# Fundamental Constants of Nature and their Possible Time Variation

Author: Guillermo Bernárdez Gil.
*Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.*

Advisor: Joan Solà Peracaula.

**Abstract:** The possibility of a slow cosmic evolution of fundamental "constants" of Nature has been discussed for a long time without being consolidated; recently, however, more and more experiments are arising whose results seem to support this idea. Apart from that, in an expanding universe the vacuum energy density $\rho_\Lambda$ is expected to be a time-evolving parameter rather than the rigid one proposed in the standard ΛCDM model; in fact, quantum field theory in curved space time suggests a slow evolution determined by the expansion rate of the universe $H$. In this work, we will try to obtain and develop some cosmological models with such a a dynamical vacuum energy density, and at the same time we will check whether they can provide an explanation for the alleged slow time variation of the fundamental constants.

## I. INTRODUCTION

Nowadays the time and space evolution of fundamental constants of Nature is a very active field of theoretical and experimental research that could provide interesting results in the near future. The idea of a cosmic time evolution of these fundamental constants of physics began with Dirac in the thirties, when he proposed a time-evolving gravitational constant $G$ [1]; since then, the possibility of different dynamical "constants" has been continuously discussed and investigated, but many technological improvements have been needed to finally notice possible evidences of these variations; see the review [2]. However, it is important to note that we still deal with low significance levels due to technical limitations on the current measurements.

For instance, there are many tests indicating the possibility that the value of the fine structure constant $\alpha_{\rm em}$ has changed over the cosmic evolution. Constraints on $\dot{\alpha}_{\rm em}/\alpha_{\rm em} \equiv (1/\alpha_{\rm em})d\alpha_{\rm em}/dt$ can be deduced from limits on the position of nuclear resonances in natural fission reactors that have been working for the last few billions years; one renowned example is the natural reactor located at the Oklo uranium mine (Gabon). Moreover, direct astrophysical observations are becoming very relevant in this course. Note that if $\alpha_{\rm em}$ does not remain constant, we could also expect a time evolution of the masses of all nucleons due to the fact that the interaction responsible for the variation of $\alpha_{\rm em}$ should couple radiatively to nucleons.

Likewise, it is also considered the proton-electron mass ratio $\mu_{\rm pe} \equiv m_p/m_e$, which is known with high accuracy, testing a possible cosmic time evolution of its value. Such a time-evolving $\mu_{\rm pe}$, thus, could be interpreted as a change of the fundamental QCD scale parameter $\Lambda_{\rm QCD}$ of the strong interaction, in the sense that a dynamical $\Lambda_{\rm QCD}$ would originate a variation of the proton mass whereas it would not affect the electron one; we refer to [5] or [6] to see how $m_p$ and $\Lambda_{\rm QCD}$ can be related. Some of these experiments consist on astrophysical tests, comparing interstellar and laboratory spectrums, but there are also laboratory experiments where a time variation of the nucleon mass is tried to be observed by monitoring molecular frequencies using atomic clocks.

What if an alternative cosmological model, instead of the standard ΛCDM one, can give us an explanation about the possible time variations of the particle physics constants mentioned above? From the cosmological point of view, the idea that both the cosmological term $\Lambda$ and the gravitational coupling $G$ could also be dynamical parameters is, intuitively, a reasonable assumption in an expanding universe. Thus, we will look for new cosmological models compatible with a time-evolving pair $(\rho_\Lambda, G)$, then trying to explain the feasible evolution of particle masses and couplings through them.

## II. COSMOLOGICAL MODELS WITH TIME-EVOLVING PARAMETERS

As we have stated in the Introduction, we propose the idea that cosmic time variations of constants of particle physics may be related with time-evolving parameters of cosmology. In order to develop this theory, the aim of this section is to obtain a set of cosmological equations compatible with time variations of the Newton ($G$) and the cosmological ($\Lambda$) constants. From this set of equations, we will finally raise some cosmological models.

We start from the General Relativity field equations $G_{\mu\nu} - g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu}$ in the presence of the cosmological term, where $G_{\mu\nu}$ is the Einstein tensor, and $T_{\mu\nu}$ the energy-momentum one of the isotropic matter and radiation of the universe. Assuming the expanding universe as a perfect fluid, with matter-radiation density $\rho_m$ and vacuum energy density $\rho_\Lambda := \Lambda/(8\pi G)$, we can rewrite them as

$$G_{\mu\nu} = 8\pi G \tilde{T}_{\mu\nu}, \qquad (1)$$

where now $\tilde{T}_{\mu\nu} := T^m_{\mu\nu} + \rho_\Lambda g_{\mu\nu}$ is a diagonal tensor called the full energy-momentum tensor of the cosmic fluid.

