CLOSURE OF HARDY SPACES IN THE BLOCH SPACE

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Abstract. A description of the Bloch functions that can be approximated in the Bloch norm by functions in the Hardy space H^p of the unit ball of $\mathbb{C}^n$ for $0 < p < \infty$ is given. When $0 < p \leq 1$, the result is new even in the case of the unit disk.

1. Introduction.

Let $\mathbb{D}$ and $\mathbb{T}$ be, respectively, the unit disk and the unit circle of the complex plane $\mathbb{C}$. For $0 < p < \infty$, recall that the Hardy space $H^p(\mathbb{D})$ is the space of analytic functions f in the unit disc such that

$$\|f\|_p^p = \sup_{0<r<1} \int_0^{2\pi} |f(re^{i\theta})|^p \, \frac{d\theta}{2\pi} < +\infty.$$

For $p = \infty$, $H^\infty(\mathbb{D})$ is the space of all bounded analytic functions in the unit disk. Recall also that the Bloch space $\mathcal{B}(\mathbb{D})$ is formed by the analytic functions f on $\mathbb{D}$ such that

$$\|f\|_{\mathcal{B}} = \sup_{z\in\mathbb{D}} (1-|z|^2)\,|f'(z)| < \infty.$$

In [10], a characterization of the closure in the Bloch norm of $H^p \cap \mathcal{B}$ for $1 < p < \infty$ was given in terms of the area of certain non-tangential level sets of the Bloch function: given a function $f \in \mathcal{B}$ and $\varepsilon > 0$ define the level set of f as

$$\Omega_\varepsilon(f) := \{z \in \mathbb{D} \ : \ (1-|z|^2)|f'(z)| \geq \varepsilon\}.$$

Recall that a Stolz angle with vertex in $\zeta \in \mathbb{T}$ is the set

$$\Gamma(\zeta) = \Gamma_\alpha(\zeta) := \{z \in \mathbb{D} : |z-\zeta| < \frac{\alpha}{2}(1-|z|)\},$$

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with $\alpha > 2$, and that

$$A_h(\Omega) := \int_\Omega \frac{dA(z)}{(1-|z|^2)^2},$$

where $dA(z)$ is the area measure in $\mathbb{D}$, represents the hyperbolic area of $\Omega \subset \mathbb{D}$. Then the result is the following:

Theorem A. *Let f be a function in the Bloch space $\mathcal{B}$ and $1 < p < \infty$. Then f is in the closure in the Bloch norm of $\mathcal{B} \cap H^p$ if and only if for any $\varepsilon > 0$ the function $A_h(\Gamma(\zeta) \cap \Omega_\varepsilon(f))^{1/2}$ is in $L^p(\mathbb{T})$.*

A basic tool in the proof of this result was the characterization of Hardy spaces in terms of the area function (a result due to Marcinkiewicz and A. Zygmund [9] for $p > 1$, and extended to the case $0 < p \leq 1$ by A. Calderón [3]), that is, for $0 < p < \infty$, a function f is in H^p if and only if its corresponding Lusin Area function

$$A(f)(\zeta) = \left(\int_{\Gamma(\zeta)} |f'(z)|^2 dA(z) \right)^{1/2}$$

is in $L^p(\mathbb{T})$. The proof of Theorem A was based on a previous result by P. Jones on the closure of $BMOA$ in $\mathcal{B}$ (see [6]). The duality argument given in the proof in [10] can not be used for $0 < p \leq 1$, so that this case requires of new techniques. In this paper we solve the case $0 < p \leq 1$. It turns out that the proof given works equally for all $0 < p < \infty$, and furthermore, it may be done in the open unit ball $\mathbb{B}_n$ of the n-dimensional complex space $\mathbb{C}^n$. The case $p = \infty$ is still an open problem, and will be discussed in the last Section.

Now we are going to introduce some notation. For $z, w \in \mathbb{C}^n$, let

$$\langle z, w \rangle = z_1 \bar{w}_1 + \cdots + z_n \bar{w}_n.$$

Hence, $|z|^2 = \langle z, z \rangle$. In this context, for $0 < p < \infty$ the Hardy space $H^p(\mathbb{B}_n)$ consists of those holomorphic functions f on $\mathbb{B}_n$ such that

$$\|f\|_p^p = \sup_{0<r<1} \int_{\mathbb{S}_n} |f(r\zeta)|^p \, d\sigma(\zeta) < +\infty,$$

where $\mathbb{S}_n$ denotes the unit sphere in $\mathbb{C}^n$ and σ is the normalized surface measure on $\mathbb{S}_n$. As in the case for $n = 1$, for $p = \infty$ the corresponding space $H^\infty(\mathbb{B}_n)$ is the space of bounded holomorphic functions defined on $\mathbb{B}_n$.
The Hardy space $H^p(\mathbb{B}_n)$ may be also characterized by means of a corresponding area function. In order to define it, let Rf denote the radial derivative of f, that is,

$$Rf(z) = \sum_{k=1}^n z_k \frac{\partial f}{\partial z_k}(z), \qquad z = (z_1, \dots, z_n) \in \mathbb{B}_n.$$

