Ibn al-Haytham and Jābir b. Aflaḥ's criticism of Ptolemy's determination of the parameters of Mercury

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1. Presentation

Andalusian astronomers of the 11th and 12th c. seem to have felt a certain interest in the critical study of Ptolemy's model for Mercury. This is clear in the case of Ibn al-Zarqālluh who, in his treatise on the construction of the equatorium, identifies the curve traced by the centre of Mercury's epicycle with an ellipse[1] and uses a non-Ptolemaic eccentricity of 2;51,26^{p}[2]. In addition, Abū Bakr b. Bājja (1070?-1138) ascribes to Ibn al-Zarqālluh an otherwise unknown short treatise on the invalidity of the method used by Ptolemy to determine the position of Mercury's apogee (*Maqāla fī ibṭāl al-ṭarīq allatī salaka-hā Baṭlīmūs fī istikhrāj al-buʿd al-*

[1] J. Samsó and H. Mielgo, "Ibn al-Zarqālluh on Mercury", *Journal for the History of Astronomy* 25 (1994), 289-296.

[2] On Ibn al-Zarqālluh's eccentricity see Willy Hartner's two papers reprinted in the volume *Oriens-Occidens* II (Hildesheim, Zürich, New York, 1984): "Ptolemy, Azarquiel, Ibn al-Shāṭir and Copernicus on Mercury. A Study of Parameters" (pp. 292-312); "The Islamic Astronomical Background to Nicholas Copernicus" (pp. 316-325); see also Mercè Comes, *Ecuatorios andalusíes. Ibn al-Samḥ, al-Zarqālluh y Abū-l-Ṣalt*, Barcelona, 1991, pp. 119-120.

ab^cad li-^cUṭārid)[3]. This led me, a few years ago, to conjecture whether the anomalous position of Mercury's apogee determined by M. Boutelle[4] from Ibn al-Zarqālluh's *Almanac* tables (210°, instead of the Ptolemaic 190°) was the result of new observations made by the Toledan astronomer: it is well known that Ptolemy's apogee for Mercury was inaccurate by about 30° in his own time,[5] and Andalusian astronomers were probably conscious of the fact that an entirely different - and, in fact, far more correct - longitude of Mercury's apogee appeared in al-Khwārizmī's *zīj*: 224;54° for the beginning of the Hijra (midday of 14th July 622)[6]. A different apogee - 198;24,17° for ca. 581 A.D., corresponding to the moment at which the value of precession was 0° -[7] appears in the *zīj*es of Ibn al-Bannā' (1256-

3 See Jamāl al-Dīn al-^cAlawī, *Rasā'il falsafiyya li-Abī Bakr b. Bājja*, Beirut-Casablanca, 1983, p. 78.

4 See Marion Boutelle, "The Almanac of Azarquiel", reprinted in E.S. Kennedy *et al.*, *Studies in the Islamic Exact Sciences* (Beirut, 1983) pp. 502-510. This paper should be read together with the important remarks by Noel Swerdlow in *Mathematical Reviews* 41 (1971), no. 5149. For a general survey of this source see J. Samsó, *Las Ciencias de los Antiguos en al-Andalus*, Madrid, 1992, pp. 166-171.

5 See O. Gingerich, "Mercury Theory from Antiquity to Kepler", first published in 1971 and reprinted in the volume by the same author *The Eye of Heaven. Ptolemy, Copernicus, Kepler* (New York, 1993), 379-387. Robert R. Newton (*The Crime of Claudius Ptolemy*. Baltimore and London, 1977, pp. 278-279) reaches a similar conclusion when he says that the longitude of Mercury's apogee should be about 219° in Ptolemy's time instead of the 190° we find in *Almagest* IX,7.

6 O. Neugebauer, *The Astronomical Tables of al-Khwārizmī. Translation with Commentaries of the Latin Version edited by H. Suter supplemented by Corpus Christi College MS 283* (Copenhagen, 1962) pp. 41, 99. Raymond Mercier ("Astronomical Tables in the Twelfth Century" in Charles Burnett [ed.], *Adelard of Bath. An English Scientist and Arabist of the Early Twelfth Century*, London, 1987, pp. 91-92) has proved the origin of this apogee longitude: with parameters of the *Brahmasphutasiddhanta* he obtains 224;53,13° for the beginning of the Hijra.

7 I.e. for a moment in which tropical and sidereal longitudes were equal. According to the *Brahmasphutasiddhānta* the sidereal and tropical longitudes of the Sun were equal in year 580 and this dating appears to have been approximately followed by Ibn al-Zarqālluh who stated that the Hindu-Iranian (sidereal) and *Mumtaḥan* systems were in agreement "about

1321) and Ibn al-Raqqām (d. 1315)[8]; this appears to be a return to the Ptolemaic tradition.

Apart from the aforementioned isolated remarks, the first complete reference to an Andalusian criticism of Ptolemy's Mercury model can be found in Jābir b. Aflaḥ's *Iṣlāḥ al-Majisṭī* (fl. ca. 1150). It is interesting, however, that another important text on the same topic circulated in al-Andalus at least from the end of the eleventh century: Ibn al-Haytham's *Shukūk ʿalā Baṭlamyūs* ("Doubts on Ptolemy") were quoted, and severely criticised, by Ibn Bājja in another passage of the same text in which he mentions Ibn al-Zarqālluh's *Maqāla fī ibṭāl*... The same work is, apparently, one of the sources used by Ibn Rushd in his *Mukhtaṣar al-Majisṭī*.[9] Ibn Bājja's passage has a certain interest and is worth translating:

> "The same attitude [as that of Ibn al-Zarqālluh] has been adopted by others who preceded him: I feel quite astonished that such is the case of Ibn al-Haytham, in spite of his fame. If you wish to consider in detail what I am telling you, read his book entitled *Shukūk ʿalā Baṭlamiyūs*, particularly the chapter in which he explains the invalidity of the method used by Ptolemy to establish the eccentricities of Venus and Mercury, and you will get a clear idea of what I am saying. If you make a detailed study of this work, you will reach the obvious conclusion that Ibn al-Haytham only studied Astronomy in a superficial way [*min ashal al-ṭuruq*] <and that he did not assimilate in due time those things which were difficult for him, either because they confirmed his idea on the lack

40 years before the Hijra, at the moment of the Prophet's birth". Ibn al-Zarqālluh's followers Ibn al-Kammād, Ibn al-Bannā' and Ibn ʿAzzūz have trepidation tables in their *zījes* which imply 581 A.D. as the year in which precession reached 0°. See J. Samsó, "Andalusian Astronomy in 14th Century Fez: *al-Zīj al-Muwāfiq* of Ibn ʿAzzūz al-Qusanṭīnī", *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 11 (1997), 73-110 (see pp. 107-110).

8 See J. Samsó and E. Millás, "The computation of planetary longitudes in the *zīj* of Ibn al-Bannā', *Arabic Sciences and Philosophy* 8 (1998), 265-270.