We will focus on solving (1) in the FLRW metric, $ds^2 = dt^2 - a(t)d\mathbf{x}^2$, being $a(t)$ the time-evolving scale factor.

The reason why we restrict our study to the spatially flat case is that it seems to be the most realistic one for both theoretical and observational points. In this context, it is important to note that a time evolution of the parameters $G$ and $\Lambda$ is allowed without violating the Cosmological Principle; this is just the framework we will contemplate from now on.

Regarding the equation of state for the matter-radiation component, it reads $p_m = \omega_m \rho_m$ with $\omega_m$ equals to 0 or 1/3 for non-relativistic matter or relativistic matter and radiation, respectively. In our analysis, since we will be only interested in the relevant epoch of the cosmic evolution where the non-relativistic matter dominates, we will take $\omega_m = 0$ (so $p_m = 0$), neglecting thus the radiation component; this is a very good approximation for the relatively present universe. We also introduce the corresponding EoS for the vacuum, $p_\Lambda = \omega_\Lambda \rho_\Lambda$; in this case, because of the assumption of $\rho_\Lambda$ that states that it is a true dynamical vacuum term whose time evolution in exclusively associated to the quantum effects on $\Lambda$, it can be seen that $\omega_\Lambda = -1$.

Therefore, on the one hand, by solving Einstein's equations under all of these assumptions, as well as including the possibility of $G = G(t)$ and/or $\Lambda = \Lambda(t)$ (and then $\rho_\Lambda = \rho_\Lambda(t)$), we obtain the following two independent gravitational field equations, which are known as Friedmann's equations. The first one,

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda), \tag{2}$$

is derived from the time component of (1), and the second one,

$$2\dot{H} + 3H^2 = 8\pi G \rho_\Lambda, \tag{3}$$

from the space components; here the overdot denotes derivative with respect to cosmic time $t$, and $H = \dot{a}/a$ the Hubble's expansion rate. Substituting (2) into (3), we can derive a expression for the rate of change of the Hubble function

$$\dot{H} = -4\pi G \rho_m. \tag{4}$$

On the other hand, the general Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$, together with (1), leads to $\nabla^\mu (G\tilde{T}_{\mu\nu}) = 0$, which induces a "mixed" local conservation law that, trading the cosmic time $t$ for the scale factor $a$ through $d/dt = aHd/da$, can be expressed as

$$a\frac{d}{da}[G(\rho_m + \rho_\Lambda)] + 3G\rho_m = 0. \tag{5}$$

We recall that $G$ and/or $\rho_\Lambda$ may be dynamical parameters. Although this conservation law is not independent of the Friedmann's equations, it is very useful in order to understand the possible transfer of energy between vacuum and matter.

Thus, it only remains to present the different cosmological models with time-evolving parameters that we will develop in further sections. We distinguish the following possibilities:

- **Model 1:** $G$ =constant, $\dot{\rho}_\Lambda \neq 0$ and it is not satisfied the local covariant conservation law of matter

$$a\rho_m' + 3\rho_m = 0, \tag{6}$$

  where the prime denotes a derivative with respect to the scale factor $a$.

- **Model 2:** $\dot{G} \neq 0$, $\dot{\rho}_\Lambda \neq 0$, but it is satisfied (6).

- **Model 3:** $G = G_0 a^q$, $|q| \ll 1$, and $\dot{\rho}_\Lambda \neq 0$; in this case, we will see that may or may not be satisfied the conservation law of matter (6).

If we take $\rho_\Lambda$ and $G$ constants, note that we recover the well-known standard case of the $\Lambda$CDM cosmological model, in which (5) transforms into (6); in this case, $\rho_\Lambda = \rho_\Lambda^0$, $G = G_0$ and it can be shown by solving (6) that $\rho_m = \rho_m^0 a^{-3}$, noting that we use the convention that the 0 in the sub- or superscript of a magnitude denotes its current value. It will be convenient to have these simple results in mind in order to evaluate the behavior of the models in the limits where both $\rho_\Lambda$ and $G$ become constants.

