}}} \geq \frac{1}{(1+\alpha b)^{\frac{\beta-\Delta\beta}{\alpha}}}$$

$$k \cdot \frac{1}{(1+\alpha a)^{\frac{\beta'}{\alpha}}} \geq \frac{1}{(1+\alpha b)^{\frac{\beta'}{\alpha}}}$$

□

So we can confirm that, if inequality (3) was true for a certain β, then it will remain true for any smaller $\beta' < \beta$. Thus, we can substitute β for the smallest possible value of it given by inequality (2):

$$\beta_0 = \frac{\alpha \cdot \ln k}{\ln \frac{1+\alpha 8}{1+\alpha 4}}$$

we can express now (5) as follows:

$$k \cdot \left[\frac{1}{(1+\alpha 7)^{\frac{\ln k}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}}} + \frac{1}{(1+\alpha 9)^{\frac{\ln k}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}}} \right] > \frac{1}{(1+\alpha 3)^{\frac{\ln k}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}}} + \frac{1}{(1+\alpha 5)^{\frac{\ln k}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}}} \quad (6)$$

Step 4

Manipulating (6) we can express all summands as having the same base k:

$$e^{\ln\left(k(1+\alpha 9)^{-\frac{\ln k}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}}\right)}+e^{\ln\left(k(1+\alpha 7)^{-\frac{\ln k}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}}\right)}-e^{\ln(1+\alpha 3)^{-\frac{\ln k}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}}}-e^{\ln(1+\alpha 5)^{-\frac{\ln k}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}}}>0$$

$$e^{\ln k-\frac{\ln k}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\ln(1+\alpha 9)}+e^{\ln k-\frac{\ln k}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\ln(1+\alpha 7)}-e^{-\frac{\ln k}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\ln(1+\alpha 3)}-e^{-\frac{\ln k}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\ln(1+\alpha 5)}>0$$

$$e^{(\ln k)\left(1-\frac{\ln(1+\alpha 9)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\right)}+e^{(\ln k)\left(1-\frac{\ln(1+\alpha 7)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\right)}-e^{(\ln k)\left(-\frac{\ln(1+\alpha 3)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\right)}-e^{(\ln k)\left(-\frac{\ln(1+\alpha 5)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\right)}>0$$

$$k^{\left(1-\frac{\ln(1+\alpha 9)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\right)}+k^{\left(1-\frac{\ln(1+\alpha 7)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\right)}-k^{\left(-\frac{\ln(1+\alpha 3)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\right)}-k^{\left(-\frac{\ln(1+\alpha 5)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\right)}>0$$

Now let me refer to the exponents in this inequality as follows:

$$a=\left(1-\frac{\ln(1+\alpha 9)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\right)$$

$$b=\left(1-\frac{\ln(1+\alpha 7)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\right)$$

$$c=\left(-\frac{\ln(1+\alpha 3)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\right)$$

$$d=\left(-\frac{\ln(1+\alpha 5)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}\right)$$

Now the proof reduces to show that

$$k^a+k^b-k^c-k^d>0 \qquad (7)$$

is impossible: first, see that:

$$\ln\left(\frac{1+\alpha 8}{1+\alpha 4}\right)>\ln\left(\frac{1+\alpha 9}{1+\alpha 5}\right)$$

$$\ln(1+\alpha 8)-\ln(1+\alpha 4)-\ln(1+\alpha 9)>-\ln(1+\alpha 5)$$

$$1-\frac{\ln(1+\alpha 9)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}>-\frac{\ln(1+\alpha 5)}{\ln(1+\alpha 8)-\ln(1+\alpha 4)}$$

$$a>d$$

Thus exponent d is equal to exponent a minus a certain amount $t>0$:

$d=a-t$

Also,

$$\ln\left(\frac{1+\alpha 4}{1+\alpha 3}\right) > \ln\left(\frac{1+\alpha 8}{1+\alpha 7}\right)$$
$$-\ln(1+\alpha 3) > \ln(1+\alpha 8) - \ln(1+\alpha 4) - \ln(1+\alpha 7)$$
$$-\frac{\ln(1+\alpha 3)}{\ln(1+\alpha 8) - \ln(1+\alpha 4)} > 1 - \frac{\ln(1+\alpha 7)}{\ln(1+\alpha 8) - \ln(1+\alpha 4)}$$
$$c > b$$

Which means that, for a certain $s > 0$,

$b = c - s$

But also note that $s > t$ because

$$\ln\left(\frac{1+11\alpha+28\alpha^2}{1+11\alpha+24\alpha^2}\right) > \ln\left(\frac{1+13\alpha+40\alpha^2}{1+13\alpha+36\alpha^2}\right)$$
$$\ln\left(\frac{(1+\alpha 4)(1+\alpha 7)}{(1+\alpha 3)(1+\alpha 8)}\right) > \ln\left(\frac{(1+\alpha 8)(1+\alpha 5)}{(1+\alpha 4)(1+\alpha 9)}\right)$$
$$\ln\left(\frac{1+\alpha 4}{1+\alpha 3}\right) - \ln\left(\frac{1+\alpha 8}{1+\alpha 7}\right) > \ln\left(\frac{1+\alpha 8}{1+\alpha 4}\right) - \ln\left(\frac{1+\alpha 9}{1+\alpha 5}\right)$$
$$-\ln(1+\alpha 3) + \ln(1+\alpha 7) > -\ln(1+\alpha 9) + \ln(1+\alpha 5) + 2\left(\ln(1+\alpha 8) - \ln(1+\alpha 4)\right)$$
$$-\frac{\ln(1+\alpha 3)}{\ln(1+\alpha 8) - \ln(1+\alpha 4)} - \left[1 - \frac{\ln(1+\alpha 7)}{\ln(1+\alpha 8) - \ln(1+\alpha 4)}\right] > 1 - \frac{\ln(1+\alpha 9)}{\ln(1+\alpha 8) - \ln(1+\alpha 4)} - \left[-\frac{\ln(1+\alpha 5)}{\ln(1+\alpha 8) - \ln(1+\alpha 4)}\right]$$
$$s > t$$

since $\alpha > 0$ by hypothesis. We can thus state there exists $v > 0$ such that

$s = t + v$

And now, we get to

$$k^a + k^{c-s} - k^c - k^{a-t} > 0$$
$$k^a + k^{c-t-v} - k^c - k^{a-t} > 0$$
$$k^a\left(1 - \frac{1}{k^t}\right) + k^c\left(\frac{1}{k^{t+v}} - 1\right) > 0$$

which is impossible for any $k > 1$ because $c > a$ by

$$\ln\left(\frac{1+\alpha 9}{1+\alpha 3}\right) > \ln\left(\frac{1+\alpha 8}{1+\alpha 4}\right)$$

$$-\ln(1+\alpha 3) > \ln(1+\alpha 8) - \ln(1+\alpha 4) - \ln(1+\alpha 9)$$

$$-\frac{\ln(1+\alpha 3)}{\ln(1+\alpha 8) - \ln 1+\alpha 4} > 1 - \frac{\ln(1+\alpha 9)}{\ln(1+\alpha 8) - \ln 1+\alpha 4}$$

$$c > a$$

and therefore,

$k^c > k^a$, which implies, in fact:

$$k^a\left(1-\frac{1}{k^t}\right) + k^c\left(\frac{1}{k^{t+v}} - 1\right) < 0,$$

□

contradicting the choice of B in Q2.

Choosing A over B in Q1 and B over A in Q2 is therefore not compatible with any standard hyperbolic discounting model. Now let me show that such preferences are also not compatible with the procedure based on similarity assessments proposed by Rubinstein, A. (2003) (see section 3.2.4). Take Rubinstein's procedure: in the time dimension there is clearly no similarity among the options. Now if the individual perceived the options as similar in the money dimension (which is hard to imagine), then clearly he would decide in both cases upon the time dimension, and choose the smaller-sooner option in both cases. In other words, Rubinstein's procedure does not explain the constant sequence effect (i.e., a choice of A over B in Q1 and a choice of B over A in Q2).

The choice proposed in Q3 aims at controlling for the magnitude effect. If individuals show a constant sequence effect in their previous choices in Q1 and Q2, this means that they are more patient for sequences than for single outcomes. Now this higher patience may come from the fact that more money is at stakes. In Q3 I thus let individuals choose among new single outcomes (€800, JAN) and (€840, MAY), to see whether a comparable amount of them incurs in an inconsistent choice (A over B in Q1 but B over A in Q3). If the constant sequence effect is *only* a different manifestation of the magnitude effect, we should observe that all subjects choosing A over B in Q1 and B over A in Q2, choose B over A in Q3.

Finally, with Q4 I wanted to check what the difference is between a sequence and a collection of outcomes (obtained all on the same date). The idea was to see the impact of the mere multiplicity of outcomes, and compare it with the single and big amount: we should expect to see the same preference for A or B in Q3 than in Q4.

Intensity of preference questions

After each question (Q1-Q4) subjects were asked to indicate how much they preferred the chosen option in a 1 to 5 scale. The (translated into English) wording was as follows:

How much better is the option you have chosen? (where 1 means 'almost equal' and 5 means 'much better')

1	2	3	4	5
○	○	○	○	○

Answering this question was compulsory, as it was answering Q1-Q4.

3.3.2. Participants

501 members of a representative sample of Spanish Internet-users participated in this study. The sample was created during 2004 under the name *Metascore* to conduct market researcher in the company YA.COM, one of the big Internet Service Providers in Spain, and adjusted to the Internet user-profile in Spain. Members of this 'online panel' participated in online surveys between April 2004 and May 2005, receiving a free subscription to a well-known Spanish publication (PC actual) as compensation. In addition to this permanent incentive, participants in this study automatically participated in a draw in which they could win up to 900 €. At the time of the experiment, Internet penetration in Spain was 33%, and main biases of such a sample with respect to general population were age, income and education (see Annex 3 for some details on Metascore members' profile). No relevant difference was

observed between the participants (sub sample n=501) and the panellists (see also Annex 3 for the relevant statistical test), which means our results can be considered representative of Spanish Internet population. Quality of responses was high: more than 94% of respondents answered thoroughly one of the most difficult tasks in the questionnaire (see again Annex 3 for the way I measured this).

3.3.3. Procedure

The experiment included 4 questions (choices) presented in four different orders: Q2Q3Q1Q4, Q4Q2Q3Q1, Q1Q4Q2Q3 and Q3Q1Q4Q2. For each question the subject had to choose among two options (A and B) and also to state the intensity of his or her preference in a scale going from 1 to 5. The web questionnaire was programmed so as to randomly rotate the order of appearance of both alternatives (A and B), and did control for single choices (either A or B, but not both). All questions were mandatory (making no choice at all impeded advancing to the next page in the questionnaire). Also, to facilitate the task, options were presented in tables in which each column corresponded to one month (see Annex 2 for the screen shots).

The first page of the questionnaire consisted of the instructions. Subjects were indicated that they should choose only according to their preferences, and reminded that there was not such thing as a correct answer in the experiment. Also, subjects were told that one randomly chosen participant would get one randomly chosen question paid for real. This 'random lottery incentive system' aims at obtaining true trade offs from the individuals. If a participant stated he preferred €800 in four months better than €840 in eight months, than, in case he won the lottery –and this particular question was selected-, he had really to wait four months to get the money paid. After the instructions page, subjects faced the four questions in the next four pages (each question in a single page); after these questions, participants answered still other questions belonging to other experiments (as the one I present in chapter 2). Between experiments, nevertheless, participants were faced with teaser questions so as to make their subsequent choices independent of the previous experiment, and a participant answered never more than 12 questions, spending usually around 10 to 15 minutes. If someone spent more than 30 minutes in a single page (in a single question), then the web questionnaire would interrupt and the respondent had to start all over again (it was not possible to leave the questionnaire having completed only part of it).

The invitation was sent October 14th 2004 via e-mail to 1.024 members of the Spanish online panel *Metascore*. In the email there was a link pointing at an online questionnaire. Once the subject had completed the questionnaire, he could not access it again anymore. A total of 501 subjects did complete the questionnaire, which means an overall response rate of 48.9% was obtained. Fieldwork data were very satisfactory: 64% of the invited panellists opened the invitation email, of which 83% clicked on the link pointing at the questionnaires, and 93% of those who started the questionnaire did also finish it.

3.3.4. Results and Discussion

Research hypothesis 1 claimed that neither hyperbolic discounting nor similarity are necessary conditions for a constant sequence effect. In other words, my prediction was to observe a constant sequence effect in experiment 1 despite its design out rules both the similarity based and the hyperbolic explanations. Figure YY shows overall results regarding H1:

		Q 2: A	Q 2: B
Q 1	A	190 (38%)	110 (22%)
	B	35 (7%)	166 (33%)

Table 3.2

As we can see, 22% of the subjects chose the smaller-sooner option in Q1 but the larger-later one in Q2, therefore revealing more patience in the choice among sequences than in the choice among single outcomes. In contrast, only 7% of the subjects chose the reverse, which seems to indicate that more subjects change from A in Q1 to B in Q2 than subjects change from B in Q1 to A in Q2. This hypothesis was confirmed by a one-tailed McNemar change test ($p<0.0001$). Also, this effect was found to be independent of the order in which questions were shown (see Table 3.3): there were 4 different forms, and results for each form can be seen in figure YY. (obtained p-values in the McNemar change test were p=0.0138 in Q2Q3Q1Q4, p=0.0007 in Q4Q2Q3Q1, p=0.0004 in Q1Q4Q2Q3 and p=0.0004 in Q3Q1Q4Q2).

		Q 2	
		A	B
Q 1	A	45 (38%)	28 (24%)
	B	13 (11%)	31 (26%)

Form order: Q2Q3Q1Q4

		Q 2	
		A	B
Q 1	A	49 (37%)	26 (20%)
	B	7 (5%)	50 (38%)

Form order: Q4Q2Q3Q1

		Q 2	
		A	B
Q 1	A	47 (36%)	27 (21%)
	B	7 (5%)	49 (38%)

Form order: Q1Q4Q2Q3

		Q 2	
		A	B
Q 1	A	49 (40%)	29 (24%)
	B	8 (7%)	36 (30%)

Form order: Q3Q1Q4Q2

Table 3.3

As can be seen, the effect is robust across the different forms, which confirms H1[78].

If we compare these results to Kirby & Guastello (2001) we find that here only about one out of every four subjects incurs in a preference reversal, while in their study almost all of them did. The reason for this is the following: Kirby & Guastello's procedure consists in first obtaining the individual's discount rate and after adapting the subsequent questions so as to maximize the probability that a subject reverses his preferences. A similar procedure was also used in the classic experiment by Kirby & Herrnstein (1995). In this case, such a procedure would have meant to strategically manipulate amounts and timings to each subject according to previously revealed individual discount rates; although this is for sure an interesting study to conduct, results could be interpreted as 'forced', in the sense that such procedures keep asking every individual until either he switches his preferences or a certain number of questions pass. I have therefore preferred to look at a one-shot choice experiment.

Comparison to Rubinstein, A. (2003)

The above results replicate the one's obtained in Rubinstein's experiment 2 (see section 3.2.3) in the sense that an almost identical percentage of subjects incurred in the predicted preference reversal:

My Experiment 1

		Q 2: A	Q 2: B
Q 1	A	**190** (38%)	**110** (22%)
	B	**35** (7%)	**166** (33%)

Rubinstein's Experiment 2

		Q 2: A	Q 2: B
Q 1	A	**23** (28%)	**19** (23%)
	B	**6** (7%)	**33** (41%)

Table 3.4

[78] A ϕ-coefficient shows, on the other hand, the following correlation measures for the different Forms: $\phi_{2314} = 0.369$, $\phi_{4231} = 0.513$, $\phi_{1423} = 0.646$ and $\phi_{3142} = 0.617$.

Also, data regarding the intensity of preferences indicate a stronger preference for the larger-later option when choosing among sequences. The results I present next in Table 3.5 refer to the average intensity (standard deviation in parenthesis) of preference declared in a 1 to 5 scale (see section 3.3.1 for the precise wording and design). Because individuals strongly tended to 'anchor' their declared intensity of preference to their answer to the first question they faced, I only consider data from the forms where questions were the first faced by the subjects.

	Q 1	*Q 2*
Intensity of preference in choices of option A	**3.04 (1.2)**	**3.07 (1.0)**
Intensity of preference in choices of option B	**3.11 (1.1)**	**3.61 (0.91)**

Table 3.5

When individuals chose the smaller-sooner option (option A), they did on average with similar intensity of preference regardless of the choice being among single outcomes (Q1) or sequences (Q2). But when they chose the larger-later option (option B), they declared a higher intensity of preference in the case of sequences.

Next I present results regarding my Hypothesis 2. Table 3.6 shows overall results in a contingency table for the choices in questions Q2 and Q3.

		Q 2	
		A	**B**
Q 3	**A**	175 (35%)	66 (13%)
	B	50 (10%)	210 (42%)

Table 3.6

In this case, the null Hypothesis as formulated before (number of subjects changing from A in Q3 to B in Q2 *smaller* than number of subjects changing from B in Q3 to A in Q2) cannot be rejected in a one-tailed McNemar test (p=0.0817>0.05). Moreover, the corresponding two-tailed McNemar test would yield p=0.1634, meaning we cannot reject the null Hypothesis (=number of subjects changing from A in Q3 to B in Q2 *different* than number of subjects changing from B in Q3 to A in Q2). Thus, in this dataset, when the single choice is among outcomes of equal magnitude as the sum of outcomes in a constant sequence, no effect can be confirmed for overall results (all Form-orders). The obtained p-value is nevertheless small; also, when you look at results for each separate Form, you seem to find an effect in two of the form-types. In my view, all this may indicate that the magnitude of a sequence does not explain *all* the constant sequence effect. I will comment more on this in the conclusions.

As I just mentioned, the result is replicated in two out of the four different Forms (orders). The corresponding one-tailed McNemar test p-values you find are p=0.9061 in Q2Q3Q1Q4, p=0.1885 in Q4Q2Q3Q1, p=0.105 in Q1Q4Q2Q3 and p=0.032 in Q3Q1Q4Q2, indicating that a significant (p<0.05) constant sequence effect is found only in the last form-type, while, possibly, also an effect occurs in Q1Q4Q2Q3 (p=0.105).

		Q 2	
		A	**B**
Q 3	**A**	**36** (31%)	**15** (13%)
	B	**22** (19%)	**44** (38%)

Form order: Q2Q3Q1Q4

		Q 2	
		A	**B**
Q3	**A**	**43** (33%)	**19** (14%)
	B	**13** (10%)	**57** (43%)

Form order: Q4Q2Q3Q1

		Q 2	
		A	**B**
Q 3	**A**	**46** (35%)	**15** (12%)
	B	**8** (6%)	**61** (47%)

Form order: Q1Q4Q2Q3

		Q 2	
		A	**B**
Q 3	**A**	**50** (41%)	**17** (14%)
	B	**7** (6%)	**48** (39%)

Form order: Q3Q1Q4Q2

Table 3.7

If we look at the intensity questions, data for Q3 also suggest that the magnitude of a sequence may be responsible for the constant sequence effect. Those individuals who chose the smaller-sooner option A in Q3 declared an average intensity of preference of 3.16 (standard deviation in parenthesis), while those choosing the larger-later option B declared 3.63, not distinguishable from the intensity declared by individuals in the choice of the larger-later sequence in Q2.[79]

[79] Recall that, for each question number Q1, Q2 or Q3, I use only data from the Form where that particular question was answered in first position in the questionnaire.

