Ibn Muʿādh on the Astrological Rays[1]

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1. Introduction

The doctrine of the astrological *aspects*, or distances between two celestial objects such as planets is sometimes defined in terms of these objects emanating rays in certain directions of astrological significance[2]. If one of these rays reaches an object at a particular angular distance, the two objects could be in conjunction (0°), sextile (60°), quartile (90°), trine (120°) or opposition (180°). This would be a simple theory if one measured these angular distances along the ecliptic, but the procedure was seldom performed this way. Since more intricate solutions gave more prestige to the astrologer who could master them, other methods were preferred, and the problem of casting these rays led to the development of several procedures that yielded a range of results and also involved a varying degree of mathematical ability. For the sake of both complexity and agreement with other astrological systems, like the division of houses or the *tasyīr* (*progressions*), the distances were most usually measured on the equator or along another great circle of the celestial sphere. Thus, the computation of

[1] The present paper has been prepared within a research programme on "Science and Society in the Western Mediterranean in the Late Middle Ages", sponsored by the Spanish Ministry of Education and Science (HUM2004-02511) and FEDER.

[2] On the projection of rays in medieval Islamic astrology see E.S. Kennedy and H. Krikorian-Preisler, "The Astrological Doctrine of Projecting the Rays", *Al-Abḥāth, 25*, 3-15, reprinted in E.S. Kennedy, colleagues and former students, *Studies in the Islamic Exact Sciences* (Beirut, 1983), 372-384; J.P. Hogendijk, "The Mathematical Structure of Two Islamic Astrological Tables for 'Casting the Rays'", *Centaurus, 32* (1989), 171-202.

these *aspects* at least required, first, a projection of the planet's longitude onto this circle and, later, a projection of the *aspected* point back to the ecliptic. Other more elaborate approaches also take ecliptic latitudes into account, but these will not be considered in this paper.

Islamic astronomers dealt with the problem in three general ways. They constructed specific astrolabe plates or instruments, composed tables for given geographical latitudes that made it possible to find the *projection of the rays* as a function of the longitudes of the ascendent and the object that casts the rays, and developed a number of algorithms for a computation of these projections.

As regards these algorithms, a valuable source from medieval Spain is the *Treatise on the projection of rays* (*Risāla fī maṭraḥ al-shu'ā'āt*[3]) of Ibn Mu'ādh al-Jayyānī (d 1093). The only extant text of this work was copied between the 10th and the 20th of March 1303 of the Hispanic Era / 1265 AD and is preserved in ff 71r - 80r of MS Or. 152 at the Biblioteca Medicea Laurenziana in Florence[4], being the third of a group of scientific treatises, and coming after another work of Ibn Mu'ādh: his treatise on trigonometry, the *Book on the Unknown of the Arcs of the Sphere* (*Kitāb majhūlāt qisī al-kura*)[5].

[3] Indeed, the extant text has no explicit title. I call it *Risāla fī maṭraḥ al-shu'ā'āt* because, by the end, one can read *tammat al-risāla*...and the first pragraph begins with the expression *Inna maṭraḥ shi'ā' al-kawākib*..., with the word *shu'ā'āt* as a marginal correction.

[4] This is one of the very few Arabic manuscripts known to have been copied at the Toledan court of Alfonso X of Castile (1252-1284). Its importance was noted for the first time by D.A. King, "Medieval Mechanical Devices", *History of Science, 13* (1975), 288-289: it includes several treatises of great interest and represents a major source for the history of Andalusian technology and science. Nevertheless, while some parts of it have been studied and published, other sections resist investigation and still await a complete analysis. On the contents of the whole MS Or 152 see, for example, J. Samsó, *Las ciencias de los antiguos en al-Andalus* (Madrid, 1992), 252-253; J. Casulleras, "El último capítulo del *Kitāb al-asrār fī natā'iŷ al-afkār*", *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet* (Barcelona, 1996), 613-615, and the references there quoted.

[5] Edition and Spanish translation: M.V. Villuendas, *La trigonometría europea en el siglo XI. Estudio de la obra de Ibn Mu'āḏ. El* Kitāb maŷhūlāt (Barcelona, 1979). See also J. Samsó, *op. cit.*, 139-144 and "Notas sobre la trigonometría esférica de Ibn Mu'āḏ", *Awrāq, 3* (1980), 60-68, reprinted in J. Samsó, *Islamic Astronomy and Medieval Spain* (Variorum,

The *Treatise* on the projection of rays consists of a small monograph dealing with the mathematical aspects of two astrological doctrines: the division of houses, and the projection of rays, starting from the principle that both subjects share the same theoretical basis. On a first reading, the work appears to be a rather disorganized dissertation. Opinions of the author and theoretical principles appear in the treatise in the shape of intermittent ideas scattered about the text[6]. Criticising the astrologers and his contemporaries, Ibn Mu'ādh censures their lack of mathematical skill, analyses the various aspects of the problem, and offers state of the art and purely technical descriptions of the computational algorithms.

