The Experimental Discovery of the Momentum of Light Quanta

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Abstract: The topic of this work is the experimental discovery of the momentum of quanta by Arthur Compton that confirmed its momentum in 1923. In his famous paper about the now called Compton Effect, he concluded that radiation have a particle nature, something that allowed Pauli, Einstein and Ehrenfest to solve the equilibrium between radiation and matter, a problem set out by Einstein in 1916.

I. INTRODUCTION

This work is a brief explanation of the Compton Effect and the experiments Compton carried out from 1919 to 1923. In order to illustrate better this topic, we begin with a different one, the thermal equilibrium between matter and radiation, developed by Einstein in 1916. This problem, which remained unsolved for years, will be reconsidered by Pauli in 1923 using Compton's discovery, the key to the solution, and also extended by Einstein and Ehrenfest in the same year. Therefore, in this work we will show the two ways in which the light quanta were discovered, the first one through Einstein's statistical way (II), and the second one through Compton's experimental way (III). Finally, we will connect both ways through Pauli, Einstein and Ehrenfest (IV).

II. EQUILIBRIUM BETWEEN MATTER AND RADIATION I

The hypothesis of light quanta, formulated by Einstein in 1905, experienced a great rejection among the scientific community since its publication. But Einstein, rather than abandoning it, achieved deeper insights into it during the following years[1]. For instance, in 1916 with his paper "On the Quantum Theory of Radiation," in which he derived Planck's radiation law with astonishing simplicity. Although this derivation is quite elegant, it is more important for our subject the answer Einstein gives to his question, "does the molecule experience an impulse when it emits or absorbs energy ϵ?"[2].

We will discuss briefly the derivation of Planck's radiation law, specifically Einstein's definition of the different transition probabilities between two different states of a molecule, and his reasoning about the motion of the molecules under the influence of radiation. We have selected this paper for introducing our main topic because the concepts appearing here are strongly connected with the Compton Effect and the subsequently reactions to it, as we will see in the following sections.

In this paper Einstein shows, by postulating a probabilistic hypothesis on the emission and absorption of radiation, "that molecules with a quantum-theoretical distribution of states in thermal equilibrium, [are] in dynamical equilibrium with Planck radiation."[3] This statement yields another conclusion: there must be a momentum transfer, associated with the emission and absorption radiation, that produces a velocity distribution which agrees with Maxwell's distribution. To obtain such a result, the process of a molecule that absorbs or emits energy in radiation form during its transition from one state to another, can be considered "perfectly directional."[4].

As a starting point, Einstein considers two quantum-theoretically possible states of a gas molecule with their belonging energies and assumes a possible transition from one state to another with a particular absorption or radiation energy, and a definite frequency. Subsequently, he defines three kinds of probabilities: the probability of emission without excitation from external causes (A), the probability of absorption under the influence of a radiation field (B) and the probability of emission of radiation under the same circumstances (B'). Once defined, he argues that "the number of elementary processes of type (B) taking place for unit time should, on average, be equal to those of type (A) and (B') taken together" [5]. After some simple calculations, he obtains Planck's radiation law.

Then, Einstein tackles the problem of the motion of the molecules under the influence of radiation. That is: the equilibrium between matter and radiation. For simplicity, he considers the motion to be unidimensional, and, in order to apply the laws of ordinary mechanics, the mass of the molecule to be large enough "so that higher powers of v/c can be neglected in comparison with lower ones"[6]. In this problem, the momentum of a molecule, Mv, suffers two different changes during an interval τ. The radiation in which the molecule is *immersed*, although constituted in all directions, will produce a friction force opposed to the motion of the molecule. This will be equal to Rv and would make the molecule stop, if there weren't irregularities of the radiative interactions which transmit momentum Δ during time τ. That is exactly the same method he applied in his successful treatment of Brownian Motion in 1905[7].