9 See Juliane Lay, "L'*Abregé de l'Almageste*: un inédit d'Averroès en version hébraïque", *Arabic Sciences and Philosophy* 6 (1996), 23-61.

of validity of the method or because he left them aside in a careless way >[10]. He [= Ibn al-Haytham] does not belong to the group of those who have persevered in the study of this science [= Astronomy] and, in this respect, he [= Ibn al-Haytham] is farther away from it than Ibn al-Zarqālluh himself.[11]

Ibn Bājja's words have led me to consider Ibn al-Haytham's text on this topic in some detail in order to ascertain whether it might have had any influence on other Andalusian astronomers.[12] The conclusion is clearly negative, at least in relation to Jābir b. Aflaḥ[13].

2. Ibn al-Haytham on Mercury

According to Ibn Bājja, the passage in question is concerned with

[10] According to the editor al-ʿAlawī the passage in angular brackets < > is an interpolation in the text and should be suppressed.

[11] al-ʿAlawī, *Rasā'il*, pp. 77-78.

[12] The passage in question has attracted the attention of only one scholar: Don L. Voss in his unpublished doctoral dissertation (*Ibn al-Haytham's doubts concerning Ptolemy. A Translation and Commentary*) presented at the University of Chicago, Illinois, in December 1985. See two brief summaries of the contents of the *Shukūk* in A.I. Sabra, "Ibn al-Haytham", *Dictionary of Scientific Biography* VI (New York, 1972), pp. 198-99 (reprinted in Sabra, *Optics, Astronomy and Logic Studies in Arabic Science and Philosophy*, Variorum, Aldershot, 1994, no. II); George Saliba, "Arabic Planetary Theories after the eleventh century AD" in R. Rashed and R. Morelon (eds.), *Encyclopedia of the History of Arabic Science*, Vol. I (London & New York, 1996), pp. 74-82. See also Sabra, "An eleventh-century refutation of Ptolemy's planetary theory", in *Science and History: Studies in Honor of Edward Rosen, Studia Copernicana 16*, Wroclaw: Ossolineum, 1978, pp. 117-131 (reprint in Sabra, *Optics* no. XIV). This latter paper contains an English translation of Ibn al-Haytham's general criticism of the five planetary models but omits the passage which interests me here.

[13] Ibn Rushd does not mention Ibn al-Haytham's criticism in the first part of his *Mukhtaṣar al-Majisṭī* which I have been able to read in the unpublished French translation by Juliane Lay, sent generously to me by the author. We find in the *Mukhtaṣar* frequent references to Jābir's commentary, including his criticism of the Ptolemaic method for determining the position of the apse line of Mercury and Venus - on this, see below §3.

Ptolemy's method to determine the position of the centre of the eccentre of Mercury and Venus as they appear in the Almagest IX,9 (Mercury) and X,3 (Venus).[14] This method, according to Ibn al-Haytham, is invalid (*fāsid*). These centres are determined on the basis of two observations of the maximum morning and evening elongations of each planet from the mean sun whose position coincides with that of the centre of the planet's epicycle (!) according to what he [i.e. Ptolemy] asserted:

> *Wa dhālika anna-hu istakhraja kull wāḥid min hādhayn al-markazayn bi-raṣadayn li 'l-kawkab ṣabāḥī wa-masā'ī kāna al-kawkab fī kull wāḥid min-humā fī ghāyat buᶜdi-hi min mawḍiᶜ al-shams al-wasaṭ alladhī huwa markaz falak al-tadwīr ᶜalā mā qarrara-hu.*[15]

This is Ibn al-Haytham's main error in this passage and it justifies Ibn Bājja's assertion that his knowledge of Ptolemaic astronomy was, apparently, superficial: in the *Almagest* the mean motions of the centre of the epicycles of the inferior planets are the same as that of the Sun, but, although the eccentricity of Venus (1;15^p) is half that of the Sun (2;30^p), that of Mercury (3^p) bears no relation to the solar eccentricity and the position of the three apogees is independent: the solar apogee is placed at 65;30° from the vernal equinox and it is fixed, while that of Venus is 55° and that of Mercury 190°, both moving at the same rate as the precession of the equinoxes. In neither case, therefore, can one assert - as Ibn al-Haytham says repeatedly - that the longitude of the centre of the planetary epicycle coincides with that of the mean sun, the only obvious thing being (see below figs. 3 and 6) that the line connecting the centre of the earth and the mean sun is parallel to the line connecting the equant and the centre of the planetary epicycle. It is true, however, that Islamic astronomers, from the ninth century onwards, were influenced by Hindu-Iranian astronomy

[14] Ibn al-Haytham, *Shukūk* ed. ᶜAbd al-Ḥamīd Ṣabra and Nabīl al-Shāhābī, Cairo, 1971, pp. 29-32. The corresponding chapters of the *Almagest* can be read in G.J. Toomer's translation: *Ptolemy's Almagest*, Springer Verlag, New York etc., 1984, pp. 456-460, 472-474.

[15] *Shukūk* ed. Ṣabra & Shāhābī pp. 29-30.

- represented, for example, by the *Zīj al-Shāh* - and introduced an important modification in the Ptolemaic model for Venus: in the Islamic tradition the apogee of the Sun is the same as that of Venus and both are subjected to the motion of precession.[16] It is understandable, therefore, that Ibn al-Haytham identifies the position of the centre of the epicycle of Venus with the mean Sun, although this cannot be considered Ptolemaic. This identification, however, does not seem to have any precedent for the case of Mercury.

I will now return to Ibn al-Haytham's text: he mentions two maximum elongations of Mercury, which are 26° (evening) and 20;15° (morning), the distance of the mean sun from the planet's apogee - which has not moved significantly in the period of time elapsed between the two observations - being, in both cases, 90°. He is, therefore, alluding to

1) an observation, made on the evening of the 4th of July 130 A.D. by a certain Theon, which yielded a maximum elongation of 26;15° (not 26° as in Ibn al-Haytham's text), the mean Sun being at 100;5°,

2) his own observation made at dawn of the 4/5th July 139 A.D. which established a maximum elongation of 20;15°[17], the mean sun being at 100;20° (*Alm.* IX, 9). As Mercury's apogee is, in Ptolemy's determination, at 190° from the vernal point, the distance between the apogee and the mean sun is, approximately, 90°.

[16] B.R. Goldstein and F.W. Sawyer, "Remarks on Ptolemy's equant model in Islamic astronomy. Appendix: On Ptolemy's determination of the apsidal line for Venus", in Y. Maeyama and W.G. Saltzer, *Prismata. Naturwissenschaft-geschichtliche Studien. Festschrift für Willy Hartner* (Wiesbaden, 1977), 165-181.

[17] Toomer, *Almagest* p. 456.

