Percolation theory and fire propagation in a forest

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Abstract: The aim of the project is to study what properties have to have a reforested forest to prevent the total propagation of a wildfire. It is going to do a simulation of the fire in two dimensions using Fortran and percolation theory will be the theoretical model to underlie the results.

I. INTRODUCTION

Many studies had been done about wildfires in the context of the percolation theory, nevertheless, in this project try to obtain a possibly useful result in the process of reforestation of the forests that many times are rapidly ignited a lot of times related to the unnatural symmetry that these forests have. These symmetry make the fire to propagate more easily.

We will obtain a result that will give a possible alternative of reforesting the forests to decrease the high rate of propagations of these fires.

Initially we present the percolation theory applied to the case we are concerning. That is the reason it will not be a general theory, but a theory that will give us useful results.

Immediately after presenting this theory, we explain with certain detail how we build the simulation of the propagation of the fire and the programming language used.

Finally, we present the first results obtained with the simulation compared with the theoretical ones to prove that our system meet the known properties of percolation. Once explained these results, we finish with the study of mixed percolation (explained at section IV.E) to get which properties that the reforested forest ought to have to prevent the total ignition of it.

II. PERCOLATION THEORY

If we look for the definition of percolation in a dictionary we find the following definition: to (cause a liquid to) pass through something that traps solid materials. So the percolation theory in these terms will explain how the liquid behaves when it passes through this porous material.

In the context of Physics or Mathematics, the percolation theory makes reference to the study of the behaviours of the clusters in a certain lattice with sites occupied with a probability of occupation. A cluster is a group of occupied sites next to its nearest neighbours. The behaviour of the cluster will depend of which neighbours we are taking into account.

In this project, we are going to use only square two-dimensional lattices, but we have to remember that the results of the percolation theory depend on the lattice and the dimension, so these results will not be, in general, extrapolatory to square lattice in three dimensions or other kind of two-dimensional lattices.

So we have a squared lattice of two dimensions of a certain size LxL with occupied sites with some probability p, as it is shown in the figure, Fig. (1).

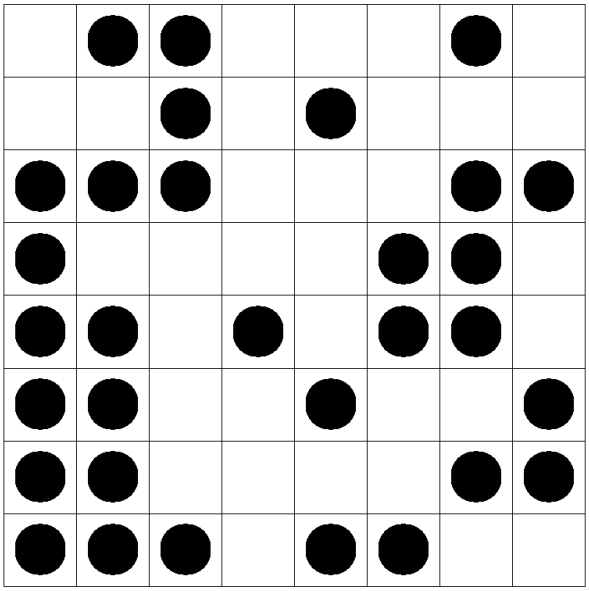

FIG. 1: Two-dimensional lattice randomly occupied.

We are going to use only nearest neighbours. In a square lattice, it means that we will have four neighbours: top, bottom, left and right.

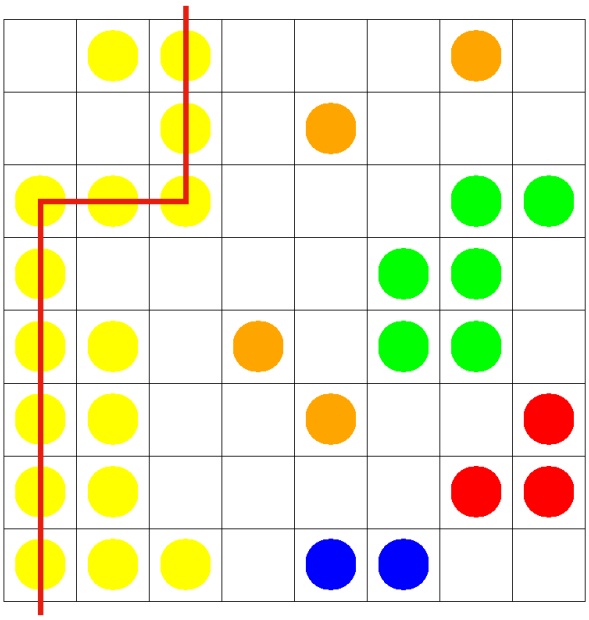

FIG. 2: Clusters of the same size are shown in the same colour. Also it is shown a path from the bottom to the top.