Besides, the hyperbolic measure in $\mathbb{B}_n$ is given by

$$d\lambda_n(z) = \frac{dv(z)}{(1-|z|^2)^{n+1}},$$

where $dv(z)$ is the normalized volume measure in $\mathbb{C}^n$. The admissible Area function is then defined as

$$Af(\zeta) = \left(\int_{\Gamma(\zeta)} |Rf(z)|^2 \, (1-|z|^2)^2 \, d\lambda_n(z) \right)^{1/2}, \qquad \zeta \in \mathbb{S}_n,$$

where $\Gamma(\zeta)$ denotes now the admissible Koranyi region, that is,

$$\Gamma(\zeta) = \Gamma_\alpha(\zeta) := \left\{ z \in \mathbb{B}_n : |1 - \langle z, \zeta \rangle| < \frac{\alpha}{2}(1-|z|^2) \right\},$$

for $\alpha > 2$ fixed. When $n = 1$ this region coincides with the usual Stolz angle in $\mathbb{D}$. The following result is the generalization of the area theorem for $\mathbb{B}_n$ and can be found, for example, in [4] or [12, Theorem 5.3].

Theorem B. *Let $0 < p < \infty$, then $f \in H^p(\mathbb{B}_n)$ if and only if $Af \in L^p(\mathbb{S}_n)$. Furthermore, if $f(0) = 0$ the norms $\|f\|_{H^p}$ and $\|Af\|_{L^p}$ are comparable.*

The Bloch space $\mathcal{B} := \mathcal{B}(\mathbb{B}_n)$ consists of those functions f holomorphic on $\mathbb{B}_n$ such that

$$\|f\|_{\mathcal{B}} = \sup_{z \in \mathbb{B}_n} (1-|z|^2)|Rf(z)| < \infty,$$

This seminorm is not conformally invariant (for more details, see [15, Chapter 3]), but it is equivalent to the seminorm defined above for the unidimensional case, and more convenient for the statements here. As in the one-dimensional case, $H^\infty(\mathbb{B}_n) \subset \mathcal{B}(\mathbb{B}_n)$.

The level sets here are defined as

$$\Omega_\varepsilon(f) := \{z \in \mathbb{B}_n \ : \ (1-|z|^2)|Rf(z)| \geq \varepsilon\}.$$

Clearly, the hyperbolic volume of any set $\Omega \subset \mathbb{B}_n$ is

$$V_h(\Omega) = \int_\Omega d\lambda_n.$$

The result proved in this paper is the following one.

Theorem 1. *Let f be a function in the Bloch space $\mathcal{B}(\mathbb{B}_n)$ and $0 < p < \infty$. Then f is in the closure in the Bloch norm of $\mathcal{B} \cap H^p(\mathbb{B}_n)$ if and only if for any $\varepsilon > 0$ the function $V_h(\Gamma(\zeta) \cap \Omega_\varepsilon(f))^{1/2}$ is in $L^p(\mathbb{S}_n)$.*

The paper is organized as follows. After some preliminaries given in Section 2, we prove Theorem 1 in Section 3. Finally, in Section 4, we disprove a conjecture of Xiao on the closure of $H^\infty(\mathbb{D})$ in the Bloch space.

2. Preliminary results

We will use the fact given in [15, p.51] that one may express a function $f \in \mathcal{B}$ as

$$f(z) = f(0) + \int_{\mathbb{B}_n} Rf(w)\, L(z,w)\, dv_\beta(w), \tag{1}$$

where the kernel

$$L(z,w) = \int_0^1 \left(\frac{1}{(1 - t\langle z,w\rangle)^{n+1+\beta}} - 1 \right) \frac{dt}{t}$$

satisfies

$$|L(z,w)| \le \frac{C}{|1-\langle z,w\rangle|^{n+\beta}}.$$

Here

$$dv_\beta(z) = c_\beta (1-|z|^2)^\beta \, dv(z),$$

where $\beta > -1$ and c_β is a normalizing constant taken so that $v_\beta(\mathbb{B}_n) = 1$.