	Q 1	Q 2	Q 3
Intensity of preference in choices of option A	3.04 (1.2)	3.07 (1.0)	3.16 (0.99)
Intensity of preference in choices of option B	3.11 (1.1)	3.61 (0.91)	3.63 (0.99)

Table 3.7

Finally, I want to present results for overall choice in Q1, Q2, Q3 and Q4 when these questions were answered in first position in the questionnaire, so that we can discard any order effects. Results just show percentages (all four Forms had very similar number of completes; see tables at end of chapter for frequences).

	Q1	Q3	Q2	Q4
A	57%	55%	50%	39%
B	43%	45%	50%	61%

Table 3.8

These results suggest an effect produced by the multiplicity of outcomes. Q1 and Q3 show similar patience, while, interestingly, Q4 shows the highest patience of all. The greater patience of Q4 with respect to Q3 is strongly significant (test of proportions p=0.008).

Let me next present a brief summary regarding the specific results in experiment 1. The first –and possibly most important- conclusion is that we do find a constant sequence effect although controlling for hyperbolic discounting and similarity. The question may remain as to what extent this effect is there for everybody. Put in other words, we may ask whether this effect is marginal (only certain special people have these preferences), or whether, on the contrary, it is an effect that would be observed for virtually everybody provided we chose the right numbers and timings for each person. A further line of empirical research would thus be to try to replicate this experiment but designed so as to present each subject the right numbers according to previously revealed degrees of impatience. Also, note that experiment 1 was designed with only four outcomes in every sequence, but more outcomes could strengthen the effect. Forced to speculate, my own prediction would be that we should find a constant sequence effect for most of the people that cannot be explained by hyperbolic discounting or similarity (also not, of course, by exponential discounting).

The confirmation of my research Hypothesis 1 thus leaves us the task of explaining why people reveal a higher patience for sequences than for single outcomes. The most natural explanation appeared to be the magnitude effect, a very robust finding in the intertemporal choice literature stating that discount rates are higher for smaller outcomes than for larger ones. The evidence obtained in experiment 1 indicates that the magnitude effect is probably the most important determinant of the constant sequence effect, but maybe not the only one. It does not seem that controlling for the magnitude dissolves the constant sequence effect completely, although more research is needed to arrive at definite results. Separate results for the different forms also suggest that only in one of them the effect clearly disappears, while low p-values are observed in the remaining three cases. The p-value obtained for the overall results is also quite small.

In sum, experiment 1 has shown that the constant sequence effect is possibly a much more interesting effect than has been thought until now. Excluding Rubinstein's experiment, all evidence of this effect has been obtained in onwards constructed sequences, and has thus been considered a mere side-effect of the hyperbolic discounting finding. But this effect should rather be considered a mere side-effect of the magnitude effect. If we believe that results in experiment 1 back this hypothesis, then the constant sequence effect just emphasizes the importance of the magnitude effect, since it shows that constant sequences are evaluated differently than single outcomes because more money is at stakes. Note that this is not a minor effect: if people's preferences are affected by the total amount in a sequence, it can mean quite different things: for example, it can mean that people discount every single outcome separately, but applying the discount rate that corresponds to the total magnitude at stakes in the sequence; but it also could mean that people integrate (add) all outcomes first, and discount after the total amount altogether. If the latter is correct, then an important further question arises: how many periods would someone discount in the case of a constant sequence as option A in Q2? (this issue is addressed in experiment 2 later in this chapter).

If, on the contrary, we consider that the above evidence does not unambiguously support the magnitude-effect explanation, then the constant sequence effect could be considered an independent effect. But, what could be a possible explanation of an eventual independent constant sequence effect? I will consider three: first, it could be that when facing a choice task comparing two sequences, an individual's preferences were affected by how many times an outcome in sequence B is preferred to an outcome in sequence A. When facing single outcomes choices, this number would always be 1, while in the case of sequences, it could be more. In fact, in the case of constant sequences, this number will obviously be equal to the number of elements in the sequence. Thus, the longer the sequence is, the longer –ceteris paribus- the effect. For a preliminary evaluation of this hypothesis, we can look into the following 2x2 contingency tables for Q1 and Q4 and for Q1 and Q3.

		Q 3	
		A	**B**
Q1	**A**	**214** (43%)	**86** (17%)
	B	**27** (5%)	**174** (35%)

		Q 4	
		A	**B**
Q1	**A**	**195** (39%)	**105** (21%)
	B	**17** (3%)	**184** (37%)

Table 3.9

Results shown in Table 3.9 suggest that the existence of multiple-comparisons may have an effect on preferences. Despite Q3 and Q4 having identical value, more subjects chose the larger later option (B) in Q4 than in Q3. The next table (Table 3.10) also suggest a possible multiple-comparison effect:

		Q 4: A	Q 4: B
Q3	A	**186** (37%)	**55** (11%)
	B	**26** (5%)	**234** (47%)

		Q 4: A	Q 4: B
Q2	A	**158** (32%)	**67** (13%)
	B	**54** (11%)	**222** (44%)

Table 3.10

In the contingency table of the left, we see that the vast majority of the people were of course consistent (420 out of 501), which should not be a surprise bearing in mind that Q3 and Q4 are in principle identical problems! But we can see also that a significant number of subjects reveal a higher preference for the larger-later outcome when it is split into four €210 outcomes than when it is presented as a single outcome (one-tailed McNemar test p=0.0008). There appears to be a preference for the splitted outcomes. And, consistent with the multiple-comparisons hypothesis, this effect vanishes when you compare Q4 with Q2, presumably because both questions deal now with four-outcomes sequences (right contingency table in Table 3.10).

A second attempt to explain the above results (constant sequence effect that is not fully due to magnitude) could be the following: in Q2 the interval between the options may be perceived as having decreased. Although the distance between the starting points of the sequences is the same as the one separating options A and B in Q3, the display of sequences in Q2 may produce the visual effect that both options are nearer than they are in Q3. This hypothesis, too, needs to be tested in a new experiment.

Finally, a third possible explanation deserves further exploration. It has to do with diminishing marginal utility. Suppose participants displayed significant diminishing marginal utility for money within the range of amounts in this experiment, so that $4 \cdot u(210) > u(840)$. And imagine, also, that people are affected by the magnitude effect in a way such that they discount the four outcomes in the sequence separately, but using the discount rate that accounts for the overall amount $4 \cdot u(210)$, while, in contrast, when they discount the single outcome they use a discount rate that corresponds to $u(840)$, and, because the latter is lower, the result is subjects do behave more patiently for the sequences than for single outcomes amounting the same total value. In such case, the particular way in which subjects integrate outcomes turns out to be determinant. If subjects 'bracket' this decision as if there is one integrated outcome, then discounting has a certain degree X. But if they treat this decision object as four separate outcomes each of which they evaluate with diminishing marginal utility, and only then discount according to the resulting overall value, it could be that this discounting was lower than X due to an extra magnitude effect.

All three alternative explanations need to be systematically explored in further experimental work. But results in this dataset already suggest several combined effects.

a. There is a strong evidence that there is a magnitude effect for sequences. People reveal more patience when choosing among constant sequences than among singles outcomes, but this effect disappears when controlling for magnitude in single outcomes. This sequence-magnitude effect suggests an interesting question. If people integrate amounts in a sequence into one single value, do they also integrate somehow the timings of the outcomes into one single timing? Do people 'map' sequences into a transformed single value object? And if yes, what would be the timing of this new 'collapsed' object?
b. There is no clear evidence for hyperbolic discounting: on one hand, if subjects were hyperbolic discounters, we should, *ceteris paribus*, observe a overall higher patience in Q3 than in Q2. But we observe the opposite (see Table 3.8). So either there is a compensating force, or there is no hyperbolic discounting at all. A compensating force could be the multiple-comparisons effect alluded before, but this effect cannot yet be considered established. But finally –and intriguinly-, if there was no hyperbolic discounting at all, why do we find more patience in Q4 than in Q2 (one-tailed McNemar p=0.13)?
c. On the other hand results have suggested a higher preference for outcomes that are splitted than for outcomes that are integrated.

So basically, results in experiment 1 yield a solid conclusion that total magnitude at stakes (and not only individual magnitudes) affects time preferences for constant sequences, and leave many unanswered questions.

3.4. Experiment 2

While in experiment 1 people chose either among sequences or among outcomes, experiment 2 was designed to test how people make choices *between* constant sequences *and* single outcomes amounting to the same total value. If there is a special preference pattern for outcomes embedded in a sequence, then this effect could appear when people are asked directly.

My research hypothesis 1 (H1) is a quite obvious one: that people's choices are sensitive to the timing of the objects of choice. I thus expect to observe more choices of an outcome when this outcome is closer, which would confirm people find the time differences in the experiment to be relevant for them. My second research hypothesis (H2) is that a constant sequence is preferred to a single amount (of equal total value) that is located at the midpoint of that sequence. Note that this hypothesis is in fact implied by both exponential and hyperbolic discounting, since, of course, moving an amount one period later (from period $i-1$ to period i) means multiplying its value by δ_i, while moving an outcome one period closer means multiplying its value by δ_{i-1}^{-1}. The result of such reallocation is positive in value in both the exponential discounting and the hyperbolic discounting models because $\delta_i \geq \delta_{i-1}$ for all $i \in I$, and therefore $\delta_{i-1}^{-1} - 1 > 1 - \delta_i$. Thus, if you split, for example, a single outcome of €700 into seven €100 outcomes so that the original timing of the outcome becomes the center point of the sequence, then you are augmenting total value[80].

On the other hand, my research hypothesis 3 (H3) is that, when evaluating constant sequences, people display what I call zero intra-sequence discounting (ZID). This hypothesis contradicts any discounted utility model since, under positive time-preference (outcomes preferred sooner rather than later), any discounting model would predict a single amount at a certain period should be preferred to a constant sequence beginning at that period.

[80] We should add to this reasoning some considerations on the type of utility function for money that individuals have. Of course, if an individual had strong enough increasing marginal utility for money, then the above statement could not be true anymore. H2 therefore assumes that utility for money is either linear or strictly concave.

3.4.1 Design

The design consisted of two treatments: in treatment I a €700 single outcome was 'split' into seven €100 outcomes to see how choices compared when subjects could instead choose a larger-later single amount of €812. In treatment II, the split single outcome was the larger-later one, in order to see whether preferences for sequences differed depending on the distance to the sequences. Let me describe both treatments graphically.

In *TREATMENT I*, questions were as follows[81]:

		OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN
Q 1	A		**€700**							
	B									**€812**
Q 2	A					**€700**				
	B									**€812**
Q 3	A		**€100**	**€100**	**€100**	**€100**	**€100**	**€100**	**€100**	
	B									**€812**

Figure 3.4

In *TREATMENT II*, questions were as follows:

[81] Here I have omitted both the verbal statement and the intensity question scale. For an exact display of a questionnaire's page see Annex 2.

Q 1

	O	N	D	J	F	M	A	M	J	J	A	S	O	N	D	J
A		€700														
B									€812							

Q 2

	O	N	D	J	F	M	A	M	J	J	A	S	O	N	D	J
A		€700														
B												€812				

Q3

	O	N	D	J	F	M	A	M	J	J	A	S	O	N	D	J
A		€700														
B									€116	€116	€116	€116	€116	€116	€116	

Figure 3.5

Let me briefly summarize my predictions.

H1: I will call it *time-sensitivity*. If TIQ1A stands for 'number of subjects choosing A in Q1 of treatment I', then we can say that this hypothesis predicts <u>TIQ1A>TIQ2A</u> and also <u>TIIQ1B>TIIQ2B</u>.

H2: I will call it *split-outcome-preference*. This hypothesis predicts <u>TIQ3A>TIQ2A</u> and <u>TIIQ3B>TIIQ2B</u>.

H3: *Zero intra-sequence discounting*. It predicts <u>TIQ1A=TIQ3A</u> and <u>TIIQ1B=TIIQ3B</u>.

Intensity of preference questions

After each question (Q1-Q3) subjects were asked to indicate how much they preferred the chosen option in a 1 to 5 scale. The (translated into English) wording was as follows:

How much better is the option you have chosen? (where 1 means 'almost equal' and 5 means 'much better')

1	2	3	4	5
○	○	○	○	○

Answering this question was compulsory, as it was answering Q1-Q3.

3.4.2. Participants

1.482 members of a representative sample of Spanish Internet-users participated in this study. The sample was created during 2004 under the name *Metascore* to conduct market researcher by the company YA.COM, one of the big Internet Service Providers in Spain, and adjusted to the Internet user-profile in Spain. Members of this 'online panel' participated in online surveys between April 2004 and May 2005, receiving both a free subscription to a well-known Spanish publication (PC actual) and discounts in several online shops as compensation. In addition to this permanent incentive, participants in this study automatically participated in a draw in which they could win up to 900 €. At the time of the experiment, Internet penetration in Spain was 33%, and main biases of such a sample with respect to general population were age, income and education (see Annex 3 for some details on Metascore members' profile). No relevant difference was observed between the participants (sub sample n=1.482) and the panellists (see Annex 3 for the relevant statistical test), which means my results can be considered representative of Spanish Internet population. Quality of responses was high: more than 94% of respondents answered thoroughly one of the most difficult tasks in the questionnaire (see also Annex 3 for the way I measured this).

3.4.3. Procedure

In every treatment the experiment included 3 questions (choices) presented in three different orders: for example, for treatment I, Q1Q2Q3, Q2Q3Q1 and Q3Q1Q2. For each question the subject had to choose among two options (A and B) and also to state the intensity of his or her preference in a scale going from 1 to 5. The web questionnaire was programmed so as to randomly rotate the order of appearance of both alternatives (A and B), and did control for single choices (either A or B, but not both). All questions were mandatory (making no choice at all impeded advancing to the next page in the questionnaire). Also, to facilitate the task, options were presented in tables in which each column corresponded to one month.

The first page of the questionnaire consisted of the instructions (see Annex 2). Subjects were indicated that they should choose only according to their preferences, and reminded that there was not such thing as a correct answer in the experiment. Also, subjects were told that one randomly chosen participant would get one randomly chosen question paid for real. This 'random lottery incentive system' aims at obtaining true trade offs from the individuals. If a participant stated he preferred €812 in seven months better than €700 next month, then, in case he won the lottery –and this particular question was selected-, the participant needed really to wait seven months to get paid €812. After the instructions page, subjects faced the three questions in the next three pages (each question in a single page); after these questions, participants answered still other questions belonging to other experiments (as the one I present in chapter 2). Between experiments, nevertheless, participants were faced with 'teaser' questions so as to make their subsequent choices independent of the previous experiment, and a participant answered never more than 12 questions, spending usually around 10 to 15 minutes. If someone spent more than 30 minutes in a single page (in a single question), then the web questionnaire would interrupt and the respondent had to start all over again (it was not possible to leave the questionnaire having completed only part of it).

The invitation was sent October 14th 2004 via e-mail to 2.850 members of the Spanish online panel *Metascore*. In the email there was a link pointing at an online questionnaire. Once the subject had completed the questionnaire, he could not access it again anymore. A total of 1.482 subjects did complete the questionnaire, which means an overall response rate of 50.2% was obtained. Fieldwork data were very satisfactory: 65% of the invited panellists opened the invitation email, 85% of them clicked on the link pointing at the questionnaires, and 94% of those who started the questionnaire did also finish it.

3.4.4 Results and Discussion

Next you can see the contingency tables showing choices in Q1 and Q2 for TREATMENT I and II:

Treatment I

		Q2	
		A	**B**
Q1	**A**	**139** (19%)	**139** (19%)
	B	**30** (4%)	**435** (58%)

Treatment II

		Q2	
		A	**B**
Q1	**A**	**249** (34%)	**27** (4%)
	B	**94** (13%)	**369** (50%)

Table 3.11

As can be seen, clearly in treatment I there are more choices of A in Q1 than in Q2. The smaller-sooner amount was chosen more often in Q1 than in Q2 (one-tailed McNemar $p<0.0001$), while in treatment II there are more choices of B in Q1 than in Q2 (one-tailed McNemar $p<0.0001$). This should be no surprise, and simply confirms that people were time-sensitive, and did care about delaying a €700 euro outcome by three months in treatment I and also about delaying €812 by three months in treatment II. Hypothesis H1 is thus confirmed; in fact it is also confirmed in any of the three different Forms (orderings) (see Annex 1) for both treatments.

Now let us look at results for hypothesis H2. This hypothesis stated that a constant sequence is preferred to a single amount (of equal total value) that is located at the midpoint of that sequence (split-outcome preference). Let me therefore present total frequencies of choices in Q3 and Q2:

Treatment I

Q2 \ Q3	A	B
A	**118** (16%)	**51** (7%)
B	**155** (21%)	**419** (56%)

Treatment II

Q2 \ Q3	A	B
A	**213** (29%)	**130** (18%)
B	**57** (8%)	**339** (46%)

Table 3.12

These data confirm that outcomes were more preferred when split into sequences as in Q3 (options A in Treatment I and B in Treatment II), than when presented as deferred single outcomes as in Q2, both when this sequence was close in time and when it was far. This result should not be confounded with the constant sequence effect as defined before in this chapter; it is not a preference that cannot be accommodated into standard discounting models as exponential or hyperbolic discounting, as I mentioned before. A further confirmation is obtained if we look into results only for the Forms where the alluded questions were answered in first position of the questionnaire (thus getting rid of any possible influence from the order of questions):

	Q 2	Q 3
TreatmentI (when first choice) Total choices of option A	**81 (30%)**	**90 (37%)**
TreatmentII (when first choice) Total choices of option B	**109 (45%)**	**177 (69%)**

Table 3.13

In sum, the confirmation of H1 and H2 means that participants in this experiment were clearly affected by the timing of options. The confirmation of H2 can be interpreted in (at least) two ways: take someone who chose, for example, option TIQ2B but also TIQ3A. It can be that he discounts outcomes as assumed by discounted utility models, and thus had a preference for the sequence in this case; alternatively, it can be that he integrated (added) all seven outcomes in the sequence together, and considered the delay to this new 'collapsed' option to be just one single month, clearly preferable to receiving the same amount in four months. This is precisely the prediction that would make zero intra-sequence discounting.

