Jābir b. Aflaḥ on the four-eclipse method for finding the lunar period in anomaly

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Abstract
The four-eclipse method was used by pre-Ptolemaic astronomers, especially Hipparchus, for finding the lunar period in anomaly. It is described by Ptolemy in *Almagest* IV.2 where he adds new considerations to be fulfilled in order to obtain a correct period in anomaly. Jābir b. Aflaḥ, who lived in early twelfth-century al-Andalus, considers this method in his *Iṣlāḥ al-Majisṭī*. In his opinion, Ptolemy did not understand the conditions stated by the ancients. Jābir b. Aflaḥ provides a complete set of conditions that makes Ptolemy's additions to the method unnecessary. In any case, the method presented by Jābir b. Aflaḥ is more coherent and elegant from a mathematical point of view that Ptolemy's.

Prolegomena

Jābir b. Aflaḥ was an Andalusian mathematician and astronomer, probably from Seville, whose work dates from the first part of the 12th century. His most notable achievement was the book *Iṣlāḥ al-Majisṭī* or *Correction of the Almagest*, in which he rewrote the *Almagest* to simplify its mathematics. He also introduced some criticisms of the original *Almagest*, although these were mainly from a mathematical perspective. In this paper we intend to study the first of these criticisms as they are listed in the introduction to the *Iṣlāḥ al-Majisṭī*. This criticism focuses on the four-eclipse method used by ancient astronomers to find the lunar period in anomaly, as described by Ptolemy. This is a main point in Ptolemy's description of the lunar theory since all his developments are based in this period initially found by Hipparchus. Even though, this issue has deserved little attention in secondary bibliography.[1] We will first describe the mathematical functions involved in this method. We will then present the method as described by Ptolemy. Lastly, we will consider the criticisms made by Jābir b. Aflaḥ in which he shows that Ptolemy did not state the conditions required clearly. This is followed by a translation of this section of the *Iṣlāḥ al-Majisṭī* from the two Arabic versions and a working edition from the three existing Arabic manuscripts in Arabic script.

1. Notation

The following notation is used in this paper. Otherwise stated, it applies to both Sun and Moon.

α	Angle of anomaly
λ	Longitude
λ	Mean longitude
λ_0	Longitude at epoch
λ_A	Apogee longitude
ω	Angular motion
a	Acceleration in longitude
A	Apogee point
c	Equation of anomaly
e	Eccentricity
i	An integer number
k	Solar true anomaly
k	Solar mean anomaly
M	Mesogee point
P	Perigee point
r	Radius of the epicycle
R	Radius of the lunar deferent and radius of the solar eccentric
t	Time
T	Period of the anomaly
v	True motion in longitude
v	Mean motion in longitude
v_a	Motion in longitude due to the anomaly

[1] As far as I know, only Neugebauer considers it in O. Neugebauer [1975], *A History of Ancient Mathematical Astronomy*, 3 vols., Berlin – Heidelberg – New York, pp. 71-3 [henceforth referred to as HAMA].

2. Functions

Before studying the work of Ptolemy and Jābir b. Aflaḥ on the four-eclipse method for finding the lunar anomaly period, some introductory information is required. Given that Jābir b. Aflaḥ bases his description of the method on the first lunar model and considers the lunar equation of anomaly and the lunar motion in anomaly, we shall describe these three functions in order to understand Jābir b. Aflaḥ's criticism of Ptolemy. It is also important to explain the solar equation as Ptolemy considers its effect on the four-eclipse method.

2.1 Lunar equation of anomaly

Vid. Figure 1. Let AM_1PM_2 be an epicycle with centre H and radius r where A is the apogee, P is the perigee, M_1 and M_2 are the points at which the true lunar motion equals its mean motion – from now on we will refer to these points as the *mesogee*. This epicycle moves along a deferent with radius R and centre Z, the centre of the ecliptic. Let the Moon be on L, so that the angle $\angle AHL$ is the anomaly of the epicycle (α) and the angle $\angle AZL$ is the equation of anomaly (c).

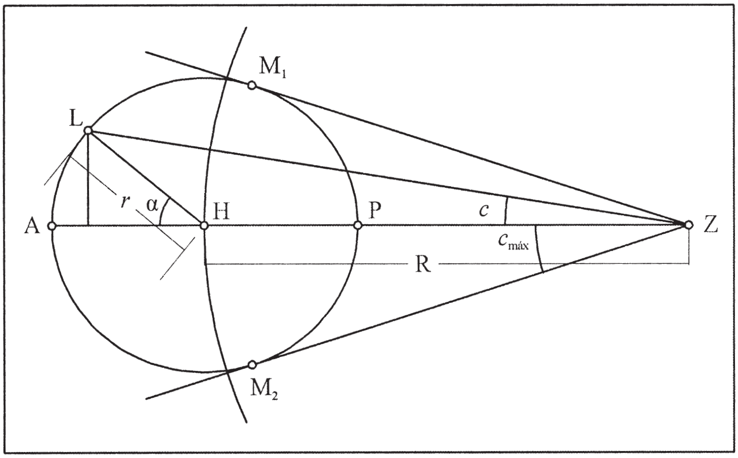

Figure 1. Equation of anomaly (c)

The lunar true longitude, λ, will be

$$\lambda = v\,t - c + \lambda_0 \qquad (1)$$

where v is the lunar mean motion in longitude, λ_0 is the lunar longitude at initial conditions and t is time. From Figure 1 we can conclude that c is

$$c = \arctan \frac{r \sin \alpha}{R + r \cos \alpha} = \text{arcsen} \frac{r \sin \alpha}{\sqrt{R^2 + r^2 + 2Rr \cos \alpha}} \qquad (2)$$

which is represented in Figure 2 assuming the values given by Ptolemy for r and R – that is, $r = 6;20^p$ and $R = 60^p$. Given that $\alpha = \omega\, t + \alpha_0$ where ω is the lunar angular motion on its epicycle and α_0 is the lunar anomaly at initial conditions, c will be

$$c = \arctan \frac{r \sin(\omega t + \alpha_0)}{R + r \cos(\omega t + \alpha_0)} = \arcsin \frac{r \sin(\omega t + \alpha_0)}{\sqrt{R^2 + r^2 + 2Rr \cos(\omega t + \alpha_0)}} \qquad (3)$$

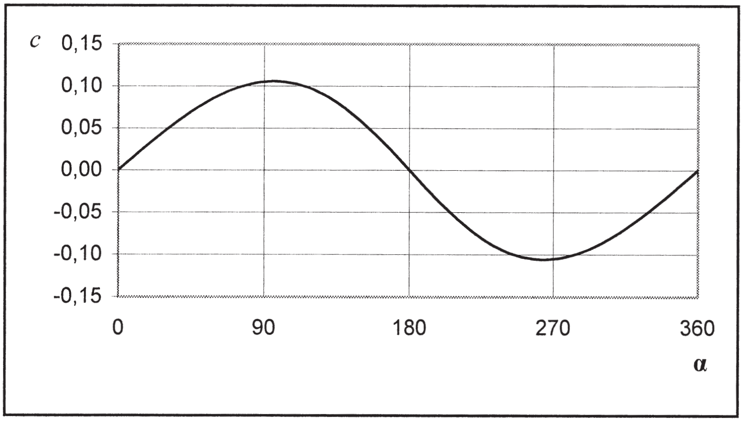

Figure 2. Equation of anomaly as a function of the anomaly

This is a periodic function with $T = 360^\circ / \omega$ and inherits the symmetry of the sinus function: that is, a 2-fold rotational symmetry about a centre $\alpha = 0^\circ$ or $\alpha = 180^\circ$, so that

$$c\,(\alpha) = -\,c\,(i360^\circ - \alpha) \qquad (4)$$

for each integer number i. In addition, given that the arcsine of x is zero whenever x is zero, c will be zero whenever $\sin \alpha$ is zero. Therefore, $c = 0$ for $\alpha = 0^\circ$, the apogee, and $\alpha = 180^\circ$, the perigee.