In the last years, this Treatise have appealed the attention of historians of science and most of its mathematical contents have been discussed by E.S. Kennedy and J.P. Hogendijk[7]. Kennedy[8] has a summary of the contents of the text, which deals with the passages on the division of houses, and analyses the two algorithms that appear in it for computing the cusps of the houses using the *Prime Vertical Method*[9]: one of them appropriate,

1994), no. VII; M.T. Debarnot, art. "Trigonometry", R. Rashed - R. Morelon (eds.) *Encyclopedia of the History of Arabic Science* (Routledge, 1996), 3 vols., II, 517-521.

[6] This disorganization may not be restricted to this work of Ibn Mu'ādh. His Treatise on trigonometry also offers the aspect of a *patchy work*; cf. M.T. Debarnot, *op. cit.*, 519.

[7] I want to express my indebtedness to Profs Hogendijk and Kennedy who both kindly provided me with precious material. Prof Hogendijk made accessible to me the preliminary version of his unpublished investigation. Prof Kennedy, for his part, spent a great deal of his valuable time explaining his opinions to me, read a particularly obscure chapter of Ibn Mu'ādh's Arabic manuscript (ff 78*v* - 80*r*) and wrote to me explaining his interpretation of the approximate rule given in it.

[8] E.S. Kennedy, "Ibn Mu'ādh on the Astrological Houses", *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften, 9* (Frankfurt am Main, 1994), 153-160 and "The Astrological Houses as defined by Medieval Islamic Astronomers", *From Baghdad to Barcelona. Studies in Honour of Prof. Juan Vernet* (Barcelona, 1996), 535-578 (specially, 543), both reprinted in E.S. Kennedy, *Astronomy and Astrology in the Medieval Islamic World* (Variorum, 1998), nos. XVI and XIX.

[9] The methods for defining the astrological houses used by ancient and medieval astrologers were classified by J.D. North, *Horoscopes and History* (London, 1986), 46-47. Two new methods were found in Islamic sources by E.S. Kennedy, "The Astrological Houses ...", cit., 537 and 544-545. See also J.D. North, "A Reply to Prof. E.S. Kennedy", *From Baghdad to Barcelona. Studies in Honour of Prof. Juan Vernet* (Barcelona, 1996), 579-582.

developed by Ibn Muʿādh, and the other erroneous, attributed to Ibn al-Samḥ (d 1035) and criticised by Ibn Muʿādh. Other methods presented by Ibn Muʿadh for the houses are the *Standard Method*, attributed to Ptolemy, and the single method which Ibn Muʿādh approves: the *Equatorial* (*fixed boundaries*) Method. Kennedy points out that this is also the first occurrence of it and that, among Muslims, it is found only in the Maghrib. In order to complete the list of methods appearing in the treatise one may add a vague reference (f 73r) to the *Single Longitude Method*.

Concerning the passages dealing with the projection of rays itself, Ibn Muʿādh, after firmly establishing the analogy between the computations for finding the houses and the rays, gives two different solutions, both based on the above mentioned *Equatorial Method*. The first one is also found in the Latin canons of Ibn Muʿādh's *Tabulae Jahen*, consists of an exact trigonometric procedure and has been analysed by Hogendijk in a recent paper[10], in which he also presents a worked example of its use together with the edition and English translation of the relevant Arabic and Latin passages. These passages are: part of chapter 26 and the last chapter from the extant Latin canons of the *Tabulae Jahen* (Nuremberg, 1549), translated by Gerard of Cremona, and ff 77v:9 - 78v:19 of the Treatise on the projection of rays from MS Medicea Laurenziana Or 152. The second solution does not occur in the *Tabulae* and is an approximate arithmetic rule that requires only the disposal of tables of right and oblique ascensions for a given locality and avoids the use of trigonometric functions. This resembles, to a certain extent, a solution given by al-Bīrūnī (973-1048) in his *Qānūn*[11], and which is found in many Arabic sources. Nevertheless, the particular computational steps used by Ibn Muʿādh are not found in other sources, and only a part of the approximate rule may be identified in a passage of Ibn al-Kammād's *Muqtabis* (12th c)[12].

I am currently preparing a complete edition and translation of the whole

[10] J.P. Hogendijk, "Applied mathematics in 11th century Spain: Ibn Mu'adh al-Jayyānī and his computation of astrological houses and aspects", to appear in *Centaurus*.

[11] Al-Bīrūnī, *Al-Qānūn al-Mas'ūdī* (Canon Masudicus) (Hyderabad, 1956), 3 vols., III, 1377-1385. Cf. E.S. Kennedy and H. Krikorian-Preisler, "The Astrological Doctrine ...", cit., 3-5; J.P. Hogendijk, "The Mathematical Structure…"., 178-180.

[12] Cf. J. Vernet, "Un tractat d'obstetrícia astrològica", *Boletín de la Real Academia de Buenas Letras de Barcelona*, 22 (Barcelona, 1949), 74-78, reprinted in J. Vernet, *Estudios sobre Historia de la Ciencia Medieval* (Barcelona-Bellaterra, 1979), 278-282.

text. For the moment, I will just present an outline of the two aforementioned procedures for finding the rays. In section 2, I will briefly summarize Ibn Muᶜadh's *modus operandi* for the trigonometric method following the material kindly provided by Hogendijk. Section 3 deals with the approximate rule and in Section 4 I present some conclusions.