The following integral estimate has become indispensable in this area of Analysis. One may find the proof in [15, Theorem 1.12].

Lemma C. *Let $t > -1$ and $s > 0$. There is a positive constant C such that*

$$\int_{\mathbb{B}_n} \frac{(1-|w|^2)^t \, dv(w)}{|1-\langle z,w\rangle|^{n+1+t+s}} \le C\,(1-|z|^2)^{-s}$$

for all $z \in \mathbb{B}_n$.

The next result may be thought as a generalized version of Lemma C, and appears in [11, Lemma 2.5].

Lemma D. *Let $s > -1$, $r, t > 0$, and $r + t - s > n + 1$. If $t, r < s + n + 1$ then, for $a, z \in \mathbb{B}_n$, one has*

$$\int_{\mathbb{B}_n} \frac{(1-|w|^2)^s}{|1-\langle z,w\rangle|^r \, |1-\langle a,w\rangle|^t} \, dv(w) \le C \frac{1}{|1-\langle z,a\rangle|^{r+t-s-n-1}}.$$

The following estimation may be found in [2] and [7], and it is the analogue in $\mathbb{B}_n$ of [8, Proposition 1].

Lemma E. *Let $0 < s < \infty$ and $b > n \max(1, 1/s)$. Then there exists a constant $C > 0$ depending on s, b and on the angle of the region $\Gamma(\zeta)$ such that*

$$\int_{\mathbb{S}_n} \left(\int_{\mathbb{B}_n} \left(\frac{1-|z|^2}{|1-\langle z,\zeta\rangle|} \right)^b d\mu(z) \right)^s d\sigma(\zeta) \le C \int_{\mathbb{S}_n} \mu(\Gamma(\zeta))^s \, d\sigma(\zeta),$$

where μ is a positive measure on $\mathbb{B}_n$.

3. Proof of Theorem 1.

Let first f be in the closure in the Bloch norm of $H^p \cap \mathcal{B}$. Then, given $\varepsilon > 0$ there exists $g \in H^p$ such that $\|f - g\|_{\mathcal{B}} < \varepsilon/2$. As in the proof of Theorem A in [10], one just needs to observe that $\Omega_\varepsilon(f) \subseteq \Omega_{\varepsilon/2}(g)$ to see that for any $\zeta \in \mathbb{S}_n$

$$V_h(\Gamma(\zeta) \cap \Omega_\varepsilon(f)) \leq \frac{4}{\varepsilon} Ag(\zeta)^2.$$

Since $Ag \in L^p(\mathbb{S}_n)$, the necessity is proved.

From now on, set $\Omega_\varepsilon = \Omega_\varepsilon(f)$. Let f be in the Bloch space and assume that $V_h(\Gamma(\zeta) \cap \Omega_\varepsilon)^{1/2}$ is in $L^p(\mathbb{S}_n)$. Given $\varepsilon > 0$, we need to find $f_2 \in \mathcal{B} \cap H^p$ such that $\|f - f_2\|_{\mathcal{B}} \leq \varepsilon$. To this end, and applying (1), f may be decomposed in the sum of two functions $f(z) = f_1(z) + f_2(z)$, where

$$f_1(z) = \int_{\mathbb{B}_n \setminus \Omega_\varepsilon} Rf(w)\, L(z,w)\, dv_\beta(w)$$

and

$$f_2(z) = f(0) + \int_{\Omega_\varepsilon} Rf(w)\, L(z,w)\, dv_\beta(w),$$

with β big enough to be fixed later. It is easy to see that

$$\begin{aligned} |Rf_1(z)| &\leq (n+1+\beta) \int_{\mathbb{B}_n \setminus \Omega_\varepsilon} |Rf(w)| \left| \int_0^1 \frac{\langle z,w\rangle}{(1 - t\langle z,w\rangle)^{n+2+\beta}}\, dt \right| dv_\beta(w) \\ &\leq \varepsilon\, C(n,\beta) \int_{\mathbb{B}_n} \frac{(1-|w|^2)^{\beta-1}}{|1 - \langle z,w\rangle|^{n+\beta+1}} dv(w). \end{aligned}$$

Now Lemma C with $t = \beta - 1$ and $s = 1$ shows that $\|f_1\|_{\mathcal{B}} \leq C\,\varepsilon$.