2.2 Lunar motion in anomaly

In order to obtain the lunar true motion in longitude, v, we have to obtain the time variation of the lunar longitude in (1). That is,

$$v(t) = v - v_a(t) \tag{5}$$

where v_a is the lunar motion in longitude due to the anomaly, i.e. the time variation of the lunar equation of anomaly. Therefore,

$$v_a(t) = \frac{\mathrm{d}c}{\mathrm{d}t} = \omega \frac{r^2 + Rr\cos\alpha}{R^2 + r^2 + 2Rr\cos\alpha} \tag{6}$$

which is represented in Figure 3 assuming Ptolemaic values for r and R.

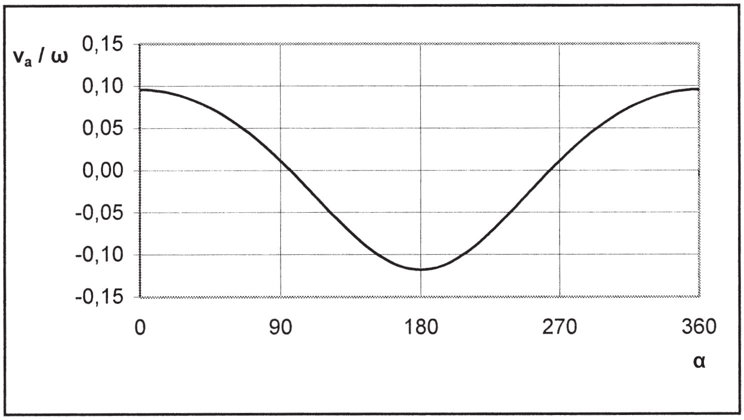

Figure 3. Lunar motion in longitude due to the anomaly (v_a) normalized by the lunar angular motion on its epicycle (ω)

As we can see, v_a has a maximum at the apogee, $\alpha = 0°$, and a minimum at the perigee, $\alpha = 180°$, but $|v_a|$ is greater at the perigee than at the apogee. As above, this is a periodic function with T = 360° / ω but with mirror symmetry about a centre $\alpha = 0°$ or $\alpha = 180°$. Therefore,

$$v_a(\alpha) = v_a(i360° - \alpha) \tag{7}$$

for each integer number i. The zero values of v_a are found for

$$\alpha = \arccos\left(-\frac{r}{R}\right) \tag{8}$$

provided that R > r. The anomalies that satisfy (8) are α_1 = 96;3,33,1,53,32° and α_2 = 263;56,26,58,6,27° and correspond to the anomalies of both mesogees (points M_1 and M_2 of Figure 1), these being the points at which the equation of anomaly is greater and the lunar true motion is equal to its mean motion, $v = v$.

2.3 Lunar acceleration

The lunar acceleration in longitude, a, is only due to its anomaly and can be obtained as the time variation of the lunar motion in longitude. Then, the acceleration function is

$$a(t) = \frac{\mathrm{d}v}{\mathrm{d}t} = \omega^2 \frac{Rr\left(r^2 - R^2\right)\sin\alpha}{\left(R^2 + r^2 + 2Rr\cos\alpha\right)^2} \tag{9}$$

and is shown in Figure 4.

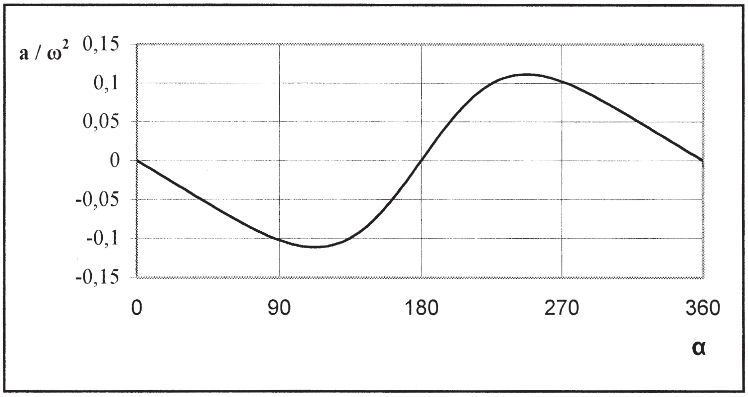

Figure 4. Lunar acceleration in longitude normalized by the quadrant of lunar angular motion in its epicycle (ω)

As above, being a function of the anomaly, the acceleration is a periodic function with T = 360° / ω. This function also inherits the symmetry of the sinus function and therefore has 2-fold rotational symmetry about a centre α = 0° or α = 180°. Therefore,

$$a(\alpha) = -a(i360^\circ - \alpha) \qquad (10)$$

for each integer number i. It can be proved that the anomalies α_1 = 112;43,11,35,9,34° and α_2 = 247;16,48,24,50,25° are the solutions to the equation

$$2rR\cos^2\alpha - (r^2 + R^2)\cos\alpha - 4rR = 0 \qquad (11)$$

These solutions render the derivative of the acceleration void and therefore indicate the anomalies in which the minimum and maximum acceleration is found.

In summary, from the above analysis of the equation of anomaly and the motion in anomaly functions, the anomaly can be divided into four sectors depending on whether the equation of anomaly, the motion in anomaly and the acceleration are positive or negative. This is shown in the following table:

Sectors of anomaly defined by c, v_a and a					
Sector	**Arc**	**Initial anomaly**	c	v_a	a
S_1	AM_1	0°	+	+	+
S_2	M_1P	96;3,33,1,53,32°	+	–	+
S_3	PM_2	180°	–	–	–
S_4	M_2A	263;56,26,58,6,27°	–	+	–

2.4 Solar equation of anomaly

Vid. Figure 5. Let APS be an eccenter with centre M and radius R where A is the apogee and P is the perigee. Let Z be the centre of the ecliptic and e = MZ, the eccentricity. Let the Sun be on S, so that the angle ∠AZS is the true anomaly (k), the angle ∠AMS is the mean anomaly (k) and the angle ∠ZSM is the solar equation (c).

Given that the eccentric model is a particular instance of the epicycle model, as shown in Figure 6 where Π represents the position of the Sun or the Moon, the next set of equivalences can be established:[2] c is either the solar equation of anomaly or the lunar equation of anomaly, R is either the deferent radius or the eccenter radius for R = ZH = MΠ, the epicycle radius is equivalent to the eccentricity, $r \equiv e$, and the lunar anomaly is equivalent to the solar mean anomaly, $\alpha \equiv k$.

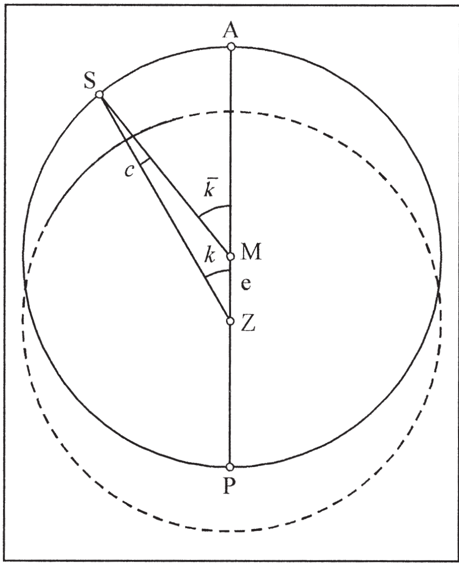

Figure 5. Solar equation of anomaly (c)

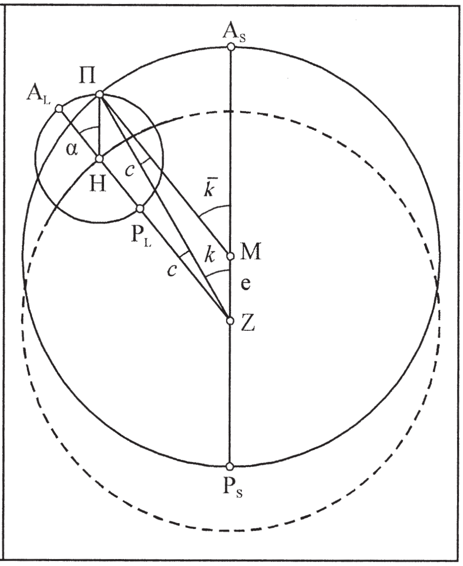

Figure 6. Correspondence between the epicycle and the eccentric model

Therefore, from (2) the solar equation (c) is obtained as

$$c = \arctan\frac{e\sin\bar{k}}{R + e\cos\bar{k}} = \arcsin\frac{e\sin\bar{k}}{\sqrt{R^2 + e^2 + 2Re\cos\bar{k}}} \qquad (12)$$

Assuming the Ptolemaic values for e and R ($e = 2;30^p$ whenever $R = 60^p$), the solar equation is as shown in Figure 7.