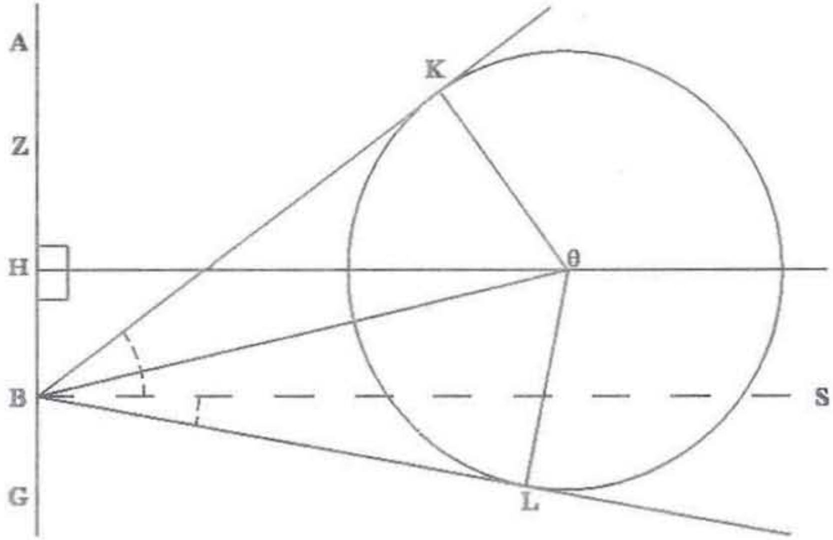

Figure 1

These observations are represented by Ptolemy with Fig. 1[18] in which AG is the apse-line, Z the centre of the eccentre, H the equant point and B the centre of the ecliptic, Θ being the centre of the epicycle. HΘ is perpendicular to AG for, in both observations, the mean Sun (S) is located at an angular distance of 90° from Mercury's apogee. Angles ∠SBL and ∠SBK correspond to the two maximum morning and evening elongations of the planet from the mean Sun and, obviously:

$$\angle SBL + \angle SBK = \angle \Theta BL + \angle \Theta BK$$

[18] Toomer, *Almagest* p. 457, fig. 9.6.

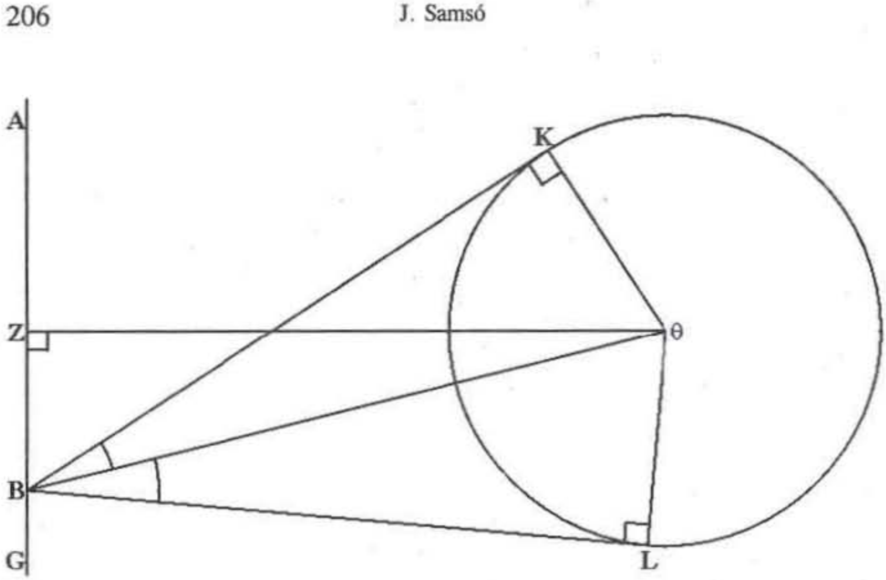

Figure 2

Fig. 2 is an attempt to reconstruct a figure which corresponds to Ibn al-Haytham's description. In it we find an important modification: the equant point (H) has disappeared from his figure and mean motion is measured from the centre of the eccentre (Z). From Z - not from the equant point as in the *Almagest* - we draw ZΘ perpendicular to AG, Θ being the centre of the epicycle in both observations[19]. Then we draw BL, BK - both tangent to the epicycle - and BΘ. The two maximum elongations correspond, according to Ibn al-Haytham, to the angle subtended by two lines: one is BΘ, which joins B, the centre of vision (*markaz al-baṣar*), with Θ, the centre of the planet's epicycle (which he identifies with the mean Sun), while the other is either BK or BL, the lines drawn from the centre of vision which are tangent to the epicycle. If the two maximum

[19] Ibn al-Haytham's text never mentions the equant point and the perpendicular is clearly drawn from the centre of the eccentre: *Wa akhraja min hādha al-markaz, aʿnī markaz al-falak al-khārij al-markaz, ʿamūd^un ʿalā al-quṭr, wa-faraḍa markaz falak al-tadwīr nuqṭat min hādhā al-khaṭṭ* (ed. Sabra & Shahābī p. 31).

morning and evening elongations are different, the corresponding angles (∠ ΘBL and ∠ ΘBK) are also different, which implies that the two centres of the epicycles - let us call them Θ and Θ' - are at different distances from the centre of vision (B) at the times of the two observations and that the two lines tangent to the epicycles (BK and BL) have different lengths. These implications contradict the fact that, in the two aforementioned observations, the positions of the centres of the two epicycles were equal: the two lines (BΘ and BΘ', as well as BK and BL) should also be equal in both observations, and the same should happen with the lines joining the centres of the two epicycles and the points of tangency (ΘK and ΘL), and with the two angles subtended by the two radii of the epicycle (∠ ΘBK and ∠ ΘBL). Ibn al-Haytham continues with this line of argument assuming that if the centres of the two epicycles are placed, in both cases, at the end of a diameter of the eccentre perpendicular to the apse line, the two maximum and evening elongations should be equal. Ptolemy claims that the addition of the two angles ∠ ΘBK and ∠ ΘBL is equal to the addition of the maximum morning and evening elongations, something which, according to Ibn al-Haytham, is clearly absurd (*muḥāl ẓāhir al-istiḥāla*). As Ptolemy's observations do not yield the equality of angles ∠ ΘBK and ∠ ΘBL, point Θ may be the position of the centre of the epicycle in one of the two observations but not in both of them. Consequently the method used by Ptolemy is invalid, and the eccentricity obtained cannot be correct. The same remarks can be applied to the case of Venus. This implies that not all computations based on the Ptolemaic parameters for the eccentricities of Mercury and Venus are reliable, and explains the frequent divergences between computed and observed positions of these two planets. The paragraph ends with the expression of Ibn al-Haytham's lack of confidence in Ptolemy's planetary models as a whole.[20]

It is unnecessary to stress the fact that Ibn al-Haytham's criticism lies in the identification of point Θ (centre of the epicycle) with the mean Sun. As a result of his confusion he misses the role played by the equation of the centre, which justifies the difference between the maximum morning and

[20] For a complete English translation of this passage see Voss, *Ibn al-Haytham's Doubts* pp. 47-52; commentary pp. 126-131.

evening elongations from the same position of the mean Sun. This is something which will be clarified by Jābir b. Aflaḥ. I have no explanation for such a mistake which I - like Ibn Bājja - find most surprising in a scientist of the category of Ibn al-Haytham[21].