Results for H3 can help us establish which of the previous alternatives is more likely to be true:

Treatment I

		Q3: A	Q3: B
Q1	A	**162** (22%)	**116** (16%)
	B	**111** (15%)	**354** (48%)

Treatment II

		Q3: A	Q3: B
Q1	A	**182** (25%)	**94** (13%)
	B	**88** (12%)	**375** (51%)

Table 3.14

No effect at all is to be found in either Treatment I or Treatment II. In the corresponding two-tailed McNemar test for treatment I you cannot reject the null hypothesis stating that there is no difference at all among the number of subjects moving from A in Q1 to B in Q3 and the number of subjects moving from B in Q1 to A in Q3 (p=0.7907). The same was true for treatment II (p=0.7110). These data show there was no difference at all in the choice of a single amount in a certain moment t and a sequence of seven *delayed* outcomes adding up to the same total value that also starts at t. Results for each different form-order do also not show any clear effect (three forms point in the direction of valuing the single outcome more, while the other three point in the direction of valuing the sequence more; see Annex 1). To make sure the order of questions is not responsible for this result, we can check the number of choices of each option (single outcome and sequence starting at the same time) only in the forms where the option was faced first. What we find is a strong confirmation of the finding that sequences were evaluated equally (or even better) than single outcomes amounting to the same total value:

	Q 1	Q 3
TreatmentI (when first choice) Total choices of option A	**92 (39%)**	**90 (37%)**
TreatmentII (when first choice) Total choices of option B	**140 (58%)**	**177 (69%)**

Table 3.15

In fact, and even more surprisingly, the larger-later sequence (option B in TIIQ3) was chosen significantly more than the corresponding single amount (option B in TIIQ1). Also, data for the intensity questions do not show any clear preference for either the sequence or the single outcome: on one hand, the declared intensity of preference after choosing A in TIQ1 was 3.48 while when choosing A in Q3 it was 3.22, suggesting a stronger preference for the singe outcome; but, on the other hand, the declared intensity of preference after choosing B in TIIQ1 was 3.63 and when choosing B in TIIQ3 it was 3.82. Thus, in general, no clear effect could be found that distinguishes both objects of choice. H3 is therefore confirmed for this experiment.

Zero Intra-Sequence Discounting (ZID)

The previous result is truly anomalous. The vast majority of participants in this study have been shown to make their choices thoughtfully. We have several signs for it: first, as can be seen in Annex 3, 94% of participants correctly performed the task that followed experiment 2, consisting of 20 decisions in a row. Second, data for a 'teaser' question seem to me clear evidence that subjects answered sincerely and, as I said, thoughtfully: people were asked the following question: "if a pen and a rubber cost €1.10, and the pen costs €1 more than the rubber, how much is the rubber?". 94% of the subjects answered either 10 cents or 5 cents, indicating that, again, only about 6% of the subjects did not read or understand the question or, in general, did not take the question seriously[82]. And third, most importantly, subjects have been found to react both to the timing of outcomes (H1) and to the splitting of outcomes (H2). So that we therefore must conclude that subjects in this experiment did evaluate single outcomes and sequences that start at the same time equally, thus displaying what I have called zero intra-sequence discounting (ZID) for this experiment's tasks.

3.5. Conclusions

Constant sequences of monetary outcomes constitute an important object of choice in individual decision making. Whenever we make choices regarding salaries, loans or services -that are paid as monthly installments, for example- our decisions involve the evaluation of constant sequences. Despite this, constant sequences have been disregarded as an important field of experimental research, probably because no departure from standard discounted utility models was to be expected in such 'simple' objects of decision. In the past years, research highlighting the effects of non-constant sequences has been much celebrated, and important findings as the preference for increasing sequences seem to have established as anomalies in discounted utility. But all these new anomalies refer to the shape of the sequences as having an influence in our evaluations. So, the motivation for the research presented in this chapter has been to explore whether only the shape of sequences produces preferences that are incompatible with discounted utility, or, rather, whether the mere fact of the object of choice being a sequence, the mere multiplicity of outcomes, influences our behaviour in a way that also contradicts the standard model proposed by Samuelson.

A few experimental studies had recently found an interesting result: people seem to behave more patiently for sequences than for single outcomes, something I have labelled the 'constant sequence' effect. Two explanations had been proposed for this effect, hyperbolic discounting and similarity-based decision making. But a more natural explanation relates the higher patience for sequences to the well-known magnitude effect. So the first objective of

[82] This question belongs to Shane Frederick's cognitive reflection test, and is known to be much more difficult than it appears to be. In this experiment a total of 37% of the subjects correctly answered 5 cents, while Fredrick reports below 50%(?) in a study with University students. I consider this a further confirmation of the quality of participation.

this chapter was to test whether the constant sequence effect persists when controlling for hyperbolic discounting and similarity, and disappears when controlling for magnitude. The finding is that, indeed, the constant sequence effect is independent of hyperbolic discounting and similarity, and seems to be strongly related to the magnitude effect: people are more patient for sequences because more money is at stakes. But I have also shown how magnitude may not be the only thing producing higher patience. Results in experiment 1 suggest that the multiplicity of outcomes itself can influence the degree of patience we display in intertemporal choices. Further experiments are needed to arrive at more conclusive results in this matter.

In a second experiment I have explored a further important question: if outcomes in a sequence are integrated so that total magnitude of the sequence affects choice, then a natural question to ask is how people discount sequences. My hypothesis has been that, at least for relatively small sequences, people in fact do not discount the value of objects within a sequence, but only the delay to the beginning of the sequence, a hypothesis I call zero intra-sequence discounting. Experiment 2 suggests this hypothesis is correct: there seems to be no difference at all between an outcome and a sequence starting at the same time and amounting to the same total value. Further experiments need to be done to confirm whether this finding is robust across designs; but experiment 2 has already shown that the effect occurs for a highly representative, large sample of people, and both when the sequence acts as the smaller-sooner reward and when it acts as the larger-later reward. A further interesting question is how such an effect relates to the sequence outcomes' sign (gains or losses).

I thus have to conclude that no, constant sequences are not discounted the same way as single outcomes are. We can be sure of two strong effects for small sequences found in experiments 1 and 2: the magnitude effect and the zero intra-sequence discounting effect. Also, it is possible that people's preferences are affected by the multiple-comparisons effect when choosing among constant sequences, i.e., by the fact that there is a repeated preference if you compare outcomes one-by-one. These three effects together with hyperbolic discounting for single outcomes constitute the best reconstruction of the results found that I can think of after these two studies. No doubt that further, systematic research needs to be conducted before these effects can be safely considered established; but one thing emerges as a definite and discouraging conclusion: the standard exponential discounting model for intertemporal choice is not able to capture people's preferences over constant sequences, one of the most natural decision objects to which, in fact, Samuelson's model could be applied.

Summary and Conclusions

In rational intertemporal choice theory anomalies are the rule rather than the exception. Descriptively, the parsimonious model proposed by Fisher and Samuelson, later underpinned by Strotz, Koopmans and many others, is not able to capture the multiple phenomena occuring whenever an individual faces an intertemporal tradeoff. The Pandora's box opened with Thaler in 1981, and has been thundering ever since. Normatively, on the other hand, the model has proven more valid. The link of discounted utility with dynamic consistency has widely been accepted as sufficient for the normative credit of the theory. Nevertheless, a closer look at this problem reveals that neither consistency can be identified with the good, nor inconsistency necessarily with the bad. As Strotz immediately realized, the good or bad for the individual is not well defined in the context of dynamic utility maximization.

However, the literature on all these anomalies has provided a good insight into intertemporal choice problems. We have learned many effects, and are able to predict behaviour much better than we used to be. Thus, to further study these anomalies is in fact to contribute to the corpus of the theory; to the present theory but also to an eventual future one. This dissertation has therefore focused on further investigating the anomalies of rational intertemporal choice. Let me next summarize the main findings.

In the first chapter, I have reviewed previous intertemporal choice theories, starting in the nineteenth century and ending in our days. I have tried to provide not only a historical perspective, but also some mathematical understanding of what foundations support the main theory and its alternatives. I believe that chapter one provides for the first time in a single text a combination of existing overall reviews together with more technical references in the field. This review will thus help anyone who wants to both get an overall picture of the discipline and also understand the foundations of intertemporal choice theory.

Also, I have shown in chapter one that among the many anomalies discovered so far, three can be considered very prominent: hyperbolic discounting, sequence effects and magnitude effects. Research into these anomalies has produced most of the literature, and will hopefully provide the clues to new, more succesful approaches in intertemporal choice. I therefore have focused the dissertation mainly on these problems. Finally, chapter one also discussed the normative validity of discounted utility, to conclude that, despite there are reasons to follow discounted utility, these are not definite ones.

Chapters two and three enter into investigating the three most relevant anomalies referred to previously. Let me begin with chapter two. The main research question that I put in chapter two is whether hyperbolic discounting is in fact method dependent. Virtually all previous experiments in intertemporal choice have asked participants to tradeoff only money amounts, while in reality most such decisions are made knowing the interest rate attached to such decisions. Thus, the answer to this research question is most relevant, because answering it we can assess how much of the well-known effects in experiments should be atributed to the fact that we are asking for tradeoffs among quantities but do not provide the subyacent interest rate to respondents. The findings I present show that, in fact, both hyperbolic discounting and excessive discounting disappear when subjects are given this little extra information. Moreover, when people are asked in terms of interest rates only, they behave 'superadditively' (ask for higher annual interest rates for longer intervals), which can be also considered rational given that this is what actually happens in the money market. In conclusion, the investigations in chapter two cast serious doubts on the robustness of the most celebrated finding in intertemporal choice while the question remains open of why an individual's preference structure does change when the choice is presented with such a slightly different framing. More research is already planned to clarify the possible causes of this effect.

Chapter three deals with two other fundamental anomalies, the sequence effect and the magnitude effect. Sequences of outcomes have been shown to produce strange preference patterns. There exists a preference for increasing sequences that contradicts the common assumption of positive time preference. And, more generally, when outcomes are embedded in a sequence, the specific shape of the sequence has an impact on preferences, contrary to what discounted utility would predict. So the question I investigate in chapter three is whether constant sequences, with no shape differences at all, can also produce some anomalous behaviour. The answer to this question is unfortunately yes. In fact, similar results had already been found in two previous papers in the literature, but none of them gave the right explanation of why such an anomalous behaviour occurred. Chapter three shows how the right explanation is that people are more patient when choosing among constant sequences basically because more money is at stakes, which constitutes a worrying finding. The constant sequence effect is a side effect of the magnitude effect, which means we cannot model preferences for sequences of outcomes based simply on the idea of adding up the discounted utilities of the different outcomes in the sequence, as Samuelson proposed. Of course, this finding immediately implies new challenges. First, it reminds us of the importance of the magnitude effect, and of the urgency of capturing it satisfactorily into the model[83]. But second, if people do consider the total amount at stakes when choosing among sequences, then, how do they integrate all single amounts, adding them up? If yes, what delay to this integrated outcome do they consider? And, more generally, how are constant sequences at all evaluated?

These questions are addressed in a second experiment. A choice task presented individuals with two options, an amount with a certain delay, or a sequence whose outcomes added up to the same amount. Manipulating the timings of these objects, I find that subjects valued the sequences and the amounts equally when delay to both objects was equal, contradicting discounting. I label this effect zero intra-sequence discounting (ZID). There are several possible explanations for ZID in these experiment: first, it can be that people use a simple heuristic consisting of adding up all outcomes in a sequence, and discounting from the delay of the beginning of that sequence. So this effect would presumably not occur if sequences were longer, and/or outcomes were not easy to add. Second, and more intriguinly, it could be that multiplicity of outcomes in itself bears some value for the individual. Be it because of a preference for commitment (I want to spread consumption and not fight against the temptation of early overconsumption), or because of the instantaneous utility/value function (diminishing marginal utility or value), or just because we process sequences essentially different in our brains than we do single outcomes; the fact is that the preferences for constant sequences cannot be accomodated into the standard, rational intertemporal choice model.

Some considerations regarding the findings in this dissertation. The experiments backing the findings I just summarized count among the largest and most reliable experiments ever

83 Recently, however, Baucells & Heucamp (2007) have proposed a model in which the magnitude of an outcome determines the 'intrinsic discount rate' of individuals within an integrated probability-time tradeoff model.

conducted in the field. More than two thousand participants, of all ages (adults), gender, education, habitat and income participated in the experiments, and results have been tested to represent the spanish Internet population as by the end of 2004. Also, tests regarding the validity of responses show that at least 94% of respondents answered the questionnaires thoroughly. We can therefore consider the findings solid from this point of view.

On the other hand, though, the question remains whether the experiments are "situationally" representative[84]. The choice tasks I presented asked for preferences over receiving certain money outcomes, but did not put them into any natural context (getting a salary payed, receiving a rent, etc.). My setting is somewhat abstract, and results observed need not be representative of actual behaviour in more realistic settings. (Chapter two has already shown how big an impact can have the mere fact of framing a question as money only or as money and interest.) Also, all experiments dealt with a quite narrow range of money outcomes, going from €200 to €840. Different results could be observed for different orders of magnitude and, of course, also for losses instead of just gains. And finally, a further critical remark that should be done is one regarding the length of sequences considered in experiments of chapter three. All sequences studied have either four or seven outcomes. The reported findings need therefore to be further tested with shorter and also longer sequences.

[84] See for instance Hogarth (2004) on 'situational representativity'.

General Discussion

Every individual is the final and absolute judge of his own interests and well-being
Frank H. Knight
Risk, Uncertainty and Profit, II.III.4.1,
1921

Consumer sovereignty bears no meaning in the context of dynamic decision making. An individual across time is an infinity of individuals, and the known problems of interpersonal comparisons of utility are there to plague us.
Robert Strotz
Myopia and Inconsistency in Dynamic Utility Maximization
Review of Economic Studies,
1956

We can now look at the results presented in the three chapters of this dissertation from a different, wider perspective, and try to find some more general intuitions on how the theory of intertemporal choice can be improved in the future. First, framing effects as shown in chapter two remind us that individuals make intertemporal choices using their brains, and the functioning of the brain is not simple. When we evaluate a distant object, thousands of different things may happen in our brain circuits depending on things such as whether the object has a high value or not, or is an 'hedonic object' like chocolate or a rather neutral one like money; or whether we are reminded of the subyacent interest rate or not, or whether the goods involved are the result of something you earned or something you won. These are all examples of decision situations that we know yield different results. Specifically, in chapter three we have seen once again how the magnitude at stakes is a fundamental aspect of intertemporal choices, even when this magnitude is made out of separated outcomes in a sequence. It seems reasonable to believe that our brain processes decisions essentially different when such decisions involve 'important' quantities or when it deals with 'peanuts', for example. In fact, we should find the opposite quite surprising –that we process them using the same neuronal paths. Sequences of outcomes have been shown also to be a sophisticated decision object. Here, too, it would be surprising that our brain follows identical processing paths when evaluating an increasing sequence than when evaluating a decreasing one, since such patterns seem inseparable of feelings (joy or sadness, for example). Also, the brain most probably treats a two-outcome sequence very differently from a forty-outcomes sequence, since, clearly, making an erroneous decision is usually more harmful in the latter case. And our mind should be considered no exception: it must be the result of evolution and thus, its design may well incorporate such different exposures to harm.

A descriptive theory of intertemporal choice thus needs to be consistent with the findings in neurobiology, or it will never be able to capture the fundamentals of behavior in a truly scientific way. Of course it can be a simplified model of the complex specific behavior observed, but we definitely need to look inside the brain functioning and induce our theories, in the spirit of McClure et. al (2004), who find some neurological backing for $\beta\delta$ -discounting (see section 1.6.3). I am sure this is what will happen in this discipline, and some day –not too far in the future- we will look at discounted utility as a descriptive theory the same way as modern physics sees today Aristotle's physics.

On the other hand, the necessary complement of such neurobiological research is, of course, more experimental work. We need a more exhaustive account of the different effects in intertemporal decision-making and their relationships. This dissertation suggests a connection between the magnitude effect and the constant sequence effect. The magnitude effect seems to me an important 'portal to discovery'. This anomaly was reported empirically more than twenty five years ago, and we still lack a good explanation of why it occurs, even though it casts important doubts on the fundamental assumption of discounted utility models, i.e. the separation between 'instantaneous' utility and time preference (discounting). We should therefore conduct much more research into the magnitude effect in the near future. For instance, receiving 700€ in one month seems not as 'instantaneous' as receiving 20€ in one month. In the first case, it is most likely that the individual will not spend all 700€ upon receiving it, and this may, in the mind of the decision-maker, turn this prospect -receiving 700€ in one month- into a decision-object similar to a sequence of –let's say- seven 100€ outcomes to be received monthly. In contrast, 20€ may well not be evaluated as a monthly sequence of seven (almost) 3€ outcomes. In my view, both a high magnitude and a long sequence indicate the decision maker that his decision will have a large impact on himself in the future, which then can trigger a different decision process, one that yields a more prudent behavior. How and when exactly such a process is triggered, or if it is triggered at all, clearly deserves a systematic and more exhaustive investigation.

On the normative side, I have a different view. First of all we don't necessarily want the same model for descriptive than for prescriptive purposes. I can quite easily predict when a young adolescent is going to take too much risk driving, and of course that does not preclude me from making a recommendation that has more normative basis than his desire of excitement. To know what he is going to do may be treated separately from evaluating what he should do. But, of course, how do we know our recommendation really is better than his own behavior? Who tells what is better for *him*? Or can we just say a certain behavior is wrong, irrational? Ultimately, we are dealing here with the principle of consumer sovereignty and its foundations, and all types of questions arise.

First, think of the recent findings showing that 'wanting is not liking', meaning that too often what we want (i.e. what we choose) does not coincide with what we like. In fact, results in neurobiology show that wanting is driven very much by the 'incentive salience' of rewards, which is processed in the brain independently from real liking (Berridge & Robinson 1998); as a result, subjects may strongly want to consume a drug they actually don't like anymore. The drug is affecting critical brain regions involved in the salience of incentives, making it impossible for the individual to resist consumption, which is driven only by this altered incentive salience, and not at all by the expected liking of consumption, that does not affect choice. These findings may well be considered the closest example of pure irrational behavior: one chooses something one expects not to like.

Second, even if we were at all able of correctly knowing what we will like in the immediate future, there is nevertheless strong evidence that we humans are very bad at predicting future utilities (see, for example, Gilbert & Wilson 2000, or Loewenstein et. al 2003) and also at remembering past experiences (see, for example, Kahneman et. al 1997, or Kahneman 1999). Kahneman therefore introduced the distinction between *experienced utility*, *remembered utility*, *predicted utility* and *decision utility* (choice). The more we learn about how all these different sources of utility operate, the lower is our confidence in the goodness of our sovereign and completely autonomous decisions.

Imagine now that we also were very good at estimating past and future utilities, and did properly incorporate that knowledge into our everyday decision-making. There would still be a third problem: what if the individual himself does disagree with a past decision of him? Should he stick to the past 'plan of consumption', or should he rather feel free to re-evaluate the situation from the current perspective, and change the plan? This is of course, again, a very problematic question, since we don't know how to compare utility among different individuals, or among the same individuals at different points in time. Economic theory has therefore made an assumption: if we consider basically stable preferences that are dynamically consistent, then we can define rationality also in the context of dynamic decision making. Why, nevertheless, should stable dynamically consistent preferences be normative? Because being stable, maintaining one's identity, is good. The question remains as whether one maintains his identity better by always sticking to past plans or rather by allowing them to be changed on an informed way. I believe the latter is correct. But, how could a normative model for intertemporal choice possibly discriminate between informed an uninformed decision-making?