In what follows, I will use the symbols below, which may represent either a value, a point on the celestial sphere, or a function if followed by a parenthesis:

δ declination.
φ terrestrial latitude.
ξ terrestrial latitude, other than φ, being $\varphi>\xi>0^\circ$ (see section 2, below).
h altitude.
α_0 right ascension.
α_φ oblique ascension. Similarly, $\alpha_{-\varphi}$ is for oblique descension, that is, the oblique ascension for a horizon of latitude $-\varphi$, and α_ξ is the oblique ascension at a horizon of latitude ξ.
α_R radial ascension, explained in section 2.
$^*\alpha_R$ approximate radial ascension, used in section 3.
$\alpha_0' = \alpha_0 + 90^\circ$, normed right ascension
$\Delta\alpha = |\ \alpha_0 - \alpha_\varphi|$, ascensional difference
λ ecliptic longitude.
λ_1, λ_4, λ_7, λ_{10} , respectively, the longitudes of the ascendent, lower midheaven, descendent, and upper midheaven (cusps of the astrological houses nos. 1, 4, 7 and 10).
λ_R longitude of an aspect. As a function, it is the inverse of α_R in Ibn Muʿādh's method.
$^*\lambda_R$ approximate longitude of an aspect, used in section 3.
$\Delta\lambda$ difference of longitudes, explained in section 3.

2. Exact method (MS Or. 152, ff 77v - 78v)

The extant copy of the *Maṭraḥ al-shuʿāʿāt* has no illustrations. However, several figures can be reconstructed using the references given in the text to letters representing points of the celestial sphere[13]. Figure 1 has been

[13] Cf. E.S. Kennedy, "Ibn Mu'ādh on the Astrological Houses", cit., 156.

slightly adapted from the reconstruction by Hogendijk[14] which corresponds to Ibn Muʿādh's passage on the trigonometrical procedure for finding the rays in MS Or 152. I will also use the same figure in next Section. It represents a zenithal view of the upper half of the sphere, the outer circle being the local horizon. Point *A* is the ascendent, points *B* and *D* are the north and south points on the horizon and point *N* is the celestial North Pole. Point *E* represents the longitude of a star (or a planet) and, as will be shown below, point *M* is its right quartile.

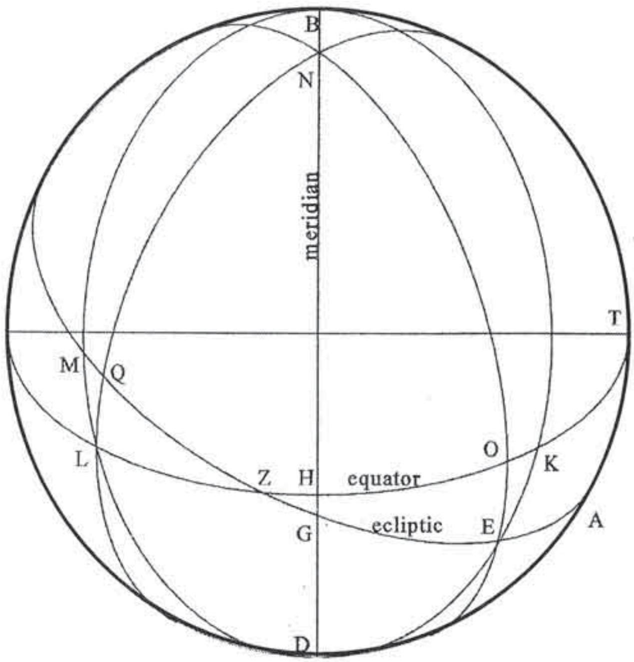

Figure 1

The most common theory for projecting the rays[15] relates the significant

[14] Cf. J.P. Hogendijk "Applied mathematics…".

[15] See J.P. Hogendijk, "The Mathematical Structure…"., 176-178.

angular distances of the aspects to the daily motion of the celestial sphere, and so these distances are measured on the equator. For this, one must *project* the ecliptic point onto the equator, add or subtract the amount in degrees of the desired aspect and, afterwards, *project* the equatorial point of the equator onto the ecliptic. When the ecliptic point is on the horizon or on the meridian the projection corresponds to its oblique or right ascension respectively. For the transitional cases, the projection of λ onto the equator and backwards must be done by means of arcs of great circles passing through the point in question and the north and south points on the horizon (arcs *DEKB* and *DLMB* in Figure 1). Other sources use the term incident horizon (*ufuq ḥādith*) for naming these arcs[16] and, in fact, they correspond to horizons that cross the local one at the north and south points, so that the problem should be posed in terms of finding an intermediate value α_ξ, corresponding to the oblique ascension of λ at a horizon of latitude ξ; $\varphi > \xi > 0°$.

Following this theory, Ibn Muʿādh uses a procedure analogous to that presented in the same treatise for the computation of the houses according to the *Prime Vertical Method*[17]. Briefly[18], it consists of two applications of Menelaos' Theorem that yield, in each case, a known ratio between the sines of two unknown arcs the sum of which (if the arcs are adjacent) or their difference (if the smaller arc is part of the other) is known. The final solution is not given in the text. Instead, there is a reference to Ibn Muʿādh's treatise on trigonometry, in which he has a particular algorithm for finding the two unknown arcs[19].