The calculation of the parameters $\overline{\Delta^2}$ and R are non-trivial and out of our topic. Nevertheless, we reproduce a fragment of Einstein's conclusion:

> If a radiation bundle has the effect that a molecule struck by it absorbs or emits a

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> quantity of energy $h\nu$ in the form of radiation (ingoing radiation), then a momentum $h\nu$ is always transferred to the molecule. For an absorption of energy, this takes place in the direction of propagation of the radiation bundle, for an emission in the opposite direction. If the molecule is acted upon by several directional radiation bundles, then it is always only a single one of these which participates in an elementary process of irradiation; this bundle alone then determinates the direction of the momentum transferred to the molecule.
>
> If the molecule undergoes a loss in energy of magnitude $h\nu$ without external excitation, by emitting this energy in the form of radiation (outgoing radiation), then this process, too, is directional. Outgoing radiation in the form of spherical waves does not exist. During the elementary process of radiative loss, the molecule suffers a recoil of magnitude $h\nu/c$ in a direction which is only determined by 'chance', according to the present state of the theory.[8]

As we clearly see, Einstein concluded that the emitted radiation is not an electromagnetic wave propagating through space, but located energy in particle form that moves in a given direction with a specific momentum. It is important to notice that, until that moment, the quanta only had an energy $h\nu$. In order to attain this result, Einstein proceeded in an statistical way. In the following section we will see another more convincing way of reaching the same conclusion using another approach: Arthur Compton's experimental work.

III. ARTHUR COMPTON AND THE COMPTON EFFECT

In this section we provide with a short biography of Arthur Compton and a schematic description of his experiments until reaching the Compton Effect. His biography has been extracted from the book "Biographical Dictionary of Scientists"[9] and the rest from the books "The Compton Effect"[10] and "The Tiger and the Shark"[11]. We have also consulted original papers from Arthur Compton (cited at the end of this work).

A. Biography

Arthur Holly Compton was born in Wooster, Ohio, in 1892. He graduated from The College of Wooster in 1913 with a Bachelor of Science degree, and then entered Princeton, where he received his Master of Arts degree in 1914. Subsequently he studied for his PhD in physics and in 1916 he earned his PhD with a dissertation under the title "The intensity of X-ray reflection, and the distribution of the electrons in atoms."

In 1919, Compton obtained a National Research Council Fellowships and went to Cambridge University's Cavendish Laboratory in England. There, he studied the scattering and absorption of gamma rays with George Paget Thomson—son of the famous J. J. Thomson—to become himself famous years later with his discovery of the electron diffraction.

In 1920 he returned to the United States, where he was appointed Professor of Physics at Washington University in St. Louis.

In 1922, he discovered the phenomenon known as "Compton effect", which earned him the Nobel Prize in Physics in 1927, namely "for his discovery of the effect named after him"[12], and what will be briefly analysed below. Later, he worked extensively on cosmic rays and, in 1941, was appointed to a US committee to explore the possibility of constructing an atomic bomb, with special responsibility himself for developing plutonium production. Under the code name "Metallurgical Project", production began at Chicago under Compton's direction in 1942, and culminated in the atomic bombs of 1945.

After the war, Compton returned to Washington University in St. Louis. He died in Berkeley, California, on March 15, 1962.

B. Experiments between 1919 and 1922

As we have said, in October 1919 the Compton family arrived in Cambridge and Arthur began to work at the Cavendish Laboratory, an extremely stimulating place where he could come into direct contact with J.J. Thomson and Rutherford. The most significant experiments Compton carried out at the Cavendish dealt with the absorption and scattering of γ-rays by thin metal plates. He intended to "investigate the nature and the general characteristics of secondary gamma rays, and to study the mechanism whereby comparatively soft secondary radiation is excited by relatively hard primary radiation."[13] After his experiments with γ-ray scattering radiation, Compton concluded that any secondary radiation *unchanged* in wavelength was "truly scattered" radiation, and any secondary radiation *changed* in wavelength was "fluorescent" radiation. As we will see, this special radiation will take significant part in the discovery of the Compton Effect.

In late summer 1920, Compton began formulating his research plans while travelling to the States. In the book "The Compton Effect"[14], by R. Stuewer, we can see that Compton made a draft entitled "Problems to be tackled at Saint Louis" about at what issues he wanted to work in his new appointment. A piece of his first problem was as follows: "Scattering of Radiation of Various hardness. (...) Study scattering at different angles for different λ's." As we see, this is an important point because

Compton decided to turn from γ-ray to X-ray experiments in a natural extension of his γ-ray work. These experiments are the ones that will bring him to discover his famous effect.