One possible solution I plan to explore in future research would be to modify our invariance assumption in rational intertemporal choice theory: instead of assuming an invariant and dynamically consistent utility function, we could require utility functions only to be 'retrospectively invariant', meaning that any departure from a previous plan should be one that is afterwards never regretted from the point of view of all subsequent 'selves'. This apparently minor modification would actually lead to a very radical alternative normative theory, since, strictly speaking, the rationality of an action would become something only *a posteriori*: is this a rational intertemporal choice? Well, let's wait and see how the same individual retrospectively evaluates this choice in the future, and then –and only then- I will tell you if that *was* a rational choice. This approach may seem absurd to us economists at first sight; why would we need a normative theory that is not able to indicate, *a priori*, what is a good decision for the individual? But a closer look may suggest that such an approach is not completely futile. We could just *define* intertemporal rationality this way, and, on the other hand, turn the normativity of intertemporal choice into a purely scientific and empirical problem. Is this a rational intertemporal choice? Well, let's see, my data show that 98% of similar people who did that, did also regret having done it afterwards, so I'm afraid not, it most probably is not rational for you. If an individual across time is an infinity of individuals, why not use their voice to inform us of what is the good of man? It is my conviction that it is perfectly possible to go one step forward in our formulation of normative intertemporal choice theory and move from the absolute monarchy of consumer-sovereignty into a more scientifically correct 'consumer-democracy', that accounts not only for the preferences of future 'selves', but also for the preferences of present and future 'others'.

Annex 1: Tables and Results

Contingency Tables (EXPERIMENT 1)

Overall Frequencies and Horizontal Percentages (EXPERIMENT1)

		Q2								
		A				B				
		Q3 A		Q3 B		Q3 A		Q3 B		
		Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	
Q1	A	140	22	10	18	36	16	9	49	300
	B	6	7	2	20	4	10	5	147	201
										501

		Q2								
		A				B				
		Q3 A		Q3 B		Q3 A		Q3 B		
		Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	
Q1	A	46,67%	7,33%	3,33%	6,00%	12,00%	5,33%	3,00%	16,33%	100,00%
	B	2,99%	3,48%	1,00%	9,95%	1,99%	4,98%	2,49%	73,13%	100,00%

Frequencies and Horizontal Percentages Form Q2Q3Q1Q4

		Q2								
		A				B				
		Q3 A		Q3 B		Q3 A		Q3 B		
		Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	
Q1	A	31	2	5	7	9	5	2	12	73
	B	1	2	1	9	1	0	2	28	44
										117

		Q2								
		A				B				
		Q3 A		Q3 B		Q3 A		Q3 B		
		Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	
Q1	A	42,47%	2,74%	6,85%	9,59%	12,33%	6,85%	2,74%	16,44%	100,00%
	B	2,27%	4,55%	2,27%	20,45%	2,27%	0,00%	4,55%	63,64%	100,00%

Frequencies and Horizontal Percentages Form Q4Q2Q3Q1

		Q2								
		A				B				
		Q3 A		Q3 B		Q3 A		Q3 B		
		Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	
Q1	A	33	9	4	3	9	3	2	12	75
	B	0	1	1	5	2	5	1	42	57
										132

		Q2								
		A				B				
		Q3 A		Q3 B		Q3 A		Q3 B		
		Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	
Q1	A	44,00%	12,00%	5,33%	4,00%	12,00%	4,00%	2,67%	16,00%	100,00%
	B	0,00%	1,75%	1,75%	8,77%	3,51%	8,77%	1,75%	73,68%	100,00%

Frequencies and Horizontal Percentages Form Q1Q4Q2Q3

		Q2								
		A				B				
		Q3 A		Q3 B		Q3 A		Q3 B		
		Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	
Q1	A	35	7	0	5	10	2	3	12	74
	B	2	2	0	3	0	3	1	45	56
										130

		Q2								
		A				B				
		Q3 A		Q3 B		Q3 A		Q3 B		
		Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	
Q1	A	47,30%	9,46%	0,00%	6,76%	13,51%	2,70%	4,05%	16,22%	100,00%
	B	3,57%	3,57%	0,00%	5,36%	0,00%	5,36%	1,79%	80,36%	100,00%

Frequencies and Horizontal Percentages Form Q3Q1Q4Q2

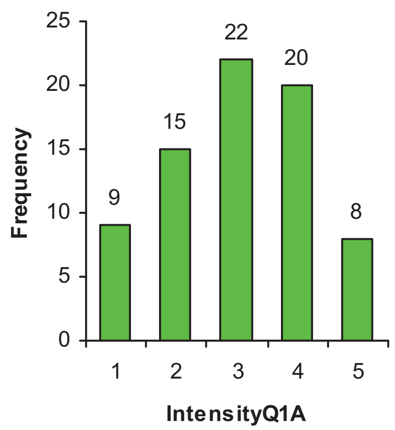
		Q2								
		A				B				
		Q3 A		Q3 B		Q3 A		Q3 B		
		Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	
Q1	A	41	4	1	3	8	6	2	13	78
	B	3	2	0	3	1	2	1	32	44
										122

		Q2								
		A				B				
		Q3 A		Q3 B		Q3 A		Q3 B		
		Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	Q4 A	Q4 B	
Q1	A	52,56%	5,13%	1,28%	3,85%	10,26%	7,69%	2,56%	16,67%	100,00%
	B	6,82%	4,55%	0,00%	6,82%	2,27%	4,55%	2,27%	72,73%	100,00%

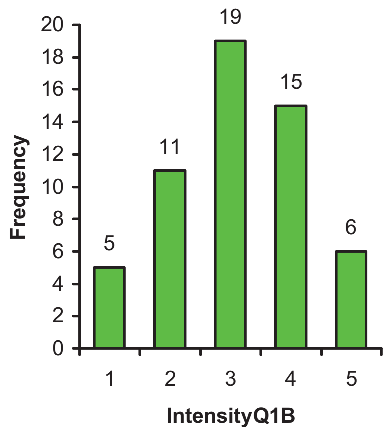
Declared Intensity-of-Preference Results (EXPERIMENT 1)

Average Intensity[85]	Q1	Q2	Q3	Q4
when choosing A	3.04 (1.2)	3.07 (1.00)	3.16 (0.99)	3.25 (1.10)
when choosing B	3.11 (1.1)	3.61 (0.91)	3.63 (0.99)	3.23 (1.23)

Declared intensity of preference when choosing A in Q1

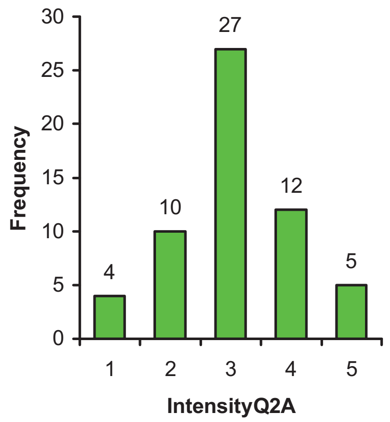


Declared intensity of preference when choosing B in Q1

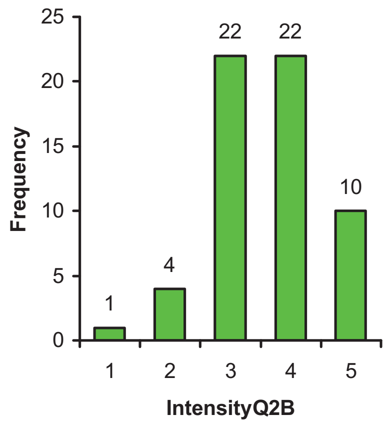


[85] Measured only in the form type where the affected question was in first position of the questionnaire.

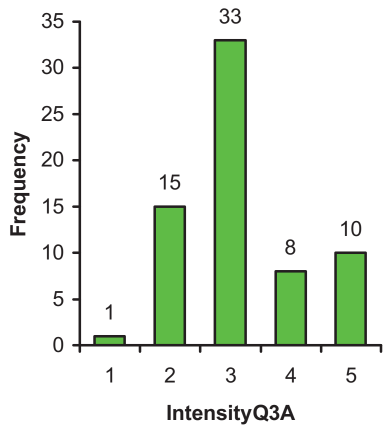
Declared intensity of preference when choosing A in Q2



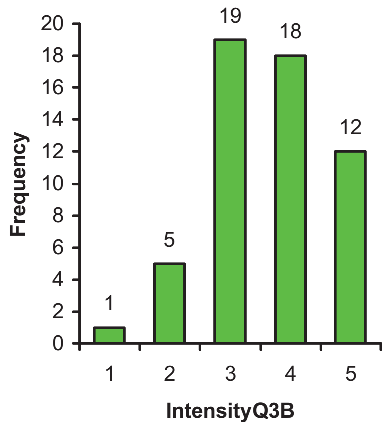
Declared intensity of preference when choosing B in Q2



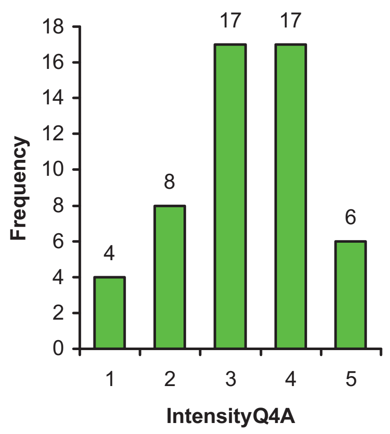
Declared intensity of preference when choosing A in Q3



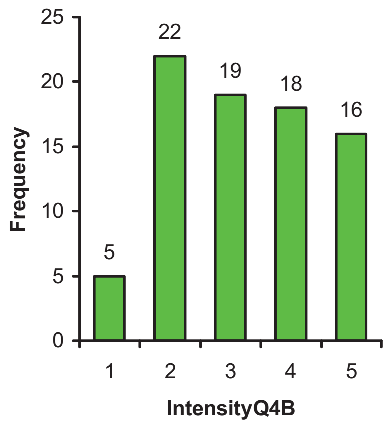
Declared intensity of preference when choosing B in Q3



Declared intensity of preference when choosing A in Q4



Declared intensity of preference when choosing B in Q4



Contingency Tables (EXPERIMENT) 2

TREATMENT I

Overall Frequencies and Horizontal Percentages (all Forms)

		Q2				
		A		B		
		Q3 A	**Q3** B	**Q3** A	**Q3** B	
Q1	A	97	42	65	74	278
	B	21	9	90	345	465
						743

		Q2				
		A		B		
		Q3 A	**Q3 B**	**Q3 A**	**Q3 B**	
Q1	A	35%	15%	23%	27%	100%
	B	5%	2%	19%	74%	100%

Frequencies and Horizontal Percentages Form Q1Q2Q3

		Q2				
		A		B		
		Q3 A	**Q3** B	**Q3** A	**Q3** B	
Q1	A	32	15	23	22	92
	B	6	2	19	115	142
						234

		Q2				
		A		B		
		Q3 A	**Q3 B**	**Q3 A**	**Q3 B**	
Q1	A	35%	16%	25%	24%	100%
	B	4%	1%	13%	81%	100%

Frequencies and Horizontal Percentages Form Q2Q3Q1

		Q2				
		A		B		
		Q3 A	**Q3** B	**Q3** A	**Q3** B	
Q1	A	51	16	22	35	124
	B	9	5	21	107	142
						266

		Q2				
		A		B		
		Q3 A	**Q3 B**	**Q3 A**	**Q3 B**	

Q1	A	41%	13%	18%	28%	100%
	B	6%	4%	15%	75%	100%

Frequencies and Horizontal Percentages Form Q3Q1Q2

		Q2				
		A		B		
		Q3 A	**Q3** B	**Q3** A	**Q3** B	
Q1	A	14	11	20	17	62
	B	6	2	50	123	181
						243

		Q2				
		A		B		
		Q3 A	**Q3 B**	**Q3 A**	**Q3 B**	
Q1	A	23%	18%	32%	27%	100%
	B	3%	1%	28%	68%	100%

TREATMENT II

Overall Frequencies and Horizontal Percentages (all Forms)

		Q2				
		A		B		
		Q3 A	**Q3** B	**Q3** A	**Q3** B	
Q1	A	173	76	9	18	276
	B	40	54	48	321	463
						739

		Q2				
		A		B		
		Q3 A	**Q3 B**	**Q3 A**	**Q3 B**	
Q1	A	63%	28%	3%	7%	100%
	B	9%	12%	10%	69%	100%

Frequencies and Horizontal Percentages Form Q1Q2Q3

		Q2				
		A		B		
		Q3 A	**Q3** B	**Q3** A	**Q3** B	
Q1	A	62	26	3	9	100
	B	9	23	13	95	140
						240

		Q2				

		A		B		
		Q3 A	**Q3 B**	**Q3 A**	**Q3 B**	
Q1	A	62%	26%	3%	9%	100%
	B	6%	16%	9%	68%	100%

Frequencies and Horizontal Percentages Form Q2Q3Q1

		Q2				
		A		B		
		Q3 A	**Q3** B	**Q3** A	**Q3** B	
Q1	A	79	30	2	3	114
	B	15	9	7	97	128
						242

		Q2				
		A		B		
		Q3 A	**Q3 B**	**Q3 A**	**Q3 B**	
Q1	A	69%	26%	2%	3%	100%
	B	12%	7%	5%	76%	100%

Frequencies and Horizontal Percentages Form Q3Q1Q2

		Q2				
		A		B		
		Q3 A	**Q3** B	**Q3** A	**Q3** B	
Q1	A	32	20	4	6	62
	B	16	22	28	129	195
						257

		Q2				
		A		B		
		Q3 A	**Q3 B**	**Q3 A**	**Q3 B**	
Q1	A	52%	32%	6%	10%	100%
	B	8%	11%	14%	66%	100%

Declared Intensity-of-Preference Results (EXPERIMENT 2)

Treatment I

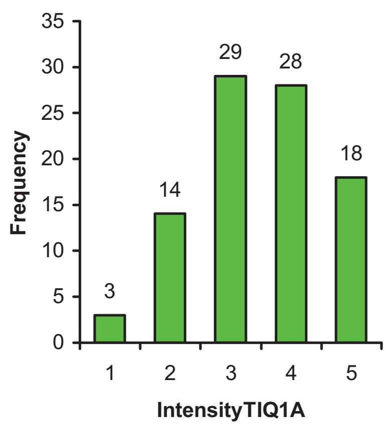
Average Intensity[86]	Q1	Q2	Q3
when choosing A	3.48 (1.07)	3.1 (0.98)	3.22 (1.07)
when choosing B	3.56 (1.08)	3.59 (1.02)	3.78 (1.06)

Treatment II

Average Intensity[87]	Q1	Q2	Q3
when choosing A	3.19 (0.95)	3.23 (1.17)	3.8 (1.14)
when choosing B	3.63 (1.02)	3.5 (1.12)	3.82 (0.96)

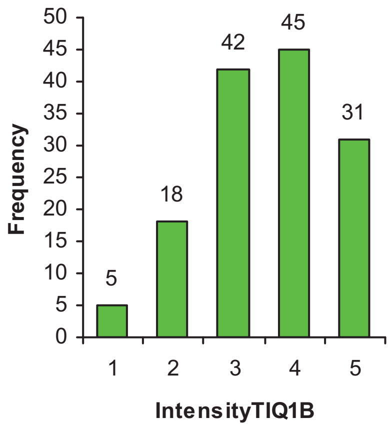
Treatment I

Declared intensity of preference when choosing A in TIQ1

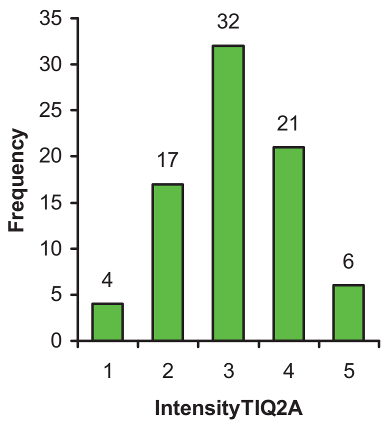


[86] Measured only in the form type where the affected question was in first position of the questionnaire. Standard deviations in parenthesis.
[87] Measured only in the form type where the affected question was in first position of the questionnaire. Standard deviations in parenthesis.

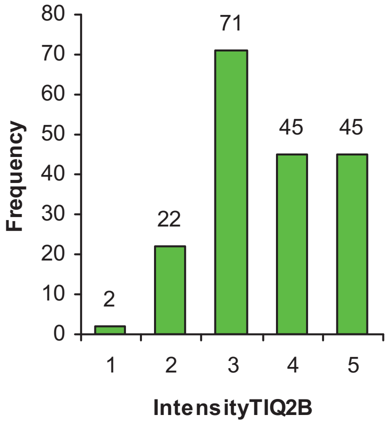
Declared intensity of preference when choosing B in TIQ1



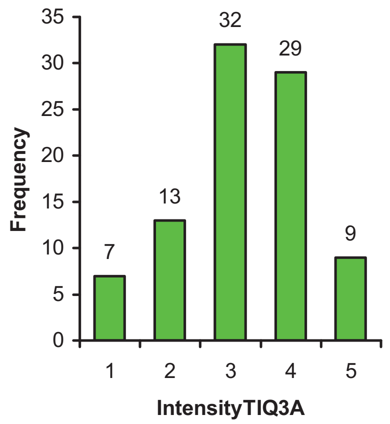
Declared intensity of preference when choosing A in TIQ2



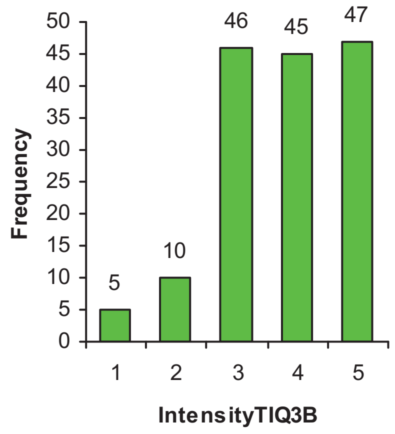
Declared intensity of preference when choosing B in TIQ2



Declared intensity of preference when choosing A in TIQ3

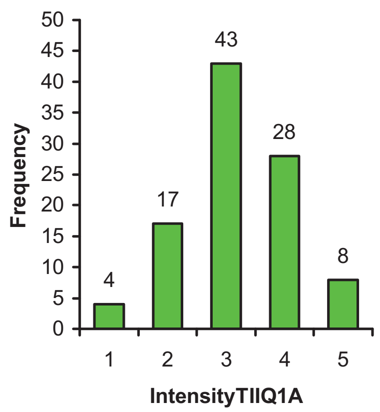


Declared intensity of preference when choosing B in TIQ3

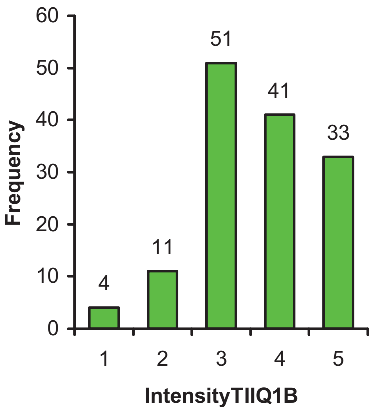


<u>Treatment II</u>

Declared intensity of preference when choosing A in TIIQ1



Declared intensity of preference when choosing B in TIIQ1



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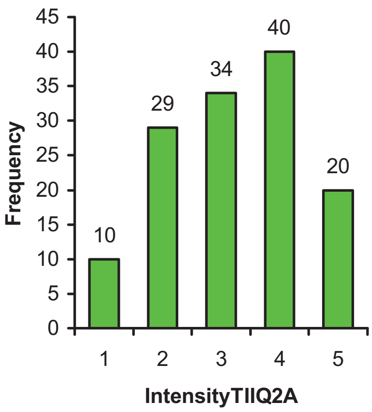
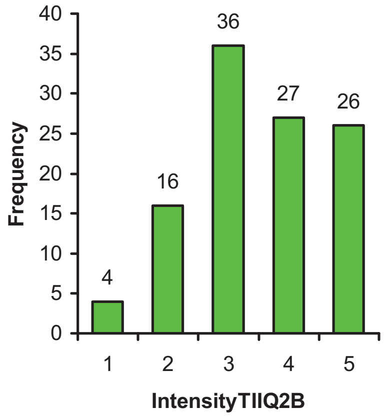
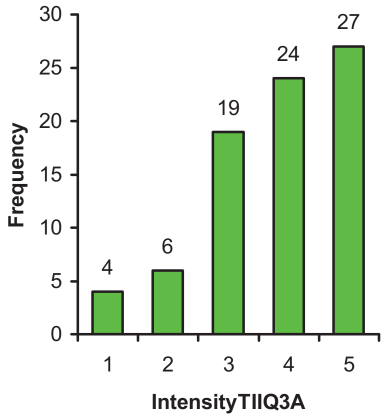