## III. RUNNING VACUUM

Intuitively, we expect that the vacuum energy density varies with the expansion of the universe, so it is reasonable to conceive theoretical proposals supporting this option; in this regard, notice that all models we are going to deal with have a common point: the cosmic time evolution of $\rho_\Lambda$. It is usually assumed in this area of study that its evolution is inherited indirectly from another dynamical variable $\mu = \mu(t)$ on which $\rho_\Lambda$ is tied of, associating then this fact with the renormalization group (RG) running of the effective charges in gauge theories; see e. g. [4]. In the cosmological context, $\mu$ is a characteristic infrared cutoff scale, and hence typically associated to the Hubble function since $H(t)$ is of the order of the energy scale associated to the non-trivial structure of the FLRW background. Generally, $\mu^2$ is in correspondence with $H^2$ and also with $\dot{H}$.

From the theoretical point of view of the renormalization group for the vacuum energy density of the expanding universe, it is proposed that the rate of change of the quantum effects on the cosmological constant as a function of the scale $\mu$ follows the form

$$\frac{d\rho_\Lambda(\mu)}{d\ln\mu} = \frac{1}{(4\pi)^2}\left[\sum_i B_i M_i^2 \mu^2 + \sum_i C_i \mu^4 + \cdots\right], \tag{7}$$

where $M_i$ are the masses of the particles contributing in the loops, and $B_i$, $C_i$,... dimensionless parameters. We recall that, because of the general covariance of the effective action, in the expression of $\rho_\Lambda$ we only expect terms with an even number of derivatives of the scale factor; due to the correspondences of $\mu^2$ stated above, it is straightforward to see that the RG equation (7) satisfies this requirement.

For the current universe, we may consider only the quadratic terms $\sim M_i^2\mu^2$ of (7); we refer to [7] for further explanations. Hence, integrating the resultant RG equation, and then expressing $\mu^2$ as a linear combination of $H^2$ and $\dot{H}$, we can finally obtain

$$\rho_\Lambda(H,\dot{H}) = \frac{3}{8\pi G_0}(c_0 + c_{\dot{H}}\dot{H} + c_H H^2), \qquad (8)$$

where $c_{\dot{H}}$ and $c_H$ ara two small dimensionless parameters, $|c_{\dot{H}}|, |c_H| \ll 1$, that control the dynamical character of $\rho_\Lambda$; this is the running vacuum energy density we are going to consider in the three proposed models. In particular, introducing equations (2) and (4) in (8), then isolating $\rho_\Lambda$ in the resulting expression, and finally defining new parameters $\nu = c_H$, $\alpha = 3c_{\dot{H}}/2$ (so $|\nu|, |\alpha| \ll 1$, too), $C_0 = 3c_0/(8\pi G_0)$, we arrive at the equivalent and more useful formulation

$$\rho_\Lambda = \frac{1}{1-\nu\frac{G}{G_0}}\left[C_0 + (\nu-\alpha)\frac{G}{G_0}\rho_m\right]; \qquad (9)$$

by normalizing it for the current values of all magnitudes involved, we find that $C_0$ can be expressed as

$$C_0 = (1-\nu)\rho_\Lambda^0 - (\nu-\alpha)\rho_m^0. \qquad (10)$$

## IV. MODEL 1

This can be interpreted as the "classical" cosmological model with time-evolving parameters: it is assumed a gravitational coupling $G = G_0$ constant, and a running vacuum energy density of the form of (9) which, consequently, simplifies to

$$\rho_\Lambda = \frac{1}{1-\nu}\left[C_0 + (\nu-\alpha)\rho_m\right]. \qquad (11)$$

Our aim is, first of all, to find the law of the matter density in terms of the scale factor, $\rho_m = \rho_m(a)$, and then to state $\rho_\Lambda = \rho_\Lambda(a)$. In this paper, whenever possible, we will provide the results in terms of $a$ because, in this way, we can easily relate them with the cosmological redshift, $z = (1-a)/a$, something very helpful for the interpretation.

In this case, the "mixed" local conservation law in the form of (5) transforms into $a(\rho_m' + \rho_\Lambda') + 3\rho_m = 0$, where $\rho_\Lambda'$ can be easily computed from (11) and replaced, thus obtaining

$$\rho_m' + \frac{3\xi}{a}\rho_m = 0$$

with $\xi := (1-\nu)/(1-\alpha)$. The corresponding normalized solution for this differential equation, i.e., the law $\rho_m(a)$, is

$$\rho_m(a) = \rho_m^0 a^{-3\xi}. \qquad (12)$$

Thereupon, replacing (10) and (12) in (11) we can also derive the law $\rho_\Lambda(a)$:

$$\rho_\Lambda(a) = \rho_\Lambda^0 + \rho_m^0(\xi^{-1}-1)(a^{-3\xi}-1) \qquad (13)$$

Note that taking $\alpha = 0$ and $\nu = 0$, and then $\xi = 1$, we recover the expressions of the standard ΛCDM model.