[2] Cf. Otto Neugebauer [1959], "The Equivalence of Eccentric and Epicyclic Motion according to Appolonius", *Scripta Mathematica*, Vol. 24 (1959), pp. 5-21 [reprint in Otto Neugebauer [1983], *Astronomy and History. Selected Essays*, New York, pp. 335-351]; HAMA, pp. 56-7.

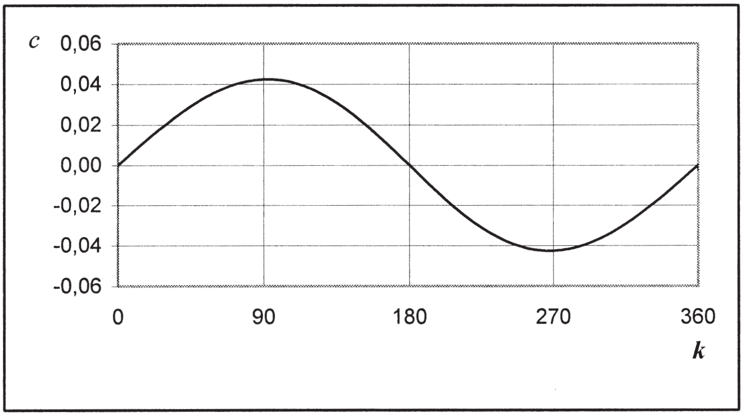

Figure 7. Solar equation (c) as a function of the solar mean anomaly (k)

This is a periodical function with T = 360º / ω and inherits the symmetry of the function: that is, a 2-fold rotational symmetry about a centre $k = 0^{\circ}$ or $k = 180^{\circ}$. Therefore,

$$c\,(k) = -\,c\,(i360^{\circ} - k) \qquad (13)$$

for any integer number i. Finally, the anomalies of the maximum and minimum equations – i.e. those corresponding to the solar mesogees – obtained from the equivalences applied to (8) are k_1 = 92;26,8,53 and k_2 = 267;33,51,6. We can obtain the solar longitude from the solar mean anomaly by considering that

$$\lambda = k + \lambda_A \qquad (14)$$

where is λ_A the longitude of the apogee, and

$$k = k - c \qquad (15)$$

3. Ptolemy on the four-eclipse method for finding the lunar anomaly period in longitude

3.1 Introduction

Ptolemy briefly introduces the four-eclipse method for finding the lunar anomaly period used by ancient astronomers – mainly Hipparchus and the

Babylonian astronomers – in *Almagest* IV.2.[3] He also adds new conditions to the method, although they do not change the final values given by Hipparchus. His main aim in presenting the four-eclipse method is to support the correction of Hipparchus' final results so that he could consider them while developing the lunar models. It must be underlined that, at this point in his work, Ptolemy has not yet provided a lunar model. Consequently, he does not refer directly to anomalies in his discussion. Instead, they are considered implicitly through lunar observed speeds. In any case, some of the statements made provide sufficient evidence to assume that he has the first lunar model in mind. For the sake of clarity we will base the discussion throughout this paper on the first lunar model, following the strategy adopted by Neugebauer.[4] Ptolemy describes the four-eclipse method as follows:

> "Hence the ancient astronomers, with good reason, tried to find some period in which the moon's motion in longitude would always be the same, on the grounds that only such a period could produce a return in anomaly. So they compared observations of lunar eclipses (for reasons mentioned above), and tried to see whether there was an interval, consisting of an integer number of months, such that, between whatever points one took that interval of [true synodic] months, the length in time was always the same, and so was the motion [of the moon] in longitude, [i.e.] either the same number of integer revolutions, or the same number of revolutions and the same arc".[5]

The ancient eclipse method is therefore based on comparing at least two intervals of the lunar motion along the ecliptic determined in each case by pairs of lunar eclipses. Consequently, at least four eclipses are needed. The method is as follows: given two lunar eclipses, E_1 and E_2, which take place in longitudes λ_1 and λ_2 and at times t_1 and t_2, an increment in the lunar longitude ($\Delta\lambda_{21}$) and a time interval (Δt_{21}) are defined. In order to verify

[3] G.J. Toomer [1984], *Ptolemy's Almagest*, London, pp. 174-9 [henceforth referred to as PtA].

[4] Cf. HAMA, pp. 71-3.

[5] Cf. PtA, 175.

that $\Delta t_{21} = n$T, the ancient astronomers considered a second pair of eclipses, E_3 and E_4, that define a second increment in longitude ($\Delta\lambda_{43}$) and a second time interval (Δt_{43}). Therefore, the following conditions must be fulfilled:

- both increments in longitude must be equal – $\Delta\lambda_{43} = \Delta\lambda_{21}$
- both time intervals must be equal – $\Delta t_{43} = \Delta t_{21}$

Given these conditions, they conclude that $\Delta t_{43} = \Delta t_{21} = n$T. Once it was determined that the interval Δt_{21} contained an integer number (n) of returns in the anomaly, they divided this interval by the number of returns enclosed in the interval and obtained the period of the anomaly, *T*. This is how Ptolemy presents the four-eclipse method as understood by the ancient astronomers.

After discussing the values obtained by Hipparchus, Ptolemy discusses the existence of certain positions of the Sun and the Moon that must be taken into account during the eclipses so that mistakes are not made when obtaining the lunar period in anomaly. He deals first with the Sun.

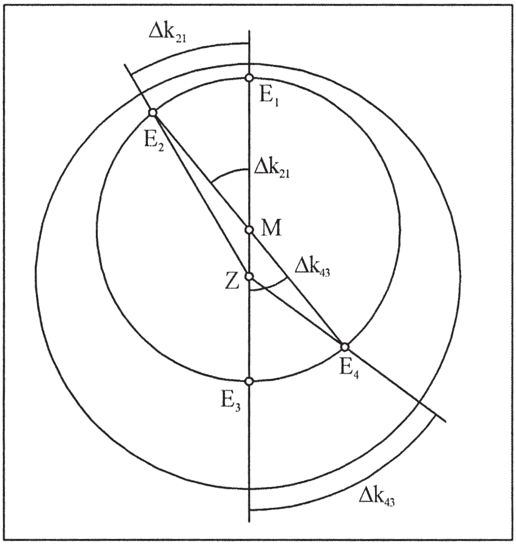

Figure 8. Equal increments of the mean anomaly, $\Delta k_{43} = \Delta k_{21}$, that not produce equal increments of longitude, $\Delta k_{43} = \Delta k_{21}$.

3.2 Positions of the Sun that provide equal increments in longitude from equal time intervals

It should be noted that the conditions added by Ptolemy simply serve to make explicit the method described by the ancient astronomers and can therefore be considered derived from it. They are not corrections of the original method but merely clarifications. Ptolemy first considers two equal time intervals determined from two pairs of eclipses. This produces two equal increments of the solar mean anomaly for $k = \omega\, t + k_0$, where ω is the solar angular motion. However, Ptolemy states that with two equal increments of the solar mean anomaly we can only infer equal increments of the solar longitude in some particular situations.