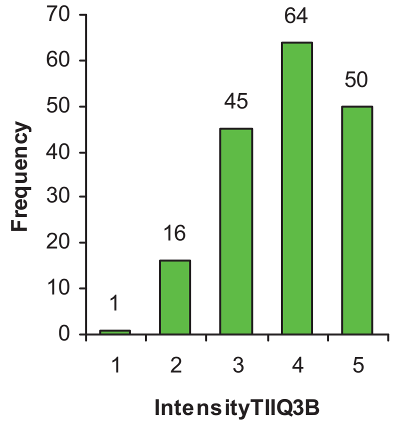
Declared intensity of preference when choosing B in TIIQ2



Declared intensity of preference when choosing A in TIIQ3



Declared intensity of preference when choosing B in TIIQ3



<u>Annex 2:</u> Materials

CHAPTER 2

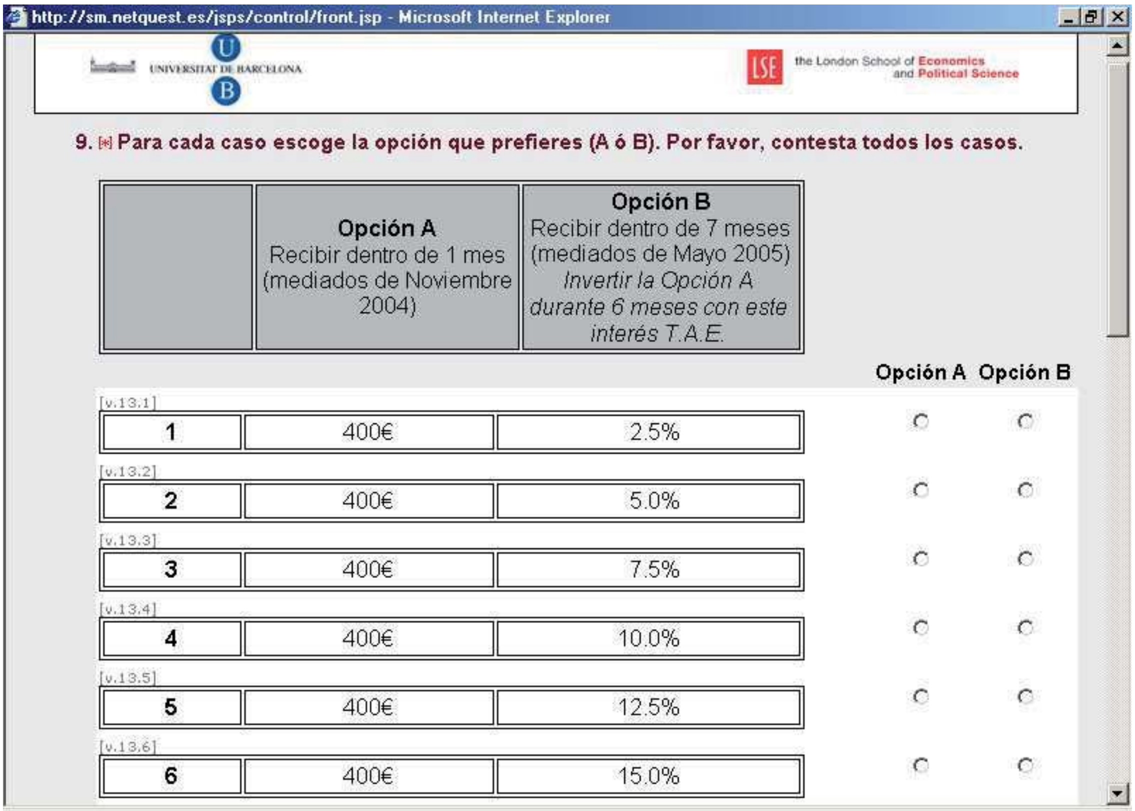
Instructions for all experimental conditions		
Condition	**SS**	**LL**
Interest-only	**OPTION A**: received in 1 month (mid November 2004).*	**OPTION B**: received in 7 months (mid May 2005). Invest Option A for 6 months at the following AER.
	€ 400	**2.5%**
Interest+Money	**OPTION A**: received in 7 months (mid May 2005).	**OPTION B**: received in 13 months (mid November 2005). Invest Option A for 6 months and receive the following at end of the investment period (AER in parentheses).
	€ 400	**€ 404 (2.5%)**
Money-only	**OPTION A**: received in 13 months (mid November 2005).	**OPTION B**: received in 19 months (mid May 2006). Invest Option A for 6 months and receive the following at end of the investment period.
	€ 400	**€ 404**
No-investment	**OPTION A**: received in 1 month (mid November 2004).	**OPTION B**: received in 19 months (mid May 2006).
	€ 400	**€ 404**
* All conditions included all intervals for a complete 4 × 4 design. To avoid redundancy, this table shows each description assigned to one interval only.		

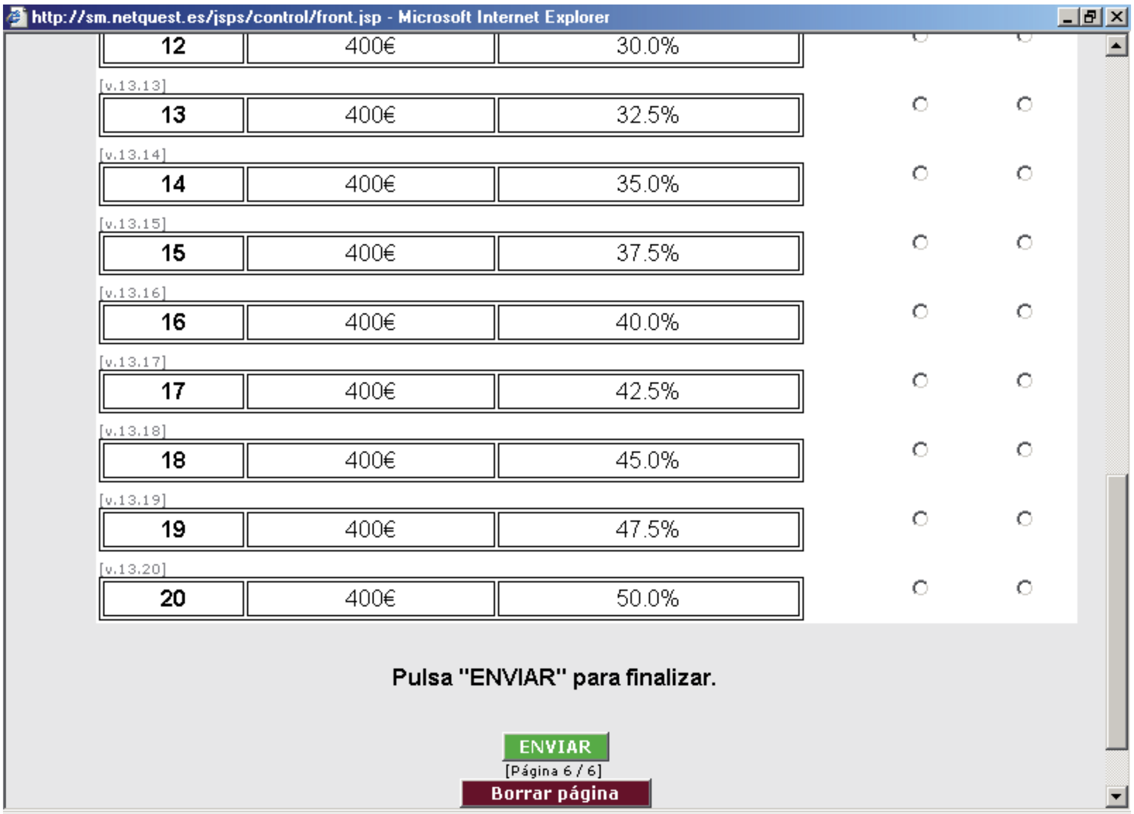
Interest rates and corresponding LL amounts. The values represent the amount received after investing € 400 for the specified period at the specified AER.

		Interval	
Payoff Alternative	AER	**6 months**	**18 months**
1	2.5	**€ 405**	**€ 415**
2	5.0	**€ 410**	**€ 430**
3	7.5	**€ 415**	**€ 446**
4	10.0	**€ 420**	**€ 461**

5	12.5	**€ 424**	**€ 477**
6	15.0	**€ 429**	**€ 493**
7	17.5	**€ 434**	**€ 509**
8	20.0	**€ 438**	**€ 526**
9	22.5	**€ 443**	**€ 542**
10	25.0	**€ 447**	**€ 559**
11	27.5	**€ 452**	**€ 576**
12	30.0	**€ 456**	**€ 593**
13	32.5	**€ 460**	**€ 610**
14	35.0	**€ 465**	**€ 627**
15	37.5	**€ 469**	**€ 645**
16	40.0	**€ 473**	**€ 663**
17	42.5	**€ 477**	**€ 680**
18	45.0	**€ 482**	**€ 698**
19	47.5	**€ 486**	**€ 717**
20	50.0	**€ 490**	**€ 735**

Sample screenshots from Experiment

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12 400€ 30.0%
[v.13.13]
13 400€ 32.5%
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14 400€ 35.0%
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15 400€ 37.5%
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16 400€ 40.0%
[v.13.17]
17 400€ 42.5%
[v.13.18]
18 400€ 45.0%
[v.13.19]
19 400€ 47.5%
[v.13.20]
20 400€ 50.0%
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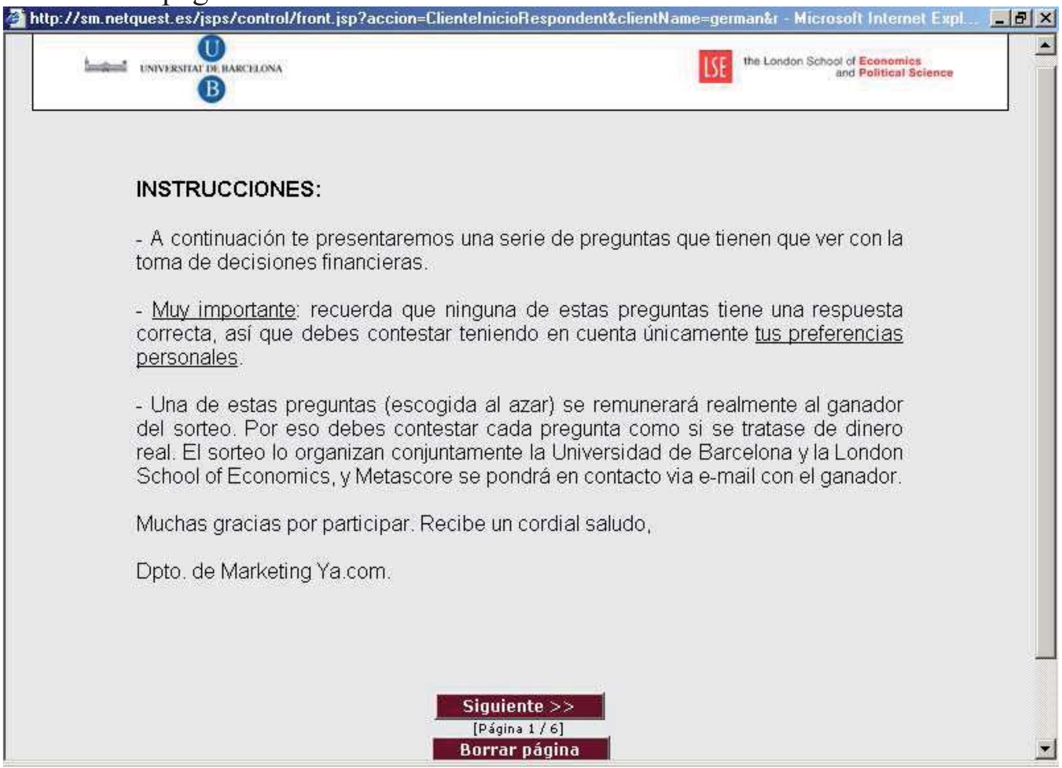
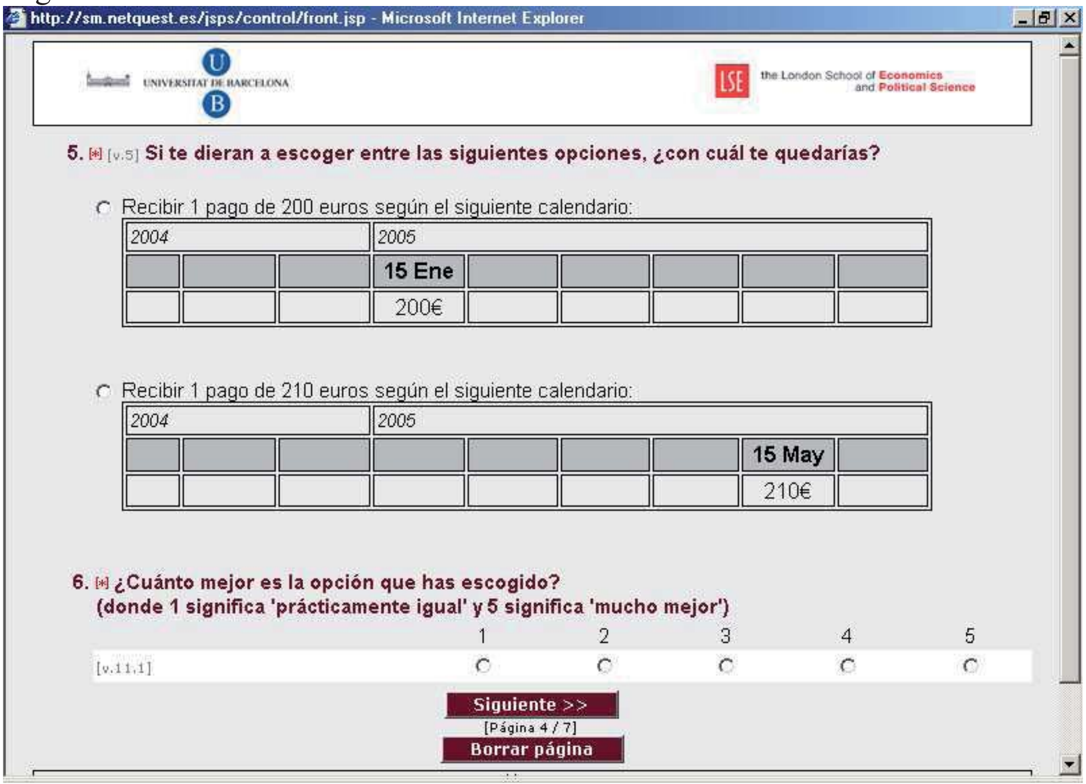
CHAPTER 3
Experiment 1 Questionnaire (screens)[88]

Instructions page



[88] The order I present here was only one possible order out of three (Q1Q2Q3, Q3Q1Q2, Q2Q3Q1)

Page 1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5. [v.5] **Si te dieran a escoger entre las siguientes opciones, ¿con cuál te quedarías?**

○ Recibir 1 pago de 200 euros según el siguiente calendario:

2004			2005					
			15 Ene					
			200€					

○ Recibir 1 pago de 210 euros según el siguiente calendario:

2004			2005					
							15 May	
							210€	

6. **¿Cuánto mejor es la opción que has escogido?**
(donde 1 significa 'prácticamente igual' y 5 significa 'mucho mejor')

	1	2	3	4	5
[v.11.1]	○	○	○	○	○

Siguiente >>
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Page 2


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UNIVERSITAT DE BARCELONA — the London School of Economics and Political Science

1. [v.1] **Si te dieran a escoger entre las siguientes opciones, ¿con cuál te quedarías?**

○ Recibir 4 pagos de 210 euros según el siguiente calendario:

2004			2005					
					15 Mar	15 Abr	15 May	15 Jun
					210€	210€	210€	210€

○ Recibir 4 pagos de 200 euros según el siguiente calendario:

2004			2005					
	15 Nov	15 Dic	15 Ene	15 Feb				
	200€	200€	200€	200€				

2. **¿Cuánto mejor es la opción que has escogido?**
(donde 1 significa 'prácticamente igual' y 5 significa 'mucho mejor')

	1	2	3	4	5
[v.2.1]	○	○	○	○	○

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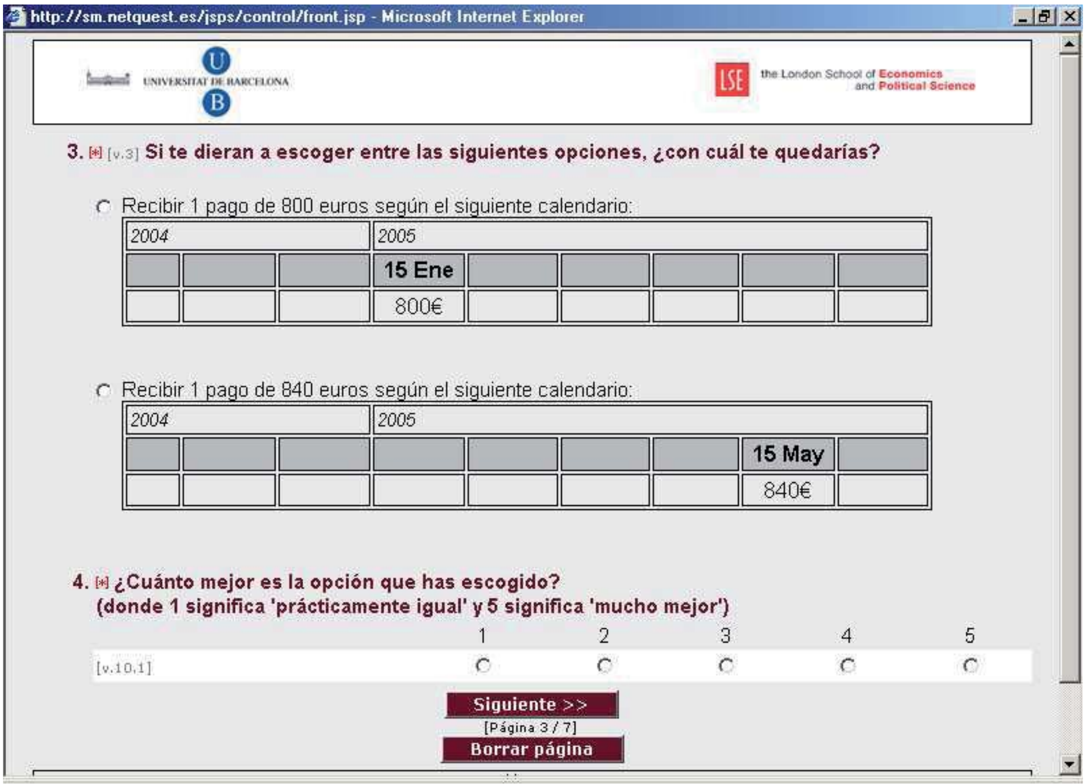
Page 3



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UNIVERSITAT DE BARCELONA

LSE the London School of Economics and Political Science

3. [v.3] Si te dieran a escoger entre las siguientes opciones, ¿con cuál te quedarías?

