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ANALYSIS OF FINANCIAL MARKETS: TECHNICAL ANALYSIS AGAINST TIME SERIES

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Abstract

Financial Markets are studied in different ways, in this piece of work we study and compare two of the most common types of analysis: Technical Analysis and Time Series.

The first one is focused on the evolution of market prices, mainly through the use of graphics. It tries to find chart patterns, with the purpose of predicting future trends of prices.

In the second type of analysis we will study the data and we will try to find a model that adjusts the best possible way to the data given. The purpose is to find a model to with we can extrapolate the data in the future, and then, make it operational in the market.

The goal of my work is to study Technical Analysis and Time Series separately. Finally, I intend to compare them and try to choose which one is more productive.

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Chapter 1

Introduction

Financial Markets are studied in different ways, but the most common types of analysis are Fundamental Analysis, Technical Analysis and Time Series.

The first one pretends to know and evaluate the authentic value of a share, called fundamental value. It uses the estimation of the share's future performance, which depends on the assumptions that have been made. If the market price is higher than the fundamental value, the share is overrated and it is advisable for us to sell. If the market price is lower than the fundamental value, the share is undervalued and we should probably buy. Fundamental Analysis can only be used in shares, but Financial Markets also include bonds, commodities,... So all the types of investing can not be studied by this kind of analysis, so that is why I rejected to include the study of Fundamental Analysis.

The second one is focused on the evolution of market prices, mainly through the use of graphics. It tries to find chart patterns, with the purpose of predicting future trends of prices. It is based on three assumptions: Market action discounts everything, prices move in trends and history tends to repeat itself.

Time Series is possibly the next step of Technical Analysis. In this part we will study the data and we will try to find a model that adjusts the best possible way to the data given. The purpose is to find a model to with we can extrapolate the data in the future, and then, make it operational in the market.

The most important difference between Time Series and Technical Analysis is that in the first we found an exact forecast price, while, in the second one we only found the future movement of the price.

Motivation and goals

The goal of my work is to study Technical Analysis and Time Series separately. Then, I intend to compare them and try to choose which one is more productive.

First I will study Time Series, from the most common models to GARCH models. Looking into all the properties and the way to estimate the weights of the models. Later, I am going to focus on computational finance using R, and I will try to analyze the way to apply Time Series to data using R.

Then I will move on to Technical Analysis, starting with the philosophical idea behind this type of analysis. I will study the three most used tools of Technical Analysis: the

Trend, the Chart Patterns and the Moving Average Indicators. And for every tool, I am going to use language C to develop a program, that selects the appropriate way to operate using these patterns and concepts.

Finally, there is a part of practical work. I will examine the data of Repsol, IBEX35, Euro/Dollar and Gold using Time Series and Technical Analysis. In the part of Time Series, I will use R and in the part of Technical Analysis I will use the programs that I will have designed.

To finish the introduction, I would like to explain my personal motivation to choose this subject as basis for my final project. When I was 5 years old, my mother asked me what I wanted to be when I grew up. She has always told me that my answer was: "I want to be the director of a company"'. Five years later, my teacher asked the students to draft an essay with the title, *What I want to be.* In my essay I wrote that I wanted to be a trader, since that day my objective has been to become a successful trader. And that is why I chose this subject, in order to begin the long road to become a successful trader.

Chapter 2

Time Series

2.1 Previous Concepts

Definition 2.1.1. *Given a probability space* $(\Omega, \mathcal{F}, \mathcal{P})$*. A random variable is a measurable function* $X : (\Omega, \mathcal{F}) \to (\mathbb{R}^n \ ; B(\mathbb{R}^n))$ *that complies*

$$\forall B \in B(\mathbb{R}^n) \ , \ X^{-1}(B) \in \mathcal{F}, \ with \ X^{-1}(B) = \{w \in \Omega, X(w) \in B\}.$$

Definition 2.1.2. *A Stochastic Process* X_t*,* $t \in T$*, is a family of random variables, defined in the probability space* (Ω, F, P)*. We normally use* $t \in \mathbb{Z}$*.*

Note 1. T *could be* $\mathbb{N}, \mathbb{R}, \mathbb{R}_+$ *or other subsets.*

Definition 2.1.3. *The joint cumulative distribution function (cdf) of a stochastic process* X_t *is defined as*

$$F_{t_1,...,t_n}(x_1, ..., x_n) = P(X_{t_1} \leq x_1, ..., X_{t_n} \leq x_n).$$

And if $t_1 < t_2 < ... < t_n$*, we have that:*

$$F_{t_n|t_{n-1},...,t_1}(x_n|x_{n-1}, ..., x_1) = P(X_{t_n} \leq x_n|X_{t_{n-1}} = x_{n-1}, ..., X_{t_1} = x_1).$$

Afterwards, we will assume that the moments exist, if this is not the case, then the corresponding function is not defined.

Definition 2.1.4. *The mean function* μ_t *of a stochastic process* X_t *is defined as*

$$\mu_t = E[X_t].$$

Definition 2.1.5. *The auto-covariance function of a stochastic process* X_t *is defined as*

$$\gamma(t, \tau) = E[(X_t - \mu_t)(X_{t-\tau} - \mu_{t-\tau})].$$

Note 2. *The auto-covariance function is symmetrical.*

Definition 2.1.6. *A stochastic process* X_t *is weakly stationary if*

- $\mu_t = \mu$.
- $\gamma(t, \tau) = \gamma_\tau$.

Note 3. *For weakly stationary, the term covariance stationary is often used.*

Definition 2.1.7. *A stochastic process X_t is strictly stationary if for any $t_1, ..., t_n$ and for all $n, s \in \mathbb{Z}$ it holds that*

$$F_{t_1,...,t_n}(x_1, ..., x_n) = F_{t_{1+s},...,t_{n+s}}(x_1, ..., x_n).$$

Definition 2.1.8. *The auto-correlation function ρ of a weakly stationary stochastic process is defined as*

$$\rho_\tau = \frac{\gamma_\tau}{\gamma_0}.$$

Definition 2.1.9. *The partial auto-correlation function of k-th order is defined as*

$$\alpha(k) = \phi_{kk} = Corr(X_t - P(X_t|X_{t+1}, \cdots, X_{t+k-1}), X_{t+k} - P(X_{t+k}|X_{t+1}, \cdots, X_{t+k-1}))$$

where $P(W|Z)$ is the best linear projection of W on Z (using the squared error minimization). In other words $P(W|Z) = \sum_{WZ} \sum_{ZZ}^{-1} Z$ where $\sum_{ZZ} = Var(Z)$ as the covariance matrix of the regressors and $\sum_{WZ} = Cov(W, Z)$ as the matrix of covariances between W and Z.

Note 4. *There are two special cases depending on k:*

$$\alpha(0) = 1$$
$$\alpha(1) = Corr(X_t, X_{t+1})$$

Definition 2.1.10. *A stochastic process X_t is a white noise if the following holds*

- $\mu_t = 0$.
- *If* $\tau = 0$, $\gamma_\tau = \sigma^2$.
- *If* $\tau \neq 0$, $\gamma_\tau = 0$.

Definition 2.1.11. *A stochastic process X_t follows a random walk, if it can be represented as*

$$X_t = c + X_{t-1} + \epsilon_t.$$

With c a constant and ϵ_t a white noise.

Note 5. *If c is not zero, we can build $Z_t = X_t - X_{t-1} = c + \epsilon_t$, so we will have a non-zero mean. We usually call it a random walk with a drift.*

Note 6. *At the beginning of the last century, the random walk was the first to represent the development of stock prices, so it is a historical stochastic process.*

Note 7. *We will simply assume that the constant c and the initial value X_0 are set to zero. And it is simple to demonstrate that, with these conditions, we have:*

- $X_t = \epsilon_t + \epsilon_{t-1} + ... + \epsilon_1$.
- $\mu_t = 0$.
- $Var(X_t) = t\sigma^2$.
- $\gamma(t, \tau)$.

$$\gamma(t,\tau) = Cov(X_t, X_{t-\tau}) = Cov(\sum_{i=1}^{t} \epsilon_i, \sum_{j=1}^{t-\tau} \epsilon_j) = \sum_{j=1}^{t-\tau} \sum_{i=1}^{t} Cov(\epsilon_i, \epsilon_j) = \sum_{j=1}^{t-\tau} \sigma^2 = (t-\tau)\sigma^2.$$

- $\rho(t, \tau)$.

$$\rho(t,\tau) = \frac{(t-\tau)\sigma^2}{\sqrt{t\sigma 2(t-\tau)\sigma 2}} = \frac{(t-\tau)}{\sqrt{t(t-\tau)}} = \sqrt{1-\frac{\tau}{t}}.$$

Definition 2.1.12. *A stochastic process has the Markov property if for all $t \in \mathbb{Z}$ and $k \geq 1$*

$$F_{t|t-1,...,t-k}(x_t|x_{t-1}, ..., x_{t-k}) = F_{t|t-1}(x_t|x_{t-1}).$$

Note 8. *That property means that a specific moment, the distribution of the process is absolutely determined by the conditions of the system at the previous moment.*

Definition 2.1.13. *The stochastic process X_t is a martingale if the following holds*

$$E[X_t|X_{t-1} = x_{t-1}, ..., X_{t-k} = x_{t-k}] = x_{t-1}$$

for every $k > 0$.

Definition 2.1.14. *The stochastic process X_t is a fair game if the following holds*

$$E[X_t|X_{t-1} = x_{t-1}, ..., X_{t-k} = x_{t-k}] = 0$$

for every $k > 0$.

Note 9. *If X_t is a martingale, then $Z_t = X_t - X_{t-1}$ is a fair game.*

Definition 2.1.15. *The lag operator operates on an element of a Time Series to produce the previous element. Given some Time Series, and L the lag operator: $X = X_1, X_2, ...$, then $LX_t = X_{t-1}$ for all $t > 1$. We also have $L^{-1}X_t = X_{t+1}$.*

2.2 Introduction to Time Series

Time Series models provide a sophisticated method for the extrapolation of Time Series. They are a bit different to the simple extrapolation. The difference is that in the series that we will predict, the model will be generated by a stochastic process. So, Time Series Models are more sophisticated than simple extrapolation. We will first study the Deterministic Models of Time Series.

2.2.1 Deterministic Models of Time Series

We start explaining how the simple models can be used to predict the future performance of the Time Series, using the observed performance in the past of the series. Most of the Time Series are not continuous, usually observations consist in discrete regular intervals, like closing prices. We denote by y_t the series values, and our objective is to build a model that describes the series. The models we will introduce, are frequently used for making informal long term predictions, like GDP or GNI.

Model 1: Linear trend model

If we believe the series will increase by a constant amount over every time period, we can predict the future with the following model:

$$y_t = c_1 + c_2 t$$

With t referring to time and $c_1, c_1 \in \mathbb{R}$.

Model 2: Exponential model

If we consider series will increase by a constant percentage, and not with absolute increases, we use:

$$y_t = ae^{rt}$$

With $a, r \in \mathbb{R}$.

Model 3: Auto-regressive trend model

In this model we consider the previous observation.

$$y_t = c_1 + c_2 y_{t-1}$$

Model 4: Auto-regressive logarithm model

In this model we consider the previous observation, but we introduce a logarithm to the observation.

$$y_t = c_1 + c_2 \log y_{t-1}$$

Model 5: Moving average model

A simple example of this model type is a monthly Time Series. We can use this model:

$$f(t) = \frac{1}{12}(y_{t-1} + y_{t-2} + ... + y_{t-12})$$

A future prediction will be:

$$\hat{y}_{t+1} = \frac{1}{12}(y_t + y_{t-1} + ... + y_{t-11})$$

This model is used when we think that the value of our series next month will be a weighting of the past twelve months. But it is not a common condition.

Model 5.1: Exponential weighted model

We usually think that the recent values are more important.

$$\hat{y}_{t+1} = \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + ... = \alpha \sum_{\tau=0}^{\infty}(1-\alpha)^{\tau} y_{t-\tau}$$

With $\alpha \in [0,1]$.

Note 10. *In the case of* $\alpha = 1$ *we have* $\hat{y}_{t+1} = y_t$.

2.2.2 A First Stochastic Time Series Model

In the models that we are going to study in this chapter, Time Series will be generated by a stochastic process. The first stochastic Time Series process we will study is random walk. Not a lot of processes are random walks , but these are a good approach for a lot of processes.

Random Walk with c = 0

Firstly, we study the process when c is zero, it means that we ignore the trend. So the process is:

$$X_t = X_{t-1} + \epsilon_t.$$

With ϵ_t a white noise.

A prediction will be

$$\hat{X}_{t+1} = E[X_{t+1}|X_t, ..., X_1]$$

With $X_{t+1} = X_t + \epsilon_{t+1}$, and ϵ_{t+1} is independent of $X_t, ..., X_1$. So the prediction will be

$$\hat{X}_{t+1} = X_t + E[\epsilon_{t+1}] = X_t$$

For the next period, the prediction will be:

$$\hat{X}_{t+2} = E[X_{t+2}|X_t, ..., X_1] = E[X_{t+1} + \epsilon_{t+2}|X_t, ..., X_1] = E[X_t + \epsilon_{t+1} + \epsilon_{t+2}|X_t, ..., X_1] = X_t$$

For all the periods is X_t.

Study of prediction error and variance

The prediction for all the periods will be X_t, but the prediction error and the variance will be different, so, we study this now:

$$e_1 = X_{t+1} - \hat{X}_{t+1} = X_t + \epsilon_{t+1} - X_t = \epsilon_{t+1}$$

With variance $E[\epsilon_{t+1}^2] = \sigma_\epsilon^2$.

For the next prediction:

$$e_2 = X_{t+2} - \hat{X}_{t+2} = X_t + \epsilon_{t+1} + \epsilon_{t+2} - X_t = \epsilon_{t+1} + \epsilon_{t+2}$$

With variance $E[(\epsilon_{t+1} + \epsilon_{t+2})^2] = E[\epsilon_{t+1}^2] + E[\epsilon_{t+2}^2] + 2E[\epsilon_{t+1}^2\epsilon_{t+2}^2] = \sigma_\epsilon^2 + \sigma_\epsilon^2 = 2\sigma_\epsilon^2$.

So for the time period $l \in \mathbb{N}$:

$$e_l = \epsilon_{t+1} + ... + \epsilon_{t+l}$$

And variance equal to $l\sigma_\epsilon^2$.

Random Walk with c $\neq$ 0

With this process we consider an increasing, or decreasing, trend. It means that the new process is:

$$X_t = c + X_{t-1} + \epsilon_t.$$

On average, the process will increase with a $c > 0$, and it will decrease with a $c < 0$.

The prediction for the next period is

$$\hat{X}_{t+1} = E[X_{t+1}|X_t, ..., X_1] = X_t + c$$

And for the $l \in \mathbb{N}$ time period will be:

$$\hat{X}_{t+l} = X_t + lc$$

The prediction error will be the same as in $c = 0$.

2.3 Auto-regressive Processes

In this model X_t is generated by a weighted average of previous observations, exactly of the last p periods.

$$X_t = \delta + \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_p X_{t-p} + \epsilon_t$$

With $\phi_i \in \mathbb{R}$ and ϵ_t a white noise for $i = 1, ..., p$, δ is a constant.

Using the Lag operator, we can rewrite:

$$(1 - \phi_1 L - \phi_2 L^2 - ... - \phi_p L^p)X_t = \delta + \epsilon_t.$$

And

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - ... - \phi_p L^p.$$

We have:

$$\phi(L)X_t = \delta + \epsilon_t.$$

2.3.1 Properties of AR(p)

- Stationary

  The process will be stationary if its mean is invariant, $E[X_t] = E[X_{t-1}] = ... = \mu$, and if $\gamma < \infty$.

  $$\mu = E[X_t] = E[\delta + \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_p X_{t-p} + \epsilon_t] = \delta + \phi_1\mu + \phi_2\mu + ... + \phi_p\mu.$$

  If we isolate μ:

  $$\mu = \frac{\delta}{1-\phi_1-\phi_2-...-\phi_p}$$

  And μ will be finite if $1-\phi_1-\phi_2-...-\phi_p \neq 0$, but this is only a necessary condition, not a sufficient condition. The necessary and sufficient condition of AR(p) stationary is often expressed by saying that the roots of the characteristic equation

  $$\phi(L) = 0$$

  must lie outside the circle of unit radius, you can look for the proof in [4].