Thus it remains to see that $f_2 \in H^p(\mathbb{B}_n)$, or by Theorem B, that $Af_2 \in L^p(\mathbb{S}_n)$. As in the previous estimation for Rf_1 one has that

$$\begin{aligned} |Rf_2(z)|^2 &\leq C(n,\beta)\, \|f\|_{\mathcal{B}}^2 \left(\int_{\Omega_\varepsilon} \frac{(1-|w|^2)^{\beta-1}}{|1 - \langle z,w\rangle|^{n+\beta+1}}\, dv(w) \right)^2 \\ &\leq \tilde{C}(n,\beta)\, \|f\|_{\mathcal{B}}^2 \left(\int_{\Omega_\varepsilon} \frac{(1-|w|^2)^{\beta-1}}{|1 - \langle z,w\rangle|^{n+\beta+1}}\, dv(w) \right) (1-|z|^2)^{-1}, \end{aligned}$$

after an application of Lemma C. Then Fubini's theorem gives

$$\begin{aligned} Af_2(\zeta)^2 &\leq C_1 \int_{\Gamma(\zeta)} \left(\int_{\Omega_\varepsilon} \frac{(1-|w|^2)^{\beta-1}}{|1 - \langle z,w\rangle|^{n+\beta+1}}\, dv(w) \right) \frac{dv(z)}{(1-|z|^2)^n} \\ &= C_1 \int_{\Omega_\varepsilon} \left(\int_{\Gamma(\zeta)} \frac{dv(z)}{(1-|z|^2)^n\, |1 - \langle z,w\rangle|^{n+\beta+1}} \right) (1-|w|^2)^{\beta-1} dv(w). \end{aligned}$$

Now, $z \in \Gamma(\zeta)$ implies that $(1-|z|^2) \simeq |1-\langle z,\zeta\rangle|$. Then, applying Lemma D with $r=t=n+\beta+1$ and $s=\beta+1$ one has that

$$\begin{aligned}\int_{\Gamma(\zeta)} \frac{dv(z)}{(1-|z|^2)^n\,|1-\langle z,w\rangle|^{n+\beta+1}} &\simeq \int_{\Gamma(\zeta)} \frac{(1-|z|^2)^{\beta+1}\,dv(z)}{|1-\langle z,\zeta\rangle|^{n+\beta+1}\,|1-\langle z,w\rangle|^{n+\beta+1}}\\ &\le \int_{\mathbb{B}_n} \frac{(1-|z|^2)^{\beta+1}\,dv(z)}{|1-\langle z,\zeta\rangle|^{n+\beta+1}\,|1-\langle z,w\rangle|^{n+\beta+1}}\\ &\le \frac{C_2}{|1-\langle w,\zeta\rangle|^{n+\beta}}.\end{aligned}$$

Hence,

$$\begin{aligned}\|Af_2\|_{L^p}^p &\le C_3 \int_{\mathbb{S}_n} \left(\int_{\Omega_\varepsilon} \frac{1}{|1-\langle w,\zeta\rangle|^{n+\beta}} (1-|w|^2)^{\beta-1}\,dv(w)\right)^{p/2} d\sigma(\zeta)\\ &= C_3 \int_{\mathbb{S}_n} \left(\int_{\Omega_\varepsilon} \frac{(1-|w|^2)^{n+\beta}}{|1-\langle w,\zeta\rangle|^{n+\beta}} \frac{dv(w)}{(1-|w|^2)^{n+1}}\right)^{p/2} d\sigma(\zeta)\\ &= C_3 \int_{\mathbb{S}_n} \left(\int_{\mathbb{B}_n} \left(\frac{(1-|w|^2)}{|1-\langle w,\zeta\rangle|}\right)^{n+\beta} d\mu(w)\right)^{p/2} d\sigma(\zeta).\end{aligned}$$

Here

$$d\mu(w) = \frac{\chi_{\Omega_\varepsilon}(w)dv(w)}{(1-|w|^2)^{n+1}},$$

where $\chi_{\Omega_\varepsilon}$ denotes the characteristic function of Ω_ε, is a positive Borel measure. Then Lemma E with $s=p/2$ and $b=n+\beta$, where β is positive and bigger than $n\cdot(2/p-1)$, shows that $Af_2 \in L^p(\mathbb{S}_n)$. This finishes the proof.

4. The case $p=\infty$ and $n=1$.

We consider only the one variable case of the unit disk in this last section. The problem of describing the closure of the space of bounded analytic functions in the Bloch norm was posed in [1], and still remains open. Remember that $H^\infty \subset \mathcal{B}$. Theorem B does not hold for $p=\infty$, so the proof given here does not work in this case. Nevertheless, it is interesting to observe that the proof given also works if one considers the class of analytic functions with area function Af in L^∞.