Compton's first X-rays experiments at Washington University were made in early 1921 and reported in April of that same year. After these experiments, several conclusions followed, but the most significant was that he had discovered a new type of X-ray "fluorescent" radiation which was entirely analogous to the new γ-ray "fluorescent" radiation he had discovered at the Cavendish. In his paper "The Spectrum of Secondary X-rays", written at the end of 1921 and published in March 1922, Compton stated explicitly that "in addition to scattered radiation there appeared in the secondary rays a type of fluorescent radiation, whose wave-length was nearly independent of the substance used as a radiator, depending only upon the wave-length of the incident rays and the angle at which the secondary rays were examined."[15]. In the same paper, Compton tentatively applied the quantum theory to the problem for the first time. He did not appeal directly to the radical lightquantum, but rather to the quantum transformation relation, using it to calculate the recoil velocity of an electron struck by the incident X-rays. He then explained the wavelength-shifted secondary X-rays as Doppler-shifted X-rays re-radiated by the recoil electron.

Throughout the summer of 1922, Compton began gathering information for a National Research Council report on the "Secondary Radiations Produced by X-rays, and some of their Applications to Physical Problems."[16] The purpose of his report was to present a critical view of the literature on the secondary radiations produced by X-rays. In this work we find a long concluding section entitled *Nature of Radiation*[17], in which Compton discusses about the validity of the quantum and classical interpretation. He argues that, even though the change of wavelength of the incident X-ray was found to have quantitative explanation if the radiation was emitted and received by each scattering electron in discrete quanta, Compton couldn't accept it to be correct because "the cogency of the argument based on interference phenomena is so great that it seems to me questionable whether the quantum interpretation of this experiment is the correct one."[18] Moreover, Compton inferred that a consideration of interference phenomena "seems to lead with certainty to the conclusion that under certain conditions radiation does not occur in a definite direction, nor in definite quanta; that radiation may be absorbed in fractions of a quantum; and that, in the process of scattering at least, radiation may be emitted in fractions of a quantum."[19]

C. The Compton Effect

At the end of 1922, after having rejected the quantum interpretation a few months earlier, Compton accomplished the discovery of the quantum theory of scattering while scrutinizing his Doppler interpretation of the secondary "fluorescent" radiation; in particular, its implications for the motion of the radiating secondary β-rays.

The context in which Compton was involved, as he described in 1924, was the following:

> The fact that the secondary rays are of greater wave-length when scattered at large angles with the primary beam suggests at once a Doppler effect as from particles moving in the direction of the primary radiation. According to the classical idea of the scattering process, however, every electron in the matter traversed by the primary X-rays is effective in scattering the rays. Thus in order to account for such a Doppler effect on this view, all of the electrons in the radiating matter would have to be moving in the direction of the primary beam with a velocity comparable with that of light, an assumption obviously contrary to fact. It was clear that if any electrons were moving in this manner, it was only a very small fraction of the whole number in the scattering material, and that it must be this small fraction which was responsible for the scattering. The idea thus presents itself that an electron, if it scatters at all, scatters a complete quantum of the incident radiation; for thus the number of electrons which move forward would just be equal to the number of scattered quanta. This suggestion that each quantum of X-rays is scattered by a single electron supplies a simple mean of accounting on quantum principles for the observed change in wave-length . For if we consider the primary rays to proceed in quanta so definitely directed that they can be scattered by individual electrons, along with their energy lip they will carry momentum $h\nu/c$.[20]

In this excerpt from 1924, a year after he made his discovery, we see the words of a certain Compton. This couldn't be further from the truth at the end of 1922, when he was still trying to find a good theoretical explanation for his experiments. Apparently, his position about the quantum interpretation changed so abruptly because he had explored every modification of the theories he considered and was still looking for the correct one.

The full text of his famous paper was sent to the editor of *The Physical Review* in mid-December 1922. As we see, Compton now joined those who were taking the lightquantum seriously: he asserted that the incident X-rays are scattered in localized lightquantum units.