If we look to the figure, Fig. (2), we can see different clusters of different size drawn in different colour to distinguish them between. We can note two orange clusters at

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the middle, reminding us that the neighbour in diagonal are not elements of the same cluster.

The following important aspect in the percolation theory will be vital to study the propagation of the fire. It is known as percolation threshold. Using again the figure, Fig. (2), we can observe that the yellow cluster has make a shape such that we can draw a line that passes through the lattice from the bottom to the top. If the occupation probability is too low, it's logical to think that we will not have any cluster as the yellow one, however, it exists certain probability from which we have the first cluster that passes through the lattice, also known as percolating cluster or infinite cluster.

This probability is known as percolation threshold and it is the probability that separate the system between a percolating phase and the no percolating. In these terms, our system have a pure geometric phase transition with the probability of occupation as the parameter. Given a lattice and a dimension, this percolation threshold is unique. It will depend also from the last thing, the kind of percolation we are studying.

There are two kinds of percolation, site and bond percolation. Until now, we have talked about probability of occupation. This percolation is what we call site percolation. Here we look at some cluster and try to find a possible path that percolates the lattice, or passes through it. In the following figure, Fig. (3), we see a lattice with a probability of occupation of $p_S = 0.5$.

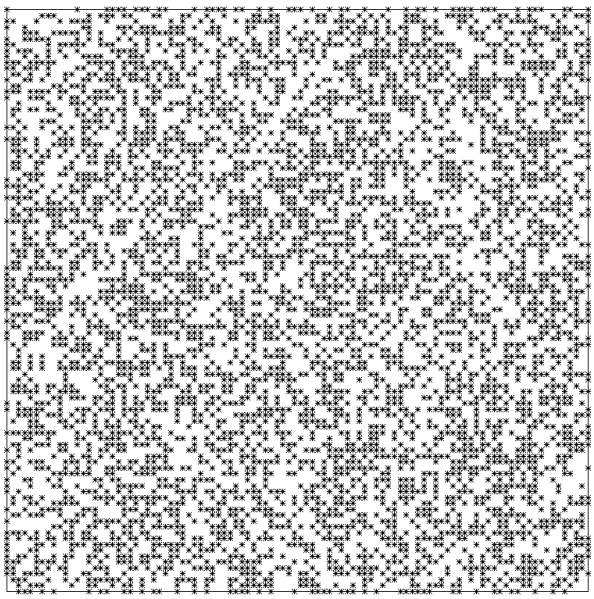

FIG. 3: Square lattice with site percolation and $p_S = 0.5$.

We can observe that the probability of occupation is not sufficiently high to have a percolating cluster. It means that we are under the percolation threshold for the square lattice in 2D.

For the case of bond percolation, the important parameter will be the probability of propagation of the path commented at the figure, Fig (2). So, here, we suppose a totally occupied lattice ($p_S = 1$), and we propagate the path to the nearest neighbours with the probability. For example, if we begin propagating the path from the bottom to the top, we can have a figure like the following, Fig. (4):

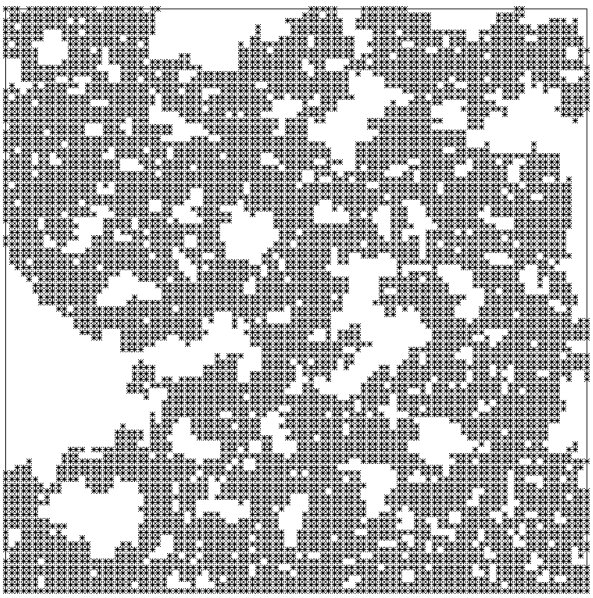

FIG. 4: Square lattice with bond percolation and $p = 0.5$.