Ptolemy describes four situations in which equal time intervals, and therefore equal increments of the mean anomaly, produce equal increments of longitude:

- "[1] [The Sun] must complete an integer number of revolutions [in both intervals]; or
- [2] traverse the semi-circle beginning at the apogee over one interval and the semi-circle beginning at the perigee over the other; or
- [3] begin from the same point [of the ecliptic] in each interval; or
- [4] be the same distance from the apogee (or perigee) at the first eclipse of one interval as it is at the second eclipse of the other interval, [but] in the other side."[6]

The first situation is immediately clear since if the Sun has completed an integer number of revolutions the anomaly plays no part and the longitude traversed during the two equal time intervals is only due to its mean motion. Therefore, both increments of longitude are equal.

The second situation is a specific case within the fourth situation, so the two will be considered together.

The third situation is also clear if we consider that each solar mean anomaly corresponds to an equal longitude, provided that the longitude of the solar apogee (λ_A) does not change significantly during the interval.[7]

[6] As translated by Toomer in PtA, p. 177.

[7] Ptolemy did not mention the motion of the lunar apogee.

Therefore, if the longitude of the initial eclipse for both intervals is the same, and given that the time intervals are equal, the longitude of the final eclipse for both intervals will be the same. That is, if $\lambda_3 = \lambda_1$, then $\lambda_4 = \lambda_2$ and $\Delta\lambda_{43} = \Delta\lambda_{21}$.

To determine the fourth situation, we must first know that, from (14), $\Delta\lambda = \Delta k$ provided that the longitude of the solar apogee (λ_A) does not change during the interval. Given that during two equal time intervals – $\Delta t_{43} = \Delta t_{21}$ – the increments of the solar mean anomaly are also equal – $\Delta k_{43} = \Delta k_{21}$ – we can consider from (15) that $\Delta\lambda_{43} - \Delta\lambda_{21} = \Delta c_{21} - \Delta c_{43}$. As a result, the equation $\Delta\lambda_{43} = \Delta\lambda_{21}$ is equivalent to the equation $\Delta c_{43} = \Delta c_{21}$. Therefore,

$$\Delta\lambda_{43} = \Delta\lambda_{21} \equiv \Delta c_{43} = \Delta c_{21}. \tag{16}$$

Whenever two increments in the equation relating to two equal time intervals are equal, the increments of longitude are also equal.

This situation indicates that the mean anomaly of the initial eclipse for one interval and the mean anomaly of the final eclipse for the other (and vice versa) are symmetrical with respect to the apsidal line. In other words, $k_1 = i360^\circ - k_4$ and $k_2 = i360^\circ - k_3$, where i is any integer number. Therefore, from (13) – $c\,(k) = -\,c\,(i360^\circ - k\,)$ – we can conclude that

$$c_4 = c(k_4) = -\,c(i360^\circ - k_4\,) = -\,c(k_1) = -\,c_1$$
$$c_3 = c(k_3) = -\,c(i360^\circ - k_3\,) = -\,c(k_2) = -\,c_2$$

Therefore,

$$\Delta c_{43} = c_4 - c_3 = -\,c_1 + c_2 = \Delta c_{21}$$

From (16) we can conclude that $\Delta\lambda_{43} = \Delta\lambda_{21}$.

The second situation is a particular instance of the fourth for values $k_1 = k_4 = 0^\circ$ and $k_2 = k_3 = 180^\circ$ or, alternatively, $k_1 = k_4 = 180^\circ$ and $k_2 = k_3 = 0^\circ$.

3.3 Positions of the Moon that invalidate the method

After discussing the solar positions, Ptolemy then points out certain lunar positions that must be avoided in order to obtain the true lunar period in anomaly. In these situations, it is possible for the Moon to cover arcs of equal longitude in equal times but without completing an integer number of returns in anomaly. To quote Toomer's translation, these situations are:

- "[1] If in both intervals the Moon starts from the same speed (either both increasing or both decreasing), but does not return to that speed; or
- [2] if in one interval it starts from its greatest speed and ends at its least speed, while in the other interval it starts from its least speed and ends at its greatest speed; or
- [3] if the distance of [the position of] its speed at the beginning of one interval is the same distance from the [position of] greatest or least speed as [the position of] its speed at the end of the other interval, [but] on the other side."[8]

With respect to the above quotation, it should be pointed out that although Ptolemy describes these situations by referring to speeds – that is, observed data – the expression "on the other side" in the third situation indicates that he has devised a model, as Toomer suggests.[9] These three situations are to some extent equivalent to three of the four situations presented for the Sun. The missing fourth lunar situation is equivalent to the first solar situation mentioned and is in fact the one that verifies the method. That is, in two equal time intervals, the Moon completes an integer number of returns in its anomaly. Since this is the desired situation, Ptolemy does not include it in his list of situations to be avoided for the Moon.

The three situations Ptolemy describes for the Moon ensure that

$$\because \Delta t_{43} = \Delta t_{21}$$

$$\because \Delta\lambda_{43} = \Delta\lambda_{21}$$

$$\therefore \Delta\alpha_{43} = \Delta\alpha_{21}$$

However, equal increments of anomaly do not imply that $\Delta t_{43} = \Delta t_{21} = n\mathrm{T}$.

The first lunar situation that Ptolemy states must be avoided is equivalent to the third solar situation mentioned above. Ptolemy states that whenever the motion of the first eclipse for both intervals is the same and the two time intervals are equal, the corresponding increments in longitude, although equal, do not imply a return in anomaly.

[8] Cf. PtA, p. 177.

[9] Cf. *ibidem* n. 15.

Firstly, Ptolemy considers the starting motion of the Moon to be the same in both intervals “either both increasing or both decreasing”. From (7) we know that whenever two different anomalies – α_a and α_b – imply the same speed in anomaly, $v_a(\alpha_a) = v_a(\alpha_b)$, it must be the case that $\alpha_b = 360^o - \alpha_a$. However, from (10) we know for the previous anomalies – α_a and α_b, from which $\alpha_b = 360^o - \alpha_a$ – that whenever the acceleration of one is positive, the acceleration for the other is negative: $a\ (\alpha_a) = -\ a\ (\alpha_b)$. Therefore, when Ptolemy states that both motions must be either both increasing or both decreasing, he is assuming that the starting anomalies of both intervals are the same: that is, $\alpha_1 = \alpha_3$. Under this condition, whenever the initial anomalies of both intervals are the same ($\alpha_1 = \alpha_3$) equal time intervals ($\Delta t_{43} = \Delta t_{21}$) imply equal increments in longitude ($\Delta\lambda_{43} = \Delta\lambda_{21}$) and anomaly ($\Delta\alpha_{43} = \Delta\alpha_{21}$). Consequently, the two intervals are exactly the same. Therefore, as stated for this first situation, whenever the final lunar motion in both intervals is different to its initial motion, the intervals do not contain an integer number of returns in anomaly, even though the increments in longitude and anomaly relative to both intervals are equal.

The second situation Ptolemy describes is a particular instance of the third, so we shall begin by studying this third situation. The third solar situation is similar to the fourth lunar situation described. In this third solar situation, Ptolemy points out the symmetry with respect to the apsidal line, as in the lunar situation. The only difference is that the description of the solar situation was based on anomalies, whereas speeds are used for the lunar situation. This situation states that the speed of the initial eclipse for one interval and the speed of the final eclipse for the other (and vice versa) are symmetrical with respect to the points of maximum and minimum motion – i.e. the perigee and the apogee respectively – that determine the apsidal line. From (7) – $v_a\ (\alpha) = v_a\ (i360^o - \alpha)$ – we know that

$$v_a\ (\alpha_4) = v_a\ (i360^o - \alpha_4\ ) = v_a\ (\alpha_1)$$
$$v_a\ (\alpha_3) = v_a\ (i360^o - \alpha_3\ ) = v_a\ (\alpha_2).$$

Therefore, the anomalies that Ptolemy indicates, although now referring to speeds, are $\alpha_1 = i360^o - \alpha_4$ and $\alpha_2 = i360^o - \alpha_3$, where i is any integer number. Once we know the anomalies, from (4) – $c\ (\alpha) = -\ c\ (i360^o - \alpha)$ – we can conclude that

$$c_4 = c\ (\alpha_4) = -\ c\ (i360^o - \alpha_4\ ) = -\ c\ (\alpha_1) = -\ c_1$$
$$c_3 = c\ (\alpha_3) = -\ c\ (i360^o - \alpha_3\ ) = -\ c\ (\alpha_2) = -\ c_2$$

and

$$\Delta c_{43} = c_4 - c_3 = -c_1 + c_2 = \Delta c_{21}$$

Therefore, $\Delta\lambda_{43} = \Delta\lambda_{21}$. In this situation we can have equal increments in longitude and anomaly for any pair of time intervals, provided that they are equal in duration. Hence these time intervals do not have to include an integer number of returns in anomaly.