3. Jābir b. Aflaḥ on the Ptolemaic determination of Mercury's apogee

Jābir b. Aflaḥ deals with the planets in book VII of his *Iṣlāḥ al-Majisṭī*[22]. This is the famous book in which he criticises the Ptolemaic order of planetary spheres and gives an interesting - though impractical - solution to the problem of determining the apse line and the eccentricity of the superior planets[23]. In this latter instance, as in the one I am going to

[21] Recent scholarship has discussed the possibility of the existence of one or two Ibn al-Haythams: see Roshdi Rashed, *Les mathématiques infinitésimales du IXe au XIe siècle. Ibn al-Haytham. Vol. II*, Al-Furqān. Islamic Heritage Foundation, London, 1993, pp. 1-28, 489-538; A.I. Sabra, "One Ibn al-Haytham or two? An exercise in reading the bio-bibliographical sources", *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 12 (1998), 1-50. In spite of this, both authors seem to agree in attributing the *Shukūk* to al-Ḥasan ibn al-Haytham, the author of the *Kitāb al-Manāẓir*.

[22] I am only using MS Escorial Ar. 910 fols. 78 v - 99 v. On Jābir see Richard Lorch, "The Astronomy of Jābir ibn Aflaḥ", *Centaurus* 19 (1975), 85-107; Lorch, "The Astronomical Instruments of Jābir ibn Aflaḥ and the Torquetum", *Centaurus* 20 (1976), 11-34. Both papers have been reprinted in Lorch, *Arabic Mathematical Sciences. Instruments, Texts, Transmission*, Variorum, Aldershot, 1995 (items VI and XVI). This latter volume contains two previously unpublished papers by Lorch: "The Manuscripts of Jābir's Treatise" (no. VII) and "Jābir ibn Aflaḥ and the Establishment of Trigonometry in the West" (no. VIII). A complete list of Arabic, Hebrew and Latin manuscripts of Jābir's *Iṣlāḥ* can be found in the aforementioned book by Lorch VI, pp. 88-94 and VII, pp. 1-2.

[23] N. Swerdlow, "Jābir ibn Aflaḥ's interesting method for finding the eccentricities and direction of the apsidal line of a superior planet" in D.A. King and G. Saliba (eds.), *From Deferent to Equant. A Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honor of E.S. Kennedy*, New York, 1987 (= *Annals of the New York Academy of Sciences* vol. 500), pp. 501-512; H. Hugonnard-Roche, "La théorie astronomique selon Jābir ibn Aflaḥ", in G. Swarup, A.K. Bag and K.S. Shukla, *History of Oriental Astronomy. Proceedings of an International Astronomical Union Colloquium no. 91*, Cambridge, 1987, pp. 207-208.

consider here, Jābir's criticism of Ptolemy is that of a teacher of Mathematics who considers that Ptolemy has assumed, without proof, the bisection of planetary eccentricity and considers the iterative method used by the Greek astronomer as an approximation which starts by considering that the centre of the deferent and the centre of the equant are the same point. Mathematical precision and proof seems to be Jābir's maximum aspiration, and reading the *Iṣlāḥ* leads me to believe that he was not, in any way, a practical astronomer and that he probably never made a single observation.

I will consider here Jābir's interesting criticism of the method used by Ptolemy to determine the position of the apogees of the inferior planets. He makes no reference to previous work done by either Eastern or Western Islamic astronomers. Here, as elsewhere in Jābir's book, our author seems to be unaware of any of the results obtained by those who lived and worked after Ptolemy. His purpose is at all times to present his own rewriting of the *Almagest*.

Like Ptolemy (*Almagest* IX, 6) he begins by proving that when the centre of the epicycle is placed symmetrically on either side of the apse line, the planet being - in both positions - also symmetrically on either side of the apogee of the epicycle, the angles corresponding to the equations of the centre and to the equations of anomaly will have, in both cases, the same absolute value. Therefore in Fig. 3 (which corresponds to Ptolemy's proof for the case of the superior planets and Venus)[24]:

AG is the apse line, A being the apogee,

E is the centre of the eccentre,

H is the centre of the equant,

Z is the centre of the ecliptic,

Epicycles CL (with centre at B) and NM (with centre at D) correspond to two positions of the centre of the epicycle such that $\angle BHA = \angle DHA$,

In Jābir's text L and M are two positions of the planet on epicycles CL and MN such that $\angle CBL = \angle NDM$, placed on symmetric sides of the apogee of the epicycle. On this point, Jābir's formulation is more general than that of Ptolemy for, in the *Almagest*, M and L are placed in the

[24] *Almagest* IX,6, trans. Toomer pp. 445-447; *Iṣlāḥ* MS Escorial fols. 83 r and v.

positions in which the planet attains its maximum elongation from the centre of the epicycle, so that ZL and ZM are tangents to the epicycle and BL and DM are perpendicular to ZL and ZM.

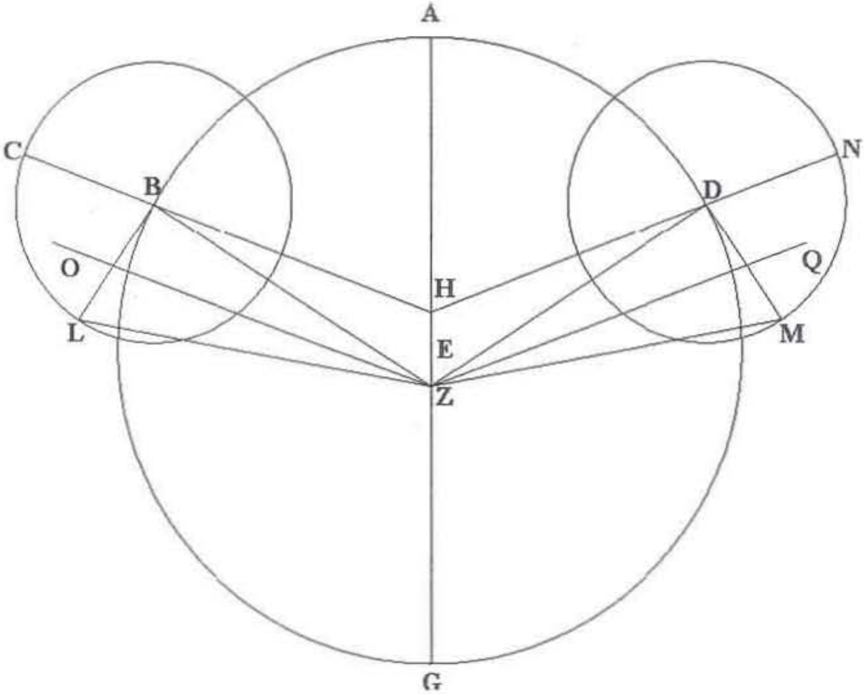

Figure 3

Both Ptolemy and Jābir seek to prove that, in the situation described, $\angle ZBH = \angle ZDH$ (equation of the centre) and $\angle BZL = \angle DZM$ (equation of the anomaly).

Jābir's proof follows that of the *Almagest*, with only trivial variants: he begins by stating, without proof, that DH = BH[25] and, consequently, the

[25] There is a proof in the *Almagest* which explains why Ptolemy's figure is slightly more complicated than Jābir's.

equalities of triangles $\triangle DZH = \triangle BZH$ and, therefore, that $\angle ZBH = \angle ZDH$.

