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Nonlinear relaxation time and the detection of weak signals

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Laser systems can be used to detect very weak optical signals. The physical mechanism is the dynamical process of the relaxation of a laser from an unstable state to a steady stable state. We present an analysis of this process based on the study of the nonlinear relaxation time. Our analytical results are compared with numerical integration of the stochastic differential equations that model this process.

Recently[1,2] it has been shown that very weak optical signals, of the same order as the intrinsic noise level but 10^8 (dye lasers) times smaller than the steady-state intensity of the laser, can be detected using the laser as a supergenerative receiver. Solid lasers can detect much weaker signals.[1] The explicit details of the detection process are given in Ref. 1. Summarizing the main aspects, the method starts with a preparation of the laser in a steady state of zero intensity, then the pump parameter is suddenly changed to a value that corresponds to a finite steady intensity. Then the laser will evolve from an initial unstable state to its final stable state. This dynamical process is very sensitive to internal fluctuations and to the presence of a small external field, which we are interested to detect. Repeating the experiments under the same conditions and making statistics with the results, one can detect clearly the effect of the external field.

Two methods have been proposed to detect these signals: the first one looks at the area under the time evolution of the mean output intensity[1] (Fig. 1), and the second one looks at the mean-first-passage time (MFPT) of the intensity.[1,2] In this paper we will pay attention to the first method. The second one has been studied in Ref. 2.

The experimental procedure works as follows. The laser working as a supergenerative receiver is periodically switched on and off. During an interval of time T_c, the laser relaxes from the initial intensity $I_0=0$ to a value very close to the final steady state I_{st}. Thus the value of T_c should be taken with care in order to ensure that the laser intensity is very close to its steady state. In Fig. 1 one can see two evolutions of the mean output intensity of the laser during this interval of time. The smaller area, A_0, corresponds to the case of the absence of external signals and the larger area, A_e, corresponds to the case of their presence. The ratio of the area under the two curves A_e/A_0 is called the receiver output. This is the quantity we are interested in because it is very sensitive to the presence of weak signals. Our purpose is to calculate this quantity in order to see its dependence on the parameters of our system and, in particular, on the internal fluctuations and the external field.

Our model of laser is the same as that of Ref. 1. The equation of motion for the complex, scaled and dimensionless laser field E is

$$\frac{dE}{dt}=-kE+\frac{FE}{1+AI/F}+q(t)+k_eE_e\ , \tag{1}$$

where

$$\langle q^*(t)q(t')\rangle=2\epsilon\delta(t-t')\ . \tag{2}$$

$I=|E|^2$ is the laser intensity, $q(t)$ is a white noise describing the internal fluctuations of intensity ϵ, E_e is the external field or weak signal, and k, k_e, A, and F, are other parameters of the model. They will take the following values:[1] $\epsilon=4\times10^{-3}\ \mathrm{s}^{-1}$, $k=1.2\times10^{7}\ \mathrm{s}^{-1}$, $k_e=6\times10^{6}\ \mathrm{s}^{-1}$, $A=2.6\times10^{6}\ \mathrm{s}^{-1}$, and $F=1.4\times10^{7}\ \mathrm{s}^{-1}$. These values correspond to a dye laser pumped about 20% above threshold.

The nonlinear relaxation time (NLRT) associated with the evolution of $\langle I(t)\rangle$ is defined by[3]

$$T_e=\int_0^\infty\frac{\langle I(t)\rangle-\langle I\rangle_{st}}{\langle I(0)\rangle-\langle I\rangle_{st}}dt \simeq\int_0^{T_c}\frac{\langle I(t)\rangle-\langle I\rangle_{st}}{\langle I(0)\rangle-\langle I\rangle_{st}}dt\ . \tag{3}$$

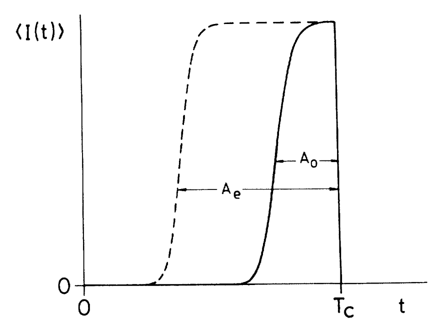

FIG. 1. Time evolution of $\langle I(t)\rangle$ during the interval of time T_c. A_0 is the area under the solid line ($E_e=0$). A_e is the area under the dashed line ($E_e\neq0$).