The second situation is a particular instance of the third, in which Ptolemy refers to an interval that begins at the greatest lunar speed and ends at its least speed, while the other begins at its least speed and ends at its greatest speed. The anomalies for these speeds are $\alpha_1 = \alpha_4 = 0^\circ$ and $\alpha_2 = \alpha_3 = 180^\circ$ or, alternatively, $\alpha_1 = \alpha_4 = 180^\circ$ and $\alpha_2 = \alpha_3 = 0^\circ$, hence they represent a particular instance of the third situation.

3.4 Best selection of eclipses

In short, these are the situations Ptolemy describes in which, for two equal time intervals, and although both increments in longitude and increments in anomaly are equal, the increments in anomaly do not contain an integer number of returns in anomaly. To correct this problem, Ptolemy considers eclipses in which the discrepancy in longitude between two intervals takes the greatest possible value when they do not contain an integer number of returns in anomaly.[10] He considers two situations in which the initial lunar motions for each interval differ greatly either 'in size' or 'in potency', which therefore provide us with an easy indication whenever the time intervals do not contain an integer number of returns in anomaly.

In referring to a great difference 'in size', Ptolemy means that the Moon in one interval begins at its least speed, while the Moon in the other begins at its greatest speed. He is therefore considering the simple concept of speed or velocity. He also remarks that the final motion of an interval cannot be the opposite of that from which it begins. That is, if for one interval the Moon begins at its maximum speed, it cannot finish at its minimum speed, and vice versa. This ensures that the second of the lunar situations mentioned above is avoided. So from (5) and (6), in one interval the Moon must begin from the perigee – that is, the point of the anomaly with the greatest speed – while in the other it must begin from the apogee

[10] Cf. PtA, p. 178.

– that is, the point of the anomaly with the least speed. Under these conditions, if the Moon begins from the apogee, it can finish close to one of the two mesogees or close to the apogee; while if the Moon begins from the perigee, it can finish close to one of the two mesogees or close to the perigee. But when the Moon finishes close to the mesogees, the intervals cannot contain an integer number of returns in anomaly, so it would in fact be preferable for the error to be as great as possible so that it can be easily identified. Whenever the Moon ends in one of the mesogees, having started from the apogee or the perigee, the increment in longitude during one time interval amounts to the maximum equation, either positive or negative. Therefore, the difference between the two increments in longitude amounts to twice the maximum equation. In order to prove this, let α_A , α_P , α_{M1} and α_{M2} be the anomalies of the lunar apogee and perigee and of both mesogees, and let the time intervals Δt_{43} and Δt_{21} be equal. Given (1) – that is, $\lambda = v\ t - c + \lambda_0$ – and that the perigee and apogee equations are void, the first increment in longitude that, for example, begins from the apogee and ends in one mesogee is

$$\Delta\lambda_{21} = v\ \Delta t_{21} - [\ c(\alpha_{M1}) - c(\alpha_A)] = v\ \Delta t_{21} - c(\alpha_{M1})$$

while the second increment in longitude that, for example, begins from the perigee and ends in a mesogee is

$$\Delta\lambda_{43} = v\ \Delta t_{43} - [\ c(\alpha_{M2}) - c(\alpha_P)] = v\ \Delta t_{43} - c(\alpha_{M2})\ .$$

Therefore, the difference between both increments in longitude is

$$\Delta\lambda_{43} - \Delta\lambda_{21} = [v\ \Delta t_{43} + c(\alpha_{M1})\ ] - [v\ \Delta t_{21} + c(\alpha_{M2})\ ] = \pm\ 2\ c_{\mathrm{Max}}$$

given that the equation of both mesogees is maximum, one positive and the other negative, and provided that the time intervals are equal, $\Delta t_{43} = \Delta t_{21}$. In conclusion, the difference is relatively large when the intervals do not contain an integer number of returns in anomaly.

By referring to a great difference ‘in potency’ between the lunar speed in both intervals, Ptolemy means that the Moon in both intervals begins at its mean speed, “not, however, from the same mean speed, but from the mean speed during the period of increasing speed at one interval, and from that during the period of decreasing speed at the other".[11] So although the values for the initial speeds are the same – that is, the actual speeds are the same – these speeds are potentially different to a maximum degree. Ptolemy relates this potential difference to the fact that one of these speeds is observed in a period with increasing speed while the other is observed in a period with decreasing speed. Thus he takes into account whether the speed is increasing or decreasing. These are time variations of speed and are related to what we now call 'acceleration'. So the Ptolemaic concept for acceleration seems to be this speed '*in potentia*', although not clearly separated from the speed itself. This interpretation seems to agree with the *Almagest* as the acceleration is almost maximum for both mesogees.

When the Moon begins from one mesogee, it can end incorrectly close to the other mesogee, thus covering approximately two quadrants of the anomaly; or incorrectly close to the apogee or the perigee, covering approximately one or three quadrants of the anomaly; or correctly close to the same mesogee and thus containing an integer number of returns in its anomaly. Ptolemy first considers the Moon beginning with its mean speed and covering one or three quadrants during the interval.[12] In other words, the Moon begins from one mesogee and ends in the apogee or the perigee, covering one or three quadrants of the anomaly provided that the mesogees are at 90° or 270° from the anomaly. This situation is similar to the one considered above for maximum difference in size, but in this case beginning from the mesogees and ending in the apogee or perigee. Therefore, the difference between the increments in longitude due to equal time intervals is equal to twice the maximum equation for

$$\Delta\lambda_{43} - \Delta\lambda_{21} = [v\ \Delta t_{43} + c(\alpha_{M1})\ ] - [v\ \Delta t_{21} + c(\alpha_{M2})\ ] = \pm\ 2\ c_{Max}\ .$$

Ptolemy then considers the Moon beginning at its mean speed and covering two quadrants during the interval. In other words, the Moon begins from one mesogee and ends in the other mesogee, covering two quadrants of the anomaly provided that the mesogees are at 90° or 270° from the anomaly. Ptolemy states that this difference is equal to four times the maximum equation, c_{Max}.

So let α_{M1} and α_{M2} be the anomalies of both mesogees and let the time intervals Δt_{43} and Δt_{21} be equal. Given (1) – $\lambda = v\ t - c + \lambda_0$ – the first

[11] *Ibidem.*

[12] This is another indication that Ptolemy was considering a model.

increment in longitude that, for example, begins from the first mesogee and ends in the second mesogee is

$$\Delta\lambda_{21} = v\ \Delta t_{21} - [\ c(\alpha_{M2}) - c(\alpha_{M1})] = v\ \Delta t_{21} \mp 2c_{Max}$$

while the second increment in longitude that, for example, begins from the second mesogee and ends in the first mesogee is

$$\Delta\lambda_{43} = v\ \Delta t_{43} - [\ c(\alpha_{M1}) - c(\alpha_{M2})] = v\ \Delta t_{43} \pm 2c_{Max}\ .$$

Therefore, the difference between both increments in longitude is

$$\Delta\lambda_{43} - \Delta\lambda_{21} = [v\ \Delta t_{43} \mp 2c_{Max}\ ] - [v\ \Delta t_{21} \pm 2c_{Max}\ ] = \pm\ 4\ c_{Max}$$

given that the equation of both mesogees is maximum, one positive and the other negative, and provided that the time intervals are equal, $\Delta t_{43} = \Delta t_{21}$. Consequently, the difference when the intervals do not contain an integer number of returns in anomaly is actually greater than for the maximum difference in size. These are Ptolemy's considerations on the four-eclipse method used by the ancient astronomers to find the lunar anomaly period.