Recibir 1 pago de 800 euros según el siguiente calendario:

2004			2005					
			15 Ene					
			800€					

Recibir 1 pago de 840 euros según el siguiente calendario:

2004			2005					
							15 May	
							840€	

4. ¿Cuánto mejor es la opción que has escogido?
(donde 1 significa 'prácticamente igual' y 5 significa 'mucho mejor')

	1	2	3	4	5
[v.10.1]	○	○	○	○	○

Siguiente >>

[Página 3 / 7]

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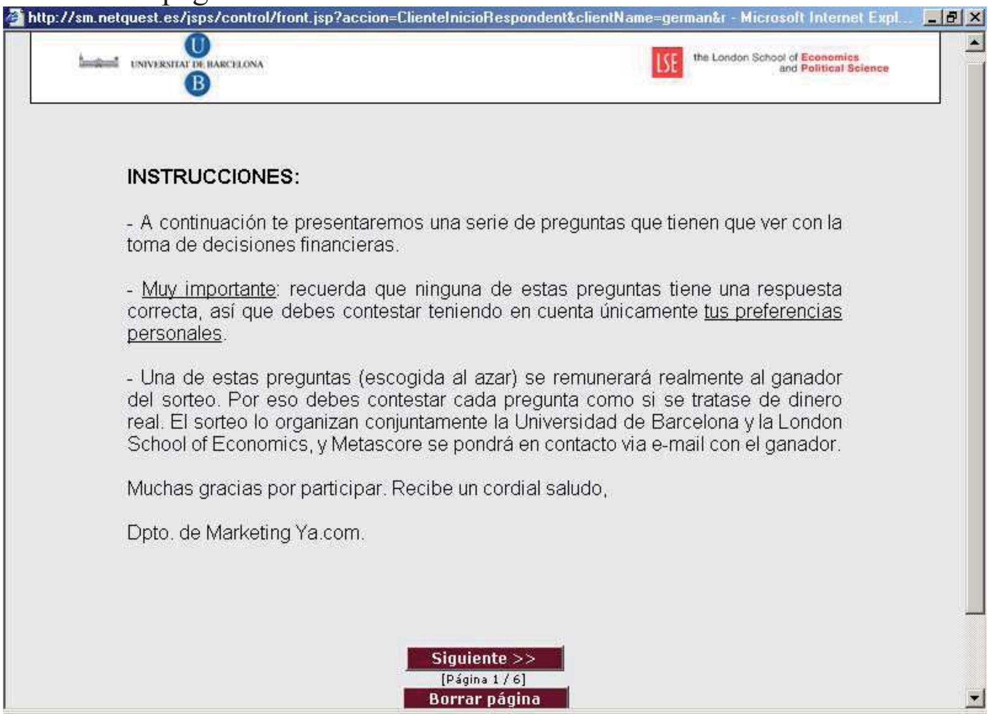
Experiment 2 Questionnaire (screens)[89]

Instructions page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INSTRUCCIONES:

- A continuación te presentaremos una serie de preguntas que tienen que ver con la toma de decisiones financieras.

- Muy importante: recuerda que ninguna de estas preguntas tiene una respuesta correcta, así que debes contestar teniendo en cuenta únicamente tus preferencias personales.

- Una de estas preguntas (escogida al azar) se remunerará realmente al ganador del sorteo. Por eso debes contestar cada pregunta como si se tratase de dinero real. El sorteo lo organizan conjuntamente la Universidad de Barcelona y la London School of Economics, y Metascore se pondrá en contacto via e-mail con el ganador.

Muchas gracias por participar. Recibe un cordial saludo,

Dpto. de Marketing Ya.com.

Siguiente >>
[Página 1 / 6]
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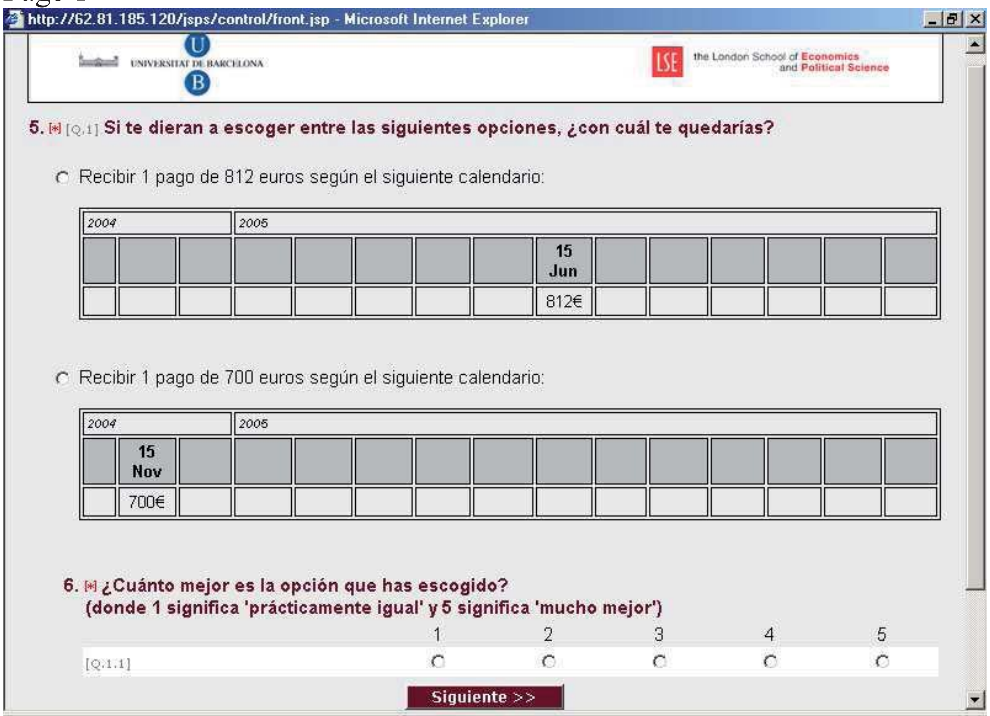
Page 1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5. [Q.1] **Si te dieran a escoger entre las siguientes opciones, ¿con cuál te quedarías?**

○ Recibir 1 pago de 812 euros según el siguiente calendario:

2004			2005											
								15 Jun						
								812€						

○ Recibir 1 pago de 700 euros según el siguiente calendario:

2004			2005											
	15 Nov													
	700€													

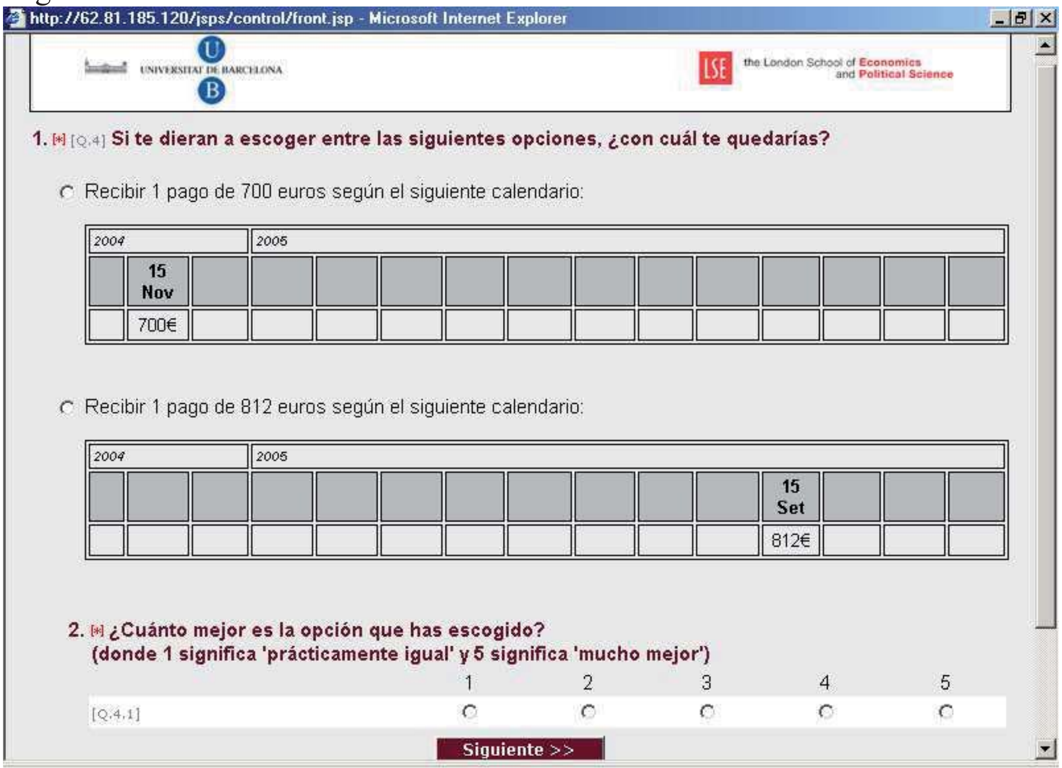
6. **¿Cuánto mejor es la opción que has escogido?**
(donde 1 significa 'prácticamente igual' y 5 significa 'mucho mejor')

	1	2	3	4	5
[Q.1.1]	○	○	○	○	○

Siguiente >>

[89] The order I present here was even so only one possible order out of three (Q1Q2Q3, Q3Q1Q2, Q2Q3Q1)

Page 2



1. [Q.4] Si te dieran a escoger entre las siguientes opciones, ¿con cuál te quedarías?

Recibir 1 pago de 700 euros según el siguiente calendario:

2004			2005											
	15 Nov													
	700€													

Recibir 1 pago de 812 euros según el siguiente calendario:

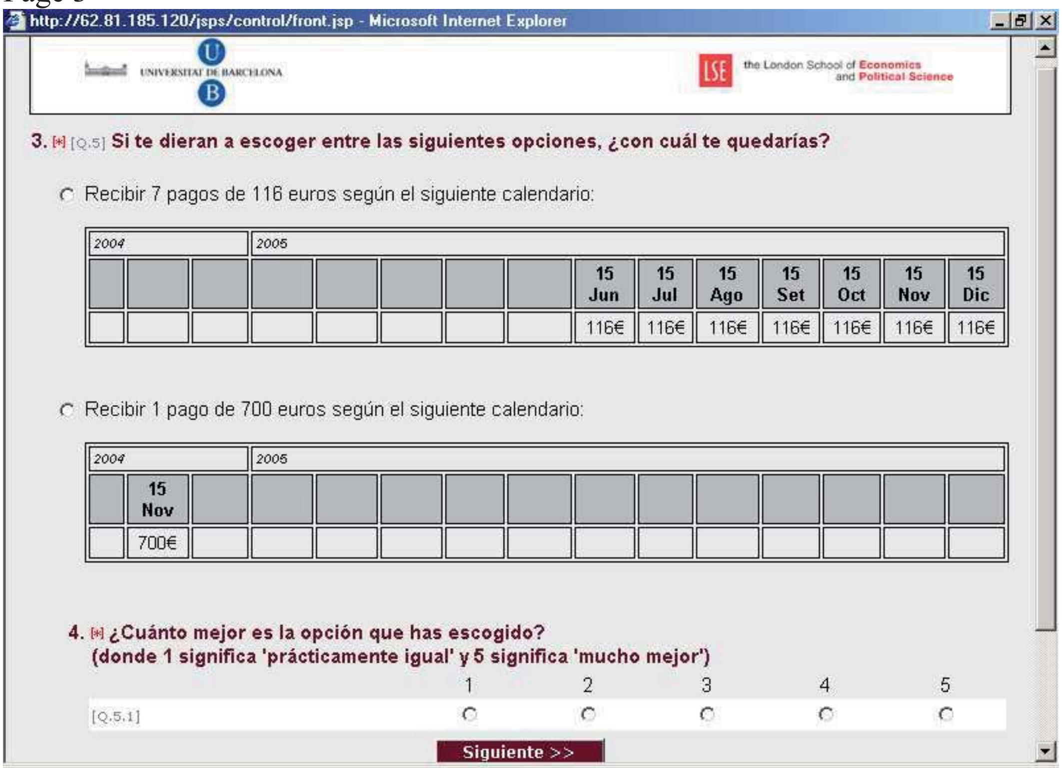
2004			2005											
											15 Set			
											812€			

2. ¿Cuánto mejor es la opción que has escogido? (donde 1 significa 'prácticamente igual' y 5 significa 'mucho mejor')

	1	2	3	4	5
[Q.4.1]	○	○	○	○	○

Siguiente >>

Page 3



3. [Q.5] Si te dieran a escoger entre las siguientes opciones, ¿con cuál te quedarías?

Recibir 7 pagos de 116 euros según el siguiente calendario:

2004			2005											
								15 Jun	15 Jul	15 Ago	15 Set	15 Oct	15 Nov	15 Dic
								116€	116€	116€	116€	116€	116€	116€

Recibir 1 pago de 700 euros según el siguiente calendario:

2004			2005											
	15 Nov													
	700€													

4. ¿Cuánto mejor es la opción que has escogido? (donde 1 significa 'prácticamente igual' y 5 significa 'mucho mejor')

	1	2	3	4	5
[Q.5.1]	○	○	○	○	○

Siguiente >>

Invitation email

Estimad@ Usuari@:

En esta ocasión, MetaScore te propone opinar en un estudio acerca de Preferencias Financieras que está realizando la Universitat de Barcelona junto con la London School of Economics.
La encuesta tiene una duración estimada de 10 minutos. El cuestionario es totalmente anónimo, utilizando las respuestas únicamente con una finalidad estadística.
Te recordamos que durante los próximos 3 meses recibirás descuentos en tiendas de la Red Ya.com y una suscripción totalmente gratuita a la revista "PC Actual", en la dirección que nos indicaste, por rellenar las encuestas que te proponemos.
Para responder al cuestionario pincha sobre el siguiente enlace:

XXXXXXXXXXXXXXXXXXXXX

Muchas gracias por participar.
Recibe un cordial saludo:
Dpto. de Marketing Ya.com.

¿Por qué recibo este E-mail?: Recibes este E-mail porque has aceptado pertenecer al proyecto MetaScore llevado a cabo por Ya.com. Si deseas darte de baja del proyecto MetaScore puedes hacerlo enviando un correo a bajasmetascore@ya.com indicando el e-mail con el que te registraste.

Annex 3: Representativity and Quality of Responses

Profile of Participants

The Metascore Online Panel was created during 2004 by the company ya.com, one of the big Internet Service Providers in Spain, and adjusted to the Internet population in Spain with respect to socio-demographical variables. To evaluate whether the sample I obtained for Experiments 1 and 2 differs from the overall Online Panel (and thus from Spanish Internet population), I will use the G-test for goodness of fit. The high response rate obtained almost already responds to this question, but I nevertheless wanted to check whether participants were truly representative.

Consider n observations within the sample in an experiment (n individuals of which we have a particular socio-demographical information, for example, age) to be grouped into k categories (the k alternatives of this information, for example, the different ages). Our objective is to decide whether the observed information is consistent with the specific frequencies that would be obtained from the overall Online Panel.

The Likelihood-Ratio test is constructed as minus 2 times the LN of the likelihood quotient of the Null hypothesis and the Alternative hypothesis. In our problem, data follow a multinomial distribution with k-categories. Thus, in the Null hypothesis H0 these categories will have the frequencies of the overall Panel as expected probabilities. In the Alternative hypothesis H1, these probabilities will be the observed frequencies. Using the following symbols:

n:	individuals who answer a particular question
k:	number of alternatives of a particular question
n_i:	individuals answering alternative *i*
$p_i = n_i/n$	observed frequency of alternative *i*
q_i:	expected frequency of alternative *i* according to the overall sample (the online panel)
$m_i = n*q_i$	expected number of individuals choosing *i*

Likelihoods:

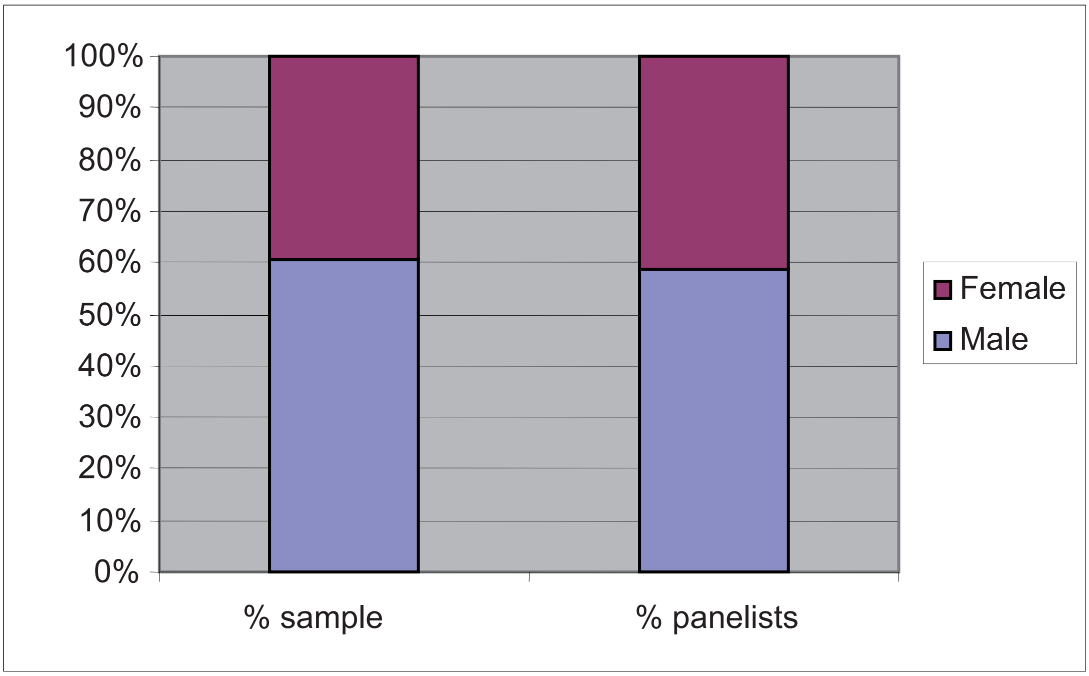
$$l(q \mid n) \propto \prod_{i=1}^{k} q_i^{n_i} \qquad l(p \mid n) \propto \prod_{i=1}^{k} p_i^{n_i}$$

G-test

$$G = -2\ln\left(\frac{l(q \mid n)}{l(p \mid n)}\right) = -2\sum_{i=1}^{k} n_i \ln\left(\frac{m_i}{n_i}\right) = -2\sum_{i=1}^{k} n_i \ln\left(\frac{q_i}{p_i}\right) \approx \chi_{k-1}^2$$

A too big G-test value would mean a deviation from the null Hypothesis of both distributions (panel and sample) being the same. I next present the charts together with p-values indicating the probability of observing an equal or higher difference if the null Hypothesis was true (panel distribution equal sample distribution). I find a small effect only for gender and for age, while no effect at all in all remaining cases.