- Covariance and auto-correlation function:

  The covariance with k delays is (we consider $\mu = 0$, it does not change anything if we consider it instead of $\mu \neq 0$, it only increases the calculus):

  $$\gamma_k = E[X_{t-k}(\phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_p X_{t-p} + \epsilon_t)].$$

  For $k = 0, 1, ..., p$:

  $$\gamma_0 = \phi_1 X_1 + \phi_2 X_2 + ... + \phi_p X_p + \sigma_\epsilon^2$$

  $$\gamma_1 = \phi_1 X_0 + \phi_2 X_1 + ... + \phi_p X_{p-1} + \sigma_\epsilon^2$$

  $$...$$

  $$\gamma_p = \phi_1 X_{p-1} + \phi_2 X_{p-2} + ... + \phi_p X_p + \sigma_\epsilon^2$$

  And for $k > p$:

  $$\gamma_k = \phi_1 X_{k-1} + \phi_2 X_{k-2} + ... + \phi_p X_{k-p}$$

Now, the auto-correlation function is:

$$\rho_1 = \phi_1 + \phi_2\rho_1 + ... + \phi_p\rho_{p-1}$$

$$...$$

$$\rho_p = \phi_1\rho_{p-1} + \phi_2\rho_{p-2} + ... + \phi_p$$

And for $k > p$

$$\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2} + ... + \phi_k\rho_{k-p}$$

So now we know that the auto-correlation function of an auto-regressive process has, mainly, infinite nonzero terms.

2.3.2 Yule-Walker equations

These equations come from multiplying the AR(p) process by $X_{t-\tau}$ and then take expectations.

$$E\left[X_tX_{t-\tau}\right] = \phi_1E\left[X_{t-1}X_{t-\tau}\right] + ... + \phi_pE\left[X_{t-p}X_{t-\tau}\right].$$

We know that $E\left[X_tX_{t-\tau}\right]$ is the auto-covariance function γ_t, so for $\tau = 1, 2, ..., p$ we obtain:

$$\gamma_1 = \phi_1\gamma_0 + \phi_2\gamma_1 + ... + \phi_p\gamma_{p-1}$$
$$\gamma_2 = \phi_1\gamma_1 + \phi_2\gamma_0 + ... + \phi_p\gamma_{p-2}$$
$$...$$
$$\gamma_p = \phi_1\gamma_{p-1} + \phi_2\gamma_{p-2} + ... + \phi_p\gamma_0$$

Now if we divide it by γ_0 we have:

$$\rho = \Gamma\phi$$

With $\rho = (\rho_1\rho_2...\rho_p)^T$, $\phi = (\phi_1\phi_2...\phi_p)^T$ and the auto-covariance matrix:

$$\Gamma = \begin{pmatrix} 1 & \rho_1 & \cdots & \rho_{p-1} \\ \rho_1 & 1 & \cdots & \rho_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{p-1} & \rho_{p-2} & \cdots & 1 \end{pmatrix}$$

These are the Yule-Walker equations.

2.3.3 Partial auto-correlation function of AR(p)

If we use the notation $\phi = (\phi_{p1}\phi_{p2}...\phi_{pp})^T$ in the Yule-Walker equations, the last coefficient, ϕ_{pp} is the partial auto-correlation of order p.

If we are only interested in this coefficient, we can solve using Cramer's Rule, so we obtain

$$\phi_{pp} = \frac{|\Gamma *|}{|\Gamma|}$$

where $|\Gamma|$ is the determinant of matrix Γ, and $\Gamma*$ is equal to Γ replacing the k-th column by ρ.

If we apply this for various orders p, we will have the partial auto-correlation function (PACF).

$$\phi_{11} = \rho_1$$

$$\phi_{22} = \frac{\begin{vmatrix} 1 & \rho_1 \\ \rho_1 & \rho_2 \end{vmatrix}}{\begin{vmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{vmatrix}} = \frac{\rho_2 - \rho_1^2}{1-\rho_1^2}$$

$$\phi_{33} = \frac{\begin{vmatrix} 1 & \rho_1 & \rho_1 \\ \rho_1 & 1 & \rho_2 \\ \rho_2 & \rho_1 & \rho_3 \end{vmatrix}}{\begin{vmatrix} 1 & \rho_1 & \rho_2 \\ \rho_1 & 1 & \rho_1 \\ \rho_2 & \rho_1 & 1 \end{vmatrix}}$$

For an auto-regressive process of order p, the partial auto-correlation function ϕ_{kk} will be non-zero for $k \leq p$ and zero for k greater than p. So, it has a cutoff after lag p.

2.3.4 Example AR(1)

In this example we study the easier AR process, that is, the case $p = 1$.

$$X_t = \phi_1 X_{t-1} + \delta + \epsilon_t$$

Properties of AR(1)

- The mean is:

$$\mu = \frac{\delta}{1-\phi_1}$$

  The process will be stationary if $\phi_1 < 1$.

- If we assume that the process is stationary and $\delta = 0$, the variance is:

$$\gamma_0 = E[(\phi_1 X_{t-1} + \epsilon_t)^2] = E[\phi_1^2 X_{t-1}^2 + \epsilon_t^2 + 2\phi_1 X_{t-1}\epsilon_t] = \phi_1^2\gamma_0 + \sigma_\epsilon^2.$$

And

$$\gamma_0 = \frac{\sigma_\epsilon^2}{1-\phi_1^2}.$$

- The covariances are:

$$\gamma_1 = E[X_{t-1}(\phi_1 X_{t-1} + \epsilon_t)] = \phi_1\gamma_0 = \frac{\phi_1\sigma_\epsilon^2}{1-\phi_1^2}$$

$$\gamma_2 = E[X_{t-2}(\phi_1^2 X_{t-2} + \phi_1\epsilon_{t-1} + \epsilon_t)] = \phi_1^2\gamma_0 = \frac{\phi_1^2\sigma_\epsilon^2}{1-\phi_1^2}$$

$$...$$

And for k delays:

$$\gamma_k = \phi_1^k\gamma_0 = \frac{\phi_1^k\sigma_\epsilon^2}{1-\phi_1^2}$$

- The auto-correlation function is simple, it starts with $\rho_0 = 1$, and then it decreases with a geometric progression:

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \phi_1^k$$

This process has an infinite memory. The current value of the process depends on all past values, although the dependence decreases with time.

2.3.5 Example AR(2)

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \delta + \epsilon_t$$

Properties of AR(2)

- The mean is:

$$\mu = \frac{\delta}{1-\phi_1-\phi_2}$$

The process is stationary if $\phi_1 + \phi_2 < 1$.

- The variance is:

$$\gamma_0 = E[X_t(\phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t)] = \phi_1\gamma_1 + \phi_2\gamma_2 + \sigma_\epsilon^2.$$

- Covariances are:

$$\gamma_1 = E[X_{t-1}(\phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t)] = \phi_1\gamma_0 + \phi_2\gamma_1.$$

$$\gamma_2 = E[X_{t-2}(\phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t)] = \phi_1\gamma_1 + \phi_2\gamma_0.$$

$$...$$

And for k delays:

$$\gamma_k = \phi_1\gamma_{k-1} + \phi_2\gamma_{k-2}.$$

Now, we solve the system of the first two equations and the last.

The second one can be written as follows:

$$\gamma_1 = \frac{\phi_1\gamma_0}{1-\phi_2}$$

And replacing the third equation with the first equation:

$$\gamma_0 = \phi_1\gamma_1 + \phi_2\phi_1\gamma_1 + \phi_2^2\gamma_0 + \sigma_\epsilon^2$$

Now replacing γ_1 with its value:

$$\gamma_0 = \frac{(1-\phi_2)\sigma_\epsilon^2}{(1+\phi_2)[(1-\phi_2)^2-\phi_1^2]}$$

- The auto-correlation function is simple, we use the previous equations:

$$\rho_1 = \frac{\phi_1}{1-\phi_2}$$

$$\rho_2 = \phi_2 + \frac{\phi_1^2}{1-\phi_2}$$

Now, using $\gamma_k = \phi_1\gamma_{k-1} + \phi_2\gamma_{k-2}$ for $k \geq 2$:

$$\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2}.$$

- If we use the Yule-Walker equations:

$$\rho_1 = \phi_1 + \phi_2\rho_1$$
$$\rho_2 = \phi_1\rho_1 + \phi_2$$

And we solve for ϕ_1 and ϕ_2

$$\phi_1 = \frac{\rho_1(1-\rho_2)}{1-\rho_1^2}$$
$$\phi_2 = \frac{\rho_2-\rho_1^2}{1-\rho_1^2}$$

2.4 Moving-Average Processes

In this model X_t is generated by a weighted average of random perturbations with q delay periods. We denote this process MA(q).

$$X_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + ... + \theta_q \epsilon_{t-q}$$

With $\theta_i \in \mathbb{R}$ for $i = 1, ..., q$, and ϵ_j a white noise for $j = t, ..., t-q$, μ is the expectation of X_t (often assumed equal to 0).

2.4.1 Properties of MA(q)

- The MA(q) process is stationary, because it is formed by a linear combination of stationary processes.
- If $\mu = 0$ the mean function is simply $E[X_t] = 0$, in other cases is $E[X_t] = \mu$.
- The variance is:

$$Var(X_t) = \gamma_0 = E[(X_t - \mu)^2] = E[\epsilon_t^2 + \theta_1^2 \epsilon_{t-1}^2 + ... + \theta_q^2 \epsilon_{t-q}^2 - 2\theta_1 \epsilon_t \epsilon_{t-1} - ...] =$$
$$\sigma_\epsilon^2 + \theta_1^2 \sigma_\epsilon^2 + ... + \theta_q^2 \sigma_\epsilon^2 = \sigma_\epsilon^2 (1 + \theta_1^2 + \theta_2^2 + ... + \theta_q^2).$$

- The covariances, with μ equal to zero, are:

$$\gamma_\tau = Cov(X_t, X_{t-\tau}) = Cov(\sum_{i=0}^{q} \theta_i \epsilon_{t-i}, \sum_{j=0}^{q} \theta_j \epsilon_{t+\tau-j}) =$$
$$\sum_{i=0}^{q} \sum_{j=0}^{q} \theta_i \theta_j Cov(\epsilon_{t-i}, \epsilon_{t+\tau-j}) = \sum_{i=0}^{q-|\tau|} \theta_i \theta_{i+|\tau|} \sigma^2.$$
$$\text{For } |\tau| \leq q.$$

- The Auto-Correlation Function is:

  1. For $k = 1, ..., q$

  $$\rho_k = \frac{-\theta_k + \theta_1 \theta_{k+1} + ... + \theta_{q-k} \theta_q}{1 + \theta_1^2 + \theta_2^2 + ... + \theta_q^2}$$

  2. For $k > q$

  $$\rho_k = 0.$$

  The auto-correlation function of a moving average process has a cutoff after lag q.

- The Partial Auto-Correlation Function is:

  In order to calculate the Partial Auto-Correlation function of a moving average process, we have to express the MA(q) process as an AR(∞) process. We will see that an invertible MA process needs the roots of $\theta(L) = 0$ lie outside the unit circle. The extended conditions are in the invertibility part of MA(q).

  $$\theta^{-1}(L) X_t = \epsilon_t$$

If we use the notation $\theta^{-1}(L) = \pi(L) = 1 - \pi_1 L - \cdots - \pi_k L^k - \cdots$, and the coefficients of $\pi(L)$ are obtained imposing $\pi(L)\theta(L) = 1$.

If we use what we know about the PACF of an AR(p) process, we have that MA(q) process is equivalent to an AR(∞) process. So, we conclude that the PACF of an MA(q) is non-zero for all lags. This happens, because in AR(p) process we have that $\alpha(k) = 0$ for $k > p$, but if p is ∞, $\alpha(k)$ will be non-zero for all k. The PACF of a MA(q) has the same structure as the ACF of an AR(q) process.

2.4.2 Example MA(1)

In this example we study the easier MA process, when $q = 1$.

$$X_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1}.$$

Properties of MA(1)

- The variance is $\gamma_0 = \sigma_\epsilon^2(1 + \theta_1^2)$.
- The covariance with one delay is:

$$\gamma_1 = E[(X_t - \mu)(X_{t-1} - \mu)] = E[(\epsilon_t - \theta_1 \epsilon_{t-1})(\epsilon_{t-1} - \theta_1 \epsilon_{t-2})] = -\theta_1 \sigma_\epsilon^2.$$

- The covariance with k delays ($k > 1$) is:

$$\gamma_k = E[(\epsilon_t - \theta_1 \epsilon_{t-1})(\epsilon_{t-1} - \theta_1 \epsilon_{t-2})] = 0.$$

  So MA(1) has only one period memory, every X_t is only related with X_{t-1} and X_{t+1}.

- The auto-correlation function is:

  1. For $k = 1$, $\rho = \frac{-\theta_1}{1+\theta_1^2}$.
  2. For $k > 1$, $\rho = 0$.

2.4.3 Example MA(2)

$$X_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2}$$

Properties of MA(2)

- $Var(X_t) = \gamma_0 = \sigma_\epsilon^2(1 + \theta_1^2 + \theta_2^2)$.
- The covariances are:

  $\gamma_1 = E[(\epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2})(\epsilon_{t-1} - \theta_1 \epsilon_{t-2} - \theta_2 \epsilon_{t-3})] = -\theta_1 \sigma_\epsilon^2 + \theta_2 \theta_1 \sigma_\epsilon^2 = -\theta_1(1 - \theta_2)\sigma_\epsilon^2$.

  $\gamma_2 = E[(\epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2})(\epsilon_{t-2} - \theta_1 \epsilon_{t-3} - \theta_2 \epsilon_{t-4})] = -\theta_2 \sigma_\epsilon^2$.

  $\gamma_k = 0$ for $k > 2$.

- The auto-correlation function is:

$$\rho_1 = \frac{-\theta_1(1-\theta_2)}{1+\theta_1^2+\theta_2^2}$$
$$\rho_2 = \frac{-\theta_2}{1+\theta_1^2+\theta_2^2}$$
$$\rho_k = 0 \text{ for } k > 2$$

MA(2) has a two period memory, every X_t is only related with X_{t-2}, X_{t-1}, X_{t+1} and X_{t+2}.

2.5 Relation among AR(p) and MA(q)

2.5.1 From MA(q) to AR(∞)

MA(q) process with $\mu = 0$ is:

$$X_t = (1 - \theta_1 L - \theta_2 L^2 - \cdots - \theta_q L^q)\epsilon_t = \theta(L)\epsilon_t$$

with $\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \cdots - \theta_q L^q$. So, we obtain:

$$\theta(L)^{-1} X_t = \epsilon_t.$$

The only condition we have to take into consideration is that the roots of $\theta(L) = 0$ must lie outside the circle of unit radius. If this is not true the reversal is not possible.

2.5.2 From AR(p) to MA(∞)

Given an AR(p) process:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + \epsilon_t$$

Using L operator we get

$$\phi(L) X_t = \epsilon_t$$

with $\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$. So,

$$Xt = \phi(L)^{-1}\epsilon_t.$$

2.6 Mixed Auto-regressive-Moving Average Models

There are a lot of processes for which we can not build a model using only AR or MA, this happens because the process has characteristics of both. When this happens, we use ARMA models, which include auto-regressive and moving average models. The equation is simple, we join both models:

$$X_t = \delta + \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_p X_{t-p} + \epsilon_t - \theta_1 \epsilon_{t-1} - ... - \theta_q \epsilon_{t-q}$$

2.6.1 Properties of ARMA(p,q)

- **Stationary**

  If the process is stationary, the mean is constant

  $$E\left(X_t\right) = \delta + \phi_1\mu + \phi_2\mu + ... + \phi_p\mu$$

  And if we isolate μ

  $$\mu = \frac{\delta}{1-\phi_1-\phi_2-...-\phi_p}$$

  So, we need the following condition to have a stationary process

  $$\phi_1 + \phi_2 + ... + \phi_p < 1$$

- **From ARMA to MA (or AR)**

  Using Lag Operator, the ARMA(p,q) process can be written as

  $$\phi(L)X_t = \theta(L)\epsilon_t$$

  with $\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$ and $\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \cdots - \theta_q L^q$.

