Unit I: The $\mathbb{R}^n$ Vector Space. Solutions

Exercise 1.

a) $(3, -2, 1)$

b) $(-4, 17, 3)$

c) $(-1, 10, -4)$

d) $(1, -4, -5)$

Exercise 2. It is not a linear combination.

Exercise 3. It is not a linear combination.

Exercise 4. For any k different from 0. That is, for $k \neq 0$.

Exercise 5. They are linearly independent.

Exercise 6. For any value of k different from 1 and $\frac{1}{2}$. For $k \neq 1$ and $k \neq \frac{1}{2}$.

Exercise 7. For no value of k.

Exercise 8.

a) True

b) False

c) False

d) False

Exercise 9. It is a spanning set of $\mathbb{R}^3$.

Exercise 10.

a) For any a that satisfies $a \neq 5$ and $a \neq -5$.

b) For any a that satisfies $a \neq 5$ and $a \neq -5$.

Exercise 11.

a)

b) $(0, 1, -1)$ are the coordinates of $\vec{u}$ with respect to $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$.

Exercise 12.

a)

b) $(5, 4, 9)$ are the coordinates of $\vec{u}$ with respect to $\{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$.

Exercise 13. $\{(7,1)\}$ is a basis (note that the answer is not unique). $dim(S) = 1$.

Exercise 14. $\{(2,1,-3)\}$ is a basis. $dim(S) = 1$.

Exercise 15. $\{(1,-1,-1)\}$ is a basis. $dim(V) = 1$.

Exercise 16. $\{(2,-3,1)\}$ is a basis. $dim(V) = 1$.

Exercise 17. $\{(3,2,2),(-4,3,1)\}$ is a basis. $dim(Span(\{\vec{u}_1,\vec{u}_2,\vec{u}_3\})) = 2$.

Exercise 18. $\{(2,-3),(1,4)\}$ is a basis. $dim(Span(\{\vec{u}_1,\vec{u}_2,\vec{u}_3,\vec{u}_4\})) = 2$.

Exercise 19. $Span(\{\vec{u}\}) = \{(x,y) \in \mathbb{R}^2 : 2x = y\}$.

Exercise 20. $Span(\{\vec{u}_1,\vec{u}_2\}) = \{(x,y,z) \in \mathbb{R}^3 : z = 2x - \frac{7}{2}y\}$.

Exercise 21.

a) True

b) False

c) False

Exercise 22.

a) True

b) True

c) False

d) False