It is shown a lattice with bond percolation and the same probability than the previous figure, Fig. (3). It can be seen that is a percolating cluster and that is why we know that we are at or above the percolation threshold.

It is legitimate to conclude that the bond percolation will describe the propagation of a fire. We will see how to simulate this propagation in the next section.

III. MODEL

Now that we have decide to proceed with the bond percolation, we ask ourselves how to build the simulation of the fire propagation with any language.

Due to the fact that the programming language it is not important regarding on the structure of the program, it is not going to explain details of the programming language, but this project have been done in Fortran.

We begin with the process of propagation generating the forest itself. Our forest will be a matrix of $1000x1000$ where every element of the matrix will be a tree that could have up to four states.

The first state will be the normal state of the tree. It is not burning but it is capable of being burned if the fire arrives at it.

The second state, the most important one regarding the fire. It is the state where the tree is burning and is able to propagate the fire to the neighbour trees with a probability of propagation. A part of that, it also will be the signal of the extinction of the fire. When there is no trees in state two, we will stop the simulation because there will not be more propagation.

The third state, corresponds to a burned tree, that have passed the state two and it has propagate the fire with a certain probability. This tree will propagate no more the fire and it is not capable of being burned again.

The last state will not be used until the last part of the project, but it corresponds to the absence of a tree, that is, there is no propagation and it cannot be burned, due to the fact that there is no tree at all.

Once we have defined the states of each element of the matrix, we explain how we decide to begin the fire.

First, we initialize the matrix making every element to the state one, so we have filled the forest with trees. To begin the fire, we make the middle tree to the state two. We define $t = 0$ to our origin in time of the fire. We show the initial state at the figure, Fig. (5), where the state one is shown in green, representing a tree, and the state two is shown in orange, representing the fire.

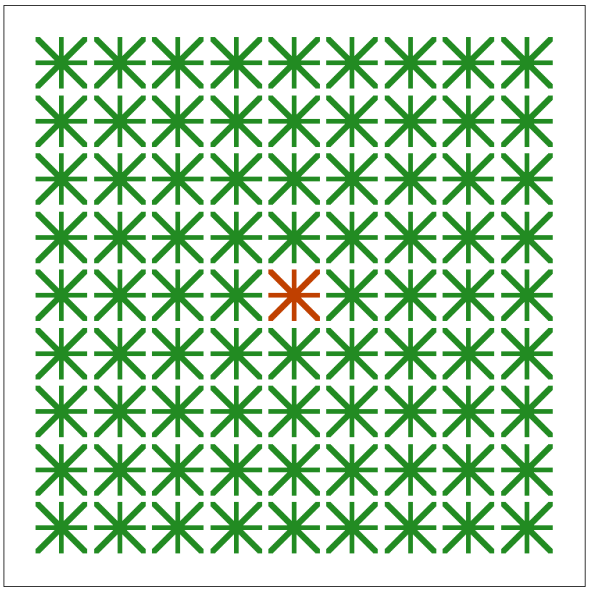

FIG. 5: Initial state of the fire. ($t = 0$)

To begin propagating the fire, first, we look for the nearest neighbours of the tree in state two and we use the generation of random numbers to propagate the state two to theses neighbours with the probability we have previously defined.

A part of that, we define the time of ignition of a tree as a unity in our characteristic time. Then, the initial tree in state two, will propagate the fire to the neighbours and then it is set to state three in the next step of time.