  Now, we can rewrite the process:

  $$X_t = \phi(L)^{-1}\theta(L)\epsilon_t.$$

  So ARMA(p,q) is equivalent to a moving-average process with p+q independent coefficients, but in order to have these, the process must satisfy some conditions.

  The convergence condition is that the roots of $\phi(L) = 0$ must lie outside the circle of unit radius.

  We can also rewrite the process as

  $$\theta(L)^{-1}\phi(L)X_t = \epsilon_t.$$

  This equation shows that ARMA(p,q) is equivalent to auto-regressive process with p+q independent coefficients.

  The convergence condition is that the roots of $\theta(L) = 0$ must be outside the circle of unit radius.

- **Auto-correlation function**

  If we multiply $X_t = \delta + \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_p X_{t-p} + \epsilon_t - \theta_1\epsilon_{t-1} - ... - \theta_q\epsilon_{t-q}$ by X_{t-k}, and we take expectations, we have

  $$\gamma_k = \phi_1\gamma_{k-1} + \phi_2\gamma_{k-2} + ... + \phi_p\gamma_{k-p} + E[X_{t-k}\epsilon_t] - \theta_1 E[X_{t-(k-1)}\epsilon_t] - \cdots - \theta_q E[X_{t-(k-q)}\epsilon_t]$$

Now we will focus on the possible variations of k.

For $k > 0$, we have $E[X_{t-k}\epsilon_t] = 0$.

For $k > 1$, we have $E[X_{t-k}\epsilon_t] = 0$ and $E[X_{t-k}\epsilon_{t-1}] = 0$.

For $k > 2$, we have $E[X_{t-k}\epsilon_t] = 0$, $E[X_{t-k}\epsilon_{t-1}] = 0$ and $E[X_{t-k}\epsilon_{t-2}] = 0$.

...

For $k > q$, we have $E[X_{t-k}\epsilon_t] = 0$, $E[X_{t-k}\epsilon_{t-1}] = 0$, $E[X_{t-k}\epsilon_{t-2}] = 0$, $\cdots$, $E[X_{t-k}\epsilon_{t-q}] = 0$.

So for $k > q$, we have $E[X_{t-k}\epsilon_t] - \theta_1 E[X_{t-(k-1)}\epsilon_t] - \cdots - \theta_q E[X_{t-(k-q)}\epsilon_t] = 0$, and

$$\gamma_k = \phi_1\gamma_{k-1} + \phi_2\gamma_{k-2} + ... + \phi_p\gamma_{k-p}$$

Obviously, if we divide by γ_0

$$\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2} + ... + \phi_p\rho_{k-p}.$$

That is:

$$\phi(L)\rho_k = 0 \text{ for } k > q.$$

So for an ARMA(p,q) process we have q auto-correlations, $\rho_1, \cdots, \rho_q$, whose values depend of the parameters of ϕ and θ.

To sum up, the ACF has $q - p + 1$ initial values with a structure that depends on the parameters, the ACF decay start after lag q as a mixture of exponential and other oscillations.

The structure of the PACF is similar to the ACF,but the decay starts after lag p, if you are interested in this part of the work, you can look for more information about it in [2].

- The variance is:

$$\gamma_k = \phi_1\gamma_{k-1} + \phi_2\gamma_{k-2} + ... + \phi_p\gamma_{k-p} + E[X_{t-k}\epsilon_t] - \theta_1 E[X_{t-(k-1)}\epsilon_t] - \cdots - \theta_q E[X_{t-(k-q)}\epsilon_t]$$

And it depends on the value of q, as we have explained in Auto-correlation function.

2.6.2 Example ARMA(1,1)

This is one of the easiest ARMA processes:

$$X_t = \phi_1 X_{t-1} + \epsilon_t - \theta_1\epsilon_{t-1}$$

This can be rewritten as

$$(1 - \phi_1 L)X_t = (1 - \theta_1 L)\epsilon_t$$

Properties of ARMA(1,1)

- Stationary and invertibility conditions

  The process will be stationary if $\phi_1 \in (-1, 1)$.

  The process will be invertible if $\theta_1 \in (-1, 1)$

- Covariances

  Using the formula of the ARMA(p,q) process explained before, we obtain

$$\gamma_0 = \phi_1\gamma_1 + \sigma^2(1 - \theta_1\psi_1)$$
$$\gamma_1 = \phi_1\gamma_0 - \theta_1\sigma^2$$
$$\gamma_k = \phi_1\gamma_{k-1}, \text{ for } k \geq 2$$

  And if we solve the system with the first two equations we obtain

$$\gamma_0 = \tfrac{1+\theta_1^2-2\phi_1\theta_1}{1-\phi_1^2}\sigma^2$$
$$\gamma_1 = \tfrac{(1-\phi_1\theta_1)(\phi_1-\theta_1)}{1-\phi_1^2}\sigma^2$$
$$\gamma_k = \phi_1\gamma_{k-1}, \text{ for } k \geq 2$$

- The Auto-correlation function is

$$\rho_1 = \tfrac{\gamma_1}{\gamma_0} = \tfrac{(1-\phi_1\theta_1)(\phi_1-\theta_1)}{1+\theta_1^2-2\phi_1\theta_1}$$

  And for $k \geq 2$ we have

$$\rho_k = \phi_1\rho_{k-1}$$

2.7 Non-stationary Models

We have just studied the stationary processes, but in practice we normally have non-stationary processes, that happens for example when we have a trend. The principal idea of the procedure is to transform the non-stationary process to a stationary process to applying what we have studied about the stationary processes.

The principal reasons to have non-stationary processes are:

- The presence of a trend.
- The variance is not constant.
- There are variations in the stationary conditions, an that is because the mean of the process changes.

2.7.1 Non-stationary homogenous processes

- When the series have a lineal trend we can transform it using

$$X_t - X_{t-1} \equiv (1 - L)X_t \equiv Z_t$$

  If X_t has a linear trend, then, Z_t will not incorporate this trend. We usually say that these Time Series are homogeneous of first order.

- If X_t has a quadratic trend, we should make a double transformation

$$W_t = \Delta X_t$$
$$Z_t = \Delta W_t$$

  In the case of Z_t being stationary, we say that X_t is homogeneous of second order. We can also write $Z_t = \Delta^2 X_t$.

  Note 11. *If we have a cubic trend, it will be the same case, although we will need to make a triple transformation.*

  Note 12. *In practice it is really difficult to know if we have done the correct number of transformations. We usually decide on the number of transformations by making a visual study of the graphic.*

- If X_t has an exponential trend, we can transform it using

$$\ln X_t - \ln X_{t-1} \equiv Z_t$$

  Now Z_t does not have a trend.

2.7.2 ARIMA Model

As we have already seen, a lot of series can be transformed into approximately stationary series after making one or more differences.

If X_t is homogeneous of order d, then

$$\Delta^d X_t = Z_t$$

is stationary.

If Z_t is MA(q) we say that X_t is an Integrated Moving Average Process, IMA(d,q).

If Z_t is ARMA(p,q):

$$Z_t = \phi^{-1}(L)\theta(L)\epsilon_t.$$

Then

$$X_t = \Delta^{-d}\phi^{-1}(L)\theta(L)\epsilon_t.$$

And we say that X_t is an Auto-regressive Integrated Moving Average Process, ARIMA(p,d,q).

Note 13. *In this case, $\phi(L)$ is called the auto-regressive operator, while $\theta(L)$ is called the moving average operator.*

Note 14. *We can replace Δ^{-1} with Σ, because*

$$\Delta^{-1} = (1 - L)^{-1} = (1 + L + L^2 + \cdots) \equiv \Sigma.$$

Special Cases of ARIMA Model

The three special cases of ARIMA models that are usually used in practice are

- ARIMA(0,1,1) or IMA(1,1)

$$\Delta X_t = \epsilon_t - \theta_1 \epsilon_{t-1} = (1 - \theta_1 L)\epsilon_t$$

- ARIMA(0,2,2) or IMA(2,2)

$$\Delta X_t = \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} = (1 - \theta_1 L - \theta_2 L^2)\epsilon_t$$

- ARIMA(1,1,1)

$$\Delta X_t - \phi_1 \nabla X_{t-1} = \epsilon_t - \theta_1 \epsilon_{t-1} = (1 - \theta_1 L)\epsilon_t$$

We can always write this:

$$(1 - \phi_1 L)\Delta X_t = (1 - \theta_1 L)\epsilon_t$$

2.8 The Box-Jenkins method

This method was developed by Box and Jenkins to find the best fit of a Time Series model to past values of a Time Series, i.e. with this method we decide the best approximation ARIMA model in accordance with the method conditions.

This method comprises three stages.

- **Model identification and model selection.** We should know if the series is stationary or not and then, by using ACF and PACF, to delimit the value of d,p and q, which we will use in ARIMA model.
- **Parameter estimation.** When we know p, d and q, we have to estimate the parameters of the model.
- **Model checking.** It consists of checking if the errors of the model are white noise.

2.8.1 Model identification and model selection

When we have a Time Series and we want to adjust an ARIMA(p,d,q), we usually have the problem of choosing the correct p,d and q. I.e. specify the model ARIMA. The selection problem is solved by using the ACF and PACF of the Time Series.

First we have to select the best d, i.e. the correct number of differentiates that we have to apply to the model to obtain a stationary process. We rely on the fact that the auto-correlation function, ρ_k, of a stationary process, is approximately zero when we increase k.

We know that in ARMA(p,q) models, the auto-correlation function associated with the Moving average part is zero for $k > q$, because the process only has a q period memory. So, for a MA(q) process, we have $\rho_k = 0$ for all $k > q$.

The Auto-regressive part of a stationary ARMA(p,q) is attenuated with a geometrical progression, so the auto-correlation plot will decay slowly.

The process to find the correct d is easy, we only have to study the auto-correlation function, and do the convenient checking to know if it is stationary or not. If it is stationary, the model will have $d = 0$. If it is non-stationary, we should make a first differentiate. Then we examine the auto-correlation function of ΔX_t. If it is stationary, we will have $d = 1$. If it is not, we will repeat the process increasing the number of differentiates. The process ends when we find a d that makes $\Delta^d X_t$ stationary. If the series have a trend, we probably have a non-zero d.

Now we have the correct d, so the process we have to study is $Z_t = \Delta^d X_t$. Fortunately, the models in practice have lower p and q, so the estimation is often done with the visual checking. But when the order of p and q is not low, the process of identification is very difficult and we must apply AIC, which is explained later.

Note 15. *In the Box-Jenkins method, once we have detected the stationary, the next step is to detect seasonality. But we are studying Time Series in Financial Markets, so the day price of the stocks can not be relevant with any seasonal past values. For example, in order to have seasonality, stock prices on Wednesday need to be related to the ones on last week's Wednesday. Another example could be 1st of March, which has to be related to 1st of February. However, this does not occur in practice. This happens when we have a monthly Time Series, but this is not the case, so we are doing to overlook it.*

Now, we know d, so we will work with a stationary model. The next step is to select the order of p and q. For every ARMA(p,q) , MA(q) and AR(p) model, we know the ACF and PACF. We can summarize:

- AR(p)
  1. The auto-correlation function has infinite nonzero terms.
  2. The partial auto-correlation function has a cutoff after lag p.
- MA(q)
  1. The auto-correlation function has a cutoff after lag q.
  2. The partial auto-correlation function has infinite nonzero terms.

- ARMA(p,q)

    1. The auto-correlation function has a slow decay after q lags.
    2. The partial auto-correlation function has a slow decay after p lags.

In cases with low p and q, the method is the analysis of ACF and PACF. We have to plot ACF and PACF and identify the correct p and q. In financial Time Series, we usually have the option of choosing the orders with the visual identification. We must take into account that Time Series do not offer only one model that allow us to estimate them correctly. The reason is that models are not perfect. What we have to try is to select the most correct model. At the end of the method, in the model checking, we will decide if the model is correct or not. If it is not correct, we will return to the first step to choose a better model.

Now we can make a table with the optimum decisions for every case of ACF.

1. Exponential decaying to zero or alternating positive and negative decaying to zero

   It is an AR, the order p is given by the cutoff of the PACF.

2. One or more spikes, and the rest are insignificant

   It is a MA, the order q is given by the number where the plot is near zero.

3. Decay after a few lags

   It is an ARMA, we know that the partial auto-correlation function has a slow decay after p lags, so that gives us the order of auto-regressive model. We also know that the auto-correlation function has a slow decay after q lags, this gives us the order of q. The orders can be chosen by other criterion that are explained later.

4. Special cases

    - All zero or close to zero
      The data can be considered as random.
    - High values at fixed intervals
      The data has a seasonal term that we did not include in the model.
    - No decay to zero
      The series is not stationary, so we have to differentiate.

The Akaike information criterion

We have studied a theory method based on the analysis of ACF and PACF to select the orders p and q. Now we will explain the method that is normally used in practice, the AIC.

The method consists on choosing the p, q, ϕ and θ that minimize

$$AIC_{p,q,\phi,\theta} = -2\ln L(\phi, \theta, \sigma^2 \tfrac{1}{n}) + 2(p+q+1)n/(n-p-q-2)$$

Where L is the likelihood function of the ARMA process.

The first step of the method is to select the two upper bounds, P and Q, these are the maximum orders, of AR and MA respectively, that we will consider. In practice we often use the values $P = 5$ and $Q = 5$. Then, for 1 to P and for 1 to Q we fit all the possible ARMA models and we will choose the one that minimizes the AIC function.

2.8.2 Parameter estimation

After estimating the weights of the models, we now focus on the following functions of the Time Series: the Mean, the Covariance, the Auto-correlation and the Partial Auto-correlation function. In that part we mainly use [4].

Estimation of the Mean Function

In this estimation we use the simple mean used in statistics

$$\bar{X}_n = 1/n \sum_{i=1}^{n} X_i$$

Properties of the Estimator of the Mean Function

- It is unbiased $E[\bar{X}_n] = \mu$
- Besides, the estimator is consistent. The variance is

$$Var(\bar{X}_n) = Var(1/n \sum_{i=1}^{n} X_i) = 1/n_2 \sum_{t=1}^{n} \sum_{s=1}^{n} Cov(X_t, X_s) = \\ 1/n^2 \sum_{t=1}^{n} \sum_{s=1}^{n} \gamma_{t-s} = 1/n \sum_{\tau=-(n-1)}^{n-1} \frac{n-|\tau|}{n} \gamma_\tau$$

We know that γ_τ is absolutely summable, so when we calculate the limit $n \to \infty$, it will be zero.

Observation 1. The last step of the process is easy to understand if we use the following table and we sum by diagonals, we have only one $\gamma_{-(n-1)}$, ..., we have n γ_0, ... and one γ_{n-1}.

$$\begin{array}{lllll} s/t & 1 & 2 & \cdots & n \\ 1 & \gamma_0 & \gamma_1 & \cdots & \gamma_{n-1} \\ 2 & \gamma_1 & \gamma_0 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n & \gamma_{-(n-1)} & \cdots & \cdots & \gamma_0 \end{array}$$

So,

$$\sum_{t=1}^{n} \sum_{s=1}^{n} \gamma_{t-s} = \sum_{\tau=-(n-1)}^{n-1} (n - |\tau|)\gamma_\tau.$$

Estimation of the Covariance Function

A possible estimator could be

$$\hat{\gamma}_{\tau,n} = \frac{1}{n} \sum_{t=1}^{n-\tau} (X_t - \bar{X}_n)(X_{t+\tau} - \bar{X}_n)$$

We use the estimator of μ as the real μ.

Properties of the Estimator of the Covariance Function

- It is not an unbiased estimator, but it is asymptotically unbiased.

$$E[\hat{\gamma}_{\tau,n}] = E[\frac{1}{n} \sum_{t=1}^{n-\tau} (X_t - \bar{X}_n)(X_{t+\tau} - \bar{X}_n)]$$

If we bracket the second order terms, it will be:

$$E[\hat{\gamma}_{\tau,n}] = \tfrac{n-\tau}{n}\gamma_\tau - \tfrac{n-\tau}{n}Var(\hat{\gamma}_{\tau,n}) + \vartheta(1/n^2)$$

But we see that, when $n \to \infty$, we have

$$lim_{n\to\infty}E[\hat{\gamma}_{\tau,n}] = \gamma_\tau$$

so it is asymptotically unbiased.

- We also have that the estimator is consistent.