One may also see that the analogue for $p=\infty$ of the condition given in Theorem 1 (that is, $A_h(\Omega_\varepsilon(f)\cap\Gamma(\zeta)) \in L^\infty(\mathbb{T})$) is not necessary for a function f to be in the closure in the Bloch norm of the space of bounded analytic functions. To this end, for $k\in\mathbb{N}$ take the points $z_k = 1-2^{-k}$, and consider the sequence $\{z_k\}$, which is a radial separated sequence. In particular, $\{z_k\}$ is an interpolating sequence for H^∞ (see [5, Chapter VII, p.279]). By Carleson interpolation theorem, there exists $\delta>0$ such that

$$(1-|z_k|^2)|B'(z_k)| \ge \delta,$$

where B denotes the Blaschke product with zeros $\{z_k\}$, which is clearly in H^∞. Given now $\varepsilon < \delta/4$ there exists $\rho > 0$ such that $(1-|z|^2)|B'(z)| \geq \varepsilon$ on each $D_h(z_k, \rho)$, that is, the hyperbolic disk with center z_k and radius ρ. Hence,

$$\bigcup_{k\in\mathbb{N}} D_h(z_k, \rho) \subset \Omega_\varepsilon(B) \cap \Gamma(1).$$

Since the sequence is separated, one can take $\rho > 0$ so that the disks $D_h(z_k, \rho)$ are pairwise disjoints. Now it is easy to see that $A_h(\Omega_\varepsilon(B) \cap \Gamma(\zeta))$ is not in $L^\infty(\mathbb{T})$, since $A_h(D_h(z_k, \rho)) \geq C$ for a certain constant $C > 0$ only depending on ρ.

In [13, Section 3.6] one finds a sufficient condition for a Bloch function to be in the closure in the Bloch norm of $H^\infty(\mathbb{D})$. The condition is the following: For every $\varepsilon > 0$ one has that

$$\sup_{w\in\mathbb{D}} \int_{\Omega_\varepsilon(f)} \frac{1}{|1-\overline{w}z|^2}\, dA(z) < \infty. \tag{2}$$

The sufficiency of this condition is checked by following the argument from [10] given in the proof of Theorem 1. Let $f \in \mathcal{B}$ satisfy (2). Without loss of generality one may take $f(0) = f'(0) = 0$. Hence the function can be expressed by the following integral (see [14, Proposition 4.27])

$$f(w) = \int_{\mathbb{D}} \frac{(1-|z|^2)f'(z)}{\overline{z}(1-\overline{z}w)^2}\, dA(z) = f_1(w) + f_2(w),$$

where f_1 and f_2 are taken as in the previous section. As before, we see that $\|f_1\|_{\mathcal{B}} \leq C\,\varepsilon$. In order to see that $f_2 \in H^\infty$ one just has to observe that

$$|f_2(w)| \leq \int_{\Omega_\varepsilon(f)} \frac{(1-|z|^2)|f'(z)|}{|\overline{z}||1-\overline{w}z|^2}\, dA(z) \leq C(\varepsilon)\,\|f\|_{\mathcal{B}} \int_{\Omega_\varepsilon(f)} \frac{dA(z)}{|1-\overline{w}z|^2},$$

and then apply the hypothesis.

Actually, in [13, p.71], J. Xiao conjectured that condition (2) is also necessary. Nevertheless, the same example as above gives a counterexample to that conjecture, just by evaluating the integral for w approaching 1 non-tangentially. Indeed, if $w_m = 1 - 2^{-m}$, then

$$\int_{\Omega_\varepsilon(B)} \frac{1}{|1-\overline{w}_m z|^2}\, dA(z) \geq \sum_{k=1}^{m} \int_{D_h(z_k,\rho)} \frac{1}{|1-\overline{w}_m z|^2}\, dA(z).$$

Now, using the estimate (2.20) in p.63 of [15], it is easy to see that there is a constant C depending on ρ such that $|1-\overline{w}_m z| \leq C(1-|z_k|)$ for $z \in D_h(z_k, \rho)$ and $k \leq m$. This clearly implies that

$$\int_{\Omega_\varepsilon(B)} \frac{1}{|1-\overline{w}_m z|^2}\, dA(z) \longrightarrow +\infty$$

as $m \to \infty$, proving that the condition (2) is not necessary.

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