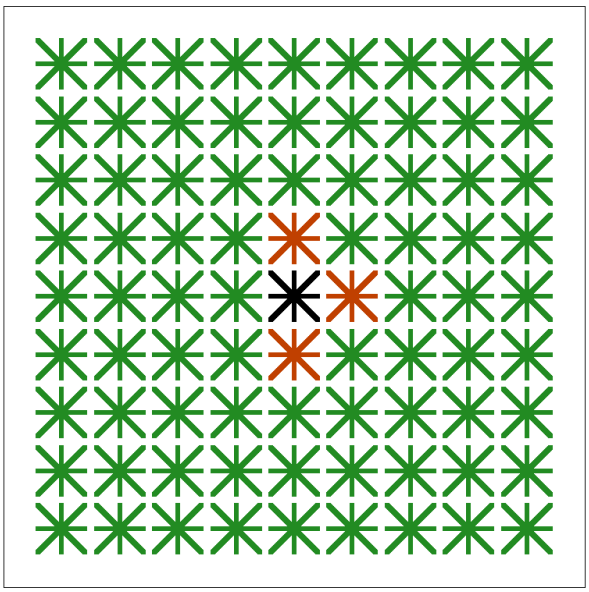

FIG. 6: Advanced state of the fire. ($t = 1$)

Once we calculate the trees that will be set to state two in the next step of time, we increase this time in one unity. Following this instructions, we will have the following figure, Fig. (6), where it is shown the state three in black, representing an ash tree.

As we can observe, the fire is only propagated in three directions. This fact is showing that the probability of propagation is less than one.

From this point, we only have to vary the probability of propagation and observe how the fire evolves following these instructions. In the next chapter we present the results obtained with this method.

We are ready now to begin to simulate and to obtain the first results of the percolation theory.

IV. RESULTS

A. Percolation threshold

As commented before, percolation threshold is a very important result in percolation theory, so we are going to begin with it. But before we recover the theoretical value of the percolation threshold in bond percolation with a two-dimensional squared lattice. A lot of books, [1] or [2], show that the percolation threshold is $p_c = 0.5$. There is some complex mathematical demonstrations like the one that presents Harry Kesten in [3].

We try to find this result representing the time of the fire as a function of probability of propagation, shown in the figure, Fig. (7).

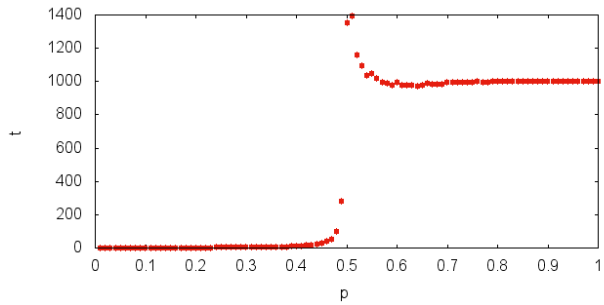

FIG. 7: Time of fire towards probability of propagation.

It agrees with the theoretical value commented before. Next we want to describe the critical behaviour observed at the percolation threshold in this phase transition.

B. Critical behaviour

The percolation theory, [1], tell us that there is a relevant critical exponent called γ with the following expression,

$$S \sim (p - p_c)^{-\gamma}$$

where S is the size of the cluster or cluster mass that in our case is the total number of burnt trees. The theoretical value of this exponent is $\gamma_{th} = 43/18 \sim 2.38$, and representing logarithmically the results of our simulation, we have found $\gamma = 2.41 \pm 0.09$, that is inside de error range.

C. Dynamical behaviour

Another way of characterize the propagation of the fire is observing how the number of burnt trees evolves with the time.

As Hong *et al* introduces in [4],

$$S \sim t^{\bar{\nu}}$$

This dynamical exponent $\bar{\nu}$ will describe how much difficulty has the fire propagating. First we do a logarithmic representation of the number of burnt trees as a function of time, as we do with the critical exponent γ in the previous subsection.

We found a dynamical exponent of $\bar{\nu} = 1.569 \pm 0.001$ with a bond probability of $p = 0.5$.

For the moment, this exponent only shows the potential law behind this propagation. Later, we will show another use more relevant.

D. Dimensionality

We know that that number of burnt trees have to behave as an area, so the behaviour should be as a radius squared. We will see that it is not always true.