The particular steps of Ibn Muʿādh's algorithm, referring to Figure 1, are:

1) Find the equatorial degree of point $K = \alpha_R(E)$, called in the text *radial*

[16] Cf. E.S. Kennedy, "The Astrological Houses ...", cit., 555, 557; J.P. Hogendijk, "The Mathematical Structure…"., 176-178.

[17] See E.S. Kennedy, "Ibn Muʿādh and the Astrological Houses", cit., 157-158.

[18] For a detailed explanation, together with the corresponding texts and translations, modern formulation and a worked example of the use of this procedure, see J.P. Hogendijk, "Applied mathematics…".

[19] See M.V. Villuendas, *op. cit.*, 11-30 (text), 106-118 (trans.), 153-164 (com.); E.S. Kennedy, "Ibn Muʿādh on the Astrological Houses", cit., 157. The problem has different solutions in various authors, cf. M.T. Debarnot, *op. cit.*, 520-521.

ascension (*maṭāliʿ šuʿāʿiyya*) of the star. To do so, take the spherical quadrilateral with outer parts *DEK*, *DHN* and inner parts *KOH*, *NOE* and apply the identity

$$(\sin KO / \sin KH) = (\sin ND / \sin DH) \cdot (\sin EO / \sin NE)\,.$$

Since we know $\sin ND = \sin \varphi$, $\sin DH = \cos \varphi$, $\sin EO = \sin \delta(E)$, and $\sin NE = \cos \delta(E)$, we find $\sin KO / \sin KH$. Beyond this, taking for granted that we have the right ascension of upper midheaven, $H = \alpha_0(\lambda_{10})$, we know the difference between the two unknown arcs, $HO = KH - KO = \alpha_0(E) - \alpha_0(\lambda_{10})$ (modulo 360°). Then, applying the alluded final algorithm found in Ibn Muʿādh's treatise on trigonometry, the two unknown arcs are determined and the equatorial degree of point $K = ZK = \alpha_0(E) + KO$ will be known. From this, we find the equatorial degree of point *L*, which is the value for point *K* minus 90°.

2) A second spherical quadrilateral, with outer parts *DLM*, *DGN* and inner parts *GQM*, *LQN* will give, by Menelaos' Theorem,

$$(\sin MQ / \sin MG) = (\sin ND / \sin DG) \cdot (\sin LQ / \sin NL)\,.$$

Now we find the ratio of sines, knowing that $\sin ND = \sin \varphi$, $\sin DG = \sin (90` - \varphi - \delta(G))$ $\sin LQ = \sin \delta(Q)$ and $\sin NL$ is the radius. Since we also know the difference between *MG* and *MQ*, which is $GQ = \lambda_{10} - \alpha_0^{-1}(L)$ (modulo 360°), the two unknown arcs can be determined and, after this, we will know the longitude of the desired quadrature, $\lambda_R(L) = M = \lambda_{10} - MG$ (modulo 360°).

3. Approximate method (MS Or. 152, ff 78v - 80r)

After giving the trigonometric algorithm for finding the rays in "the most accurate and correct way ... for a special anniversary or a matter which deserves an exact investigation"[20], Ibn Muʿādh ends the treatise considering how "to know it in an approximate way *using the ascensions between the meridian and the horizon* (*bi-maṭāliʿ mā bayna wasaṭ al-samāʾ wa-l-ufq*)".

The text proposes two possible cases: to find $^*\alpha_R$, the approximate *radial*

[20] This is the end of Hogendijk's translation. Cf. J.P. Hogendijk, "Applied mathematics...".

ascension, given an ecliptic degree of longitude λ, and to find the approximate longitude, $^{*}\lambda_R$, which corresponds to a given α_R. To these ends, the text presents two functions, each one supposedly the inverse of each other.

The complete procedure for finding the longitude of a corresponding aspect given the longitude of an ecliptical degree is not explicitly described in this part of the treatise, but it may be deduced from the previous explanation for the trigonometric algorithm. On the whole, this consists of:

1) Find $^{*}\alpha_R$ corresponding to λ (point *E* in fig. 1). That is, using the first function, find an approximation to the degree on the equator that corresponds to λ projected by means of an arc of a great circle passing through λ and points north and south of the local horizon (refer to point *K* in Figure 1).

2) To this, apply the amount of the desired aspect. This consists of an addition or subtraction of the aspect, depending on its sign (left: increasing; right: decreasing), thus obtaining another equatorial point (for the right quadrature represented in Figure 1, refer to point *L*).

3) Find the ecliptic point $^{*}\lambda_R$ that corresponds to this equatorial degree. Operating with the second function, this time we find an approximation for the longitude of the projection of the equatorial degree onto the ecliptic, again, by means of a great circle passing through the north and south points of the horizon and, in this case, the given equatorial degree (refer to point *M* in Figure 1).

The use of the first function, $^{*}\alpha_R(\lambda)$, given λ, assumes that the longitudes of the four intersections of the ecliptic with the local meridian and horizon (λ_1, λ_4, λ_7, λ_{10},) are known. These are points of the ecliptic with special significance in astrology and are called in Arabic *watad* (pl. *awtād*). The operative algorithm is:

1) Subtract 90° from the $\alpha_0'(\lambda)$ (normed right ascension) of the given λ, "in order to have it reckoned from the beginning of Aries". Ibn Muʿādh calls these *mean ascensions* (*maṭāliʿ wasaṭiyya*). In the medieval tradition, right ascensions were reckoned from 0° Capricorn, whereas oblique ascensions were counted from 0° Aries; by subtracting 90° Ibn Muʿādh intends to use the same origin of coordinates.