We can see that the parameter $\xi$ defined above determines the evolution of both densities $\rho_m$ and $\rho_\Lambda$. Since $|\nu|, |\alpha| \ll 1$, it is commonly used the approximation $\xi = 1 - \nu_{\text{eff}}$, where $\nu_{\text{eff}} = \nu - \alpha$; thus, remarkably, we notice that if in this model we consider a running vacuum energy density of the form of (8) with $c_{\dot{H}} = 0$, i.e., $\alpha = 0$, the final results (12) and (13) will be formally the same simply exchanging $\xi = 1 - \nu_{\text{eff}}$ by a new parameter $\zeta := 1 - \nu$.

## V. MODEL 2

Let $\rho_\Lambda$ be of the form of (9), $\dot{G} \neq 0$, and suppose that the standard local conservation law of matter (6) is verified. Our objective now is to find the expression $G = G(a)$.

The conservation law of matter implies that matter density follows the normal law of the ΛCDM model

$$\rho_m(a) = \rho_m^0 a^{-3}; \qquad (14)$$

it can be proved by solving (6) in terms of the scale factor. Thus, the vacuum energy density yields

$$\rho_\Lambda = \frac{1}{1-\nu\frac{G}{G_0}}\left[C_0 + (\nu-\alpha)\frac{G}{G_0}\rho_m^0 a^{-3}\right], \qquad (15)$$

where we don't forget that $G = G(a)$. Nevertheless, due to the fact that $|\nu|$ and $|\alpha|$ are very small, and that we are focusing in the the relatively present universe, where the possible values of $G$ must be closed to $G_0$ (otherwise it should have been appreciated the evolution of $G$ by the observational data), we can approximate $1/(1-\nu G/G_0) \approx 1 + \nu G/G_0$ by Taylor series, in such a way (15) may be estimated as

$$\rho_\Lambda = \left(1 + \nu\frac{G}{G_0}\right)C_0 + (\nu-\alpha)\frac{G}{G_0}\rho_m^0 a^{-3}, \qquad (16)$$

where we have also neglected terms with two or more products between the parameters $\nu$ and $\alpha$.

Therefore, taking into account (6), the conservation law (5) simplifies to

$$G'(\rho_m + \rho_\Lambda) + G\rho_\Lambda' = 0. \qquad (17)$$

It remains to compute $\rho_\Lambda'$ from (16). To this end, we consider the reasonable assumption that the normalized variation of the gravitational coupling is very small, i.e., $|G'/G_0| \ll 1$. Hence, removing terms with two or more products between $\nu$, $\alpha$ and $G'/G_0$, we get

$$\rho_\Lambda' = -3(\nu-\alpha)\rho_m^0\frac{G}{G_0}a^{-4}.$$

Putting all together in (17) and then making some simplifications, we finally obtain the following Bernoulli differential equation for $G = G(a)$

$$G' + \left[-3\frac{\rho_m^0}{G_0}\nu_{\rm eff}\frac{a^{-4}}{C_0 + \rho_m^0 a^{-3}}\right] G^2 = 0.$$

After doing the change of variable $u := G^{-1}$ and multiplying the equation by $-G^{-2}$, it reduces to a linear differential equation for $u = u(a)$ that, in particular, can be solved by direct integration. If we undo the change of variable, we isolate $G$ and we normalize the resulting expression imposing $G(a = 1) = G_0$, then we obtain

$$G(a) = \frac{G_0}{1 - (\nu - \alpha)\ln\left(\frac{C_0+\rho_m^0}{C_0+\rho_m^0 a^{-3}}\right)}, \tag{18}$$

with $C_0$ given in (10), just what we were looking for.

## VI. MODEL 3

Whereas Models 1 and 2 had been proposed previously in the literature (see for instance [5]), this one is presented here for the first time. As in the other models, we assume $\rho_\Lambda$ of the form of (9), but now we state that the gravitational coupling $G$ follows the law

$$G(a) = G_0 a^q \tag{19}$$

with $|q| \ll 1$. Thus, we look for the corresponding law $\rho_m = \rho_m(a)$.