4. Jābir b. Aflaḥ on the four-eclipse method for finding the lunar anomaly period in longitude

4.1 Introduction

After Ptolemy, Thābit b. Qurra (836-931) also studied the four-eclipse method. The method was the subject of his treatise *On the motion of the two luminaries* which has been edited by Régis Morelon.[13] Though Thābit b. Qurra tries to systematize the four-eclipse method in this text in order to clarify it, he does not improve the method, as Jābir b. Aflaḥ does. In any case, Jābir b. Aflaḥ knew of Thābit b. Qurra, and wrote two commentaries

[13] Thābit ibn Qurra, *Oeuvres d'Astronomie. Text établi et traduit par Régis Morelon*, Paris, 1987, pp. 85-92. Cf. also pp. LXXX-XCII for the introductory study. The title of this text is *Fī īḍāḥ al-wahj alladhī dhakara Baṭlamiyūs anna bihi istakhraja man taqaddamuhu masīrāt al-qamar al-dawriyya wa-hiya al-mustawiya*.

on two of the latter's minor mathematical treatises.[14] Therefore, he may have known the treatise we have just mentioned. Nonetheless, Jābir b. Aflaḥ's improvements are not based on Thābit b. Qurra, since his criticisms do not appear in this short text.

Jābir b. Aflaḥ[15] studies the Ptolemaic lunar models in the fourth book or *maqāla* of his *Iṣlāḥ al-Majisṭī*. In the initial part of this book, Jābir b. Aflaḥ presents the four-eclipse method for determining the lunar anomaly period. As he states in his introduction, the *Iṣlāḥ al-Majisṭī* is a reedited version of the *Almagest* in which he considers only the theoretical contents of the *Almagest*, introduces some additional demonstrations and corrects some of Ptolemy's statements.

[14] See R.P. Lorch [2001], *Thābit ibn Qurra, On the Sector-Figure and Related Texts. Edited with Translation and Commentary*, Frankfurt am Main, pp. 387-90.

[15] On Jābir b. Aflaḥ, see R.P. Lorch [1975], "The Astronomy of Jābir b. Aflāḥ", *Centaurus*, Vol. 19, pp. 85-107, which is an abridgement of his doctoral thesis read at Manchester University in 1971: *Jābir ibn Aflaḥ and his Influence in the West*. Lorch has written other papers on the work of Jābir b. Aflaḥ, such as R.P. Lorch [1976], "The Astronomical Instruments of Jābir ibn Aflaḥ and the Torquetum", *Centaurus*, Vol. 20, pp. 11-34 [reprint in R.P. Lorch [1995a], *Arabic Mathematical Sciences: Instruments, Text, Transmission*, Aldershot, XVI]; R.P. Lorch [1995c], "Jābir ibn Aflaḥ and the Establishment of Trigonometry in the West" in Lorch (1995a), VIII; R.P. Lorch [1995b], "The Manuscripts of Jābir's Treatise" in Lorch (1995a), VII; R.P. Lorch [2001b], *Thābit ibn Qurra, On the Sector-Figure and Related Texts. Edited with Translation and Commentary*, Frankfurt am Main, pp. 387-90. Other scholars have also studied additional points on Jābir b. Aflaḥ, such as N.M. Swerdlow [1987], "Jābir ibn Aflaḥ's interesting method for finding the eccentricities and direction of the apsidal line of superior planets" in D.A. King and G. Saliba (eds.) [1987], *From Deferent to Equant. A Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honour of E.S. Kennedy*, New York, pp. 501-12; H. H. Hugonnard-Roche [1987], "La théorie astronomique selon Jābir ibn Aflaḥ", in G. Swarup, A.K. Bag and K.S. Shukla (1987), *History of Oriental Astronomy. Proceedings of an International Astronomical Union Colloquium nº 91 (1985)*, Cambridge, pp. 207-8; J. Samsó [2001], "Ibn al-Haytham and Jābir b. Aflaḥ's Criticism of Ptolemy's Determination of the Parameters of Mercury", *Suhayl*, Vol. 2 (2001); and my own Ph.D. thesis read at the University of Barcelona: J. Bellver, *Críticas a Ptolomeo en el s. XII: El caso del* Iṣlāḥ al-Maŷisṭī *de Ŷābir b. Aflaḥ*. There are some abridgements of Jābir b. Aflaḥ's *Iṣlāḥ al-Majisṭī*, for example in M. Delambre [1819], *Histoire de l'Astronomie du Moyen Age*, Paris, 1819 [reprint New York – London, 1965] pp. 179-85; M. Cantor, *Vorlesungen über Geschichte der Mathematik, vol. 2 Vom Jahre 1200 bis zum Jahre 1668*, 2ª ed. (Leipzig, 1900; reprint New York – Stuttgart, 1965), p. 404; P. Duhem [1913-1959], *Système du monde*, 10 vols., Paris, Vol. II, p. 172; G. Sarton [1927-48], *Introduction to the History of Science*, 3 vols., Baltimore, Vol. II, p. 206; F.J. Carmody [1952b], *Al-Biṭrūjī, De Motibus Celorum*, Berkeley and Los Angeles, pp. 29-32; J. Samsó [1992], *Las ciencias de los antiguos en al-Andalus*, Madrid, pp. 317-320 and 326-330.

As Lorch has pointed out, there are two Arabic versions of the *Iṣlāḥ al-Majisṭī*.[16] Basing his work on the in-depth study of Jābir b. Aflaḥ's trigonometry, Lorch mentioned the existence of two versions of the *Iṣlāḥ al-Majisṭī* which differ in certain sections, such as the trigonometric introduction. These two Arabic versions were the one extant in Ms. Berlin 5653, which was translated by Gerard of Cremona, and the one preserved in Mss. Escorial 910 and Escorial 930. Although the content of the trigonometric section differs in the two versions, there are other sections of the *Iṣlāḥ al-Majisṭī* in which Ms. Escorial 930 follows the Ms. Berlin 5653 version. This is the case of the section of the *Iṣlāḥ al-Majisṭī* dealing with the four-eclipse method. From indications in the text, we now assume that the version of Ms. Berlin 5653 is the one that is closer to Jābir b. Aflaḥ's original. We will see later in this paper that, on this point at least, Ms. Escorial 910 seems to depend on material from the Ms. Berlin 5653 version. In any case, the ideas presented in the two versions are almost the same: the versions differ mainly in the order adopted and, to some extent, the terminology used.

Following the work of Ptolemy, Jābir b. Aflaḥ first mentions the difficulty of finding the true lunar longitude due to the lunar parallax. This problem can be solved by using lunar eclipses. He then shows that the lunar motion values are not related to particular longitudes. Instead, for any particular longitude, the true lunar motion can be any value between its maximum and minimum. Next, Jābir b. Aflaḥ assumes the existence of a particular lunar orbit along which the Moon's motion takes place. The two versions differ on this point. The Berlin manuscript refers to this orbit as the *al-falak al-khāṣṣ* or 'particular orbit', while the Escorial 910 manuscript calls it the *falak al-tadwīr* or 'epicycle', thus basing its description on the first as yet undescribed lunar model. The Berlin manuscript is therefore closer to the *Almagest*, as it does not at this point refer to this orbit as an epicycle. It infers four significant points from the lunar motion (*nuqṭa*, pl. *nuqaṭ*) that belong to the particular lunar orbit, while the Escorial 910 manuscript deduces the lunar motion from the significant points of the epicycle such as the apogee (*al-buʿd al-abʿad*), the perigee (*al-buʿd al-abʿad*) and both mesogees (*al-majāz al-awsaṭ*).