Following this expression,

$$S \sim R_S^d$$

where d is the dimensionality and R_S is what is known as the gyration radius. In [5] is presented and defined as the following way,

$$R_S = \frac{1}{S}\sum_{i=1}^{S} |r_i - r_{cm}|^2$$

where S is the number of burnt trees, as always, r_i is the position of each tree and r_{cm} is the position of the center of mass of the cluster. The center of mass is defined as usually,

$$r_{cm} = \frac{1}{S}\sum_{i=1}^{S} r_i$$

that means that R_S is the average quadratic distance from its center of mass.

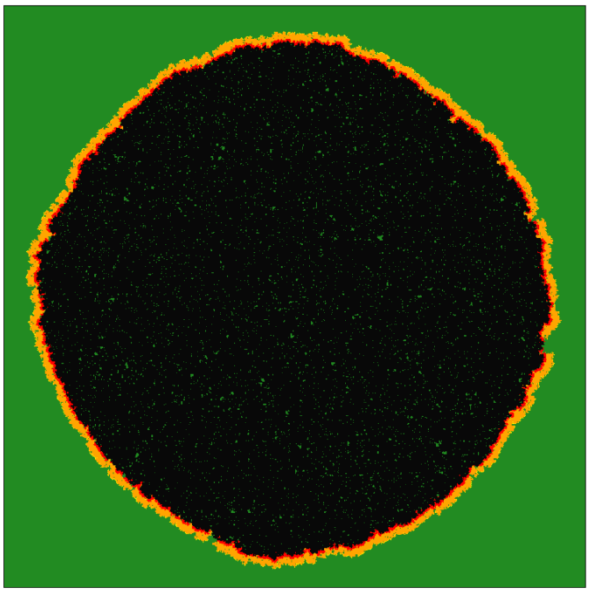

FIG. 8: Burnt area for $p = 0.7$

Firstly, we calculate the dimensionality d for $p = 0.7$, where we expect a squared behaviour where $d_{th} = 2$.

Effectively, we have found $d = 2.00 \pm 0.05$. It means that, graphically, the burnt area will have a squared geometric shape, as the circle shown in the figure, Fig. (8).

However, for $p = 0.5$, the theory tell us that $d_{th} = 91/48 \sim 1.896$, that is approximately what we have found $d = 1.90 \pm 0.01$. This value tell us that the area of burnt trees will have a fractal behaviour, as shown in figure, Fig. (9).

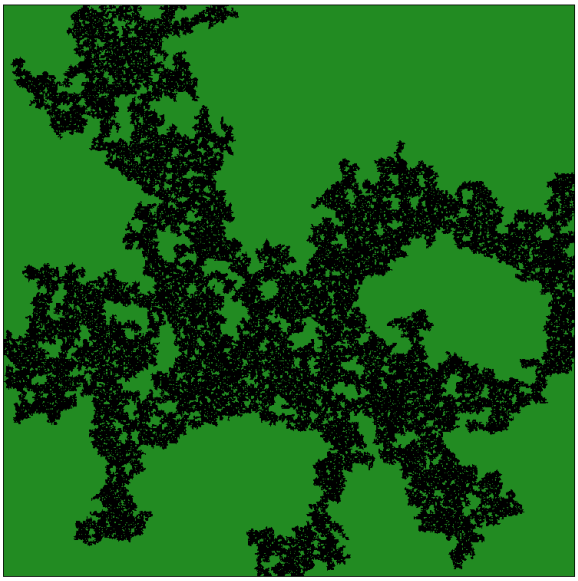

FIG. 9: Burnt area for $p = 0.5$

E. Mixed percolation

In this last section, we are going to mix the two kinds of percolation, what is known as mixed percolation. In our case, it will mean that we have, a part of the propagation probability, the occupation probability, that is, the probability that a tree is or not on a site of the forest. The propagation will be done as before, but we will have some holes in the forest, which corresponds to the fourth state that the elements of the matrix could have. This state cannot be burnt and does not propagate the fire. A figure, Fig. (10), is showing an example.