Replacing (19) in (9), we get in this case

$$\rho_\Lambda = \frac{1}{1-\nu a^q}\left[C_0 + (\nu-\alpha)a^q\rho_m\right]. \tag{20}$$

Since $|\nu|, |q| \ll 1$ and we consider the relatively present universe, when the scale factor is comparable to the unity, the approximations $1/(1-\nu a^q) \approx 1 + \nu a^q$ and $a^q \approx 1 + q\ln(a)$ at first order of Taylor series are good enough. In the same line, from now on we will neglect the terms that contain two or more products between $\nu$, $\alpha$ and $q$ because of the smallness of all three parameters. Therefore, the vacuum energy density and its derivative respect to the scale factor can be written as:

$$\rho_\Lambda = (1+\nu)C_0 + (\nu-\alpha)\rho_m$$
$$\rho'_\Lambda = (\nu-\alpha)\rho'_m$$

On the other hand, it is easy to compute $G'(a)$ from (19)

$$G'(a) = qG_0 a^{q-1}.$$

So we have all the ingredients in order to obtain a differential equation for $\rho_m = \rho_m(a)$ from the local conservation law (5); once we replace all the expressions collected, expanding some resulting formulations in Taylor series as well as removing the terms we have specified above, eventually it yields the non-homogeneous linear differential equation

$$\rho'_m + \frac{1}{a}\left[3(1-(\nu-\alpha)) + q\right]\rho_m = -qC_0\frac{1}{a},$$

which can be solved by the variation of constants method. Thus, after some calculations, we get at last the following properly normalized expression for the matter density

$$\rho_m(a) = \left(\rho_m^0 + \frac{qC_0}{\gamma}\right)a^{-\gamma} - \frac{qC_0}{\gamma}, \tag{21}$$

where $\gamma = 3(1-(\nu-\alpha)) + q$. Taking into account (10), and neglecting again terms with two or more products between the parameters $\nu$, $\alpha$ and $q$, (21) may be rewritten as

$$\rho_m(a) = \left(\rho_m^0 + \frac{q\rho_\Lambda^0}{\gamma}\right)a^{-\gamma} - \frac{q\rho_\Lambda^0}{\gamma}. \tag{22}$$

Note that, in the general case, the conservation law of matter (6) is not satisfied in this model. Nevertheless, if we focus on the particular case with $q \approx 3(\nu-\alpha)$; then we have $\gamma \approx 3$, so $\rho_m(a)$ can be approximated with the law $\sim \rho_m^0 a^{-3} + (\nu-\alpha)\rho_\Lambda^0(a^{-3}-1)$; this may be interpreted, at first approximation in our relatively present universe, as the usual law $\sim \rho_m^0 a^{-3}$, i.e., it may be followed the matter conservation law (6). Finally, it is straightforward to check out that for $q \to 0$, i.e., $G \to G_0$, we get (13), the result of Model 1, and also that when all three parameters $\nu$, $\alpha$ and $q$ tends to zero together, i.e., $G \to G_0$ and $\rho_\Lambda \to \rho_\Lambda^0$, we recover the normal law $\rho_m = \rho_m^0 a^{-3}$ of the standard $\Lambda$CDM model. In addition, see that for a vacuum energy density constant, i.e. $\nu = \alpha = 0$, the final expression of $\rho_m$ would be formally the same changing $\gamma$ by $\gamma' := 3 + q$.

## VII. VACUUM DYNAMICS AND MATTER NON-CONSERVATION: THE MICRO AND MACRO CONNECTION

Once we have developed the three proposed models in the previous sections, it is time to analyze the usefulness of each of them for our purpose: to explain the possible time variation of fundamental constants of particle physics. We are going to see that there is a condition which a cosmological model has to verify in order to be appropriate for this micro and macro connection: the usual local conservation law of matter (6) should no longer be satisfied.

Let $\rho_B$ be the (baryonic) matter density in the universe; since it is essentially the mass density of protons, we can write it as $\rho_B = n_p m_p$, where $n_p$ is the number density of protons and $m_p^0$ will denote the current proton mass. Since the non conservation of $\rho_B$ in the usual sense implies that it does not follow the law $\rho_B = \rho_B^0 a^{-3}$ anymore, we have that, either $n_p$ does not follow the strictly