[16] R.P. Lorch [1975], pp. 88-90.

4.2 The sectors of the Moon

Jābir b. Aflaḥ therefore considers four points related to lunar motion: one for the Moon's maximum motion (the perigee of its particular orbit); one for its minimum motion (the apogee of its particular orbit); and two for its true motion when this is the same as its mean motion (the mesogees of its particular orbit). These last two points differ in that one is part of an increasing motion interval and the other is part of a decreasing motion interval.

These four significant points divide the lunar orbit into four sectors (*qiṭʿa*, pl. *qitaʿ*):[17]

- The one from the Moon's fastest motion to its mean motion. The lunar motion (*ḥaraka*) in this sector is fast but decreasing (*surʿat mutanāqiṣa*).
- The one from its mean motion to its slowest. The lunar motion in this sector is slow and decreasing (*buṭū' mutanāqiṣ*).
- The one from its slowest motion to its mean motion. The lunar motion in this sector is slow but increasing (*buṭū' mutazāyid*).
- The one from its mean motion to its fastest. The lunar motion in this sector is fast and increasing (*surʿat mutazāyida*).

Jābir b. Aflaḥ points out that whenever we know the lunar speed and whether it is increasing or decreasing, we will know where the Moon is in its particular orbit. Jābir b. Aflaḥ therefore considers two aspects of lunar speed: its actual value and its potential value – or, in our terminology, its acceleration.[18]

4.3 Jābir b. Aflaḥ's conditions for the four-eclipse method

The method is almost the same as the one presented by Ptolemy. It is based on two pairs of lunar eclipses that determine two intervals. Jābir b.

[17] The sector theory has a long tradition in Islamic astronomy. For a brief history of the sector theory and its introduction in al-Andalus, especially in Ibn Muʿadh's *Tabulae Jahen*, see J. Samsó [1996], "Al-Bīrūnī in al-Andalus" in J. Casulleras and J. Samsó (eds.) [1996], *From Baghdad to Barcelona. Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*, 2 vols., Barcelona, pp. 583-612.

[18] In a forthcoming paper I will discus this issue at length.

Aflaḥ states that the intervals must fulfil a set of conditions in order to contain an integer number of lunar returns in its anomaly, which are the conditions originally stated by Ptolemy and two additional ones. Given two pairs of eclipses, E_1-E_2 and E_3-E_4, that define two increments in the lunar longitude, $\Delta\lambda_{21}$ and $\Delta\lambda_{43}$, and two time intervals, Δt_{21} and Δt_{43}, the conditions Ptolemy considers are:

- both increments in longitude must be equal – $\Delta\lambda_{43} = \Delta\lambda_{21}$
- both time intervals must be equal – $\Delta t_{43} = \Delta t_{21}$

Jābir b. Aflaḥ adds two new conditions based on the lunar speed for each eclipse (v_i for each E_i):

- the lunar speeds of the initial and final eclipses of an interval must be equal, i.e. $v_1 = v_2$ and $v_3 = v_4$
- the lunar speeds of the initial eclipses (and, therefore, of the final eclipses) of both intervals must be different, i.e. $v_1 \neq v_3$ and $v_2 \neq v_4$

In the Berlin version, Jābir b. Aflaḥ then states that these two additional conditions are self-evident, since they are a logical consequence if they have to provide us with a time interval that contains an integer number of returns in anomaly:

> Ptolemy mentioned this method from the ancient astronomers, but he did not clearly state the conditions on these lunar motions (*ḥarakāt*) during the desired eclipses, as included here. Nevertheless, although he did not state this directly, the meaning itself (*nafs al-maʿnà*) implies that these conditions are required for the eclipses. Were it not as he described, it would not be possible [for the Moon] to complete an integer number of returns [in its anomaly].[19]

This can be clearly inferred as Jābir b. Aflaḥ states that if the time intervals must contain an integer number of returns in anomaly, where the lunar speed is only a function of the lunar anomaly (α), as pointed out in (5) and (6), the speed of the initial and final eclipses of an interval must be

[19] Ms. Berlin 5653, 39r.

the same. Similarly, in order to provide two different intervals for verifying the lunar period in anomaly, the initial speeds of the two intervals – and, therefore, the final speeds – must be different; otherwise, both intervals would be the same and it would not be possible to verify the period.

There are also points to be made in relation to the terminology used in the two versions. In the text, we find two different terms for speed. The first is *ḥaraka*, which is usually translated as 'motion'. It gives the actual value of speed, although when dealing with time increments it can give the increments in longitude in these time intervals. The second term is *sayr*, which in Isḥāq b. Ḥunayn's translation of the *Almagest* – the one quoted here by Jābir b. Aflaḥ – is usually associated with motions that may be fast, medium or slow and vary over time, for example, fast and increasing or fast and decreasing. Therefore, it can be given as 'variable speed', which takes into account the concept of acceleration. The last term is *masīr*, which always appears in relation to an ecliptic point with a given variable speed.

These terms appear in both versions with the meanings explained above. Nevertheless, there are occasions on which one manuscript uses *ḥaraka* while the other uses *sayr*. For example, when describing the additional conditions established by Jābir b. Aflaḥ, Ms. Berlin 5653 uses *ḥaraka* whereas Ms. Escorial 930 uses *sayr*: on this occasion the version of Ms. Escorial 910 uses *sayr*.

Demonstration

After describing the conditions that must be met as part of the method, Ptolemy included a brief description of a demonstration. Jābir b. Aflaḥ extends this demonstration but does so in more formal terms and with consideration of all four conditions. He also bases it on an epicycle. This is one of the situations mentioned by Jābir b. Aflaḥ in his introduction to the *Iṣlāḥ al-Majisṭī*, in which he provides additional demonstrations that do not appear in the *Almagest* or clarifies those which are described only briefly.

There are certain differences between the two versions relating to the letters used and to the fact that Ms. Escorial 910 gives some examples that do not appear in the Berlin version. We will summarize the demonstration as it appears in the Berlin version. E_1, E_2, E_3 and E_4 refer to first, second, third and fourth eclipses.

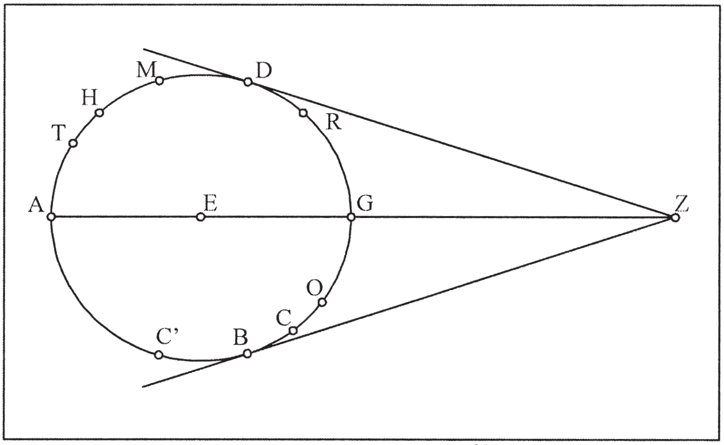

Figure 9: Ms. Berlin 39v[20]

Let us consider Figure 9. The circle ABGD is the lunar epicycle with point E as its centre. Point Z is the centre of the ecliptic. Point A is the apogee, point G the perigee and points B and D are the mesogees. The Moon is at E_1 on point H and at E_3 on point C. So, the following conditions apply:

- the lunar variable speed (*sayr*) of the initial and final eclipses of an interval must be equal, i.e. $v_1 = v_2$ and $v_3 = v_4$
- the lunar variable speed of the initial eclipses of both intervals must be different, i.e. $v_1 \neq v_3$
- both increments in longitude must be equal, i.e. $\Delta\lambda_{43} = \Delta\lambda_{21}$
- both time intervals must be equal, i.e. $\Delta t_{43} = \Delta t_{21}$

Jābir b. Aflaḥ intends to prove that the Moon returns in E_2 to exactly point H (as in E_1) and in E_4 exactly to point C (as in E_3). To do so, he uses the method of stating the opposite of what he wishes to prove. Therefore, he considers that in E_2 the Moon returns to point T (i.e. different to H) and in E_4 to point O (i.e. different to C). The demonstration is as follows:

$$\because \Delta t_{43} = \Delta t_{21}$$

[20] In the manuscript, point C appears where point C' is now located. The sense of this is clear when discussed further. However, in the present demonstration point C should be where it is currently located.