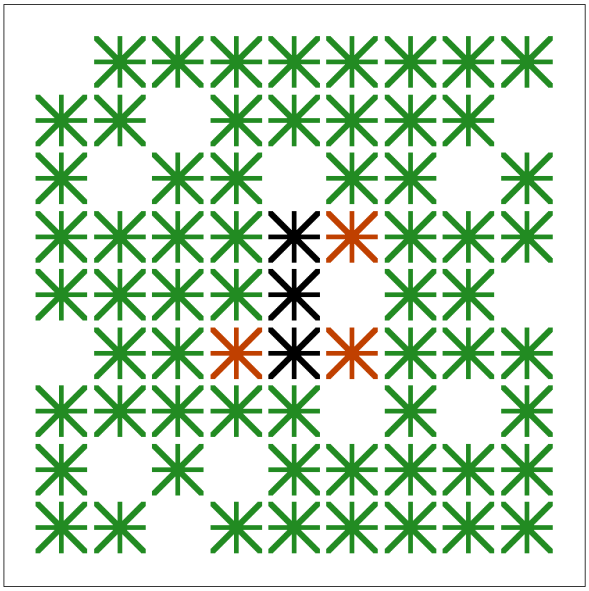

FIG. 10: Possible state of a fire with mixed percolation.

The following step is to calculate the percolation threshold for this system. In this case we will have two parameters, p, probability of propagation, and p_S, the probability of occupation.

We have obtained this figure, Fig. (11), in 3D.

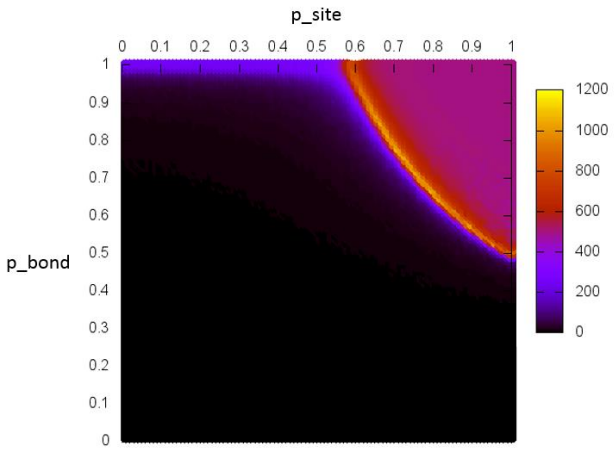

FIG. 11: Phase diagram for mixed percolation

The yellow zone shows the maximum of the percolation surface that determines the combination of (p, p_S) that are a percolation threshold. This line split the diagram between two phases. The black zone is the non-percolating phase, and the purple zone is the percolating phase.

The percolating phase will be the zone where the fire will propagate all over the forest, burning almost every tree. In the non-percolating zone, we have the other case where the fire have difficulties to spread and it extinguishes itself very fast.

As the two kinds of percolation have the same universality class, they have to have the same critical behaviour, so the exponent γ will have the same value in the two percolations including the mixed one. We have calculated anyway to prove it and we have obtained $\gamma = 2.35 \pm 0.05$, that again is inside the error range. The difference of this system will be in the dynamical exponent $\bar{\nu}$.

We have calculate this exponent $\bar{\nu}$ for the case $(p, p_S) = (0.51,1.00)$ obtaining $\bar{\nu} = 1.753 \pm 0.001$, with a total of burnt trees of $S = 711072$. Next we remove the two per cent of the trees and calculate the exponent again. So, in this case, $(p, p_S) = (0.51,0.98)$ and we obtain $\bar{\nu} = 1.524 \pm 0.001$, and a number of burnt trees of $S = 269959$.

In both cases, it has passed the same time, due to this fact, we can observe that in the second case, there is no many burnt trees as the first one. This indicates that the higher the value of the exponent $\bar{\nu}$, the faster the fire will propagate.

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V. CONCLUSIONS

Our aim was to know the parameter or properties that has to have a reforested forest to prevent the fast propagation of a fire. After these simulations we can conclude that we have to have two factors taken into account regarding the security of the forest.

The first one is the exponent $\bar{\nu}$, that we have to maintain as low as possible because a part of making more difficult the fire propagation, it slows the velocity of the propagation as we have seen previously, allowing the firefighters have more time to plan the action or to extinguish before the situation gets worse.

The last important element is the phase diagram shown in the figure, Fig. (11), that tell us that if we can calculate the probability of propagation that will depend on physical properties of the forest, we can know what have to be the percentage of reforestation to be in a non-percolating phase or what is the same, to have a safer forest.

Acknowledgments

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I also want to thank specially my family and my girlfriend, because I would not be here if they had not been by my side when I needed it.

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