Estimation of the Auto-correlation function

Obviously we will estimate the Auto-correlation function using the Covariance functions that we have estimated before. So we have

$$\hat{\rho}_{\tau,n} = \tfrac{\hat{\gamma}_{\tau,n}}{\hat{\gamma}_{0,n}}$$

Properties of the estimator of the Auto-correlation Function

- We have a bias of order 1/n, because:

$$E(\hat{\rho}_{\tau,n}) = \rho_\tau + \vartheta(1/n)$$

- The estimator is asymptotically unbiased.
- The estimator is consistent.

Estimation of the Partial Auto-correlation Function

In this estimation we use the predictions used before. Recall that $\hat{\phi}_{hh}$ is the last component of $\hat{\phi} = \hat{\Gamma}^{-1}\hat{\gamma}$.

Estimation of AR(p) process

We estimate the AR(p) process using the Yule-Walker equations that we have explained before. In the matrix Γ we will replace all the ρ_i for the estimated ρ_i, i.e. $\hat{\rho}_i$. In the vector ρ we do the same, we replace the ρ_i by the estimated ρ_i. Finally replace ϕ by $\hat{\phi} = (\hat{\phi}_1, \cdots, \hat{\phi}_p)$.

So the matrix and vectors are:

$$\hat{\Gamma} = \begin{pmatrix} 1 & \hat{\rho}_1 & \cdots & \hat{\rho}_{p-1} \\ \hat{\rho}_1 & 1 & \cdots & \hat{\rho}_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\rho}_{p-1} & \hat{\rho}_{p-2} & \cdots & 1 \end{pmatrix}$$

$$\hat{\phi} = (\hat{\phi}_1, \cdots, \hat{\phi}_p)$$

$$\hat{\rho} = (\hat{\rho}_1, \cdots, \hat{\rho}_p)$$

And the new Yule-Walker equations will be:

$$\hat{\Gamma}\hat{\phi} = \hat{\rho}$$

If we solve this system, we will have the estimated weights.

Note 16. *If you want to know more about this subject, you can look for information about another algorithm method (Burg's Algorithm) , in [3]. This algorithm is used to estimate AR models.*

Estimation of MA(q) and ARMA(p,q) processes

The estimation of the weights of MA(q) and ARMA(p,q) processes are more difficult than the estimation of the weights of AR(p). This is because, when we have to estimate the weights of the moving average part of the model, we can not apply the Yule-Walker equations, and we have to curry out another type of estimation.

We know that MA(q) and ARMA(p,q) processes can be represented like an AR(∞), so the process is:

$$X_t = \sum_{j=1}^{\infty} \pi_j X_{t-j} + \epsilon_t$$

In order to apply the estimation of the weights that we want to do, we have to assume some conditions:

1. The process is stationary.
2. The process is invertible.
3. $\epsilon_t \sim N(0, \sigma^2)$.
4. $X_0 = 0$.

Note 17. *The way to check the first two conditions is explained in the part of MA(q) and ARMA(p,q) processes.*

Under these conditions, we will see that $X = (X_1, \cdots, X_n)^T$ has multivariate normal distributions.

If we see the representation of X_1, is:

$$X_1 = \pi_1 X_0 + \epsilon_1 = \epsilon_1$$

In the second step, we assume that $X_0 = 0$, so $X_1 = \epsilon_1$ and X_1 have a normal distribution.

If we now consider the representation of X_2, is:

$$X_2 = \pi_2 X_1 + \epsilon_2$$

So X_2 has a normal distribution because X_1 and ϵ_2 are independent, so the sum of two independent normal distributions results in a normal distribution.

For the other orders is exactly the same, so we have that $X = (X_1, \cdots, X_n)^T$ has multivariate normal distributions with a density

$$\eta(x|\lambda) = (2\pi\sigma^2)^{-n/2}|\Gamma|^{-1/2}exp(-\frac{1}{2\sigma^2}x^T\Gamma^{-1}x)$$

Where Γ is the covariance matrix and $\lambda = (\phi_1, \cdots, \phi_p, \theta_1, \cdots, \theta_q, \sigma^2)^T$ the vector with the parameters we have to estimate.

Now we can calculate the likelihood function L, that is the density function interpreted as a function of the parameter vector λ.

$$L(\lambda|x) = \eta(x|\lambda)$$

We take the logarithm of $L(\lambda|x)$

$$\log L(\lambda|x) = -\frac{n}{2}\log(2\pi\sigma^2) - \frac{1}{2}\log(\Gamma) - \frac{1}{2\sigma^2}x^T\Gamma^{-1}x.$$

Then, we have to derive $\log L(\lambda|x)$ and equal it to zero. We do this to find the correct estimators, and check whether if it is a maximum. This is better done with a computer algorithm.

Finally, the ML estimator is $\hat{\lambda} = \arg\max_{\lambda\in\Theta} L(\lambda|x)$.

2.8.3 Model checking

In the last part of the method we are going to check the model. We will study if the error term ϵ_t is a white noise. We have to test the white noise assumptions.

These assumptions are:

- $E(\epsilon_t) = 0$
- $E(\epsilon_t^2) = \sigma^2$
- $E(\epsilon_t\epsilon_{t+s}), \neq 0 \ \forall s \neq 0$

This analysis can also be made graphically. First, we need to plot the errors, and decide if the error is a white noise.

If the assumptions are not satisfied, we should change the model, so we should go back to the first step of the method and change p,d and q.

Note 18. *There is no need to be extremely accurate when we are trying to prove an exact white noise. Sometimes we have that the assumption $E(\epsilon_t) = 0$ is false, but $E(\epsilon_t) \approx 0$. In this cases we consider that the error is white noise.*

2.9 ARCH and GARCH

Sometimes in practice, we have Time Series that can not be represented by an ARIMA model. In that cases the next step is to think about conditional heteroscedastic models. When we have an ARMA process, $\phi(L)X_t = \theta(L)\epsilon_t$, we assume that ϵ_t has a zero mean and a constant variance, i.e. the variance of ϵ_t is a constant that does not hinge on the past. But if we think that the errors depend on the past errors we have to change the model. The assumption we make is based on the fact that a tendency of large deviations may be followed by moments with large deviations. On the other hand, moments with small deviations may be followed by moments of small deviations.

Thanks to his introduction of the ARCH models, Engle won the Economics Nobel prize in 2003. In this part of my work, the definitions are basically based on [4]. I have focused on this book to study the ARCH and GARCH models, because these models are quite different depending on where you study them. However, other books have also come in handy.

2.9.1 The Auto-regressive Conditional Heteroscedastic Model

The process ϵ_t is ARCH(q) when $E[\epsilon_t|\mathcal{F}_{t-1}] = 0$ and

$$\sigma_t^2 = \omega + \alpha_1\epsilon_{t-1}^2 + \cdots + \alpha_q\epsilon_{t-q}^2$$

with $\omega > 0$, $\alpha_1 > 0$, $\cdots$, $\alpha_q > 0$.

We distinguish three ARCH.

1. Strong ARCH: when $Var(\epsilon_t|\mathcal{F}_{t-1}) = \sigma_t^2$ and the process $Z_t = \epsilon_t/\sigma_t$ is i.i.d.
2. Semi-Strong ARCH: when $Var(\epsilon_t|\mathcal{F}_{t-1}) = \sigma_t^2$.
3. Weak ARCH: when $\mathcal{P}(\epsilon_t^2|1, \epsilon_{t-1}, \epsilon_{t-2}^2, \cdots, \epsilon_{t-1}^2, \epsilon_{t-2}^2, \cdots) = \sigma_t^2$.

In the Weak ARCH, $\mathcal{P}$ is the best linear projection, but this type of ARCH is not studied in this work.

Note 19. *The ARCH process often refers to Semi-Strong ARCH. We will use ARCH to refer to Semi-Strong ARCH.*

Theorem. *Unconditional variance: If ϵ_t is an ARCH(q) process with $Var(\epsilon_t) = \sigma^2 < \infty$, if $\alpha_1 + \cdots + \alpha_q < 1$ we have that*

$$\sigma^2 = \tfrac{\omega}{1-\alpha_1-\cdots-\alpha_q}$$

Proof.

$$\sigma^2 = E[\epsilon_t^2] = E[E(\epsilon_t^2|\mathcal{P}_{t-1})] = E[\sigma_t^2] = \omega + \alpha_1\sigma^2 + \cdots + \alpha_q\sigma^2$$

Now if we isolate σ^2 we have $\sigma^2 = \frac{\omega}{1-\alpha_1-\cdots-\alpha_q}$.

□

Theorem. *ARCH representation : If ϵ_t is a stationary strong ARCH(q) process with $E[\epsilon_t^4] < \infty$ and we assume that Z_t follows a $N(0,1)$, we have that:*

- *$\eta_t = \sigma_t^2(Z_t - 1)$ is white noise.*
- *$\epsilon_t^2 = \omega + \sum_{i=1}^{q} \alpha_i\epsilon_{t-i}^2 + \eta_t$, so ϵ_t^2 is an AR(q) process.*

Proof.

- To show proof of this, first we have to prove that the expected value of η_t is zero. Then, we need to proof that the variance is constant and independent of time and, finally, that the covariance is zero for all pair.

  1. $E[\eta_t] = E[\sigma_t^2]E[Z_t^2 - 1] = 0$
  2. $Var(\eta_t) = E[\sigma_t^4]E[(Z_t^2-1)^2] = E[(\omega+\alpha_1\epsilon_{t-1}^2+\cdots+\alpha_q\epsilon_{t-q}^2)^2]E[Z_t^4+1-2Z_t^2] = 2E[(\omega+\alpha_1\epsilon_{t-1}^2+\cdots+\alpha_q\epsilon_{t-q}^2)^2]$
     This will be constant since, if we do the square of the values that we have in parenthesis, we will get a polynomial function of ϵ's and the expected values of all these are constant and independent of t.
  3. $Cov(\eta_t, \eta_{t+s}) = E[\sigma_t^2(Z_t^2-1)\sigma_{t+s}^2(Z_{t+s}^2-1)] = E[\sigma_t^2(Z_t^2-1)\sigma_{t+s}^2]E[(Z_{t+s}^2-1)] = 0$. For all $s \neq 0$.
     The last step , $E[(Z_{t+s}^2 - 1)] = 0$ is because $E[Z_{t+s}^2] = 1$.

- The rest of the proof is really easy if we consider that $\eta_t = \sigma_t^2(Z_t - 1)$ is a white noise:

$$\epsilon_t^2 = \sigma_t^2 Z_t^2 = \sigma_t^2 - \sigma_t^2 + \sigma_t^2 Z_t^2 = \sigma_t^2 + \sigma_t^2(Z_t^2 - 1) =$$
$$\omega + \alpha_1\epsilon_{t-1}^2 + \cdots + \alpha_q\epsilon_{t-q}^2 + \sigma_t^2(Z_t^2 - 1) = \omega + \alpha_1\epsilon_{t-1}^2 + \cdots + \alpha_q\epsilon_{t-q}^2 + \eta_t$$

□

Selection of the correct q for an ARCH(q) process

The selection of the best q of an ARCH process is easy if we use the theorem of the ARCH representation. This theorem claims that ϵ_t^2 is an AR(q). So the problem becomes the selection of the order of an AR process (this is explained in the AR estimation). The method uses ACF and PACF plots (of ϵ_t^2) to determinate the correct order.

Estimation of ARCH(q)

We can do the estimation of the parameters by means of some methods.

We have seen in the last theorem that an ARCH(q) process can be represented by an AR(q) in X_t^2, so one possibility of estimation is to do what we have studied in the part of AR estimation; using the Yule-Walker equations to select appropriate parameter estimators. The way to do this is explained before, but this estimation often leads to a problem: ϵ_t^2 may not be normal, and this causes the Yule-Walker estimation to be inefficient.

When we have to estimate the parameters of an ARCH model, the most used method is the maximum likelihood method. For this estimation we assume that ϵ_t has a normal distribution. So the likelihood function of the observation t is

$$\rho(\epsilon_t|\mathcal{F}_{t-1}) = \frac{1}{\sqrt{2\pi}\sigma_t}\exp\left\{-\frac{1}{2}\frac{\epsilon_t^2}{\sigma_t^2}\right\}$$

If we do the logarithm of $\rho(\epsilon_t|\mathcal{F}_{t-1})$

$$l_t = \log\rho(\epsilon_t|\mathcal{F}_{t-1}) = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\sigma_t^2) - \frac{1}{2}\frac{\epsilon_t^2}{\sigma_t^2}$$

Now we can obviate the constants, because they do not affect the result when we are doing the maximum. So we have

$$l_t = \log \rho(\epsilon_t|\mathcal{F}_{t-1}) = -\tfrac{1}{2}\log(\sigma_t^2) - \tfrac{1}{2}\tfrac{\epsilon_t^2}{\sigma_t^2}$$

Finally, the log-likelihood function, noted as l, for T observations is:

$$l = \tfrac{1}{T}\sum_{t=1}^{T} l_t.$$

Once we have the log-likelihood function the process is nearly finished. We only have to calculate the maximum of this function and get the estimated parameters. To calculate the maximum we have to derive the log-likelihood function depending on each parameter and equalize it to zero. To check it is a maximum we have to derive one more time. This analytic process is long but easy, so sometimes it is better to calculate it with a computer.

ARIMA(p,d,q)-ARCH(w)

Sometimes we approximate X_t by an ARIMA process, but we think that the approximation is not as good as we intended. One option is to consider ARCH models and reject ARIMA process, but often this is not a good idea because, if we have done the Box-Jenkins method like we have studied, we may have passed the model checking, and this means that the ARIMA process is a good approximation. So it is a bad idea to reject all we have done. We can use both of this models together. The way to do this is to construct a model based in ARIMA process introducing the ARCH models to the errors. Now, we introduce an example with an AR(p) model.

Definition 2.9.1. *An AR(p) process with ARCH(w) model errors is* $X_t = \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p} + \epsilon_t$, *where* $E[\epsilon_t|\mathcal{F}_{t-1}] = 0$ *and*

$$Var(\epsilon_t|\mathcal{F}_{t-1}) = \sigma_t^2 = \omega + \alpha_1\epsilon_{t-1}^2 + \cdots + \alpha_w\epsilon_{t-w}^2$$

with $\omega > 0$, $\alpha_1 > 0\ , \cdots\ , \alpha_w > 0$.

2.9.2 The Generalized Auto-regressive Conditional Heteroscedastic Model

The process ϵ_t is GARCH(p,q) when $E[\epsilon_t|\mathcal{F}_{t-1}] = 0$ and

$$\sigma_t^2 = \omega + \alpha_1\epsilon_{t-1}^2 + \cdots + \alpha_q\epsilon_{t-q}^2 + \beta_1\sigma_{t-1}^2 + \cdots + \beta_p\sigma_{t-p}^2 = \omega + \sum_{i=1}^{q}\alpha_i\epsilon_{t-i}^2 + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2$$

with $\omega > 0$, $\alpha_1 > 0\ , \cdots\ , \alpha_q > 0$, $\beta_1 > 0\ , \cdots\ , \beta_p > 0$.

We distinguish three GARCH.

1. Strong GARCH: when $Var(\epsilon_t|\mathcal{F}_{t-1}) = \sigma_t^2$ and the process $Z_t = \epsilon_t/\sigma_t$ is i.i.d.
2. Semi-Strong GARCH: when $Var(\epsilon_t|\mathcal{F}_{t-1}) = \sigma_t^2$.
3. Weak GARCH: when $\mathcal{P}(\epsilon_t^2|1, \epsilon_{t-1}, \epsilon_{t-2}^2, \cdots, \epsilon_{t-1}^2, \epsilon_{t-2}^2, \cdots) = \sigma_t^2$.

Theorem. *Unconditional variance: If* ϵ_t *is a GARCH(q) process with* $Var(\epsilon_t) = \sigma^2 < \infty$, *if* $\alpha_1 + \cdots + \alpha_q + \beta_1 + \cdots + \beta_p < 1$ *we have that*

$$\sigma^2 = \frac{\omega}{1-\alpha_1-\cdots-\alpha_q-\beta_1-\cdots-\beta_p}$$

Proof.

The proof of the theorem is exactly the same as in ARCH process.

