Unit IV: Optimization. Solutions

(Extra Material)

Exercise 1.

a) The Extreme Value Theorem can be applied because f is continuous in A and A is a compact set. Then, $\exists p, q \in A$ such that $\forall x \in A$, $f(p) \leq f(x) \leq f(q)$, in other words, we know that there are both a global maximum and a global minimum in A.

b) The Extreme Value Theorem cannot be applied because f is not continuous in B.

c) The Extreme Value Theorem cannot be applied because f is not continuous in C and C is not compact (it is not bounded).

Exercise 2.

a) The Extreme Value Theorem can be applied because f is continuous in A and A is a compact set. Then, $\exists p, q \in A$ such that $\forall x \in A$, $f(p) \leq f(x) \leq f(q)$, in other words, we know that there are both a global maximum and a global minimum in A.

b) The Extreme Value Theorem can be applied because f is continuous in B and B is a compact set. Then, $\exists p, q \in B$ such that $\forall x \in B$, $f(p) \leq f(x) \leq f(q)$, in other words, we know that there are both a global maximum and a global minimum in B.

c) The Extreme Value Theorem cannot be applied because C is not compact (it is not closed).

Exercise 3.

a) The Extreme Value Theorem can be applied because f is continuous in A and A is a compact set. Then, $\exists p, q \in A$ such that $\forall x \in A$, $f(p) \leq f(x) \leq f(q)$, in other words, we know that there are both a global maximum and a global minimum in A.

b) The Extreme Value Theorem cannot be applied because f is not continuous in B and B is not compact (it is not bounded).

c) The Extreme Value Theorem cannot be applied because f is not continuous in C.

Exercise 4. The only stationary point is $(-1/4, 0, 0)$.

Exercise 5. The point, p, is not stationary. Therefore it cannot be a local extreme point. The only thing we can say is that f is (strictly) convex at point p.

Exercise 6. There is no stationary point because the partial derivative with respect to z never cancels. Thus, there is no local extreme point.

Exercise 7. $(q_1, q_2) = (9, 12)$.

Exercise 8.

a) $(15/16, -99/16)$ is a saddle point.

b) $(11/7, 8/7)$ is a local maximum.

c) $(1, 3)$ is a local minimum.

d) $\forall n \in \mathbb{N}$, $(2\pi n, 0)$ are local maxima and $(\pi + 2\pi n, -2)$ are saddle points.

e) $(0, 0, 0)$ are saddle points.

Exercise 9. *a),d)* are true and *b),c)* are false.

Exercise 10. No, it is a saddle point.

Exercise 11. *c)* is the only true statement.

Exercise 12. The point is a local minimum.

Exercise 13. If $a \in [-2, 2]$, f is convex. If $a \in (-2, 2)$, f is strictly convex. Otherwise, f is neither convex nor concave.

Exercise 14. $(0, 0)$ is the global minimum of f in $\mathbb{R}^2$.

Exercise 15. $(2, -2)$ is the global minimum of f in $\mathbb{R}^2$.

Exercise 16.

a) $x = 100$ and $y = 200$

b) $x = 70$ and $y = 210$

Exercise 17.

a) $q_1 = 136$ and $q_2 = 74$.

b) $q_1 = 81$ and $q_2 = 19$.