2) Determine the *oblique ascension or descension* (*maṭāliʿ* or *magārib ufuqiyya*) of λ, depending on whether it is, respectively, on the eastern or the western half of the celestial sphere. This last detail may be known if λ_{10} and λ_4 are known. To find the oblique descension $\alpha_{-\varphi}$ of a degree the text instructs us to take "the oblique ascension of its nadir and add to this 180°". Taking as reference the western horizon, the *setting time* of λ corresponds to the *rising time* of the same degree for a latitude of $-\varphi$[21] ,but the implementation of this reasoning requires a table of oblique ascensions for $-\varphi$. Ibn Muʿādh avoids the use of this secondary table, using

$$\alpha_{-\varphi}(\lambda) = \alpha_{\varphi}(180° \pm \lambda) \pm 180° \text{ (modulo } 360°) .$$

3) In any case, the result of the last step is finally called *oblique ascension*, α_{φ}. Then, find out which of the ascensions is greater and find its difference from the other:

$$|\Delta_{\alpha}| = |\alpha_0(\lambda) - \alpha_{\varphi}(\lambda)|$$ [22].

4) Calculate what Ibn Muʿādh calls the *imām*. Kennedy points out that this term, proper to the Andalusian and Maghribi tradition, usually refers to a divisor[23]. Here, the *imām* is defined as 90° + or $-|\Delta_{\alpha}|$, the operator depending on the sign of the declination and the altitude of λ. The sign of the altitude can be determined if λ_1 and λ_7 are known. Then

imām = $90° + |\Delta\alpha|$, when both $h(\lambda)$ and $\delta(\lambda)$ are positive or negative, and

imām = $90° - |\Delta\alpha|$ in the other cases.

Although Ibn Muʿādh does not use the expression, this *imām* corresponds to the semidiurnal arc of λ, when it is above the horizon, and to the

[21] See J.P. Hogendijk, "The Mathematical Structure…", 178.

[22] Al-Bīrūnī, *op. cit.*, 1381-1382 explicitly calls this value *equation of daylight* (*taʿdīl al-nahār*). An equivalent expression is *ascensional difference.*

[23] Cf. E.S. Kennedy, "Ibn Muʿādh on the Astrological Houses", cit., 158 and "The Astrological Houses ...", cit. 560.

seminocturnal arc of λ, when it is below the horizon, for, using real values instead of absolute values for $\Delta\alpha(\lambda)$, this would have the sign of $\delta(\lambda)$ and

imām = $90° + \Delta\alpha$, when $h>0$, and
imām = $90° - \Delta\alpha$, when $h<0$[24].

5) Find the *buʿd* (*distance*). Hogendijk noted that this term is invariably translated as *longitudo* in the Latin canons of the *Tabulae Jahen*. The concept of *buʿd* is here defined as the time (expressed in degrees) in which the given λ will reach the nearest of the four *awtād*, following the sense of the diurnal movement (f 79r). The computation of this value consists of determining the equatorial arc between the corresponding positions of λ and the *watad* involved. For this, oblique ascensions or descensions (according to step 2) will be used if the *watad* is *horizontal* (λ_1, λ_7). If the *watad* is *meridian* (λ_{10}, λ_4), right ascensions will be used. The four cases are:

if the next *watad* is λ_1, then *buʿd* = $\alpha_\varphi(\lambda) - \alpha_\varphi(\lambda_1)$;
if the next *watad* is λ_7, then *buʿd* = $\alpha_{-\varphi}(\lambda) - \alpha_{-\varphi}(\lambda_7)$;
if the next *watad* is λ_{10}, then *buʿd* = $\alpha_0(\lambda) - \alpha_0(\lambda_{10})$;
if the next *watad* is λ_4, then *buʿd* = $\alpha_0(\lambda) - \alpha_0(\lambda_4)$.

For the last case, the literal expression (f 70v) is *buʿd* = $\alpha_0(\lambda + 180°) - \alpha_0(\lambda_{10})$, which is equivalent to the expression I give, because $|\alpha_0(\lambda) - \alpha_0(\lambda + 180°)| = 180° = |\alpha_0(\lambda_{10}) - \alpha_0(\lambda_4)|$.

6) Find a value $e = |\Delta\alpha| \cdot$ *buʿd* / *imām*, which corresponds to what Bīrūnī[25] calls *equation* (*taʿdīl*). Ibn Muʿādh does not use this term but at the end of the explanation (f 80r) he states that "this is the procedure by which the rays are obtained in a correct way, by means of the *equation* (*taʿdīl*) between the two ascensions", revealing that he is also thinking of an *equation*.

7) Finally, use e in the adequate manner to obtain $^*\alpha_R(\lambda)$. To this end, the

[24] Al-Bīrūnī, *op. cit.*, 1381 and 1387, uses the same value as a divisor in his algorithm for finding the equation, using the expression semidiurnal / seminocturnal arc instead of *imām* and without giving instructions as to its computation.