In the second stage Jābir demonstrates the equality of triangles $\triangle BZL$ and $\triangle DZM$ due to the fact that $\angle ZBH = \angle ZDH$, and that $\angle CBL = \angle NDM$, from which he deduces that $\triangle LBZ = \triangle MDZ$; he has also proved that $BZ = DZ$ and, obviously, $BL = DM$. Therefore $\angle BZL = \angle DZM$. Ptolemy's proof for this stage is slightly simpler for he assumes that $\angle BLZ$ and $\angle DMZ$ are right angles.

Jābir, however, proceeds one step further, for he draws lines ZO (parallel to HB) and ZQ (parallel to HD) and states that $\angle BZO = \angle DZQ$ and that $\angle OZL = \angle QZM$. No proof is given but it is easy to see that

$\angle BZO = \angle AZO - \angle AZB$

$\angle DZQ = \angle AZQ - \angle AZD$

From which we have $\angle BZO = \angle DZQ$ for

$\angle AZB = \angle AZD$, due to the equality of triangles $\angle ZBH = \angle ZDH$. And

$\angle AZO = \angle AHB$

$\angle AZQ = \angle AHD$

$\angle AHB = \angle AHD$ by hypothesis

Therefore $\angle BZO = \angle DZQ$.

As for the equality $\angle OZL = \angle QZM$, we have that

$\angle OZL = \angle BZL - \angle BZO$

$\angle QZM = \angle DZM - \angle DZQ$

He has already proved that $\angle BZL = \angle DZM$ and we have also seen that $\angle BZO = \angle DZQ$.

Once Jābir has established this, he states that, in the case of Venus, straight lines ZO and ZQ link the centre of the Universe with the position of the mean Sun. Therefore, in this configuration, the two distances between Venus and the mean Sun will be equal, a phenomenon that can also be applied to the maximum elongations of the planet from the mean Sun.

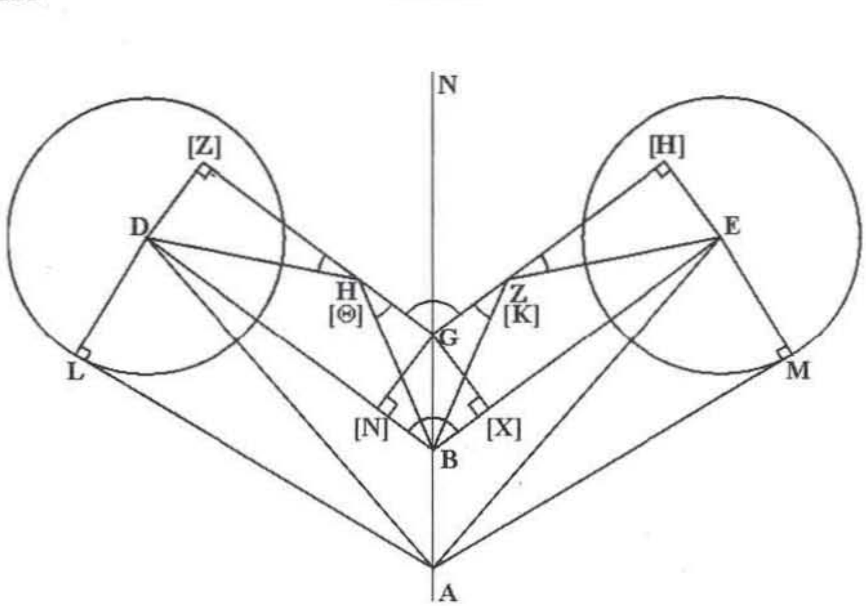

Figure 4

These trivial differences between the *Almagest* and the *Iṣlāḥ* are not so trivial in the case of Mercury[26]. Fig. 4 corresponds to the *Almagest*, and Fig. 5 is that of the *Iṣlāḥ*. In Fig. 4 letters in square brackets are those of the *Almagest* which do not appear in the figure of the *Iṣlāḥ*: for the points which are common to both figures I have added to the letters in square brackets the letters used by Jābir in Fig. 5.

In both figures:

A is the centre of the Universe

B is the centre of the equant

G is the centre of the small circle in which the centre of Mercury's deferent rotates (the *mudīr*).

DL and EM correspond to two positions of Mercury's epicycle such that ∠ DBG = ∠ EBG

L and M are symmetrical on both sides of the apogee of the epicycle. In

[26] See *Almagest* IX, 6 (Toomer pp. 447-448); *Iṣlāḥ* fols. 83 v - 84 v.

Fig. 4, L and M correspond to maximum elongations from the centre of the epicycle and, therefore, AL and AM are perpendicular to the epicycle radii DL and EM.

GH = GZ = GB = BA and

∠ NGH = ∠ NGZ = ∠ DBG = ∠ EBG

Z, therefore, is the centre of Mercury's deferent when the centre of the epicycle is in D and H is the centre of the deferent when E is the centre of the epicycle.

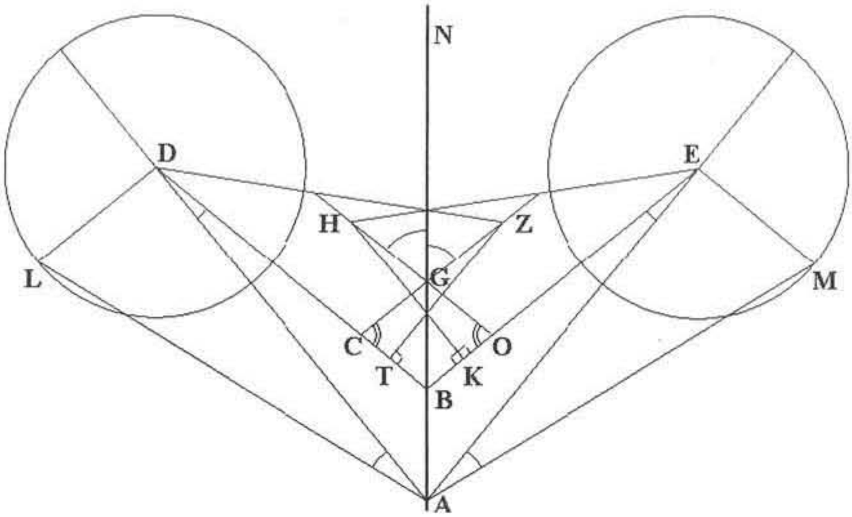

Figure 5

Both Ptolemy and Jābir seek to prove that

∠ ADB = ∠ AEB (equation of the centre)

∠ DAL = ∠ EAM (equation of the anomaly; in Fig. 4 it is also the maximum elongation of Mercury from the centre of the epicycle).

Ptolemy (Fig. 4) drops the perpendiculars:

G[N] and G[X] to BD and BE

D[Z] and E[H] to G[Z] and G[H]

He then begins by proving that right-angled triangles △GB[N] and △GB[X] are equal for they have a common side (GB) and ∠GB[N] =

∠ GB[X] by hypothesis. Therefore G[N] = G[X].