□

Theorem. *GARCH representation: If ϵ_t is a stationary strong GARCH(p,q) process with $E[\epsilon_t^4] < \infty$ and we assume that Z_t follows a $N(0,1)$, we have that:*

- $\eta_t = \sigma_t^2(Z_t - 1)$ *is white noise.*
- $\epsilon_t^2 = \omega + \sum_{i=1}^q \gamma_i \epsilon_{t-i}^2 - \sum_{j=1}^p \beta_j \eta_{t-j}^2 + \eta_t$*, so ϵ_t^2 is an ARMA(m,p) process, with $m = max(p,q)$ and $\gamma_i = \alpha_i + \beta_i$.*

Proof.

The proof is the one in the ARCH process.

□

Selection of the correct p and q for a GARCH(p,q) process

The selection of the best p and q of a GARCH process is easy if we use the theorem of the GARCH representation. This theorem establishes that ϵ_t^2 is an ARMA(m,p) with $m = max(p,q)$. So, the next problem is the selection of the order of an ARMA process, and this is explained in the ARMA estimation. The method consists of using the ACF and PACF plots (of ϵ_t^2) to determinate the correct orders. The only problem of this selection is that m is a maximum. Therefore, we may not know the order of q. But this is not a big problem because we usually have low orders, for example if $m = 1$, it means that q is 0 or 1. So to select the final model we should have to estimate both and select the one with fewer errors.

Estimation of GARCH(p,q)

Like in ARCH processes, the estimation of the parameters can be done by means of several methods. One of them is the same used in ARCH. We should apply what we have seen in the last theorem: a GARCH(p,q) process can be represented like an ARMA(m,p) with $m = max(p,q)$. So, the estimation of the parameters can be done using what we know about the estimation of ARMA models. Usually we do not use this method of estimation, we normally use the maximum likelihood method.

For the maximum likelihood method, the likelihood function of the GARCH(p,q) is the same as the ARCH(q), the difference is that now we have q more parameters to estimate. We have the same function to maximize, but with more parameters. So, the maximum likelihood method for GARCH entails the same difficulties as the maximum likelihood ARCH method. However, the first one is longer.

Chapter 3

Technical Analysis

3.1 Introduction to Technical Analysis

Technical Analysis is the study of the price, the volume and the open interest, mainly through the use of graphics, with the purpose of predicting future price trends.

To understand the definition of Technical Analysis we must know what volume and open interest are. Open interest refers to the total number outstanding of derivative contracts that have not been settled. Volume indicates the amount of securities contracted in financial markets in a session.

Technical Analysis can be used with any financial instrument. Due to this, it is applied to price stocks, futures, foreign money exchange,...

Note 20. *If you want to know more about the meaning of open interest and volume, you can look for information in [7]. In my work I do not study hard both concepts.*

3.1.1 Basic assumptions of Technical Analysis

Before we start the study of the technical chart patterns we have to make the basic assumptions of Technical Analysis.

It can be summarized in three points:

1. **Market action discounts everything.**

   This is surely the most important fact that we have to assume. If we disagree on this statement, everything else is meaningless.

   Technical analysts believe that all the relevant information that might influence price should be reflected in the price. For example, an increase in food taxes will be reflected in the market price of Mercadona.

   Price action should reflect the movements in the supply and demand. If the demand is larger than the supply, the price will increase. Analysts claim that, if the price is increasing, it is because the demand is larger than the supply. They are not interested in the reasons why prices rise, they only worry about the consequences.

   I.e. everything that affects the price, is reflected in the market price in the long term. So the only thing we have to study is the price action.

2. **Prices move in trends.**

   Analysts assume that markets forms trends. As in the first point, if we do not accept that assumption, there is no need to go any further.

   The purpose of technical analysts is to detect the price trend in the first phases of the trend, in order to operate in the market in the right direction.

   Analysts often suppose that it is more likely for a trend to continue in motion than to changing its tendency.

3. **History tends to repeat itself.**

   It is based on the fact that the future is a repetition of the past. In order to be able to predict the future, we have to study the past.

3.1.2 Dow theory

Charles Dow was an economist and a journalist. He was born in 1851, and he died in 1902. He did not write any book, but he wrote a lot of articles for his own newspaper, *The Wall Street Journal*. His economic principles are extracted from his articles. He is considered the forefather of Technical Analysis, and for this reason we study his proposals.

We summarize the basic outlines of his theories:

- Averages discount everything.

  This assumption is exactly the same that we have used in the basic assumptions appearing in Technical Analysis.

- The Market has three trends.

  The primary, the secondary and the minor trend (In order of importance).

- Primary trends have three phases.

  Those trends have an accumulation phase, a public participation phase and a distribution phase. His basic definitions of phases are:

  – The first one is the one which we want to operate, i.e. operate on the beginning of the trends.
  – The second phase is when more investors are able to distinguish the new trend.
  – The last one is when everybody perceives the trend.

- Averages must confirm each other.

  Dow was referring to the industry and rail averages, so we are not interested in that aspect.

- The trend is confirmed by volume.

  Dow said that the volume must confirm trends generated by graphics. So volume must expand in the trend's same direction.

- Trends exist until definitive signals prove that they have ended.

  This is the base of Technical Analysis used today. It is the same as saying that the trend in movement may be in movement.

The most important difference between the Dow Theory and Technical Analysis is that most investors who follow the Dow Theory operate only in the principal trend, i.e. in long term. In Technical Analysis we operate equally in long and short term.

3.1.3 Work Distribution

I have divided the Technical Analysis study into three separated parts, the trend and canals, the chart patterns and moving average indicators.

In the part of trend and canals I analyze the different types of trends, the influence of the trend on market prices and how to operate with each trend. Then, I explain how to draw the correct trend line. Finally, I work with Canals. A canal is a figure that appears when all prices fluctuate between two parallel lines.

A chart pattern is a distinct formation on a stock chart that creates a trading signal, or a sign of future price movements. Analysts use the patterns to identify current trends and trend reverse.

We will study some of the most used patterns like Head and Shoulders, triple tops and bottoms, triangles, diamonds,... We distinguish two types of patterns, the reversal patterns and the continuation patterns. The most important difference between both is that return patterns change the trend.

Finally I examine Moving average indicators. A moving average is an average of the last prices. There, I study the different types of Moving averages and how to operate on it.

3.2 Trend and Canals

3.2.1 Trend

To study the evolution of prices, it is essential to understand the importance of the trend. The most important idea is to operate in the same direction as the trend, i.e. if the trend is ascendant (descendent) we should buy (sell).

In this part of the work we study the ways to measure and predict the trend.

Markets usually show zig-zags moves, and obviously every zig-zag has a maximum and a minimum. The main direction of zig-zags is what we know as the trend.

When ridges are increasing (decreasing) we call it uptrend (downtrend). If we have a trend with ridges that are not increasing or decreasing (horizontal ridges), we call it lateral trend.

As we will see, we use patterns to operate in the markets. Those patterns are usually based on uptrends or downtrends. So if we have a lateral trend, we rarely operate in the market, because nothing happens and we can not recognize any pattern.

Now, we have taken into consideration the direction of the trend, but we also have to study the three trend types. That characterization is done by dividing the trend by time frame. Time frames are slightly different to each analyst, but we rely on what is more accepted.

1. Principal trend: it is the trend with a time frame of more than one year.

2. Intermediate trend: it is the trend with a time frame from one to many months.

3. Short-term trend: it is the trend with a time frame of less than two or three weeks.

So, for example if we have an ascendant principal trend, we have an ascendant trend for more than a year, but in this case we can also have a decreasing trend of the last two weeks, and it means that the short-term trend is decreasing.

This reveals that trends by time frame are not exclusive. We can have different principal, intermediate and short-term trends.

Note 21. *We denominate support (resistance) to the minimum (maximum) of the zig-zag, In a support, or a resistance, the movement of the price stops and reverses.*

Once a support (resistance) is broken, it becomes a resistance (support).

Exact numbers have often a tendency to become resistances and supports. Numbers are usually 10, 20 , 25, 50, 75 and 100.

Note 22. *Some analysts suggest a 45 degree trend is needed in order to have a consistent trend. They point out that, if the trend is above (below) the degree, it will decay (grow) to 45 degree trend.*

3.2.2 Trend line

To study the trend, first we have to select the correct trend, and this process depends on the characteristics of the trend placed before us.

We know that the trend is ascendant because the final price is higher than the first price. In these cases, the trend line must be below the prices all the time. If the price is lower than the trend line at any time, the trend line is not the correct. First we have to select the lower support. Then, we build for all the other supports a line trend through both. We select the trend line with the lower slope.

If we have a downtrend, we do the same but with the resistances. We select the first resistance and we build a line for all the others resistances, and we select the one with the higher slope.

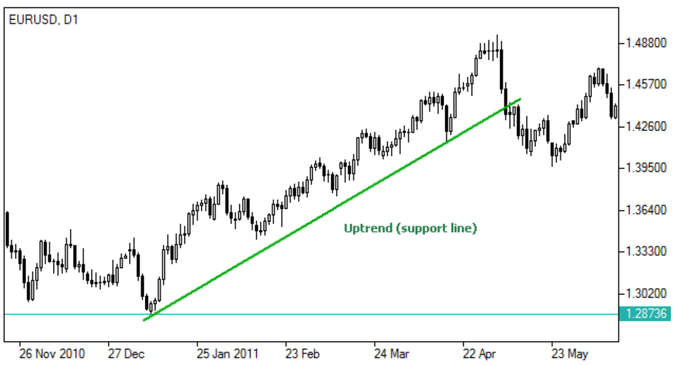

Figure 1: Trend Line.

Observation 2. To be sure that the trend line has been broken, analysts often use two different criterion. The first, when we have two straight days below the trend line. The second is that the difference between the price in the trend line and the actual price is bigger than 2%.

Observation 3. Sometimes when an uptrend line is broken it becomes a descendent trend line.

Observation 4. Technical Analysis suggest that when an uptrend (downtrend) line is broken, the price tends to go down (up) really fast. So predicting a future break or distinguishing a real break from an isolated event is very decisive in our operations.

Observation 5. Some analysts suggest that in a zig-zag of an uptrend, if we have a resistance A, a support B and a resistance C, the difference between B and C is often 1/3, 1/2 or 2/3 of the difference between A and B. And the same happens in a downtrend, but the other way around.

Program 1. *Trend.c*

Basically the program computes the trend and elucidates whether the trend is broken or not using both criteria.

The program works as follows:

1. First it asks for the data and it calculates whether the trend is ascendant or descendent.
2. If the trend is descendent, the program searches the maximum of the data and names it maxA, and saves the maximum time as tA. Then, from tA to the end of the data, the program calculates all the maximums. For every maximum, it calculates the slope of the line through this maximum and maxA. It selects the maximum with a higher slope and it saves it as maxB. If the slope of the trend line is larger than -0.05, the program considers the descendent trend as lateral.
3. Until the trend line is not broken, the program asks for new data. It checks for the new data the two break conditions. If one of the conditions is satisfied, the output is that the trend line is broken.
4. For uptrends it is analogue.
5. In the cases with lateral prices, the program says that the data given do not have any trend.

3.2.3 Canal

If we consider the case of an uptrend, sometimes prices are between the ascendant (descendent) trend line and another parallel upper (lower) line. The other line is called the canal line. And it is used to operate between both. If the price is near the trend (canal) line, we are supposed to buy (sell).

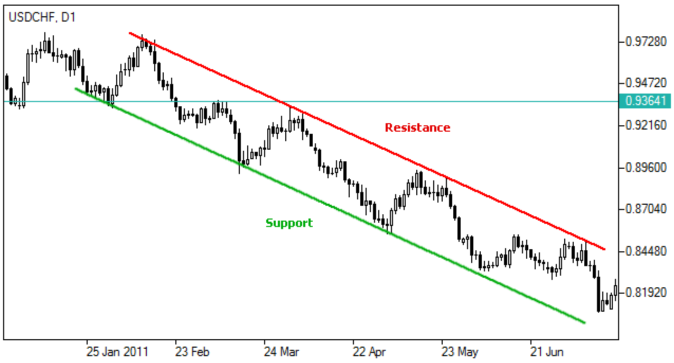

Figure 2: Canal.

Observation 6. Like in observation 4, if a canal line of an ascendant (descendent) trend is broken, Technical Analysis suggest that the price will go up (down) quickly.

Program 2. *Canal.c*

The program searches the trend line and the canal line. It predicts for future prices, whether the lines are broken or not, and it advises us on buying or selling.

The program works as follows:

1. The first part of the program is the same as the first two points of 1trend.c.
2. We consider a downtrend. The program search the two farthest prices of the trend line, and it calculates the line connecting these two points. This is the canal line.
3. The program checks if the canal line and the trend line are parallel.
4. Finally while the price is between both lines, the program suggests if we have to buy or sell.
5. For uptrend, the same applies.

3.3 Reversal Patterns

The most important properties of the Reversal Patterns are, on the one hand, the existence of a previous trend and, on the other hand, the fact that bigger patterns imply bigger trend changes.

3.3.1 Head and Shoulders

The pattern is based on the fact that we have an ascendant trend. The figure is formed by 3 maximums and 2 minimums. We note the maximums by temporal order A,C and E, and the minimums by temporal order B and D. The sequence of the figure must be A-B-C-D-E. The figure must be as follows:

- The prices on A and E are similar.

- The prices on B and D are similar.
- B price is the lowest mark between A and C.
- D price is the lowest mark between C and E.
- C price must be the highest price between A and E.

When this happens, we have a HS. The name is given after the body parts, where A and E are the shoulders and A the head.

We call the line through B and D the neckline, and it has an important roll in the operate system.

The way to operate with this pattern is to calculate the neckline. When the price after E is lower than that neckline, we have to sell. HS is a signal of a change of trend. Sometimes after the neckline breaks, a reverse movement appears, and the price increases up to the neckline. After that reverse movement, the decay of the price will be sharper.

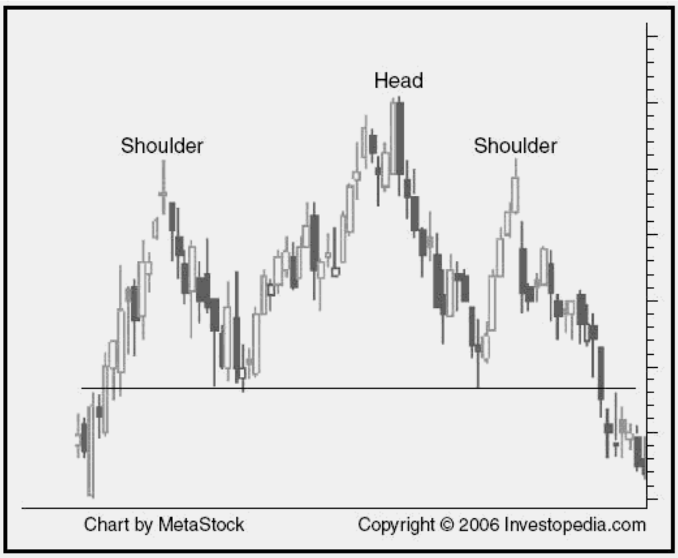

Figure 3: Head and Shoulders.

Note 23. *Head and Shoulders (HS) is possibly the most famous pattern.*

Program 3. *Hch.c*

The first one is based on the fact that an analyst thinks he has found a HS. The program distinguishes the correct HS from the incorrect, i.e. if the HS that we give to the program is really a HS or it is not. If the series given is a HS, the program proposes the way to operate, sell or wait in relation to every price and each moment.

The program works this way:

1. It calculates the line through A and E to check that they are at the same height. To check it, we use the slope of the line.
2. Then, it checks whether C is sufficiently higher than A and E. We use the slope of the line through A and C, as well as through E and C.

3. It calculates the neckline to check if B and D are at the same height. To check it, we use the slope of the line.

4. The program makes us enter future prices, and recommends us if we have to sell or not, by using the neckline. In this part, the program informs us that the neckline is totally broken if the price is lower than the neckline plus 0.05 times the difference between E and D. This is done to ensure that the neckline is broken.

If you are interested in the exact algorithms and the code, you can find the program in the appendix.

Program 4. *Buscarhch.c*

The second one is based on the idea that the analyst is not in front the computer, i.e. the program does all the work for the analyst and looks for possible HS. The program searches HS in the prices given, and then we apply the program hch.c to operate with the found HS.

This is how the program works:

1. First it finds the maximum of the series (this price is C).

2. We do not care what happens after the HS, so we establish the research of A on the preceding time period of C, but under the condition that it is, at most, 1.5 times the distance between C and the end of the series. The first time is called k.