[25] Al-Bīrūnī, *op. cit.*, 1384.

equation is applied to the value of the ascension of λ which is analogous (right or oblique) to that of the *watad* taken as reference when finding the *buʿd*, so,

if the *watad* is meridian $(\lambda_{10}, \lambda_4)$, then
  if $\alpha_0(\lambda)>\alpha_\varphi(\lambda)$, then $^*\alpha_R(\lambda) = \alpha_0(\lambda) - |e|$;
  otherwise, $^*\alpha_R(\lambda) = \alpha_0(\lambda) + |e|$;
if the *watad* is horizontal (λ_1, λ_7), then
  if $\alpha_0(\lambda)<\alpha_\varphi(\lambda)$, then $^*\alpha_R(\lambda) = \alpha_\varphi(\lambda) - |e|$;
  otherwise, $^*\alpha_R(\lambda) = \alpha_\varphi(\lambda) + |e|$.

In sum, the first function consists of finding an approximation to the exact result by means of an interpolation coefficient: the *equation e*. This value involves a fraction that takes, as a numerator, the *distance* (*buʿd*), measured on the equator, between the corresponding ascension of λ and its next intersection with the horizon or the meridian following the diurnal movement (arc *OH* in Figure 1) and, as a denominator, the whole equatorial arc corresponding to the successive passage of λ from the one *watad* to the other, that is, the *imām*. The value (between 0 and 1) of this fraction, which is proportional to the time elapsed of the passage of λ between the two *awtād*, is then multiplied by the difference between the two kinds of ascension (projections onto the equator) available in medieval tables for a given λ: right ascensions and oblique ascensions for the latitude of the specific place. Thus, one obtains an *equation* or *correction* to be applied in order to approximate the value of one of the two ascensions to the required oblique ascension of λ for an *incident horizon* of latitude ξ.

As to the second function, $^*\lambda_R(\alpha_R)$, given α_R, the algorithm to be used is as follows:

1) Determine in which quadrant α_R is, the given equatorial degree that is to be projected onto the ecliptic. For this, $\alpha_0(\lambda_{10})$, $\alpha_0(\lambda_4)$, $\alpha_\varphi(\lambda_1)$ and $\alpha_{-\varphi}(\lambda_7)$ must be known.

2) Find two values for $\lambda(\alpha_R)$ by simple transformation of coordinates using both right, $\alpha_0^{-1}(\alpha_R)$, and oblique ascensions, $\alpha_\varphi^{-1}(\alpha_R)$. In the latter case, observing that $\alpha_{-\varphi}$ will be used instead of α_φ if the degree is in the western half of the sphere.

3) Find the difference between the two longitudes obtained:

$$|\Delta\lambda| = |\alpha_0^{-1}(\alpha_R) - \alpha_\varphi^{-1}(\alpha_R)| .$$

4) Find the *distance* (*bu'd*) on the equator between α_R and the corresponding ascension of the *watad* to which it goes first. That is, the absolute difference between α_R and the ascension of the *watad*, right or oblique, depending on whether it is meridian or horizontal. The concept of *bu'd* is the same as in the above function.

5) Find a value $e = |\Delta\lambda| \cdot bu'd / 90°$, analogous to the *equation* used in the first function. In this case the divisor is always 90°, presumably because a proportion is sought between the time elapsed of the passage of α_R from the ascension of one *watad* to the next, that is, the obtained distance (*bu'd*), and the whole equatorial distance between these two ascensions, which is always 90°, because it is the difference between α_0 of a meridian *watad* and α_φ of the horizontal *watad* that follows : $|\alpha_0(\lambda_{10}) - \alpha_\varphi(\lambda_7)| = 90°$, etc…

6) Operate with e in the appropriate manner to find $^*\lambda_R$:

if the *watad* is horizontal (λ_1, λ_7) , then
 if $\alpha_0^{-1}(\alpha_R) > \alpha_\varphi^{-1}(\alpha_R)$, then $^*\lambda_R = \alpha_\varphi^{-1}(\alpha_R) + |e|$;
 otherwise, $^*\lambda_R = \alpha_\varphi^{-1}(\alpha_R) - |e|$;
if the *watad* is meridian (λ_{10}, λ_4) , then
 if $\alpha_0^{-1}(\alpha_R) < \alpha_\varphi^{-1}(\alpha_R)$, then $^*\lambda_R = \alpha_0^{-1}(\alpha_R) + |e|$,
 otherwise, $^*\lambda_R = \alpha_0^{-1}(\alpha_R) - |e|$.

As for the formal aspect of the extant text, Ibn Mu'ādh announces the two functions, but after a description of the first one he then presents a *Summary* (*talkhīs*) which repeats the description in similar terms and defines the second function for the first and only time. This is only one instance of the disorganized style of the manuscript. This disorder, together with the absence of figures, suggests that this single copy of the treatise is a draft rather than a definitive work. On the other hand, the only appearance of a description of an algorithm known to me that suggests the idea of a transmission in another author – found in Ibn al-Kammād's *Muqtabis*[26] –

[26] Cf. Vernet, *op. cit.*.

does not include the second function. This may indicate that the approximate solution for finding the radial ascension given an ecliptical degree was more widely diffused than the inverse function, probably because if one has tabulated values for a concrete function, it goes without saying that data for the inverse function are available using the same table, entering the column of the values in order to find the corresponding arguments, without any need for tabulating a second function.