As G[N][Z]D and G[X][H]E are rectangles, D[Z] = G[N] = G[X] = [H]E. Ptolemy assumes, then, that right-angled triangles △ HD[Z] and △ ZE[H] are equal because D[Z] = E[H] and HD = ZE. The latter equality is given without proof;[27] this is the basis of Jābir's criticism and the cause of his using a different approach to reach the same conclusion.

Triangles △HGB and △ZGB are also equal because they have a common side (GB), GH = GZ (by hypothesis) and ∠HGB = ∠ZGB (by hypothesis). Therefore ∠GHB = ∠GZB and, as he has just proved that ∠DH[Z] = ∠EZ[H], it is now obvious that ∠DHB = ∠BZE.

Ptolemy now deals with the equality of triangles △HDB and △ZEB in which we can see that

∠HBD = ∠GBD - ∠GBH

∠ZBE = ∠GBE - ∠GBZ

As ∠GBD = ∠GBE (by hypothesis)

∠GBH = ∠GBZ (because he has just proved the equality of triangles △HGB and △ZGB)

Therefore ∠HBD = ∠ZBE

As he has also established that ∠DHB = ∠BZE and has stated, without proof, that DH = ZE, he now assumes that BD = BE.

This allows him to prove the equality of triangles △BAD and △BAE which have

Side BA in common

∠DBA = ∠EBA (by hypothesis)

BD = BE

Therefore ∠ADB = ∠AEB (equation of the centre)

and AD = AE

Finally, right-angled triangles △ADL and △AEM are equal because DL = EM and, consequently ∠DAL = ∠EAM (maximum elongation of the planet from the centre of the epicycle).

Jābir's proof is different (Fig. 5). He joins ZD and HE (deferent radii

[27] Toomer (*Almagest* p. 448 n. 50) remarks: "Although one can see that this must be so by symmetry, the proof is quite intricate".

which correspond respectively to D and E centres of the epicycle), and extends ZG and HG until they intersect BD and BE at points C and O. From Z and H he also drops perpendiculars ZT (to BD) and HK (to BE).

As ∠ ZGN = ∠ GBC (by hypothesis), ∠ GBC = ∠ BGC and CB = GC in triangle △GBC. We can also prove, in the same way, that GO = BO [and that △GBO = △GBC (their angles are respectively equal and side GB is common to both triangles), GO being, therefore, equal to GC].

Jābir now proves that right-angled triangles △ZTC and △HKO are equal, because:

OH = CZ

[OH = OG + GH

CZ = GC + ZG

and we have just proved that OG = GC, while GH = ZG (by hyp.)]

∠ ZCT = ∠ HOK (△GBO = △GBC)

Therefore ZT = HK

and BK = BT

[BK = OB - KO

BT = CB - CT

and OB = CB, KO = CT]

[He then considers right-angled triangles △DZT = △EHK in which] DZ = HE (both are deferent radii) and ZT = HK. Therefore DT = EK. As he has already proved that BK = BT, BD = BE.

Jābir's demonstration now joins Ptolemy by proving (as in the *Almagest*) that △BAD and △BAE are equal and, therefore, that ∠ ADB = ∠ AEB (equation of the centre) and AD = AE. Finally he proves that △ADL = △AEM because DL = EM and ∠ ADL = ∠ AEM (by hyp.). Therefore ∠ DAL = ∠ EAM, *q.e.d.*

Jābir has shown a certain degree of ability by avoiding the need to prove that (in Fig. 4) HD = ZE. As a matter of fact he states[28] that, here, Ptolemy makes the mistake of considering that HD and ZE are two radii of the deferent when the centres of the epicycles are, respectively, in D and E. He could not have said, otherwise, that HD = ZE, an equality which

[28] Jābir, *Iṣlāḥ* fol. 84 v.

can only be proved if we previously demonstrate that DB = EB. To this he adds (omitting the unnecessary proof) that, as he has shown for Venus, the two distances of Mercury from the mean Sun are necessarily equal.

This first remark on Ptolemy's determination of Mercury's apogee is characteristic of Jābir's standard attitude of criticising the *Almagest* on account of what we might call his "mathematical scruples". The important part of his argument appears later[29]: Ptolemy has proved that two maximum and opposite (i.e. morning and evening) elongations of the planet from the mean Sun, which take place symmetrically in relation to Mercury's apogee, are necessarily equal. The author of the *Almagest* claims, however, that the reciprocal formulation is also true: two equal maximum and opposite elongations of the planet from the mean Sun will necessarily take place symmetrically on both sides of the apogee.[30] Jābir states repeatedly that this is not true and that a planet may have many equal maximum morning and evening elongations from the mean Sun without the planet's apogee being at the midpoint between the two positions of the centre of the epicycle.

Jābir's argument can be better explained with Fig. 6 (not in the manuscript) which corresponds to a Ptolemaic standard planetary model, like that of Venus: A is the apogee, E the centre of the equant, C the centre of the deferent and T the centre of the Universe. D_1 and D_2 are two positions of the centre of the epicycle on both sides of the apse line, P_1 and P_2 the two positions of the planet at the moment of two maximum morning elongations from the mean Sun (S_{m1} and S_{m2}). Jābir states correctly that the maximum elongation of the planet from the mean Sun ($\angle P_1TS_{m1}$ or $\angle P_2TS_{m2}$) will be equal to the angle subtended by the radius of the epicycle ($\angle r = \angle P_1TD_1$ or $\angle r = \angle P_2TD_2$) plus or minus the equation of the centre ($\eta = \angle ED_1T = \angle D_1TS_{m1}$, or $\eta = \angle ED_2T = \angle D_2TS_{m2}$). Obviously at the apogee or perigee of the deferent $\eta = 0$ and the maximum

[29] Jābir, *Iṣlāḥ* fols. 84 v - 85 r.

[30] See the interesting remarks made by Sawyer in Goldstein & Sawyer, "Remarks on Ptolemy's equant model" pp. 169-173. His observations for Venus can also be applied to the case of Mercury.

elongation will be equal to ∠ r.