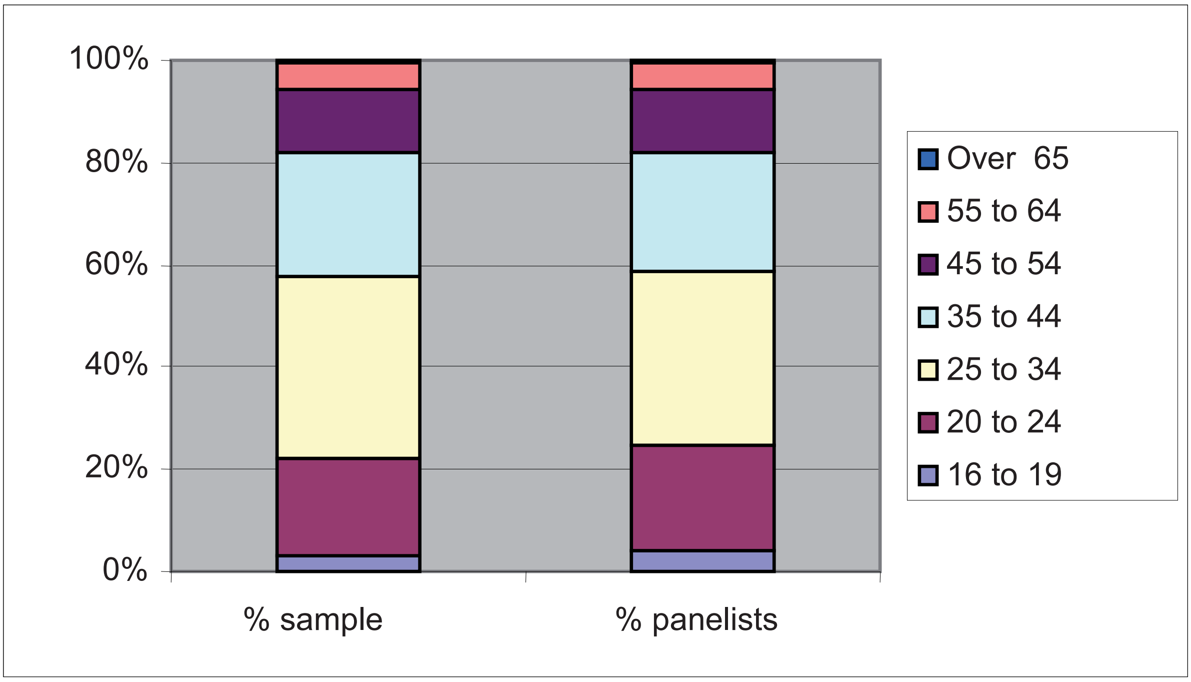
GENDER



G-test (we can reject null Hypothesis with $p<0.1$)

$\chi_1^2 = 3.33$ (right tail p=0.068)

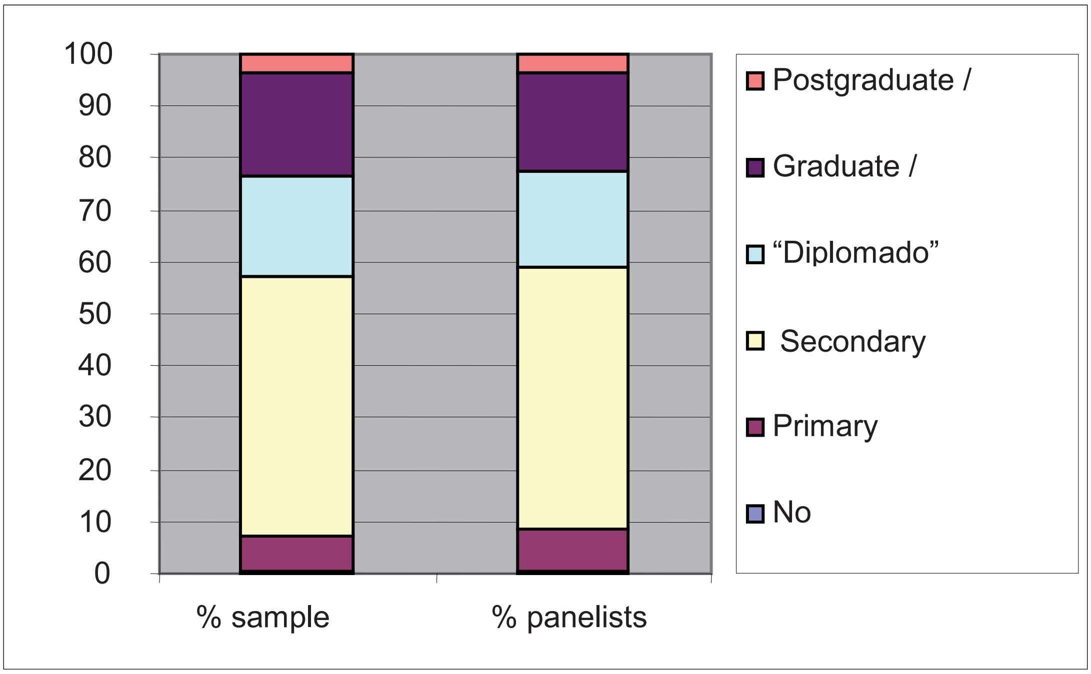
AGE



G-test (we can reject null Hypothesis with $p<0.05$)

$\chi_6^2 = 13.36$ (right tail p=0.0377)

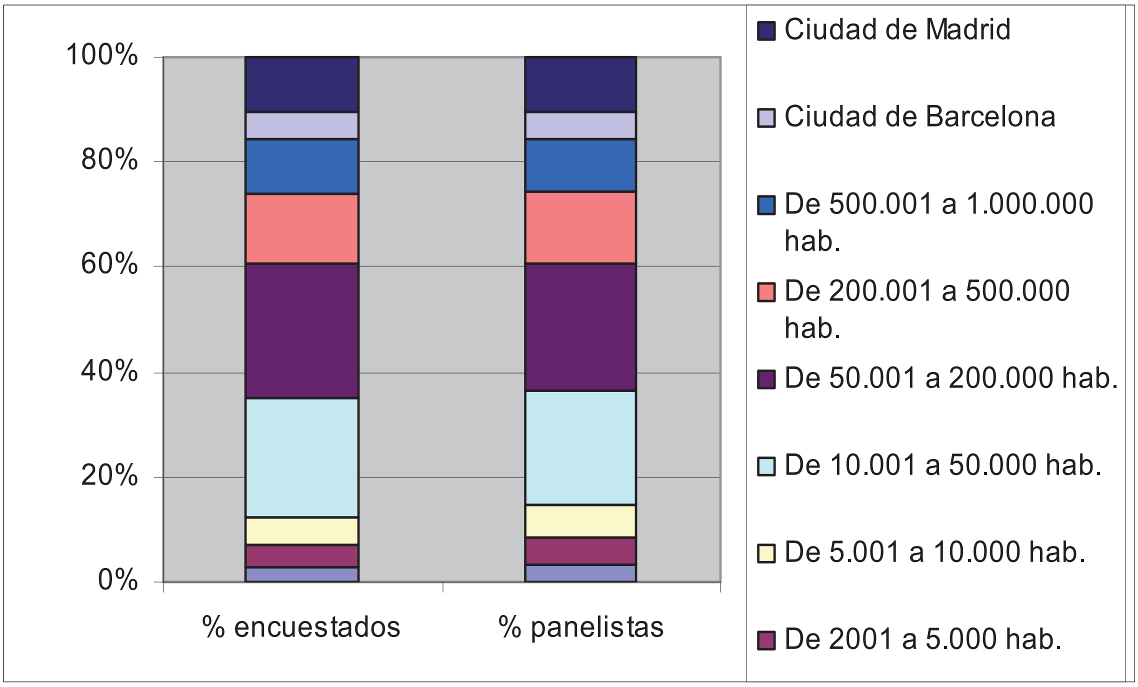
EDUCATION



G-test (we cannot reject null Hypothesis)

$\chi_5^2 = 6.73$ (right tail p=0.2415)

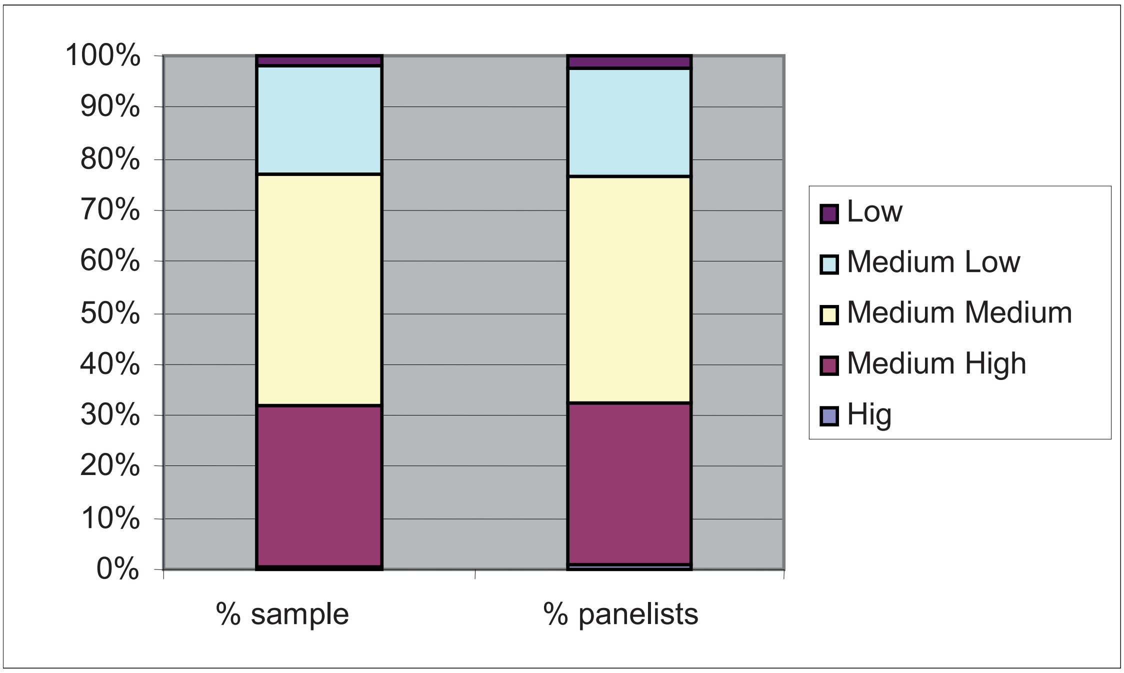
HABITAT



G-test (we cannot reject null Hypothesis)

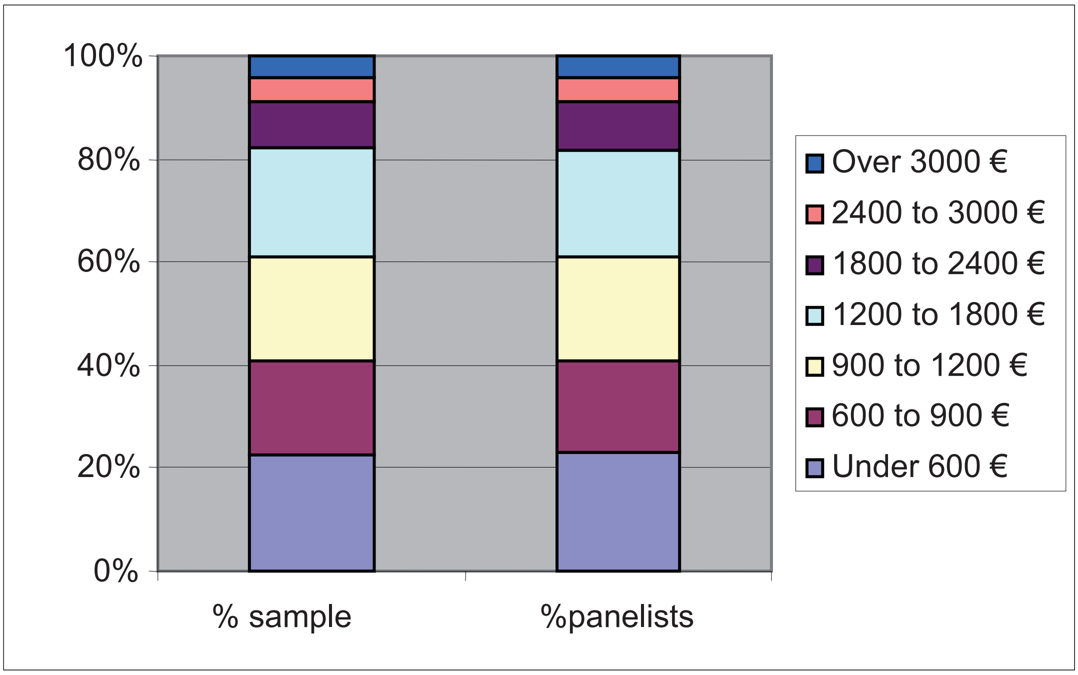
$\chi_7^2 = 8.95$ (right tail p=0.3465)

SUBJECTIVE SOCIAL CLASS



G-test (we cannot reject null Hypothesis)
$\chi_4^2 = 3.36$ (right tail p=0.4995)

MONTHLY INCOME



G-test (we cannot reject null Hypothesis)
$\chi_6^2 = 2.28$ (right tail p=0.89)

Quality of Responses

In order to estimate the quality of the obtained participation, I used the following device: in a series of consecutive questions subject had to indicate their preference among a smaller-sooner amount and a larger later amount. Always the next question would ask subjects to choose among the same smaller-sooner amount, and an increased larger-later amount, so that, normally, subjects would switch preference at a certain point. A typical response looked as follows (see Figure YY):

	A	B	Opción A	Opción B
1	400€	405€	✓	
2	400€	410€	✓	
3	400€	415€		✓
4	400€	420€		✓
5	400€	424€		✓
6	400€	429€		✓

But certain subjects made odd choices, such as switching more than once from option A to B, or switching only once from B to A. I consider that virtually all of these choices indicate that the individual was not doing the task properly; also, subjects not incurring in such inconsistencies can be regarded as mostly having done the task properly, since if a subject only cared about finishing the questionnaire and getting paid, the most natural thing to do is to switch several times. Thus, I believe the percentage of odd responses to be a reasonably good estimate of the quality of responses in experiments 1, 2 and 3. Here are the results:

	Participants	*Rate*
Consistent	1.844	94,08 %
Inconsistent	116	5,91 %

BIBLIOGRAPHY

Ainslie, G. (1975). Specious Reward: A Behavioral Theory of Impulsiveness and Impulse Control. *Psychological Bulletin, 82,* 463-469.

Ainslie, G. (1986). Beyond Microeconomics: Conflict Among Interests in a Multiple Self as a Determinant of Value. In *The Multiple Self*, John Elster (ed). New York: Cambridge University Press.

Ainslie, G. (1992). *Picoeconomics: The Strategic Interaction of Successive Motivational States Within the Person*. Cambridge: Cambridge University Press.

Ainslie, G. and Haendel, V. (1983) The Motives of Will. In E. Gottheil, K. Druley, T. Skolda and H. Waxman (eds.), *Etiologic Aspects of Alcohol and Drug Abuse* (Springfield, IL: Charles C. Thomas).

Ainslie, G. and Haslam, N. (1992). Hyperbolic Discounting. In G. Loewenstein and J. Elster. (eds), *Choice over Time*, Russel Sage Foundation, pp. 57–92

Ainslie, G. and Herrnstein, R. J. (1981). Preference Reversal and Delayed Reinforcement. *Animal Learning and Behavior*, 9, 476-482.

Ainslie and Monterosso. (2003). Building Blocks of Self-Control: Increased Tolerance for Delay with Bundled Rewards. *Journal of The Experimental Analysis of Behaviour* 79: 37-48.

Ahlbrecht, M. and Weber, M. (1995): Hyperbolic Discounting Models in Prescriptive Theory of Intertemporal Choice. *Zeitschrift für Wirtschafts- und Sozialwissenschaften*, 115, 535-568.

Ahlbrecht, M., and Weber, M. (1997). An Empirical Study on Intertemporal Decision Making Under Risk. *Management Science*, 43, 813–826.

Anand, P. (1993). *Foundations of Rational Choice Under Risk*. Clarendon Press.

Angeletos, G.M., Laibson, D. Tobacman, J., Repetto, A., and Weinberg, S. (2001). The Hyperbolic Consumption Model: Calibration, Simulation, and Empirical Evaluation. *Journal of Economic Perspectives*, 15(3), pp. 47-68.

Ariely, Daniel and Ziv Carmon. (2003). Preferences Over Sequences of Outcomes. In *Time and Decision: Economic and Psychological Perspectives on Intertemporal Choice*. George Loewenstein, Daniel Read, and Roy Baumeister eds. New York: Russell Sage Foundation Press.

Baucells, M. and Heukamp, F. H. (2007). Probability and Time Tradeoff. IESE Business School Working Paper.

Baron, J. (2000). Can We Use Human Judgments to Determine the Discount Rate? *Risk Analysis*, 20, 861-868.

Benartzi, S. and Thaler, R.. H.. (1995). Myopic Loss Aversion and the Equity Premium Puzzle. *Quarterly Journal of Economics*, vol.110, 73-92.

Benzion, U., Rapoport, A., & Yagil, J. (1989). Discount Rates Inferred from Decisions - an Experimental-Study. *Management Science, 35*(3), 270-284.

Berridge, K.C. and Robinson, T.E. (1998). What is the Role of Dopamine in Reward: Hedonic Impact, Reward Learning or Incentive Salience? *Brain Research Reviews* 28 (3), 309-369.

Böhm-Bawerk, Eugen von. (1890) [1884]. *Capital and Interest: A Critical History of Economical Theory*. London: Macmillan Company.

Bowman, D., Minehart, D., and Rabin, M. (1999). Loss Aversion in a Consumption-Savings Model. *Journal of Economic Behavior and Organization*, 38, pp. 155-178.

Camerer, C., Loewenstein, G., Prelec, D. (2005). Neuroeconomics: How Neuroscience can Inform Economics. *Journal of Economic Literature* , XLIII, 9-64.

Carrillo, J.D. (1999). Self-control, Moderate Consumption and Craving. CEPR discussion paper 2017.

Chapman, G. B. (1996). Temporal Discounting and Utility for Health and Money. *Journal of Experimental Psychology-Learning Memory and Cognition, 22*(3), 771-791.

Chapman, G. (2000). Preferences for Improving and Declining Sequences of Health Outcomes. *Journal of Behavioral Decision Making*, 13, pp. 203-18.

Chapman, G. B. and Elstein, A. S. (1995). Valuing the Future - Temporal Discounting of Health and Money. *Medical Decision Making, 15*(4), 373-386.

Chapman, G. B. and Winquist, J. R. (1998). The Magnitude Effect: Temporal Discount Rates and Restaurant Tips. *Psychonomic Bulletin & Review, 5*(1), 119-123.

Coller, M., and Williams, M. B. (1999). Eliciting Individual Discount Rates. *Experimental Economics*, 2, 107-127.

Cubitt, R., Sudgen R. (2001). On Money Pumps. *Games and Economic Behavior* 37, S. 121-160.