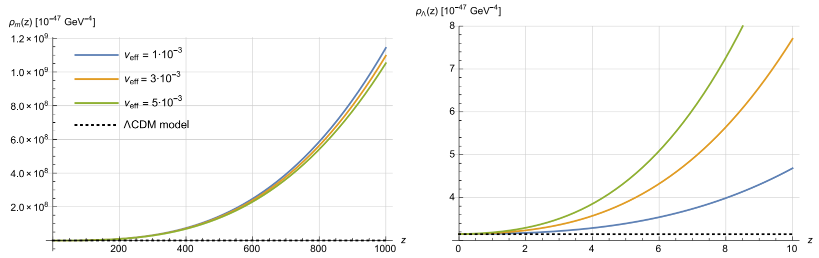


FIG. 1: Left: Evolution of the matter density $\rho_m$ obtained in Model 1, in terms of the redshift, for different values of the parameter $\nu_{\rm eff}$. Right: The corresponding evolution of the vacuum energy density $\rho_\Lambda$.

normal dilution law $n_p \sim a^{-3}$ but an anomalous one, and/or the proton mass $m_p$ suffers a dynamical evolution and has no longer a constant value; in all cases it is assumed that the vacuum absorbs the difference. In fact, we are particularly interested in the second possibility, which would imply a dynamical proton mass, inasmuch as it may provide us a direct explanation of the possible variations of the fundamental constants mentioned in the Introduction.

However, let us consider a cosmological model which admits time-evolving parameters but at the same time satisfies the local conservation law of matter (6); in this case, $\rho_B$ follows its normal law $\rho_B = \rho_B^0 a^{-3}$, and we cannot repeat the same argument as in the previous paragraph in order to relate the cosmological model with the possible variations of fundamental constants of nature. Thus, such a model would explain the possible dynamical character of the cosmological parameters $\rho_\Lambda$ and $G$ in a way which is independent from the microphysical phenomena in particle and nuclear physics.

We can conclude, then, that the exchange of energy between matter and vacuum presents in Models 1 and 3 may be straightly linked to the time-evolving masses in the Standard Model of particle physics, while Model 2 can't be used to explain this problem due to its local conservation law of matter. The next step would be to estimate the bounds of the parameters involved in both Models 1 and 3 confronting each of them with the observational data we referred in the Introduction; a thorough review of this procedure in the case of Model 1 can be found in [7]. For instance, using the results of the mentioned paper, which give us that $\nu_{\rm eff} \sim 10^{-3}$, we present in Figure 1 a representation of the results of Model 1 with different values of the parameter $\nu_{\rm eff}$; note that we have approximated $\xi = 1 - \nu_{\rm eff}$ and represented the densities in terms of the redshift by using $a = (z+1)^{-1}$.

## VIII. CONCLUSIONS

In this work we have attempted to explain the possible cosmic time evolution of fundamental constants of particle and nuclear physics through the context of cosmology, by letting alternative cosmological models compatible with a time-evolving pair $(\rho_\Lambda, G)$; this is what we called "the micro and macro connection" as in [6]. We first have obtained a set of cosmological equations that allow time variations of both $\rho_\Lambda$ and $G$, realizing that now the Bianchi identity provides us a "mixed" local conservation law which generalizes the usual local covariant conservation law of matter so it could be a transfer of energy between vacuum and matter. From this new set of equations, we have proposed and developed three different models, based on the ones presented in [5][6][7][8], with only one feature in common: a running vacuum energy density whose evolution is motivated by the quantum fiel theory in curved space time. In this development, they deserve special mention the analytical results of Model 3 since it has been suggested here for the first time. The search for the relation between these models and the change of the basic quantities of the Standard Model of particle physics has finally led us to see that this micro and macro connection can only be achieved if, besides a mild dynamical behavior of $\rho_\Lambda$ and/or $G$, the considered cosmological model does not satisfy the usual conservation law of matter. Thus, whereas Models 1 and 3 could be considered by our purpose, Model 2 could not.

**Acknowlegments:** I would like to thank my advisor Joan Solà for all his support and implication, as well as for introducing me to this amazing field. Of course, I also thank my parents, my grandparents and my girlfriend, who are always by my side

[1] P. A. M. Dirac, Nature **139** (1937) 323.
[2] J. P. Uzan. Rev. Mod. Phys. **75** (2003) 403.
[3] J. Solà. Mod. Phys. Lett. **A30** (2015) 22, 1502004.
[4] J. Solà, J. Phys. Conf. Ser. **453** (2013) 012015.
[5] H. Fritzsch and J. Solà. Class. Quant. Grav. **29** (2012) 215002.
[6] H. Fritzsch and J. Solà. Mod. Phys. Lett. **A30** (2015) 22, 1540034.
[7] A. Gómez-Valent, J. Solà and S. Basilakos. JCAP **1501** (2015) 004.
[8] J. Solà, A. Gómez-Valent and J. de C. Pérez. Astrophys. J. **811** (2015) L14.