3. For k to C, we calculate all the maximums, and with every maximum we calculate all the minimums between the maximum and C. We select the maximum and the minimum with a higher difference, and in the case with two maximums with the same difference, we select the closest to C. The maximum will be A and the minimum B.

   Note 24. *If we do not use this algorithm and we select the maximum and minimum between k and C, probably we will not find a HS. This is because in a HS, we have a lot of maximums between B and C, and they are probably higher than A. The only case when this algorithm will be correct is if the data do not have any maximum between B and C. However, this is almost impossible.*

4. For C to the end, we calculate all the maximums, and with every maximum we calculate all the minimums between the maximum and C. We select the maximum and the minimum with a bigger difference, and in the case with two minimums with the same difference we select the one closest to C. The maximum will be E and the minimum D.

5. Finally the program output is A-B-C-D-E, and we have to introduce that into the hch.c. The other program will tell us if the HS is correct or not, and if we have to sell or not in future prices.

If you are interested in the exact algorithms and the code, you can find the program in the appendix.

3.3.2 Inverted Head and Shoulders

The inverted HS is the same as HS, but inverting the HS.

This pattern is based on the fact that we have a descendent trend. The figure is formed by 3 minimums and 2 maximums. We note the minimums by temporal order A,C and E, and the maximums by temporal order B and D. The sequence of the figure must be A-B-C-D-E. The figure must be as follows:

- The prices on A and E are similar.
- The prices on B and D are similar.
- Price B is the highest price between A and C.
- Price D is the highest price between C and E.
- Price C must be the lowest price between A and E.

As in HS we call the line through B and D the neckline.

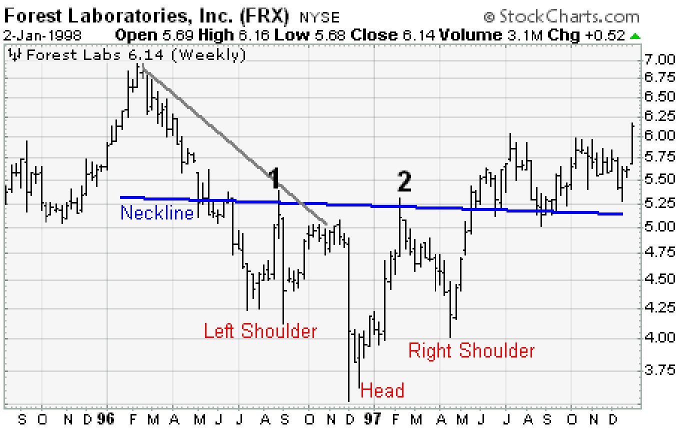

Figure 4: Inverted Head and Shoulders.

The way to operate in this pattern is to calculate the neckline. When the price after E is higher than that neckline, we have to buy. Inverted HS is a signal for a change of trend. Sometimes after breaking the neckline, a reverse movement occurs and the price falls down to the neckline. After that reverse movement, the increase of the price will be higher.

Program 5. *inverthch.c*

In the development of this program I had the idea to use hch.c program. I thought on multiply all the prices by -1. This method makes minimums become maximums, and maximums become minimums. After that transformation the inverted HS becomes a HS, and we can use part of the hch.c code.

Program 6. *Buscarhchinvertit.c*

The idea is the same as in inverthch.c, we multiply the prices by -1. Then, the research is almost the same as in buscarhch.c. I had some troubles in the development of this program, because the negative prices affect to some algorithms.

3.3.3 Triple Top

This pattern is the same as HS but with a small difference, the three maximums are in a similar height. All the other assumptions are the same.

Program 7. *Tripletops.c*

The program is the same as in HS but excluding the conditions of the different heights, so we do not have to check that C is higher than A and E. We have to include another condition. This condition is to check that A, C and D are at the same height. We check this by calculating three lines, the one through A and C, the one through C and E and the one through A and E. If the slopes of two of them are almost zero, the condition is correct.

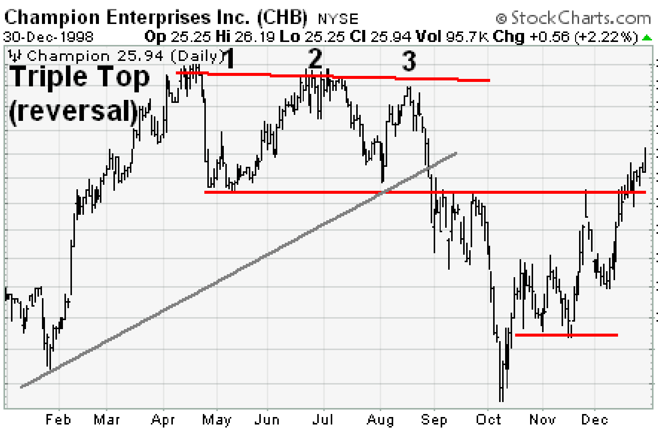

Figure 5: Triple Top.

Note 25. *To search possible tripe tops we use buscarhch.c, because in that program there is not any condition of different height in respect of shoulders and head.*

3.3.4 Triple Bottom

This pattern is the same as inverted HS but with a little difference, the three minimums are in a similar height. All the other assumptions are the same.

Figure 6: Triple Bottom.

Program 8. *Triplebottom.c*

The programs are the same as in Triple Top, but after introduce the prices, we have to multiply all of them by -1. The argument is the same used in inverted HS.

As in triple tops, we use buscarhchinvertit.c to search possible triple bottoms.

Observation 7. Some analysts use Double tops (bottoms), which are the same that in triple tops (bottoms) but with only two maximums (minimums). We might venture to say that this pattern is very ambiguous. According to the experience gathered, frequently the figure is perfect, but the prediction is totally false. So we are not going to introduce this pattern.

3.4 Continuation Patterns

Continuation patterns are patterns on which the trend will continue once the pattern is complete, i.e. the pattern will not change the trend. This does not mean that we can not operate with them. These figures are usually used on a short-term basis.

3.4.1 Triangles

Triangles are one of the most known figures in Technical Analysis. Essentially the triangles are figures that, for a period of time, the prices range between two lines. Those lines may not be parallel, so they must have an intersection. If the intersection of the lines is in the right part of the figure, we have a triangle, and if it is in the left part of the figure, it is a broadening formation. There are three types of triangles: symmetrical, ascendants and descendents.

3.4.2 Symmetrical triangle

These triangles are the ones with an upper line descending and a lower line ascending. It represents a pause on the trend, so after the triangle the trend will have the same original trend. We need two points to draw a line. So we note the points of the upper line, in temporal order, A and C. The two points of the lower line, in temporal order, are B and D.

To have a symmetrical triangle the points must be as follows:

1. Price A must be higher than C.
2. Price B must be lower than D.
3. Price A must be highest.
4. Price C must be higher than B and D.

Figure 7: Symmetrical Triangle.

The way to operate in these figure is to purchase when the price is next to the lower line, and to sell when the price is near the upper line.

The triangle is a continuation pattern, so the most probable event is that the price will break the triangle in the direction of the previous trend. If the opposite happens and we had a descendent (ascendant) trend, once the price breaks the upper (lower) line, it means that the price will increase (decrease) quickly.

Program 9. *Triangles.c*

The program is based on the fact that the analyst believes he has found a triangle.

The program works as follows:

1. The analyst must introduce the price and the time following the order A-B-C-D, independently of the temporal order.

2. It makes some checks on the conditions of a symmetrical triangle.

3. The program calculates the upper and lower lines.

4. If tA is previous than tB, it calculates the difference between B and the price in the upper line on tB. If tB is previous than tA, it calculates the difference between A and the price in the upper line on tA. This difference is multiplied by 0.05 and named percentaux. This will be used to do an interval of lines. Because we will say that the upper line is broken when the price is higher than the upper line plus percentaux. We act the same way with the lower line.

5. The program asks for future prices and it resolves if we have to operate or not. We will operate if the future time is previous to tintersecmaxim, this is the intersection of the lower line plus percentaux and the upper line less percentaux. The idea behind this is that near the vertex of the triangle, we can not use the pattern because it is too close to the edge.

6. The program establishes how we have to operate. If the price is between upper line plus percentaux and upper line less percentaux, we will sell. If the price is between lower line less percentaux and lower line plus percentaux, we will buy.

7. If the triangle is totally broken, the program says if it is broken by the upper or the lower line.

Program 10. *Buscartriangles.c*

The second one is based on the idea that the analyst is not in front of the computer, i.e. the program does all the work for him and it searches for possible triangles. The program searches triangles among the prices given. Then we need to apply the program triangles.c to operate with the found triangle.

This is how it works:

1. The analyst introduces the data in temporal order, beginning with the oldest.
2. The program searches the maximum and the minimum of the data, and it always selects the newest values.
3. We are searching symmetrical triangles, so when we are looking for C, we are only interested on the prices upper B plus the difference between the price A and B.

   For every maximum after tA it searches the slope of the line through A and the maximum. The program selects the maximum with lower slope, and that maximum is C.

   The research of D is the same as C, but with minimums and the maximum slope.
4. The program calculates tintersecmaxim as in triangles.c. Then it checks if the triangle is in the past, or if it can even be applied. The way is to prove if tintersecmaxim is bigger than the final time of the data.
5. Finally the output is A-B-C-D. And we have to apply triangles.c to know if we have to operate or not when we encounter future prices.

3.4.3 Ascendant triangle

It is a variation of the symmetrical triangle. The difference is that the upper line is almost horizontal. The lower line is ascendant. It almost always ends breaking the upper line and increasing even more. It is considered an ascendant figure, i.e. the price tends to increase.

This type of triangle entails the problem that, if we know that the price will increase, why should we want to sell. So analysts are very careful with this pattern.

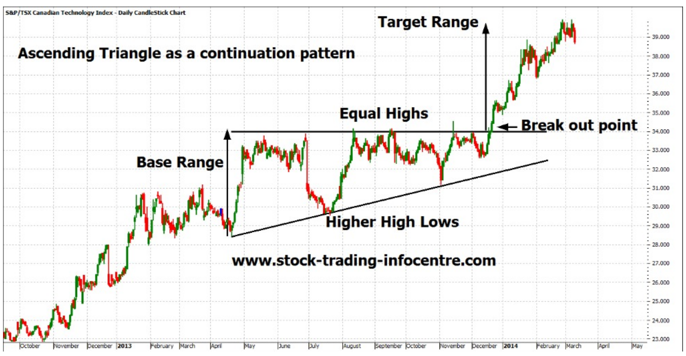

Figure 8: Ascendant Triangle.

The programs

The programs will be the same as in Symmetrical triangles, we can introduce a condition to differentiate the ascendant of the symmetrical, which could be: checking if the slope of the upper line is between -0.1 and 0.1. The programs will be the same because I do not introduce any strong condition to the slopes in the symmetrical triangles programs.

3.4.4 Descendent triangle

This triangle is the same as Ascendant Triangle, but inverting the Triangle.

The difference with the symmetrical is that the lower line is almost horizontal. The upper line is ascendant. It almost always ends breaking the lower line and decreasing. It is considered a descendent figure, i.e. the price tends to decrease.

Figure 9: Descendent Triangle.

The programs

We have the same as in Ascendant triangles, the programs will be the same as in symmetrical triangles. We can differentiate the descendent by introducing this condition: the slope of the lower line must be between -0.1 and 0.1.

Program 11. *Quintriangle.c*

It requests a possible triangle and it tells if is symmetrical, ascendant or descendent and how to operate with future prices.

Program 12. *Buscarquinstriangles.c*

The program searches triangles and expresses if they are symmetrical, ascendant or descendent.

3.4.5 Broadening Formation

The Broadening Formation are figures showing that, over a certain period of time, the prices range between two lines. The intersection of the lines is in the left part of the figure, i.e. in the past. So it is like an inverted triangle, the vertex is on the start of the figure. This figure is not as common as triangles.

The way to operate in the Broadening Formation is to buy when the price is close to the lower line, and to sell when the price is next to the upper line. If the price breaks the upper (lower) line, it is a signal that the price will increase (decrease).

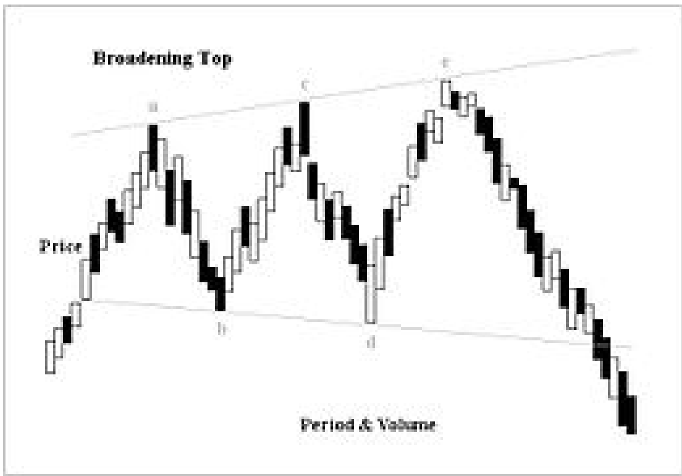

Figure 10: Broadening Formation.

Program 13. *Ensanch.c*

The program requests data that we think that is Broadening Formation and it decides if it is correct or not. Then, determines the best way to operate at future values.

Note 26. *A program that searches Broadening Formations, with data that we can provide, is almost impossible. We have tried a lot of things, but is hardly ever possible to find Broadening Formations. The conditions of that pattern are very ambiguous, and if we do not introduce any more conditions, that type of program is not prepared to be used to operate.*

3.4.6 Diamonds

The diamond is an extremely rare figure which only appears on the top of prices. It is a combination of a Broadening Formation and a symmetrical Triangle. It starts with a Broadening Formation and finishes with a Symmetrical Triangle,is named after the gem it resembles.

Figure 11: Diamonds.

Apparently, this figure does not introduce anything new. When we study triangles, we do not care about what happens after the triangle.

We treat the diamond like as a triangle, because we do not care about the past, so the programs will be the same ones we use symmetrical triangle.

3.4.7 Rectangles

Rectangles are figures that represent a pause on the trend, prices oscillate between two horizontal lines. The break on the rectangle points at the trend direction. We have to be careful to distinguish these from the double or triple tops and bottoms, because the figures are really similar. We differentiate them according to the line that it breaks.

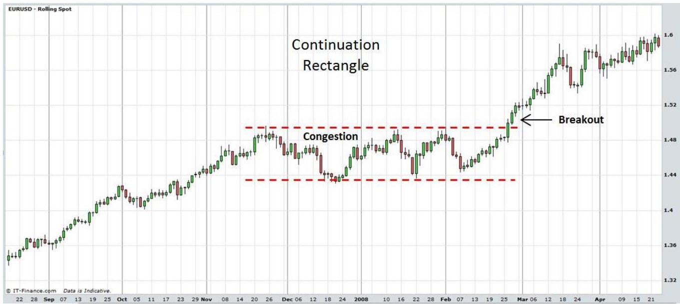

Figure 12: Rectangles.

Program 14. *Rectangles.c*

The program is the same as triangles.c but it introduces two new conditions:

1. The upper line slope must be between -0.1 and 0.1.

2. The lower line slope must be between -0.1 and 0.1.

3.5 Moving Average Indicators

Moving average is probably one of the most used indicators. This is because moving average is very easy to program, and the results are totally objective.

The indicator is based on the idea of doing an average of the last n prices. If the weight is the same for the last n prices, we call it simple moving average. If the weight is different depending on the temporal position, we call it weighted moving average.

The purpose of the moving average indicator is to identify the end of a trend or the beginning of another. We can understand the Moving average as a curve trend line. It is important to understand that the MA does not anticipate anything, it only reacts to trend changes that have already occurred.

We have to be careful with the choice of the correct n, i.e. the number of lags we will consider. An analyst has to make that decision considering the past of the stock he is going to analyze, because the best MA is different in every market. The most used short-term MA are 10, 20 and 60. For long term MA, we normally use 100 or 200.

I will distinguish two types of Moving average: the simple Moving average and the differences of Moving average. The difference between them is that in the first we only consider one moving average, and in the second we consider the subtraction of two moving averages.

3.5.1 Simple Moving Average

If we consider the last n prices, $(P_1, P_2, \cdots, P_n)$, with P_n the actual price. The Simple Moving Average is:

$$MA_n(0) = \frac{1}{n}\sum_{i=1}^{n} P_i$$

The first MA is the $MA_n(0)$.