The explanation of the effect is as follows:

> According to the classical theory, each X-ray affects every electron in the matter traversed, and the scattering observed is that due to the combined effects of all electrons. From the point of view of the quantum theory, we may suppose that any particular quantum of X-rays... spends all of its energy upon some particular electron. This electron will in turn scatter the ray in some definite direction, at an angle with the incident beam. This bending of the path of the quantum of radiation results in a change in its momentum. As a consequence, the scattering electron will recoil with a momentum equal to the change in momentum of the X-ray. The energy in the scattered ray will be equal to that the incident ray minus the kinetic energy of the recoil of the scattering electron; and since the scattered ray must be a complete quantum, the frequency will be reduced in the same ratio as is the energy. Thus on the quantum theory we should expect the wave-length of the scattered X-rays to be greater than that of the incident rays.[21]

In order to explain the scattering process, Compton drew the quantum-electron collision and the momentum conservation diagram, then set up the corresponding equations for conservation of energy and momentum. Solving both equations simultaneously, Compton obtained the angular variation of frequency of the scattered quantum and the angular variation of the velocity of the recoil electron:

$$\frac{\nu_\theta}{\nu_0} = \frac{1}{1 + 2\alpha sin^2(\theta/2)} \quad (1)$$

$$\beta = 2\alpha sin(\theta/2)\frac{\sqrt{1 + (2\alpha + \alpha^2)sin^2(\theta/2)}}{\sqrt{1 + 2(\alpha + \alpha^2)sin^2(\theta/2)}}, \quad (2)$$

where $\beta = v/c$ and $\alpha = h\nu_0/mc^2 = h/mc\lambda_0$.

The scattered radiation therefore undergoes a discrete change in wavelength given by the famous relation

$$\Delta\lambda = \lambda_\theta - \lambda_0 = \frac{h}{mc}(1 - cos\theta). \quad (3)$$

Compton, besides checking expression (3) with his X-ray spectroscopic measurements, had to confirm the existence of recoil electrons. At the time he presented his theory, there wasn't any recognised experimental evidence of recoil electrons, but he presumed their existence to calculate the angular distribution $I(\theta,\alpha)$, and the "scattering absorption coefficient"$\sigma(\alpha)$ of the *secondary quanta*. The final expression for $I(\theta,\alpha)$ and its integral, which corresponds with the "scattering absorption coefficient", were displayed graphically and compared with Compton's Cavendish γ-ray data. He concluded that "The beautiful agreement between the theoretical and the experimental values of the scattering is the more striking when one notices that there is not a single adjustable constant connecting the two sets of values."[22]

At the end of the paper, Compton summarized his past work and came to a remarkable conclusion. It is interesting the comparison between this final consideration and the one he wrote for the National Research Council Bulletin (reproduced a few lines above). Below we quote his words:

> This remarkable agreement between our formulas and the experiments can leave but little doubt that the scattering of X-rays is a quantum phenomenon... The present theory accounts satisfactorily for the change in wave-length due to scattering... [and] depends essentially upon the assumption that each electron which is effective in the scattering scatters a complete quantum. It involves also the hypothesis that the quanta of radiation are received from definite directions and are scattered in definite directions. The experimental support of the theory indicates very convincingly that a radiation quantum carries with it directed momentum as well as energy.[23]

IV. EQUILIBRIUM BETWEEN MATTER AND RADIATION II

Although Einstein explained the thermal equilibrium between matter and radiation successfully in 1916, in that moment no experiment confirmed that quanta had a specific momentum. In 1923, Compton's discovery spread and many physicists presented their responses to it. In this section we consider Pauli's response and, as a consequence, Einstein and Ehrenfest's reply to Pauli.

Compton's discovery allowed Pauli to provide an appropriate interaction mechanism leading to the establishment of thermal equilibrium between radiation and free electrons[24], which had been an open problem for years. Guided by Einstein's 1916 paper, discussed at the beginning of this paper, he hypothesised that the probability per unit time associated with the light quantum-electron interactions was of the form $dW = A\rho + B\rho\rho'$ where ρ and ρ' are the black-body special distributions corresponding to the incident and scattered frequencies ν and ν'. Pauli obtained this particular expression after a long and complex mathematical development. Notwithstanding, we will explain the main reflections.

Pauli considers a function F that depends on variables of the incident and emitted radiation (such as energy, momentum, etc.) and tries to determine its form. Using different kinds of arguments and applying some theoretical restrictions he guesses that F only depends

—classically— on the incident radiation density. This result leads to Wien's radiation density, which is, obviously, wrong. In order to obtain a better result, Pauli considers that F depends on ρ^2, which is the suitable dependence of F in wave mechanics, and assuming there is an incident and a dispersed photon, writes $\rho'\rho$ instead. Adding this term to the one he used before, he obtains Planck's radiation density. Pauli suspected that this squared term was related to the wave-particle dualism.