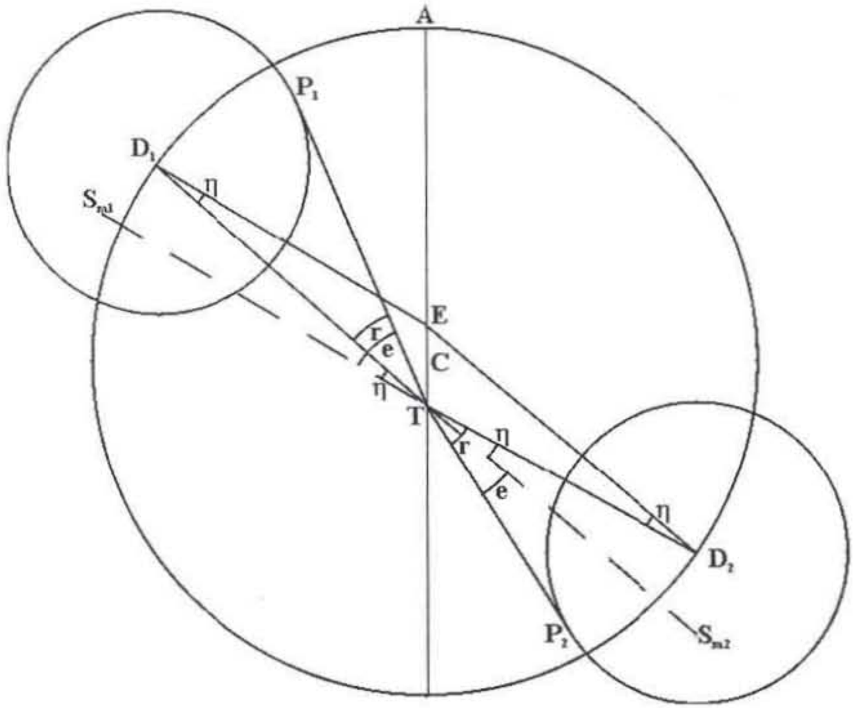

Figure 6

When the centre of the epicycle moves between the apogee and the first mean distance (in which TD = 60^p) the maximum morning elongation increases because both ∠ r and η increase, and η has to be added to ∠ r. In the same way, when the centre of the epicycle moves between the perigee and the second mean distance, the maximum morning elongation decreases because ∠ r decreases, while η (which has to be subtracted from ∠ r) increases. According to Jābir there is an unlimited number of morning elongations (*abʿād ṣabāḥiyya ghayr mutanāhiya fī 'l-ʿidda*) which have another equal morning elongation on the other side of the apse-line. If we divide the planet's deferent into four unequal quadrants determined by the apse-line and the line which joins the first and second mean distances we will have Fig. 7 in which A is the apogee, G the perigee, B and D the first and second mean distances. Each maximum morning elongation in AB will have an equal maximum evening elongation in GD (and the same can be said of BG and DA): in such cases the apogee will not be placed in the midpoint between the two positions of the centre of the epicycle. According to Jābir, in the case of Mercury Ptolemy established, by observation, that:

- 19;3° is the maximum morning and evening elongation when the centre of the epicycle is in A (apogee),
- 23;15° is the maximum morning and evening elongation when the centre of the epicycle is in G (perigee),
- 26;15° is the maximum morning elongation in B (first mean distance),
- 20;15° is the maximum evening elongation in B,
- 20;15° is the maximum morning elongation in D (second mean distance),
- 26;15° is the maximum evening elongation in D.

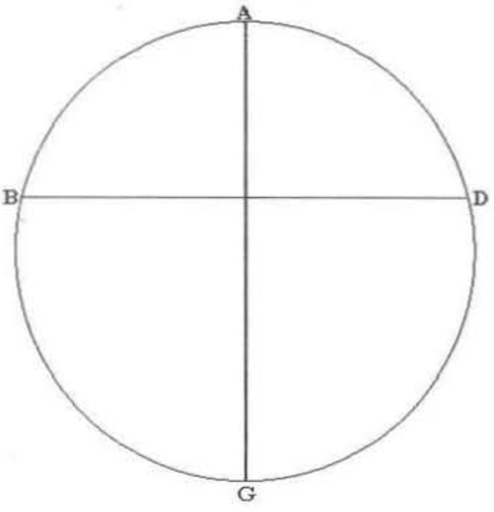

Figure 7

The aforementioned values correspond to the following four observations quoted in the *Almagest* ([3] and [4] are the same ones quoted by Ibn al-

Haytham):

Almagest	Date	Long. of Mercury	Mean solar long.	Max. elongation	Remarks
[o.1] IX,8 (Toomer 454)	2/3 Oct 134	170;12°	189;15°	19;3°	Morning
[o.2] IX,8 (Toomer 454)	5 Apr 135	34;20°	11;5°	23;15°	Evening
[o.3] IX,9 (Toomer 456)	4 Jul 130	126;20°	100;5°	26;15°	Evening
[o.4] IX,9 (Toomer 456)	4/5 Jul 139	80;5°	100;20°	20;15°	Morning

Before going further, I should make a remark: 19;3° and 23;15° correspond approximately to Mercury's maximum elongations from the mean Sun when the latter is in Ptolemy's apogee (190°) and perigee (10°), but the approximation does not seem so good for the mean distances: although the computation of the mean distance in the case of Mercury is not straightforward, Pedersen has calculated that it should be placed at 67;45° from the apogee[31]. 100;5° and 100;20° for the mean solar longitude correspond to the second mean distance which should be 190° - 67;45° = 122;15°.

Jābir also gives an equivalent list for Venus:
- 44;48° is the maximum morning and evening elongation in A,
- 47;20° is the maximum morning and evening elongation in G,
- 48;20° is the maximum morning elongation in B,
- 43;35° is the maximum evening elongation in B,
- 43;35° is the maximum morning elongation in D,
- 48;20° is the maximum evening elongation in D.

These figures correspond to

[31] O. Pedersen, *A Survey of the Almagest*, Odense, 1974, pp. 325-326; O. Neugebauer, *A History of Ancient Mathematical Astronomy*, Berlin, Heidelberg, New York, 1975, I, p. 169.

Almagest	Date	Long. of Venus	Mean solar long.	Max. elongation	Remarks
[o.5] X,2 (Toomer 471)	19/20 May 129	10;36°	55;24°	44;48°	Morning
[o.6] X,2 (Toomer 471)	18/19 Nov. 136	282;50°	235;30°	47;20°	Evening
[o.7] X,3 (Toomer 473)	18/19 Feb. 140	13;50°	325;30°	48;20°	Evening
[o.8] X,3 (Toomer 472-473)	17/18 Feb. 134	281;55°	325;30°	43;35°	Morning

As the longitude of the apogee of Venus established by Ptolemy is 55°, it is quite reasonable that Jābir should ascribe the maximum elongations corresponding to [o.5] and [o.6] to the planet's apogee and perigee respectively. The mean distance should be reached at 91;47° on both sides of the apogee[32]: therefore it should be at 55° + 91;47° = 146;47° and 360° + 55° - 91;47° = 323;13°, which agrees well with Ptolemy's maximum elongations in [o.7] and [o.8].

Jābir's analysis of these data allows him to exemplify his ideas on the subject. In the case of Mercury, the morning elongation of the planet increases between A (19°, [o,1]) and B (26°, assumed in [o.3]), and decreases between B (26°, assumed in [o.3]) and G (23°, assumed in [o.2]). Therefore we will be able to find an unlimited number of maximum morning elongations between A and B identical to other maximum morning elongations between B and G. In the same way, the maximum morning elongations decrease from G (23°, assumed in [o.2]) to D (20°, [o.4]) and we will also find another number of maximum morning elongations in this sector equal to others which take place between A and B. As each of these morning elongations will have an equal maximum evening elongation on the other side of the apse-line, one should be very careful when selecting the observations in order to establish in which of the four sectors each observation takes place, in order to determine accurately the position of the apogee.

[32] See Pedersen, *Survey* p. 293.

For that purpose Jābir states the following criteria[33]:

- Between A and B the morning elongations (e_m) increase clearly, for

$$e_m = \angle r + \eta \ [1]$$

and this is a sector in which both $\angle r$ and η increase[34].