In order to give an idea as to the precision of the procedure, Figure 2 shows the results of a test consisting of applying both the approximate rule and the trigonometric exact method to obtain the radial ascension at a sample situation, latitude 36°, ascendent 330°. The graph[27] represents

$$^*\alpha_R(\lambda) - \alpha_R(\lambda)\text{, for } \lambda = 1° \text{ to } 360° \text{ (differences in degrees).}$$

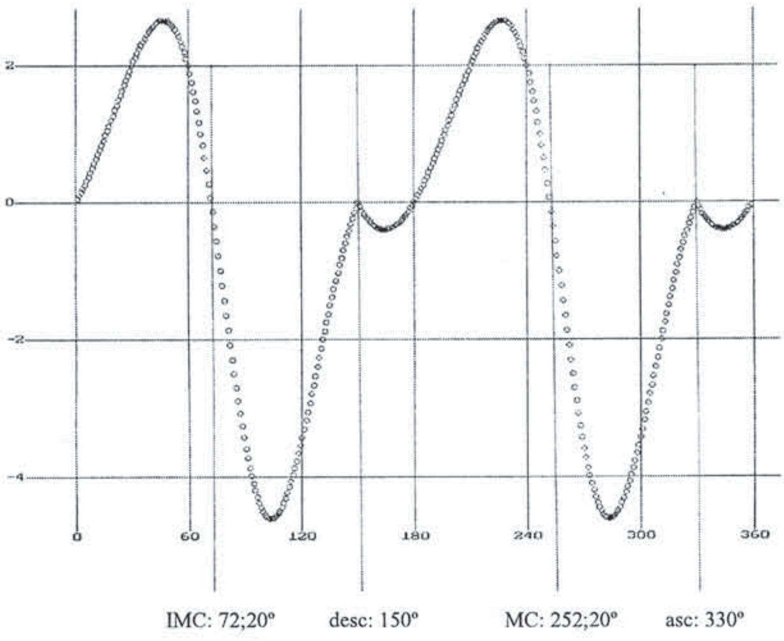

Figure 2

[27] For the plot I use a function of the computer program *Table Analysis*, by Benno van Dalen.

Regarding the reciprocity that two inverse functions are expected to have, Figure 3 represents, at the same situation,

$$\lambda - {}^{*}\lambda_R[{}^{*}\alpha_R(\lambda)]\text{, for } \lambda = 1° \text{ to } 360°.$$

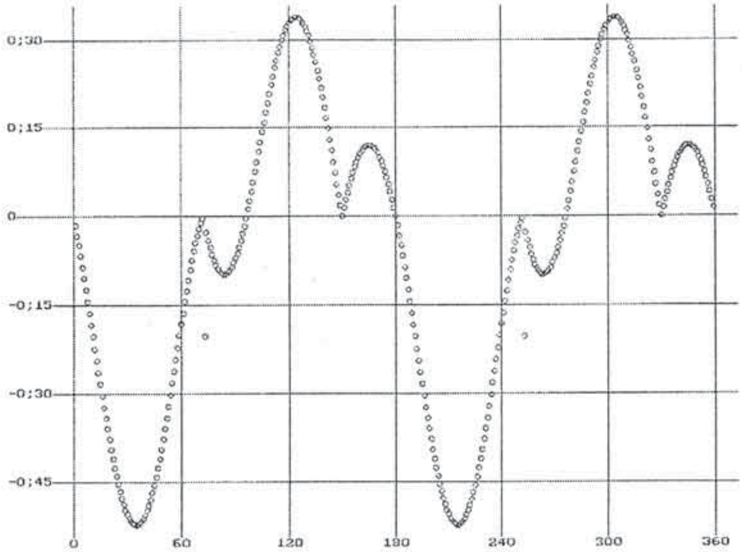

Figure 3

The results, using the approximate rule, coincide with the exact ones and have a correct reciprocity (difference = 0) only when the argument is 0°, 180° or the longitude of one of the four *awtād*. The errors do not seem large enough to invalidate an astrologer's work but, of course, one should think that a mathematician of the ability of Ibn Muʿadh must have been aware of the procedure's inaccuracy. In any case, for the record, he ends the treatise insisting that "this method is somewhat approximate, God willing." (f 80r).

4. Conclusions

Though when dealing with the division of houses, the idea of the coexistence of two different methods of varying exactitude is found, for example, in Ibn Isḥāq's *Zīj*[28], what is disturbing about the whole, on my opinion, is the very occurrence of the description of a rudimentary and approximate rule after the presentation of an exact method. If there is a reason for presenting such a procedure, it may be due to the author's awareness of the lack of mathematical knowledge by his Andalusian contemporaries, especially when dealing with trigonometry. Nothing in the computation requires the use of a single table of sines, nor any particular trigonometric aptitude. The two approximate functions require only – in addition to some means of previously knowing the longitude of the four *awtād* and the sign of the declination of the degree – a method for converting both α_0 and α_φ into λ, and vice versa. Normally, tables for α_0 and α_φ for the latitude of the place were available and sufficient for this purpose. A similar motivation, a desire to be understood by his contemporaries, must be the reason for the use of Menelaos' Theorem in the trigonometric method by a mathematician who had mastered far more recent techniques in his previous treatise on trigonometry.