With those assumptions, he found that an electron gas with the Maxwell-Boltzmann energy distribution is in statistic equilibrium with a radiation field distributed according Planck's spectral distribution.

Pauli's work was quickly answered by Einstein and Ehrenfest in 1923[25]. They pointed out, using Einstein's molecule gas, that since the scattering process involves the disappearance (absorption) and the appearance (emission) of a light quantum, both of whose directions are fixed by the conservation laws, the interaction probability should be of the form $dW = (b\rho)(a' + b'\rho')$, where the first factor involves an induced term only, while the second factor involves both a spontaneous and an induced term.

Therefore it is important to notice that while Pauli derived the formula in an heuristic way, Einstein and Ehrenfest expanded and included Pauli's and Compton's work and and obtained the same formula from the concrete principles and concepts showed in the first section of this paper. Another relevant difference which has to be emphasized is that Einstein and Ehrenfest used molecules or atoms with two quantum states Z and Z^* which absorb or emit light quanta, something Pauli couldn't apply because of his electron gas example, which was formed by electrons only provided with translational energy.

V. CONCLUSIONS

During the writing of this paper some questions may have arisen. From my point of view, one of the most interesting is why Compton changed his mind so abruptly. We can imagine he decided to use a quantum interpretation of his scattering experiments because he had tried all the other possible theories without any success. However, what seems more surprising is that, through all the documents and papers checked, it seems that Compton hardly knew something about Einstein's 1916 paper.

Although many bibliography and papers had not been consulted in order to focus in a concrete topic, we know that this work can be widely improved complementing it with other topics. For instance, how other physicists received the fact that quanta have momentum, such as Einstein, and what kind of responses obtained the Compton Effect, such as the Bohr-Krammers-Slater paper, de Broglie contribution to the wave-particle theory or Debye's discovery of the quantum theory of scattering.

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[1] Navarro Veguillas, Luis, *Einstein, Profeta y Hereje*, (Tusquets Editores S.A., 1990)
[2] A. Einstein, "*On the Quantum Theory of radiation,*" (1916) in Van der Waerden, B. L., *Sources of Quantum Mechanics*, (Dover Publications, 2007)
[3] Ibid., p. 64
[4] Ibid., p. 65
[5] Ibid., p. 68
[6] Ibid., p. 69
[7] Einstein, A. , "*Investigations on The Theory of the Brownian Movement,*" (Dover Publications, 1956)
[8] Ibid., p. 76
[9] Porter, R., "*Biographical Dictionary of Scientists,*" (Oxford University Press, 1994)
[10] Stuewer, Roger H., *The Compton Effect*, (Science History Publications, 1975)
[11] Wheaton, Bruce R., *The Tiger and the Shark: Empirical roots of wave-particle dualism*, (Cambridge University Press, 1991)
[12] "The Nobel Prize in Physics 1927," *Nobelprize.org*, Nobel Media AB 2013. Web
[13] A. Compton, "*The Degradation of Gamma-Ray Energy,*" Phil. Mag., 41 (1912), p. 749
[14] Stuewer, Roger H., *The Compton Effect*, (Science History Publications, 1975), p. 160
[15] A. H. Compton, "*The Spectrum of Secondary X-rays,*" Phys. Rev., 19 (March 1922), p. 267
[16] A. H. Compton, "*Secondary Radiations Produced by X-rays, and some of their Applications to Physical Problems,*" National Research Council Bulletin, 4 (1922), pp. 1-56
[17] Ibid., pp. 52-55
[18] Ibid., p. 55
[19] Ibid., p. 54
[20] A. Compton, "*The Scattering of X-rays,*" J. Franklin Inst., 198 (1924), pp. 61-63
[21] A. Compton, "*A Quantum Theory of the Scattering of X-rays by Light Elements,*" Phys. Rev., 21 (1923), p. 484
[22] Ibid., p. 498
[23] Ibid., p. 501
[24] Pauli, W., "*Uber das Termische Gleichgewicht zwischen Strahlung und freien Elektronen,*" Z. Phys., 17 (1923), pp. 272-286
[25] Einstein, A. and Ehrenfest, P., "*Zur Quantentheorie des Strahlungsgleichgewichts,*" Z. Phys., 19 (1923), pp. 301-306