This increase is not so conspicuous in the case of evening elongations (e_e) for

$$e_e = \angle r - \eta \ [2]$$

- Between B and G, [1] and [2] are still valid: as $\angle r$ increases and η decreases with the motion of the centre of the epicycle towards the perigee, e_m will not increase substantially, while the growth of e_e will be clear.

- Between G and D we will have

$$e_m = \angle r - \eta \ [3]$$

$$e_e = \angle r + \eta \ [4]$$

As $\angle r$ will decrease while η increases with the progression of the centre of the epicycle from the perigee to the second mean distance, e_m will suffer a clear decrease, while e_e will not vary perceptibly.

- Between D and A both $\angle r$ and η diminish and, applying [3] and [4] again, we will conclude that e_m does not vary much, while e_e is notably reduced.

As a consequence, Jābir recommends making two observations of maximum morning elongations at two points of the ecliptic near to each other. If the second observation shows a clear increase of the maximum

33 Jābir, *Iṣlāḥ* fol. 86 r.

34 This implies that mean distances (points B and D) should coincide, approximately, with the positions of the centre of the epicycle for which the equation of the centre reaches its maximum. This is clearly not the case for Mercury. Mean distances, as we have seen, are 67;45° and 122;15°. In the tables of equations for Mercury of the *Almagest* (XI, 11, Toomer p. 553) the maximum equation of the centre is reached for an argument comprised between 90° and 96°, 264° and 270°.

elongation when compared to the first one, we may be sure that we are in sector AB. These two observations should be coupled with another two, corresponding to maximum evening elongations, in sector DA. Here the second observation should show a clear decrease of e_e in relation to the first one. Another possibility is to select two evening elongations which show a clear increase (sector BG) and two morning elongations which diminish (sector GD). In the first case half the distance between the two mean positions of the Sun will give us the planet's apogee, while in the second case we will obtain the perigee.

It is important, therefore, to make a good selection of the observations used to determine the position of the planet's apogee. A maximum elongation will be well selected (*mukhtār*) when its value increases or decreases rapidly in relation to a previous observation of the same kind. Without going into details, Jābir refers now (fol. 87 v) to an analysis of the observations used by Ptolemy to establish the longitude of the apogee in *Almagest* IX,7:

Almagest	Date	Long. of Mercury	Mean solar long.	Max. elongation	Remarks
[o.9] IX,7 (Toomer 449)	2/3 Feb. 132	331°	309;45°	21;15°	Evening
[o.10] IX,7 (Toomer 449)	3/4 June 134	48;45°	70°	21;15°	Morning
[o.11] IX,7 (Toomer 449-450)	4/5 June 138	97°	70;30°	26;30°	Evening
[o.12] IX,7 (Toomer 450)	1/2 Feb. 141	283;30°	310°	26;30°	Morning

Assuming a Ptolemaic apogee at 190° from Aries and a first mean distance placed at 67;45° from the apogee, the beginnings of the four sectors used by Jābir are:

A: 190°
B: 257;45°
G: 10°
D: 122;15°

and this explains why Jābir considers that the two first observations used by Ptolemy were well selected, for they correspond to a maximum evening elongation in sector BG ([o.9]) and to a maximum morning elongation in GD ([o.10]). The other two are not *mukhtār*, for they are a maximum evening elongation in GD ([o.11]) and a maximum morning elongation in BG ([o.12]). The same criticism can be applied to Ptolemy's selection of observations made in the third century B.C., which are:

Almagest	Date	Long. of Mercury	Mean solar long.	Max. elongation	Remarks
[o.13] IX,7 (Toomer 450)	11/12 Feb. -261	292;20°	318;10°	25;50°	Morning
[o. 14] IX,7 (Toomer 450-451)	25/26 April -261	53;40°	29;30°	24;10°	Evening
[o. 15] IX,7 (Toomer 451-452)	28/29 May -256	89;20°	62;50°	26;30°	Evening
[o.16] IX,7 (Toomer 452)	23 Aug. -261	169;30°	147;50°	21;40°	Evening
[o.17] IX,7 (Toomer 452)	29/30 Oct. -236	194;10°	215;10°	21°	Morning
[o.18] IX,7 (Toomer 452-453)	18/19 Nov. -244	212;20°	234;50°	22;30°	Morning

These six observations are used by Ptolemy in two sets of three: a maximum evening elongation of 25;50° is obtained by interpolation from [o.14] and [o.15], to match observation [o.13]. In the same way a maximum morning elongation of 21;40° is the result of an interpolation betweeen [o.17] and [o.18] to match the evening elongation of [o.16]. As the apogee obtained from these observations is 186°, the limits of the four sectors will be, in this case:

A: 186°
B: 253;45°
G: 6°
D: 118;15°

Jābir only refers to four observations out of the six mentioned by Ptolemy ([o.13]-[o.18]), for he seems to consider the two couples used to obtain a result by interpolation as two real observations. He states that two of them are well selected as they are a morning elongation in sector AB (this applies to [o.17] and [o.18]) and an evening elongation in DA ([o.16]). The two others, however, are not *mukhtār*, for they are a morning elongation in BG ([o.13]) and an evening elongation in GD ([o.14] and [o.15]).

Actually, the results obtained by Ptolemy's use of two pairs of observations in two opposite sectors do not confirm the method Jābir advocates: between [o.17] and [o.18] the morning elongation increases clearly from 21° to 22;30°, but an equivalent increase - taking into account the increment in the mean solar longitude - of the evening elongation from 24;10° to 26;30° also appears between [o.14] and [o.15]. Jābir does not say a word about this.

Jābir makes similar criticisms of Ptolemy's choice of the observations leading to the determination of the apogee of Venus. His conclusion is quite harsh: Ptolemy "has no feeling of the [real] implication of these elongations" (*inna-hu lam yashᶜur bi shay' min hādhihi 'l-maᶜānī al-lāḥiqa fī hādhihi 'l-abᶜād*, fol. 87 v). Ptolemy's knowledge of the subject was mistaken, but the error he made led him to a correct conclusion (*fa-kāna ᶜilmu-hu dhālika khaṭaᵓan addā ilā ṣawāb*, fol. 88 r), although his determination of the apogees of Mercury and Venus was only "by accident and not by the nature of the thing itself" (*wa-kāna wujūdu-hu li-mawḍiᶜ al-buᶜd al-abᶜad bi 'l-ᶜaraḍ, lā bi 'l-dhāt*, fol. 88 r). A final casual remark is absolutely correct: Ptolemy generalised, in a rather abusive way, his conclusions about the displacement of Mercury's apogee with a velocity of 1° per century (from 186°, ca. -250, to 190°, ca. 140) to an equivalent displacement of the apogee of Venus (and of the other planets).

In brief, this interesting set of remarks made by Jābir show his mathematical ability and confirm what we already know through other indirect sources, i.e. that there was a certain awareness of the existence of an error in Ptolemy's determination of the longitude of Mercury's apogee. Jābir's criticisms have a few points in common with modern analysis of the same topic[35]. He does not offer us, however, a new set of observations or an attempt to analyse critically those made or mentioned by Ptolemy, except for a few general remarks which he does not apply to the data available to him.

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[35] See Sawyer's paper mentioned above.

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