Cubitt, R., Starmer, C. and Sugden, (1998). On the Validity of the Random Lottery Incentive System. *Experimental Economics*, 1, 115-31.

Damasio, A. (1994). *Descartes' Error*. G.P. Putnam's Sons, New York.

Debreu, G. (1960). Topological Methods in Cardinal Utility Theory. In Arrow, Karlin and Suppes (eds.) *Mathematical Methods in the Social Sciences*, 1959. Stanford: Stanford University Press, pp 16-20.

Duesenberry, J. S. (1952). *Income, Saving and the Theory of Consumer Behavior*. Cambridge, MA: Harvard University Press.

Elster, J. (1979). *Ulysses and the Sirens*. Cambridge: Cambridge University Press.

Elster, J. (1982). Sour Grapes: Utilitarianism and the Genesis of Wants in Amartya Sen and Bernard Williams (eds.), *Utilitarianism and Beyond*, London: Cambridge University Press, pp. 219-238.

Elster, J. (1985). Weakness of Will and the Free-Rider Problem. *Economics and Philosophy* 1, 231-65.

Fischer, C. (1999). Read This Paper Even Later: Procrastination With Time-Inconsistent Preferences. Discussion Paper 99-20, Resources for the Future.

Fishburn, P. and Rubinstein, A. (1982). Time Preference. *International Economic Review*, 23, 677-694.

Fisher, Irving. (1927). A Statistical Method for Measuring "Marginal Utility" and Testing the Justice of a Progressive Income Tax. In J.H. Hollander (ed.), *Economic Essays Contributed in Honor of John Bates Clark*. New York: Macmillan Company.

Fisher, Irving. (1930). *The Theory of Interest*. New York: Macmillan Company.

Frederick, S., Loewenstein, G., and O'Donoghue, T. (2002). Time Discounting and Time Preference: A Critical Review. *Journal of Economic Literature, 40*(2), 351-401.

Frederick, S. and Loewenstein, G. (2002). The Psychology of Sequence Preferences. Working paper, Massachusetts Institute of Technology, Sloan School of Management: Cambridge.

Gigliotti, G. and Sopher, B. (2004). Analysis of Intertemporal Choice: A New Framework and Experimental Results. *Theory and Decision*, 55, 209-233.

Green, L., Fisher, E.B., Jr., Perlow, S. and Sherman, L. (1981). Preference Reversal and Self-Control: Choice as a Function of Reward Amount and Delay. *Behaviour Analysis Letters*, 43-51

Green, L., Fristoe, N., and Myerson, J. (1994). Temporal Discounting and Preference Reversals in Choice Between Delayed Outcomes. *Psychonomic Bulletin & Review, 1*(3), 383-389.

Green, L., Fry, A. F., and Myerson, J. (1994). Discounting of Delayed Rewards: A Life-Span Comparison. *Psychological Science*, 5, 33-36.

Green, L., Myerson, J. and McFadden, E. (1997). Rate of Temporal Discounting Decreases With Amount of Reward. *Memory & Cognition, 25*(5), 715-723.

Gruber, J. and Koszegi, B. (2000). Is Addiction Rational? Theory and Evidence. NBER Working Paper 7507.

Gul, F. and Pesendorfer, W. (2001). Temptation and Self-Control. *Econometrica*, 69, 1403-1435.

Halley, E. (1761). Of Compound Interest. In Henry Sherwin, *Sherwin's Mathematical Tables* (published posthumously after Halley's death in 1742, London: W. and J. Mount, T. Page and Son, 1761).

Harrison, G. W., Lau, M. I., and Williams, M. B. (2002). Estimating Individual Discount Rates in Denmark: A Feld Experiment. *American Economic Review*, 92, 1606–1617.

Herrnstein, R. J. (1981). Self Control as Response Strength. *Quantification of Steady-State Operant Behavior*. In Bradshaw, Szabadi & Lowe eds. (1981). Amsterdam: Elsevier, North Holland.

Hoch, S. J., Loewenstein, G. (1991). Time-Inconsistent Preferences and Consumer Self-Control. *The Journal of Consumer Research*, Vol. 17, No. 4, pp. 492-507.

Holcomb, J. H. and Nelson, P. S. (1992). Another Experimental Look at Individual Time Preference. *Rationality and Society*, 4, 199-220.

Jevons, William S. (1888) [1871]. *The Theory of Political Economy*. London: McMillan.

Kahneman, D. (1999). Objective Happiness. In *Well-Being: The Foundations of Hedonic Psychology*, pp 3-25. New York, NY: Russell Sage Foundation.

Kahneman, D. and Tversky, A. (1979). Prospect Theory: An Analysis of Decision Under Risk. *Econometrica*, 47, 263-291.

Kahneman, D. and Tversky, A. (1984). Choices, Values and Frames. *American Psychologist*, 39, 341-350.

Kahneman, D., Ritov, I., Jacowitz, K. E., and Grant, P. (1993). Stated Willingness to Pay for Public Goods: A Psychological Perspective. *Psychological Science*, 4, 310-315.

Kahneman, D., Wakker, P. and Sarin, R. (1997). Back to Bentham? Explorations of Experienced Utility. *Quarterly Journal of Economics*, 112, pp. 375-405.

Kelsey, D. and Milne, F. (1997). Induced Preferences, Dynamic Consistency and Dutch Books. *Economica*, 1997, 64, 471-81.

Kirby, K. N. (1997). Bidding On the Future: Evidence Against Normative Discounting of Delayed Rewards. *Journal of Experimental Psychology-General, 126*(1), 54-70.

Kirby, K. N. and Herrnstein, R. J. (1995). Preference Reversals Due to Myopic Discounting of Delayed Reward. *Psychological Science, 6*(2), 83-89.

Kirby, K. N. and Guastello, B. (2001). Making Choices in Anticipation of Similar Future Choices Can Increase Self-Control. *Journal of Experimental Psychology: Applied*, 7(2), 154-164.

Kirby, K. N., and Marakovic, N. N. (1996). Delay-Discounting Probabilistic Rewards: Rates Decrease As Amounts Increase. *Psychonomic Bulletin & Review*, 3(1), 100-104.

Kirby, K. N., Petry, N. M., and Bickel, W. K. (1999). Heroin Addicts Have Higher Discount Rates for Delayed Rewards Than Non-Drug Using Controls. *Journal of Experimental Psychology: General*, 128, 78-87.

Koopmans, T. C. (1960). Stationary Ordinal Utility and Impatience. *Econometrica*, *28*, 287-309.

Krantz, D.H., Luce, R.D., Suppes, P. and Tversky, A. (1971). *Foundations of Measurement, Vol. I: Additive and Polynomial Representations*. New York: Academic Press.

Laibson, D. (1997). Golden Eggs and Hyperbolic Discounting. *Quarterly Journal of Economics*, 112, 443-477.

Laibson, D., Repetto, A., and Tobacman, J. (1998). Self-Control and Saving For Retirement. *Brookings Papers on Economic Activity*, 191-196.

Lancaster, K. (1963). An Axiomatic Theory of Consumer Time Preference. *International Economic Review* 4, 2210-231.

Leland, J.W. (2002). Similarity Judgements and Anomalies in Intertemporal Choice. *Economic Inquiry* 40 (4), 574–581.

Loewe, G. and Read, D. (2005). Magnitude and Hyperbolic Discounting in the Choice of Constant Sequences. Working Paper.

Loewenstein, G. (1987). Anticipation and the Valuation of Delayed Consumption. *Economic Journal*, 97, 666-684.

Loewenstein, G. (1988). Frames of Mind in Intertemporal Choice. *Management Science*, 34, pp. 200-14.

Loewenstein, G. (1996). Out of Control: Visceral Influences on Behavior. *Organizational Behavior and Human Decision Processes,* Volume 65, Issue 3, Pages 272-292.

Loewenstein, G. and Angner, E. (2003). Predicting and Indulging Changing Preferences. In Loewenstein, G., Read, D. and Baumeister, R. (Eds.), *Time and Decision: Economic and Psychological Perspectives on Intertemporal Choice* (pp. 351-391). New York: Russell Sage Foundation Press.

Loewenstein, G., O'Donoghue, T. and Rabin, M. (2003). Projection Bias in Predicting Future Utility. *The Quarterly Journal of Economics*, MIT Press, vol. 118(4), pages 1209-1248.

Loewenstein, G. and Prelec, D. (1992). Anomalies in Intertemporal Choice - Evidence and an Interpretation. *Quarterly Journal of Economics, 107*(2), 573-597.

Loewenstein, G. and Prelec, D. (1993). Preferences for Sequences of Outcomes. *Psychological Review, 100*(1), 91-108.

Loewenstein, G., Read, D. and Baumeister, R. (ed). (2003). *Time and Decision*: *Economic and Psychological Perspectives on Intertemporal Choice.* Russell Sage Foundation, New York.

Loewenstein, G. and Sicherman, N. (1991). Do Workers Prefer Increasing Wage Profiles? *Journal of Labor Economics*, 9(1), 67-84.

Loewenstein, G. and Thaler, R. (1989). Anomalies: Intertemporal Choice**.** *The Journal of Economic Perspectives*, Vol. 3, No. 4, pp. 181-193.

Luce, D. (1956). Semiorders and a Theory of Utility Discrimination. *Econometrica*, 24, 178-191.

Luce, D. (1966). Two Extensions of Conjoint Measurement. *Journal of Mathematical Psychology, 3*, 348–370.

Machina, M. (1989). Dynamic Consistency and Non-Expected Utility Models of Choice Under Uncertainty. *Journal of Economic Literature,* 27, 1622-1668.

MacKeigan, L.D., Larson, L.N., Draugalis, J.R., Bootman, J.L., and Bruns, L.R. (1993). Time Preference for Health Gains versus Health Losses. *Pharmaco Economics*, 3, 374-386.

Macmillan, M. (2000). *An Odd Kind of Fame.* MIT Press: Cambridge.

Madden, G.J., Bickel, W.K. & Jacobs, E.A. (1999). Discounting of Delayed Rewards in Opioid-Dependent Outpatients: Exponential or Hyperbolic Discounting Functions. *Experimental and Clinical Psychopharmacology*, 7, 284-293.

Malkoc, S. and Zauberman, G. (2005). Deferring versus Expediting Consumption : The Effect of Temporal Framing on Sensitivity to Time Horizon. Working Paper.

Manuck, S.B., Flory, J.D., Muldoon, M.F. and Ferrell, R. (2003) A Neurobiology of Intertemporal Choice. In: *Time and Decision*: *Economic and psychological perspectives on intertemporal choice*. Ed. Loewenstein G, Read D, Baumeister R. Russell Sage Foundation, New York. pp. 139-172.

Mazur, J. E. (1984). Tests of an Equivalence Rule for Fixed and Variable Delays. *Journal of Experimental Psychology: Animal Behavior Processes*, *10*, 426-436.

Mazur, J. E. (1987). An Adjusting Procedure for Studying Delayed Reinforcement. In M. L. Commons, J.E. Mazur, J. A. Nevin, and H. Rachlin (Eds.). *Quantitative analyses of behavior*. 5. The Effect of Delay and of Intervening Events on Reinforcement Value. Hillsdale, NJ: Erlbaum.

McClennen, E. (1990). *Rationality and Dynamic Choice*. Cambridge, England: Cambridge University Press.

McClure, S.M., Laibson, D.I., Loewenstein, G., Cohen, J.D. (2004). Separate Neural Systems Value Immediate and Delayed Monetary Rewards. *Science* 306:503–507.

Millar, A., Navarick, D. J. (1984). Self-Control and Choice in Humans: Effects of Video Game Playing as a Positive Reinforcer. *Learning and Motivation* 15, 203-218.

Mischel, W., Grusec, J., and Masters, J. C. (1969). Effects of Expected Delay Time On the Subjective Value of Rewards and Punishments. *Journal of Personality and Social Psychology, 11*, 363–373.

O'Donoghue, T. and Rabin, M. (1999a). Addiction and Self Control, in Jon Elster (ed.), *Addiction: Entries and Exits*, Russel Sage Foundation.

O'Donoghue, T. and Rabin, M. (1999b). Incentives for Procrastinators. *Quarterly Journal of Economics*, 114(3), August 1999, 769-816.

O'Donoghue, T. and Rabin, M. (2000a). Addiction and Present-Biased Preferences, Cornell University, Ithaca, and University of California, Berkeley. Center for Analytic Economics Working Paper 02-10, Cornell University.

O'Donoghue, T., and Rabin, M. (2000b). The Economics of Immediate Gratification. *Journal of Behavioral Decision Making*, 13 (2), 233-250.

O'Donoghue, T. and Rabin, M. (2001). Choice and Procrastination. *Quarterly Journal of Economics, 116*(1), 121-160.

Ok, E.A. and Masatlioglu, Y. (2003). A General Theory of Time Preferences. Levine's Bibliography 234936000000000089, UCLA Department of Economics.

Peart, S. (2000). "Irrationality and Intertemporal Choice in Early Neoclassical Thought." *Canadian Journal of Economics* 33 (1).

Pender, J.L. (1996). Discount Rates and Credit Markets: Theory and Evidence From Rural India. *Journal of Development Economics*, 50, 257-296.

Phelps, E.S. and Pollack, R. (1968). On Second-Best National Saving and Game-Equilibrium Growth. *The Review of Economic Studies,* 35:2, pp.185-199.

Pollak, R. (1970). Habit Formation and Dynamic Demand Functions. *Journal of Political Economy* 78 (4), 745-763.

Prelec, D. and Loewenstein, G. (1998). The Red and the Black: Mental Accounting of Savings and Debt. *Marketing Science*, 17, 4-28.

Rachlin, H. (1989). *Judgment, Decision, and Choice: A Cognitive/Behavioral Synthesis*. New York: Freeman.

Rae, John. (1905) [1834]. *The Sociological Theory of Capital*. London: McMillan.

Raineri, A. and Rachlin, H. (1993). The Effect of Temporal Constraints On the Value of Money and Other Commodities. *Journal of Behavioral Decision Making*, 6, 77–94.

Read, D. (2001). Is Time-Discounting Hyperbolic or Subadditive? *Journal of Risk and Uncertainty, 23*(1), 5-32.

Read, D. (2003). Subadditive Intertemporal Choice. In *Time and Decision: Economic and Psychological Perspectives on Intertemporal Choice*. George Loewenstein, Daniel Read, and Roy Baumeister eds. New York: Russell Sage Foundation Press.

Read, D. (2004). Intertemporal Choice. In D. Koehler & N. Harvey (Eds.), *The Blackwell handbook of judgment and decision making*. Oxford: Blackwell.

Read, D., Airoldi, M., Loewe, G. (2005). Intertemporal Tradeoffs Priced in Interest Rates and Amounts: A Study of Method Variance. LSE Working Paper LSEOR 05.77.

Read, D., Frederick, S., Orsel, B., and Rahman, J. (2005). Four Score and Seven Years From Now: The "Date/Delay Effect" in Temporal Discounting. *Management Science*, 51 (9), 1326-1335.

Read, D., Loewenstein, G. and Rabin, M. (1999). Choice Bracketing. *Journal of Risk and Uncertainty* 19: 171-197.

Read, D., and Powell, M. (2002). Reasons for Sequence Preferences. *Journal of Behavioral Decision Making*, 15, 433–460.

Read, D. and Roelofsma, P. H. (2003). Subadditive Versus Hyperbolic Discounting: A Comparison of Choice and Matching. *Organizational Behavior and Human Decision Processes*, 91, 140–153.

Redelmeier, D.A. and Heller, D.N. (1993). Time Preferences in Medical Decision Making and Cost-Effectiveness Analysis. *Medical Decision Making*, 13, 212-217.

Rosenberg, A. (1992). *Economics: Mathematical Politics or Science of Diminishing Returns?* Chicago: Chicago University Press.

Ross, W. T., and Simonson, I. (1991). Evaluation of Pairs of Experiences: A Preference for Happy Endings, *Journal of Behavioral Decision Making*, 4, 273-282.

Rubinstein, A. (1988). Similarity and Decision-Making Under Risk. *Journal of Economic Theory*, 46 (1988), 145-153.

Rubinstein, A. (2003). Economics and Psychology? The Case of Hyperbolic Discounting. *International Economic Review*, Vol. 44, pp. 1207-1216.

Rubinstein, M. (2003). Great Moments in Financial Economics: I. Present Value. *Journal of Investment Management* (first quarter).

Ryder, H.E. and Heal, G.M. (1973). Optimal Growth with Intertemporally Dependent Preferences**.** *The Review of Economic Studies*, Vol. 40:1, pp. 1-33.

Samuelson, P. (1937). A Note On the Measurement of Utility. *Review of. Economic Studies*, vol. 4, no. 2, feb., pp. 155-161.

Samuelson, P. (1938). A Note on the Pure Theory of Consumers' Behaviour. *Econometrica* 5: 353-54.

Schelling, T.C. (1984). Self-Command in Practice, in Policy, and in a Theory of Rational Choice. *American Economic Review*, 74:2, pp. 1-11.

Shea, J. (1995a). Myopia, Liquidity Constraints, and Aggregate Consumption: A Simple Test. *Journal of Money, Credit and Banking*, 27:3, pp. 798-805.

Shea, J. (1995b). Union Contracts and the Life-Cycle/Permanent-Income Hypothesis. *American Economic Review*, 85:1, pp. 186-200.

Shelley, M. K. (1993). Outcome Signs, Question Frames and Discount Rates. *Management Science, 39*(7), 806-815.

Solnick, J. V., Kannenberg, C. H., Eckerman, D. A., & Waller, M. B. (1980). An Experimental Analysis of Impulsivity and Impulse Control in Humans. *Learning and Motivation*, 11, 61-77.

Strotz, R H. (1956). Myopia and Inconsistency in Dynamic Utility Maximization. *Review of Economic Studies*, 23, 165-180.

Sugden, R. (1991). Rational Choice: a Survey of Contributions From Economics and Philosophy. *The Economic Journal*, 101: 751–85.

Thaler, R. (1981). Some Empirical Evidence of Dynamic Inconsistency. *Economics Letters*, *8*, 201-207.

Thaler, R. (1989). Anomalies: The Ultimatum Game. *Journal of Economic Perspectives*, 2195-206.

Thaler, R.H. and Shefrin, H.M. (1981). An Economic Theory of Self-Control. *Journal of Political Economy*, 89:2, pp. 392-410.

Tversky, A. (1977). Features of Similarity. Psychological Review, 84, 327-352.

Winston, G.C. (1980). Addiction and Backsliding: A Theory of Compulsive Consumption. *Journal of Economic Behavior and Organization*, 1, pp. 295-324.

Wakker, P. (1988). The Algebraic versus the Topological Approach to Additive Representations. *Journal of Mathematical Psychology* 32, 421-435.

Wakker, P. (1989). *Additive Representations of Preferences, A New Foundation of Decision Analysis*. Kluwer Academic Publishers, Dordrecht.

Yaari, M. (1998). On the Role of 'Dutch Books' in the Theory of Choice under Risk. In D. Jacobs, E. Kalai, and M. Kamien, eds. Frontiers of Research in Economic Theory. Cambridge: CUP, pp. 33-46.

Yates, J.F., Watts, R.A. (1975). Preference for Deferred Losses. *Organizational Behavior and Human Performance*, 13, 294-306.