With respect to the question of sources and possible transmission from the East of these and similar techniques, one should consider al-Bīrūnī as a representative exponent of the scientific generation that made possible the Eastern astronomical and mathematical development between the end of the 10th century and the beginning of the 11th[29]. Though Ibn Muʿādh does not mention any of these sources, the theoretical basis behind the application of the approximate rule may be detected in a passage of al-Bīrūnī's *Qānūn*[30] where he states that "the people (*al-qawm*) construct a procedure based on that the ratio of *AT*, the *distance* (*buʿd*) from the meridian, to *AC*, half the arc of daylight, is like the ratio of *HT* to *DT* ..." (see figure 4).

[28] In this case, for important affairs the *Standard Method* is required whereas the *Dual Longitude Method* suffices for ordinary purposes. Cf. E.S. Kennedy, "The Astrological Houses ...", cit., 554-555; A. Mestres, "Maghribī Astronomy in the 13th Century: a Description of Manuscript Hyderabad Andra Pradesh State Library 298", *From Baghdad to Barcelona. Studies in Honour of Prof. Juan Vernet* (Barcelona, 1996), 400-401.

[29] See J. Samsó, "'al-Bīrūnī' in al-Andalus", *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet* (Barcelona, 1996), 583-612.

[30] Al-Bīrūnī, *op. cit.*, 1382.

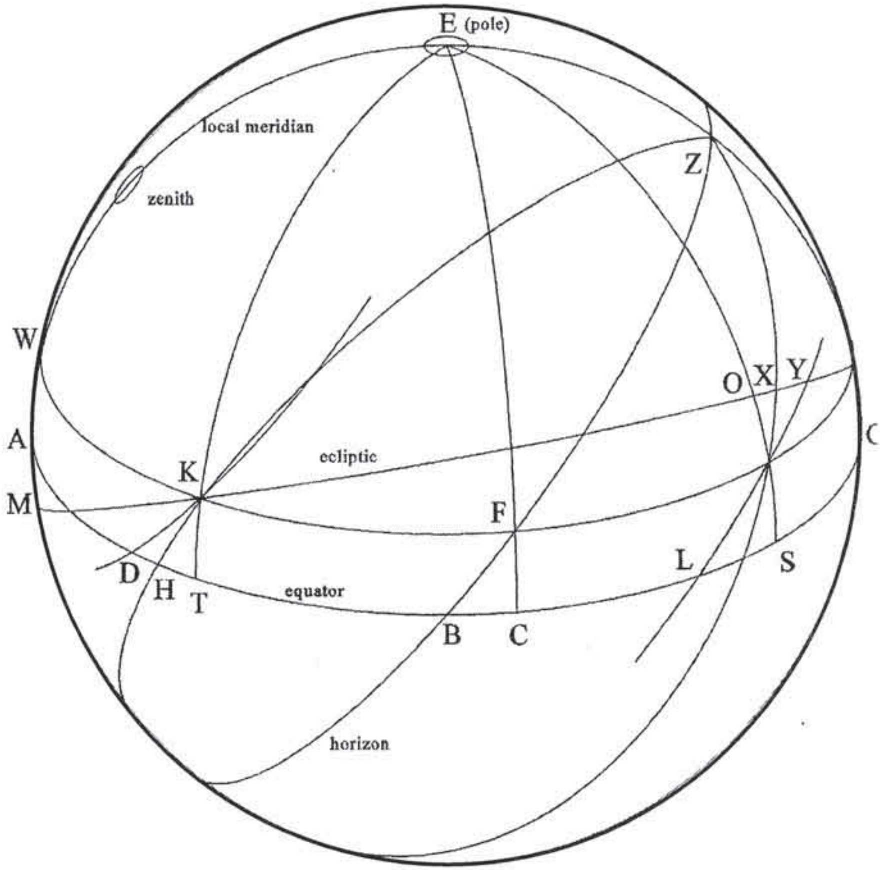

Figure 4

The arc here referred to as *HT* is the ascensional difference of an ecliptical degree λ (*K* in figure 4) for an *incident horizon* of latitude ξ. For its part, arc *DT* in al-Bīrūnī's account is the ascensional difference of λ for the local horizon. Moreover, recalling that the *imām* used by Ibn Muʿādh is the semidiurnal arc of λ, we have, merging the approaches of al-Bīrūnī and Ibn Muʿādh, the identity

$$(AT \equiv bu\text{ʿ}d) \,/\, (AC \equiv im\bar{a}m) = (HT \equiv e) \,/\, (DT \equiv \Delta\alpha)\,,$$

which is Ibn Muʿādh's expression for the equation $e = \Delta\alpha \cdot bu\text{ʿ}d \,/\, im\bar{a}m$. Nevertheless, considering the folk and arcane character of a discipline like astrology, it is very risky to argue for a possible chain of transmission without a clear textual evidence, such as a direct quotation of a source in an original work.