If we introduce the next price P_{n+1}, the last n prices are $(P_2, P_3, \cdots, P_{n+1})$. So the Simple Moving Average is:

$$MA_n(1) = \frac{1}{n}\sum_{i=2}^{n+1} P_i$$

So, the second MA is $MA_n(1)$.

And for every time $t > 0$ the moving average is:

$$MA_n(t) = \frac{1}{n}\sum_{i=t}^{n+t} P_i$$

Finally, the k MA is $MA_n(k)$.

If the actual price, P_t, is above (below) the $MA_n(t)$, the signal is to buy (sell).

Analysts usually work with two MA, with different number of lags (usually 20 and 50 or 10 and 20), and they operate if both MA signals are the same.

Program 15. *Media.c*

The program requires two numbers of lags, n and m, with $m > n$. Then it asks for the last m prices. Finally, it determines if we had better buy or sell in the future.

3.5.2 Weighted Moving Average

It works as Simple Moving average, but it actually ponders the prices. The most used way to weight is to give more importance to the last prices and less to the older. For every time $t > 0$ the weighted moving average is:

$$WMA_n(t) = \frac{2}{n(n-1)}\sum_{i=t}^{n-1+t}(i + 1 - t)P_i$$

We multiply the oldest price by 1, the second oldest by 2 ... and the last price by n. Then, we divide that addition by $1 + 2 + \cdots + n$.

The way to operate is the same as in Simple Moving Average.

Program 16. *Ponderatmedia.c*

The program asks for two number of lags, n and m, with $m > n$. Then you need to provide the last m prices. In the end, it says if we have to buy or sell in future cases.

3.5.3 Difference of Moving Averages

The difference of Moving Averages is the subtraction of two Moving Averages of different number of lags. The first Moving Average has lower number of lags than the second.

For every time $t > 0$ we consider the value of

$$X_{n,m}(t) = MA_n(t) - MA_m(t)$$

With $n < m$.

Then we have to introduce the next price, and calculate $X_{n,m}(t+1)$:

$$X_{n,m}(t+1) = MA_n(t+1) - MA_m(t+1).$$

The operate system is:

- If $X_{n,m}(t) \cdot X_{n,m}(t+1) > 0$ we do not have to operate.
- If $X_{n,m}(t) \cdot X_{n,m}(t+1) < 0$ we operate following:
  1. We buy if $X_{n,m}(t) < 0$ and $X_{n,m}(t+1) > 0$.
  2. We sell if $X_{n,m}(t) > 0$ and $X_{n,m}(t+1) < 0$.

Program 17. *Mediasmo.c*

The program works as follows:

1. The program asks you to fill in the n, m and the last m prices. Then it checks if the lags are correct.
2. It calculates $X_{n,m}(1)$ and demands for the next price. While $X_{n,m}(t) \cdot X_{n,m}(t+1) \geq 0$ the program asks for next prices.
3. Once $X_{n,m}(t) \cdot X_{n,m}(t+1) < 0$ the program should advise us to buy or sell. Once the program has done that, in future situations the program resolves that we do not have to operate because we have already made a purchase.

Chapter 4

Practical Work

In this part of the work, I intend to study Repsol shares, IBEX35 stock exchange, the EUR/USD exchange rates and Gold price. The reason why I chose these four is that each one belongs to a different type of Financial Markets categories. The first one belongs to Capital Markets, the second is the mean of a Capital Market, the third one belongs to Foreign exchange markets and the last one belongs to Commodity markets. I study the data with both types of analysis. The idea is to forecast prices and then compare the decisions made using Technical Analysis with the ones made using Time Series.

The data is updated to 19/06/2015 but I study the data until 19/05/2015. I do this to check if the forecast we will make is correct or not. So the forecast is at most one month beyond. I do the forecast using the last 20, 60 and 240 prices. Since markets do not operate on weekends I select 20 in order to represent a month average. I select 60 in order to represent 3 months, and finally, 180 to represent 9 months. The data used is in the folder, so you can use it to check the results.

4.1 Time Series using R

In this part, there is an explanation of how we have worked with R. I curry out different types of study for every data. The first I will is to adjust an ARIMA to the prices. Then, I will adjust an ARIMA to the logarithms of the prices. Once I have done both analysis, if I have an ARIMA(0,1,0), the next step is to adjust an ARCH to the prices or the logarithms of the prices. If we have an ARIMA(p,d,q) with p or q different of zero, we adjust an ARMA-GARCH of the log returns, exactly an ARMA(p,q)+GARCH(1,1). I select GARCH(1,1) because, in practice, we hardly ever need long lags.

I study the forecast for the last 20,60 and 180 prices. When I use the last 20 prices, I try to forecast the fifth next future price. With the last 60 prices I try to forecast the next fifteenth future prices. Finally, for the last 180 prices I try to forecast the next thirty future prices.

4.1.1 ARIMA with the prices

In the ARIMA with the prices, we do the following commands: (we use data1 as repsol,ibex35,...)

data1< −scan()

I introduce the last 20,60 or 180 prices.

acf(data1)

pacf(data1)

With these plots, as we have already seen, we can select the order of the ARMA. But I use it to check if the model selected using the minimum AIC is correct or not. The plots can be found in the folders with the data. The way they have been named can be inferred from this example: if we are studying 20 prices of repsol, the name will be 20ACFrepsol and 20PACFrepsol.

auto.arima(data1)

It selects the model with the lower AIC, and the output is the weights.

predict(auto.arima(data1),n)

We get the forecast of the future n prices.

Finally, we have to check the model testing white noise conditions, we use the Ljung-Box test and is done with the command:

Box.test(resid(data1))

Resid(data1) are ϵ_t, i.e. the difference between the real prices and the forecast prices. If the result is lower than 0.05, the model is not correct, in other cases the model is precise.

4.1.2 ARIMA with the logarithms of the prices

Before working with the data, we have to set the logarithms of prices.

lndata1< −log(data1)

After that command, we apply the same method.

4.1.3 ARCH with the log returns

If the model selected is an ARIMA(0,1,0), it is an indicator of an ARCH model. Then, we have to plot the PACF for the square log returns, we select the order of the ARCH with that plot. First of all, we have to install the fGarch package and introduce the command: library(fGarch).

After that, the commands are

data< −scan()

fr=diff(log(data))

This command computes the log returns.

ts< −fr ˆ 2

We know that the order of ARCH is the order of AR(ts), so we plot:

pacf(ts)

And we select the order of the ARCH. For example p.

archdata=garchFit(formula = ˜ garch(p,0),data=ts) predict(archdata,k)

With k representing the number of forecast prices that we want.

4.1.4 ARMA-GARCH with the log returns

We do an ARMA model selecting the one with lower AIC, and then we apply a GARCH(1,1), we apply GARCH(1,1) because the order is almost always equal or lower than 1, in other cases we only have to change the values. The commands are:

data1$<-$scan()

ts=diff(log(data1))

auto.arima(ts)

In that moment, the output is nearly always a model like ARIMA(p,0,q). In the case of ARIMA(p,d,q), with $d \neq 0$, we should have to differentiate on another occasion.

Then we have to check if we have to apply the GARCH or not, with the Test Ljung-Box of the squared residuals.

Box.test(ts^2)

If the p-value is below 0.05, we have to continue, in other cases we stop this model adjust.

armaGarch=garchFit(formula = ~ arma(p,q)+garch(1,1), data=ts)

predict(armaGarch,k)

With k representing the number of forecast prices that we want.

4.2 The investment decision

In the case of Technical Analysis, the risk averse decision will be the points of union between all patterns. Sometimes, I also take into consideration a slightly more risky decision.

The Time Series investment decision depends on the ARIMA models. If we have an ARIMA(0,d,0), first we do not have to operate. But the final investment decision could be to operate if the ARCH with the log returns decision is very confident. If we have an ARIMA(p,d,q) with p or q different to zero, and the first decision is to operate. The final investment decision could only change if ARMA-GARCH with the log returns decision is the opposite of ARIMA(p,d,q).

4.3 Repsol

The Repsol data goes from 04/01/1994 to 19/06/2015.

4.3.1 Repsol with the last 20 prices

The data used figures on the folder Repsol with the name CTRepsol.txt or CTRepsol.xlsx.

Using Technical Analysis

- Trend.c considers the trend as lateral.
- Canal.c considers the trend as lateral.
- Buscarhch.c finds a possible HS on times 0, 1, 5, 11 and 16, but when we introduce the possible HS into hch.c the result is that the head is lower than the first shoulder. When this happens it is an indicator of a possible Triple Top. We introduce to the tripletops.c and, after the verification, we see that we found a Triple Top. So, we introduce the next price 18.12, and we are advised not to buy or sell. But after introducing the 20/05/2015 price, the signal is to sell.
- Buscarhchinvertit.c finds a possible inverted HS on times 1, 5, 11, 16 and 18. If we introduce the possible inverted HS to inverthch.c it tells us that the head is not high enough. So, as in HS it is an indicator of a possible triple bottom. According to the program, we have a Triple Bottom. When we introduce the next three prices the program makes us the recommendation to buy. Then, we do not operate in the next period. Nevertheless, after 20/05/2015 we have to sell always.
- Buscartriangles.c finds a possible triangle in the past.
- Media.c with n=10 and m=20 urges us to sell after 25/05/2015.
- Ponderatmedia.c with n=10 and m=20 urges us to sell for all the prices, regardless of all prices.
- Mediasmo.c output is that we do not have to operate.

The risk averse decision is to sell after 25/05/2015. The difference between that price and the fifth price is -0.18, so the benefit was 1.0084%. A slightly more risky decision is to sell after 20/05/2015. The difference between that price and the fifth price is -0.31, so the benefit is 1.7108%.

Using Time Series

- ARIMA with prices.

  It selects an ARIMA(1,1,0) with the weight ϕ_1 = -0.4663. The forecast is that we should sell, because the fourth and fifth prices are 17.94061 and 17.93775.
- ARIMA with the logarithms of the prices.

  It selects an ARIMA(1,1,0) with the weight ϕ_1 = -0.4668. The forecast determines that it would be more profitable to sell, because the fourth and fifth prices are 17.94049 and 17.93762.
- ARMA-GARCH of the log returns

  The model that we have to compute is ARMA(1,0)+GARCH(1,1). The meanForecast is always negative, so the prediction is that the prices will decrease.

The decision is to sell at once. The difference between the first and the fifth price is -0.31, it grants a benefit of 1.7241%.

4.3.2 Repsol with the last 60 prices

The data used is on the folder Repsol with the name MTRepsol.txt or MTRepsol.xlsx.

Using Technical Analysis

- Trend.c considers the trend as lateral.
- Canal.c considers the trend as lateral.
- Buscarhch.c finds a possible HS. But when we introduce the data into hch.c, the head is lower than the first shoulder.

  If we introduce the possible HS found into the tripletops.c, the checking says that we have a Triple Top. So we introduce the next price 18.12, and the program drives us to sell right away because the neckline is totally broken.
- buscarhchinvertit.c searches for a possible inverted HS but it does not pass the checking of inverthch.c. If we introduce it into triplebottom.c, it advises against operating until 25/05/2015, then we have to sell, independently of all future prices.
- Buscartriangles.c finds a possible triangle in the past.
- Media.c with n=30 and m=60 dissuades us from operating until 26/05/2015, then we have to sell for all always.
- Ponderatmedia.c with n=30 and m=60 tells us to ell at any future price.
- Mediasmo.c claims it is unadvisable to operate.

The risk averse decision is to sell after 25/05/2015. The difference between that price and the fifteenth price is -1, so the benefit was 5.6022%. A slightly more risky decision is to sell after 20/05/2015. The difference between that price and the fifteenth price is -1.27, so the benefit was 7.0088%.

Using Time Series

- ARIMA with prices.

  It selects an ARIMA(1,1,1) with the weights $\phi_1 = -0.9404$ and $\theta_1 = 0.8262$.The forecast renders it better to wait, because the fourth and fifth prices are 17.98458 and 18.01647, extremely similar to the actual value.
- ARIMA with the logarithms of the prices.

  It selects an ARIMA(1,1,1) with the weights $\phi_1 = -0.9434$ and $\phi_2 = 0.8360$.The forecast considers it better to wait, because the forecast of the log returns are positive and negative.
- ARMA-GARCH of the log returns

  We have to compute the model ARMA(1,1) + GARCH(1,1). The p-value of α_1 is not lower than 0.05, so the final model is:

$$P_t = -0.952P_{t-1} + 0.836\epsilon_{t-1}$$
$$\sigma_t^2 = 1.668 \times 10^{-5} + 0.8722\sigma_{t-1}^2$$

The forecast considers it better to wait, because the meanForecast of the log returns are positive and negative.

If we do not operate, the benefit is zero.

4.3.3 Repsol with the last 180 prices

The data used appears in the folder Repsol with the name LTRepsol.txt or LTRepsol.xlsx.

Using Technical Analysis

- Trend.c considers the trend as lateral.
- Canal.c considers the trend as lateral.
- Buscarhch.c does not find any possible HS.
- Buscarhchinvertit.c finds a possible inverted HS on times 32, 59, 93, 125 and 136. But the head is not low enough. The triplebottom.c considers it like a triple bottom, but on the past.
- Buscartriangles.c finds a possible triangle in the past.
- Media.c with n=90 and m=180 considers it advisable to buy until 01/06/2015, then we do not have to operate for two days, and finally sell the stock, regardless of all future prices.
- Ponderatmedia.c with n=90 and m=180 says that we have to buy the three first future prices but then from 29/05/2015 until the end we have to sell.
- Mediasmo.c says that we do not have to operate.

The decision is to buy from the first price until 22/05/2015 and then sell after 03/06/2015. The benefit of the first operation is 1.1123%, while the benefit of the second is 7.4541%.

Using Time Series

- ARIMA with prices.

  It is an ARIMA(0,1,0) with drift, the forecast of the future prices is that they will decrease.
- ARIMA with the logarithms of the prices.

  It is an ARIMA(0,1,0) with non-zero mean. The forecast of the future prices is that they will decrease.
- ARCH with the prices.

  We select an ARCH(4), but all the p-values are lower than 0.05, so we do not have to take into consideration an ARCH model.

The decision is to sell, the difference is 1.84 and the benefit is 10.23%.

4.4 IBEX35

The IBEX35 data goes from 02/01/1996 to 19/06/2015.

4.4.1 IBEX35 with the last 20 prices

The data used is in the folder IBEX35 with the name CTIBEX35.txt or CTIBEX35.xlsx.

Using Technical Analysis

- Trend.c finds an ascendant trend and the trend is broken on 25/05/2015 and totally broken on 26/05/2015, then we have to sell.
- Canal.c finds a canal, but not parallel enough.
- Buscarhch.c finds a possible HS on times 0, 1, 4, 9 and 19. But the shoulders are not at the same height. It is not a Triple Top either.
- Buscarhchinvertit.c finds a possible inverted HS, on times 1, 4, 9, 13 and 17. But the shoulders are not at the same height and it is not a Triple Bottom.
- Buscartriangles.c does not find any triangle.
- Media.c with n=10 and m=20 determines that we have to buy for the next three prices and then sell for the next 2.
- Ponderatmedia.c with n=10 and m=20 determines that we have to sell always.
- Mediasmo.c recommends that we do not have to operate for all prices except for the second, whenn we have to buy.

The risk averse decision is to sell after 25/05/2015. The difference between that price and the fifth price is -82 points, so the benefit was 0.7242% . A slightly more risky decision is to buy in the first moment until 22/05/2015 and then sell for the next two periods. The difference of the first operation was 56.5 points so the benefit was 0.4914%. The difference of the second operation is 82 points, so the benefit was 0.7242%.

Using Time Series

- ARIMA with prices.

  It is an ARIMA(0,0,1) with $\theta_1 = 0.5817$ and constant 11390.4145. The forecast prices for the fourth and the fifth are 11390.41 and 11390.41, so we have to sell. Because the forecast prices are above the actual price.
- ARIMA with the logarithms of the prices.

  It is an ARIMA(0,0,1) with $\theta_1 = 0.5776$ and constant 9.3405. The forecast prices for the fourth and the fifth are 11389.68 and 11389.68, so we have to sell, because the forecast prices are above the actual price.