This approximation makes sense if $T_c \geq 1.5T_0$, T_0 being the NLRT in the absence of external field. In our calculations we will use $T_c = 7\times 10^{-6}$ s, as in Ref. 1(a). From this equation the area A_e can be expressed as

$$A_e = T_e[\langle I(0)\rangle - \langle I\rangle_{st}] + \langle I\rangle_{st}T_c \ . \tag{4}$$

The same expression applies for A_0 as a function of T_0. As $\langle I(0)\rangle \approx 0$, we obtain for the receiver output

$$\frac{A_e}{A_0} = \frac{T_e - T_c}{T_0 - T_c} = 1 + \frac{T_0 - T_e}{T_c - T_0} \ . \tag{5}$$

We see that the receiver output is a function of the nonlinear relaxation times T_e and T_0. The theory of NLRT has been presented in several papers.[3-5] In order to avoid this straightforward but lengthy calculation we will make use of a result of Ref. 4. It states that in the decy of an unstable state the MFPT calculated using the quasideterministic theory[2] and the NLRT for the same process differs only by a constant that is equal to $1/[2(F-k)]$. Then we have

$$T_e = \langle t\rangle_{QDT} - \frac{1}{(2(F-k)} + \Delta C_{NL} \ . \tag{6}$$

The term $\Delta C_{NL} = 7/[12(F-k)]$ comes from nonlinear contributions calculated deterministically ($\epsilon \to 0$), and $\langle t\rangle_{QDT}$ is calculated with linear terms only. Nevertheless, the presence of the external field E_e makes the calculation a bit more difficult. Following the same procedure as in Ref. 2 we consider the linear approximation to Eq. (1), which, for the components of E, reads

$$\frac{dE_i}{dt} = aE_i + k_e E_{ei} + q_i(t), \quad i = 1,2 \tag{7}$$

where $a = F - k$. The solution can be expressed as

$$E_i(t) = h_i(t)e^{at} \ , \tag{8}$$

where $h_i(t)$ will play the role of the initial condition, because for long times it becomes a Gaussian random variable $h_i(\infty) = h_i$. The marginal probability corresponding to the modulus h is then obtained

$$P(h) = \frac{h}{\sigma} I_0 \left[\frac{ch}{a\sigma}\right] e^{-(1/2\sigma)[h^2 + (c/a)^2]} \ , \tag{9}$$

where $c = k_e|E_e|$, $\sigma = \epsilon/a$, and $I_0(z)$ is the modified Bessel function of zeroth order.[6]

The MFPT to get the final value $I_r = F(Fk)/kA$ is evaluated from Eq. (7) by doing the following average with the probability (9):

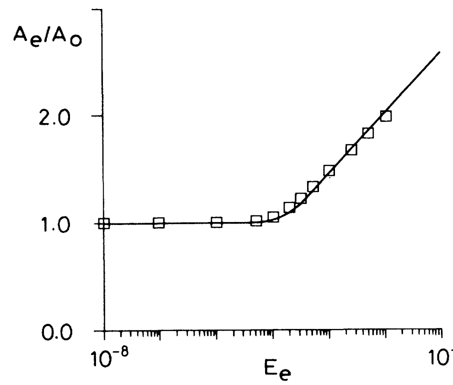

FIG. 2. Receiver output A_e/A_0 vs the modulus of the external field E_e. The line corresponds to the theoretical result (12). Squares are the numerical results of Ref. 1(a).

$$\langle t\rangle_{QDT} = \frac{1}{2a}\left\langle \ln\frac{I_r}{h_2}\right\rangle$$
$$\cong \langle t\rangle_0 - \frac{1}{2a}[E_1(b\alpha) + \gamma + \ln(b\alpha)] \ , \tag{10}$$

where $E_1(z)$ is the integral exponential function,[6] γ is the Euler constant, $\alpha = (2\sigma)^{-1}$, and $b = c^2/a^2$. $\langle t\rangle_0$ is the MFPT in the absence of external field

$$\langle t\rangle_0 \simeq \frac{1}{2a}\ln\left[\frac{aI_r}{2\sigma}\right] + \frac{\gamma}{2a} \ . \tag{11}$$

From all these results we get for the receiver output (5)

$$\frac{A_e}{A_0} = 1 + \frac{E_1(b\alpha) + \gamma + \ln(b\alpha)}{2a(T_c - T_0)} \ . \tag{12}$$

The variable $b\alpha$ is proportional to $|E_e|^2/\epsilon$; thus it compares the intensity of the external signal with the intensity of internal noise. In Fig. 2 we can check our theoretical prediction (12) with the numerical data of Ref. 1(a). The agreement is quite remarkable. As a conclusion we want to stress the following aspect. Our results manifest clearly that the NLRT technique can be used to detect weak signals of the same order or even smaller than the intensity of the internal (quantum) fluctuations.

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