- ARMA-GARCH of the log returns

  We have to compute the model ARMA(0,1)+GARCH(1,1). But the p-values are not the expected, so we can not introduce GARCH errors.

The decision is to sell. The difference is 257 points, so the benefit is 2.2352%.

4.4.2 IBEX35 with the last 60 prices

The data used is in the folder IBEX35 with the name MTIBEX35.txt or MTIBEX35.xlsx.

Using Technical Analysis

- Trend.c finds an ascendant trend, and future prices does not break it.
- Canal.c does not find any canal.
- Buscarhch.c finds a possible HS on times 6, 12, 34, 49 and 59. But when we introduce the data into hch.c, it says that the shoulders are not equally high. The problem is the same as in tripletops.c, so we do not have a triple top.
- buscarhchinvertit.c does not find any inverted HCH.
- Buscartriangles.c does not find any triangle.
- Media.c with n=30 and m=60 determines we have to buy beginning at the first price up to 22/05/2015, and then sell from 25/05/2015 until 26/05/2015, after that one buying period, and finally for all the next prices we have to sell.
- Ponderatmedia.c with n=30 and m=60 urges us to sell for all future prices we come across.
- Mediasmo.c determines that we do not have to operate for any time.

The decision is to sell only in 25/05/2015 and after 28/05/2015. The first operation difference is -56.5 points, with a benefit of 0.4990%. The difference of the second operation is -438.5 points. It is a sale, so the benefit is 3.8479%.

Using Time Series

- ARIMA with prices.

  It is an ARIMA(0,1,0) with drift. The forecast is that the prices will increase.
- ARIMA with the logarithms of the prices.

  It is an ARIMA(0,1,0), so we do not have to operate.
- ARCH with the prices.

  The p-values are not correct.

We do not operate. Therefore, the benefit is zero.

4.4.3 IBEX35 with the last 180 prices

The data used is the folder IBEX35 with the name LTIBEX35.txt or LTIBEX35.xlsx.

Using Technical Analysis

- Trend.c says that we have an ascendant trend, and it is not broken for any future prices.
- Canal.c does not find any canal.
- Buscarhch.c finds a possible HS. But the shoulders are not at the same height. So, it is not a triple top either.
- Buscarhchinvertit.c finds a possible inverted HS, but applicable in the past.
- Buscartriangles.c does not find any possible triangle.
- Media.c with n=90 and m=180 urges us to to buy in all future prices.
- Ponderatmedia.c with n=90 and m=180 deems it advisable to buy only for the first two prices. Then, we do not have to operate.
- Mediasmo.c assures that we do not have to operate.

The decision is to buy for the first two prices. The difference is 76.4 points, the benefit we obtain is 0.6644%.

Using Time Series

- ARIMA with prices.

  It is an ARIMA(0,1,0), with drift. The forecast of the future prices is that they will increase.
- ARIMA with the logarithms of the prices.

  It is an ARIMA(0,1,0) with non-zero mean, and the forecast of the future prices is that they will increase.
- ARCH of the log returns

  We have to adjust an ARCH(4). This gives us the following ARCH(4) model

  $$P_t = \epsilon_t$$
  $$\sigma_t^2 = 7.602 \times 10^{-4} + 0.453\epsilon_{t-3}$$

  The meanForecast is always positive, so we have to buy.

The final decision is to buy. The difference is -129.5, with a loss of 1.126%.

4.5 Euro/Dollar

The Euro/Dollar data goes from 04/01/1999 to 19/06/2015.

4.5.1 Euro/Dollar with the last 20 prices

The data used is in the folder EURODOLLAR with the name CTEURODOLLAR.txt or CTEURODOLLAR.xlsx.

Using Technical Analysis

- Trend.c considers the trend as lateral.
- Canal.c considers the trend as lateral.
- Buscarhch.c does not find any possible HS.
- Buscarhchinvertit.c does not find any possible inverted HS.
- Buscartriangles.c does not find any triangle.
- Media.c with n=10 and m=20 deems it prudent to sell in all future prices.
- Ponderatmedia.c with n=10 and m=20 says that we have to sell for all future prices.
- Mediasmo.c says that we do not have to operate.

The decision is to sell for all future prices. The difference between the first price and the fifth price is -0.02443, so the benefit was 2.1912%.

Using Time Series

- ARIMA with prices.

  It is an ARIMA(0,1,1) with $\theta_1 = 0.5715$. The forecast is 1.109839, so we have to sell.
- ARIMA with the logarithms of the prices.

  It is an ARIMA(0,1,1) with $\theta_1 = 0.5715$. The forecast is 1.109823, so we have to sell.
- ARMA-GARCH of the log returns

  The test of Ljung-Box is negative, so we do not have GARCH errors.

The decision is to always sell, so the benefit is the same as in Technical Analysis.

4.5.2 Euro/Dollar with the last 60 prices

The data used is in the folder EURODOLLAR with the name MTEURODOLLAR.txt or MTEURODOLLAR.xlsx.

Using Technical Analysis

- Trend.c finds a descendent trend, but it is almost lateral.
- Canal.c does not find a canal.
- Buscarhch.c does not find any possible HS.
- buscarhchinvertit.c finds a possible inverted HS, but it is in the past.
- Buscartriangles.c does not find any triangle.
- Media.c with n=30 and m=60 says we have to buy for the first two prices but then we do not operate during two prices. After, we have to sell two times.
- Ponderatmedia.c with n=30 and m=60 urges us to sell for always.
- Mediasmo.c says that we do not have to operate.

The decision is to sell after 25/05/2015. The difference between that price and the fifteenth price is 0.03507, so the loss was 3.1947%.

Using Time Series

- ARIMA with prices.

  It is an ARIMA(0,1,0), so we do not have to operate.
- ARIMA with the logarithms of the prices.

  It is an ARIMA(0,1,0), so we do not have to operate.
- ARCH with the prices.

  The p-values are lower than 0.05.

The decision is not to operate. The benefit is zero.

4.5.3 Euro/Dollar with the last 180 prices

The data used is in the folder EURODOLLAR with the name LTEURODOLLAR.txt or LTEURODOLLAR.xlsx.

Using Technical Analysis

- Trend.c considers the trend as lateral.
- Canal.c considers the trend as lateral.
- Buscarhch.c does not find any possible HS.
- Buscarhchinvertit.c finds a possible inverted HS but the head is not high enough. It is a triple bottom, but it is applicable on the past.

- Buscartriangles.c finds a possible triangle on times 1, 132, 73 and 153. But when we introduce the future prices we can not apply the triangle.
- Media.c with n=90 and m=180 says that we have to sell for the first eight future prices, then we do not have to operate.
- Ponderatmedia.c with n=90 and m=180 says that we have to sell for the first nine future prices, then we do not have to operate.
- Mediasmo.c says it is preferable not to operate.

The decision is to sell until the eight period. The difference between the first price and the eight price is 0.020345, so the loss was 1.8248%.

Using Time Series

- ARIMA with prices.

  It is an ARIMA(0,1,1) with drift, the forecast of the future prices is a decay. So we have to sell.
- ARIMA with the logarithms of the prices.

  It is an ARIMA(0,1,0).
- ARCH of the log returns.

  The model that we have to adjust is ARCH(5). This gives us the following model

$$P_t = \epsilon_t$$
$$\sigma_t^2 = 2.347 \times 10^{-5} + 0.3268\epsilon_{t-5}^2$$

  The forecast of the future prices is a decay. So we have to sell.

The decision is to sell, the loss is the same as in Technical Analysis.

4.6 Gold

The Gold data goes from 19/04/2002 to 19/06/2015.

4.6.1 Gold with the last 20 prices

The data used is in the folder OR with the name CTOR.txt or CTOR.xlsx.

Using Technical Analysis

- Trend.c finds an ascendant trend. When we introduce the 4th next price, it says that we have to sell because the trend is totally broken.
- Buscarhch.c does not find any possible HS.

- Buscarhchinvertit.c finds a possible inverted HS, but the head is not high enough. It is not a Triple Bottom either.
- Buscartriangles.c does not find any triangle.
- Media.c with n=10 and m=20 asserts that we have to buy the first two future prices, then we do not have to operate for one period. Finally we sell for the rest periods.
- Ponderatmedia.c with n=10 and m=20 says that we have to sell in all future prices.
- Mediasmo.c says that we do not have to operate.

The decision is to sell after 22/05/2015, the difference is 18 points. The benefit is 1.495%.

Using Time Series

- ARIMA with prices.

  It is an ARIMA(1,0,0) with non-zero mean, the weight is $\phi_1 = 0.5559$ and the constant is 1198.7342. The fourth and fifth forecast future prices are 1199.523 and 1199.173, so we should sell.
- ARIMA with the logarithms of the prices.

  It is an ARIMA(1,0,0) with non-zero mean, the weight is $\phi_1 = 0.5527$ and the constant is 7.0889. The fourth and fifth forecast future prices are 1199.4 and 1199.051, so we should sell.
- ARMA-GARCH of the log returns.

  The p-values are not lower than 0.05.

The decision is to sell. So the difference is 21 points. The benefit represents 1.739%.

4.6.2 Gold with the last 60 prices

The data used is in the folder OR with the name MTOR.txt or MTOR.xlsx.

Using Technical Analysis

- Trend.c consider a lateral trend.
- Canal.c does not find a canal.
- Buscarhch.c does not find any possible HS.
- buscarhchinvertit.c finds a possible inverted HS, but the shoulders are not at the same height. So, we do not have a triple bottom too.
- Buscartriangles.c does not find any triangle.
- Media.c with n=30 and m=60 says we have to buy for the first three prices but then we have to sell for all the future prices.

- Ponderatmedia.c with n=30 and m=60 says that we have to sell for all future prices.
- Mediasmo.c says that we do not have to operate.

The decision is to sell after 22/05/2015, the difference is 18 points. The benefit is 1.495%.

Using Time Series

- ARIMA with prices.

  It is an ARIMA(1,0,0) with non-zero mean, the weight is $\phi_1 = 0.7890$ and the constant is 1195.0332. The forecast of future prices is that will decay, so we have to sell. The first forecasts are 1204.475, 1202.483, 1200.911, 1199.671, 1198.692, 1197.920, 1197.311 and 1196.831.
- ARIMA with the logarithms of the prices.

  It is an ARIMA(1,0,0) with non-zero mean, the weight is $\phi_1 = 0.7913$ and the constant is 7.0858. The forecast of future prices is that they will decay, so we have to sell. For instance, the first four are 1204.468, 1202.469, 1200.887 and 1199.639.
- ARMA-GARCH of the log returns.

  The p-value of the Ljung-Box test is 0.8537, so we reject GARCH errors.

The decision is to sell. The difference is 21 points, and the benefit is 1.739%.

4.6.3 Gold with the last 180 prices

The data used is in the folder OR with the name LTOR.txt or LTOR.xlsx.

Using Technical Analysis

- Trend.c says that we have a descendent trend.
- Canal.c does not find any canal.
- Buscarhch.c finds a possible HS but the shoulders are not at the same height. So, we do not have a triple top too.
- Buscarhchinvertit.c finds a possible inverted HS but its application is exclusive for past times.
- Buscartriangles.c finds a possible triangle on times 103,57, 178 and 136. But with the future prices we can not apply the triangle.
- Media.c with n=90 and m=180 determines that we have to sell for all the prices except the last two prices of the data.
- Ponderatmedia.c with n=90 and m=180 urges us to sell always.
- Mediasmo.c tells us that we do not have to operate.

The decision is to sell for all the prices except the last two. The difference is 31 points. The benefit is 2.568%.

Using Time Series

- ARIMA with prices.
  It is an ARIMA(0,1,0) with drift. The future prices will decrease, so we have to sell.
- ARIMA with the logarithms of the prices.
  It is an ARIMA(0,1,0) with non-zero mean. The decision is to sell.
- ARCH with prices.
  The PACF determines that we do not have an ARCH model.

The decision is to sell, the difference is 7 points and the benefit is 0.58%.

4.7 Analysis of the practical work

We analyze the results of this part using the ratio benefit/non-operate/loss.

- With Repsol data we obtain 7/1/0, with IBEX35 data we obtain 5/1/1, with Euro/Dollar data we obtain 2/1/3 and finally with Gold data we obtain 6/0/0.
- With the last 20 prices we obtain 10/0/0, with the last 60 prices we obtain 5/3/1 and with the last 180 prices we obtain 5/0/3.
- Using Technical Analysis we obtain 13/0/2 and using Time Series we obtain 7/3/2.

With these ratios we can draw some conclusions.

The investment decisions based on the last 20 prices are totally profitable. Because we always take the decision to operate and we always obtain benefits. Exactly the same happens with Gold.

Excluding Euro/Dollar, all the ratios are lucrative.

The Technical Analysis investment decisions are more profitable than the ones using Time Series. Time Series has more non-operate decisions than Technical Analysis. The perception we have is that to generate Time Series operating decisions we have to be more secure than with Technical Analysis.

The conclusion of this part is that we should take more consideration to Technical Analysis decisions, and we also recommend to use, more than anything else, the only last 20 prices.

Chapter 5

Conclusions and Further Work

Throughout this work, we have studied two of the most used types of analysis of Financial Markets. The first objective of my thesis was to make a rigorous study of both analysis.

In the part of Time Series we have focused on the properties, on how to estimate the weights and how to adjust models using R. We started with stationary models. After that, we moved forward and we studied the non-stationary models and we finished with Time Series Analysis with the Auto-regressive Conditional Heteroscedastic Models.

The work of Technical Analysis has been much more complicated than what we expected. First we work on the definitions and the patterns, one complication was to determine how to approach the patterns to a mathematical vision. Then, I focused on the development of 17 computer programs, which turned out to be much more complicated than what it may seem at first sight. The purpose was to make Technical Analysis decisions something objective, leaving aside subjectivity.

The practical part was really surprising, the results were really unexpected. The decisions using both types of analysis with Repsol, IBEX35 and Gold prices were spectacular. They all produced great benefits, with the exception of the three non-operate decisions (out of twenty-one) and one loss. So, we have benefits in practically all the operate decisions. The study of the Euro/Dollar exchange has thrown the results we were expecting on the beginning, benefits and losses for equal. My conclusion is that, probably, the bad results with the Euro/Dollar exchange are caused by the actual unsteadiness of the Euro, due to the possible euro exit of Greece and the impact of this hypothetical event to European currency.

The second objective of my thesis was to choose the analysis that I consider to be more accurate. This decision is a bit subjective, because the practical part's results are satisfactory for both types of analysis.

Firstly I would like to comment that Technical Analysis has an interesting self-fulfilling prophecy. Part of the patterns studied do not have any logic, but they are eventually accomplished day after day. I think that these patterns are met because a lot of investors believe in them and make use of them. For example, if all the people link together a HS with a decay on the prices, when this patterns come about in practice, all the investors that believe in HS pattern will make the decision of selling. Those decisions will increase the volume and decrease the price.

Despite what it may seem, both types of analysis are based on past prices, errors, volatility, among others, in order to forecast future prices. So, both types could be considered to be blood-related and opposed Fundamental Analysis.

In regard to Technical Analysis, it is more prone to consider changes on trend and to execute investment orders than Time Series. This could be seen as an advantage or a disadvantage, depending on the risk averse of the investor. So, the decision depends on the risk averse of what we are supposed to be. For example, if we work for an investment fund, we are more likely to be liable for all of our investing decisions.

Although results of Technical Analysis are better that the ones using Time Series, the conclusion is that we may use both types of analysis to make decisions, because both generate good results in practice. If we are supposed to be risk averse, we probably should operate if the investment decisions are the same for both types of analysis. If we take into consideration a slightly more risky decision, we should only consider the decisions using Technical Analysis.

We would like to conclude this thesis by discussing some possible lines of future work.

My intention is to write a complete program for Technical Analysis. The idea would be to introduce the data into this program, and then, it can search for all possible patterns, both the ones studied in this thesis and the ones that we have not contemplated, and it selects the right investment decision. The complication of the program would certainly be the decision of the way to operate. This is because we often have opposed decisions.

Another path of future work would be to study the correlations between different shares of a Capital Market using Technical Analysis. For example, whether a buy signal in Repsol using Technical Analysis echoes in the Technical Analysis of Telefonica, or it does not.

Finally, I would like to study a way to join Technical Analysis and Time Series. The idea would be to introduce the patterns studied to Time Series. It would be incredible to create a new type of analysis based on both.

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