$\therefore \text{HT} = \text{CO}$

$\because$ so $v(\text{H}) \neq v(\text{C})$ and $v(\text{T}) \neq v(\text{O})$

$\therefore \Delta\lambda_{21} = \Delta\lambda(\text{HT}) > \Delta\lambda_{21}$ and $\Delta\lambda_{43} = \Delta\lambda(\text{CO}) < \Delta\lambda_{43}$

where $\Delta\lambda_{21}$ is the mean motion in the first time interval, $\Delta\lambda_{43}$ is the mean motion in the second, $\Delta\lambda(\text{HT})$ is the increment in longitude as the Moon traverses from point H to point T, and $\Delta\lambda(\text{CO})$ is the increment in longitude during the Moon traversing from point C to point O. However,

$\because \Delta t_{43} = \Delta t_{21}$

$\therefore \Delta\lambda_{43} = \Delta\lambda_{21}$

$\therefore \Delta\lambda_{43} \neq \Delta\lambda_{21}$ because $\Delta\lambda_{21} = \Delta\lambda(\text{HT}) > \Delta\lambda_{21}$ and $\Delta\lambda_{43} = \Delta\lambda(\text{CO}) < \Delta\lambda_{43}$

$\therefore \Delta\lambda_{43} - \Delta\lambda_{21} = \pm\, \Delta c_{43} \mp \Delta c_{21}$ from (1), that is, $\lambda = v\,t - c + \lambda_0$

where the sign depends on the semi-epicycle as determined by the apsidal line the Moon is on. However, this violates the premise that both increments in longitude during both equal time intervals must be equal. Therefore, the supposition that $v(\text{H}) \neq v(\text{C})$ and $v(\text{T}) \neq v(\text{O})$ is false. Hence,

$v(\text{H}) = v(\text{C})$ and $v(\text{T}) = v(\text{O})$

This is the demonstration described by Jābir b. Aflaḥ as it appears in the Berlin version, although there is the possibility that from $v(\text{H}) \neq v(\text{C})$ and $v(\text{T}) \neq v(\text{O})$ it can be concluded that $\Delta\lambda_{43} = \Delta\lambda_{21}$. This holds only when $v_1 = v(\text{H}) = v(\text{T}) = v_3$ and $v_2 = v(\text{C}) = v(\text{O}) = v_4$. However, Jābir b. Aflaḥ clearly set the additional condition that the variable speeds of the initial eclipses, v_1 and v_3, must be different. Therefore, his final conclusion – that $v_1 = v_2$ and $v_3 = v_4$, provided that $v_1 \neq v_3$ – is sufficient for the four-eclipse method to vouch for the lunar anomaly period.

The Ms. Escorial 910 version extends the above demonstration to some particular cases. Firstly, it considers separately whether the lunar anomaly of the initial eclipses for both intervals is greater or smaller than that of the final eclipses, although this does not alter the final conclusion. It also takes into account the possibility that the lunar eclipses are in the same semi-epicycle as determined by the apsidal line or in opposite ones.

4.4 Jābir b. Aflaḥ's best eclipse-selection method

We mentioned above that in order to avoid the situations in which equal increments in longitude during equal time intervals do not guarantee that the time intervals include an integer number of returns in anomaly, Ptolemy considered two different methods for selecting eclipses. In the first, the difference between the initial variable speeds of each interval must be as large as possible. Ptolemy described this as the 'maximum difference in value' between initial speeds. This situation corresponds to intervals in which one initial eclipse is close to the apogee and the other is close to the perigee. In the second method, the difference between the initial variable speeds in potency of each interval must be as large as possible. Ptolemy described this as the 'maximum difference in potency' between initial speeds. This situation corresponds to intervals in which the initial eclipses are close to each mesogee.

Jābir b. Aflaḥ considers that the best method for selecting eclipses is the one Ptolemy described as 'maximum difference in value', in which one initial eclipses must be close to the apogee and the other close to the perigee. For Jābir b. Aflaḥ, this is a reliable way of achieving the greatest increments when there is no return in anomaly. In contrast, he criticizes the selection described as 'maximum difference in potency'. We will consider his criticisms in the next section.

The Berlin version places this section on the best method for selecting eclipses just after the demonstration of the method, while the Ms. Escorial 910 version places it after Jābir b. Aflaḥ's criticisms of the lunar and solar situations that must be considered. The other notable difference is that the Berlin version quotes the *Almagest* at length,[21] while the Ms. Escorial 910 version is more concise.[22]

4.5 Additional calculations

Having obtained the time interval that fulfils the conditions stated, Jābir b. Aflaḥ then mentions some additional calculations in order to derive

[21] Cf. Ms. B. fols. 39v-40v and *infra* p. 208 and ff.

[22] Cf. Ms. Es[1] fols. 41v-42r and *infra* p. 201 and ff.

different lunar periods (closely following material from *Almagest* IV.2 and IV.3). The two versions differ in where they situate these additional computations in the text, in the number and order of the variables derived and whether these magnitudes are given.

In the Ms. Escorial 910 version, this section follows the demonstration and precedes Jābir b. Aflaḥ's criticisms of Ptolemy.[23] Firstly, the lunar anomaly period is obtained by dividing the time interval between one pair of eclipses by the number of lunar returns in anomaly. This is not found in the Berlin version, perhaps because it is obvious. The text continues with the method for calculating other variables (although the final values are not given) such as the arc of the epicycle traversed by the Moon in one day; the mean synodic month; and the longitudes traversed by the Sun and Moon during one mean month.[24]

In the Berlin version, this is the last section devoted to the four-eclipse method after Jābir b. Aflaḥ's criticisms of Ptolemy.[25] In this version, the Ptolemaic values for the different variables are provided. The only difference is found in the correction of the mean month, probably due to al-Ḥajjāj, who gives 29;31;50,8,9,20 days instead of the Ptolemaic value of 29;31;50,8,20 days.[26] The lunisolar elongation in one day must be added to the previous values given in the Ms. Escorial 910 version.

Jābir b. Aflaḥ also considers the lunar anomaly period in latitude.[27] The two versions are almost identical on this point.[28] The additional conditions posited for the four eclipses in order to supply the anomaly period in latitude are that

- the magnitudes of the initial and final eclipses of each interval must be the same
- its obscured lunar sector to both the north and south must be the same

[23] Cf. Ms. Es[1] 41r.

[24] Cf. Ms. Es[1] 42r and *infra* p. 204.

[25] Cf. Ms. B. 41r-41v.

[26] Cf. J.L. Mancha [2002-03], "A note on Copernicus' 'correction' of Ptolemy's mean synodic month" in *Suhayl*, Vol.3 (2002-03), pp. 221-230.

[27] Cf. PtA, p. 176.

[28] Cf. Ms. Es[1] and Ms. B. 41r-41v.

- they must be next to exactly the same node

Under these conditions, the lunar nodal distance for the initial and final eclipse of one pair will be the same. Given two pairs of eclipses that meet these conditions, the lunar anomaly period in latitude would be obtained. The text adds the lunar anomaly period in latitude and the daily mean arc in latitude traversed by the Moon to the different periods obtained in longitude. All of these values are Ptolemaic.

4.6 On the positions of the Moon that invalidate the method

Ptolemy considers three situations that invalidate the four-eclipse method since, even with equal time intervals and equal increments in longitude, the time intervals do not contain an integer number of returns in anomaly. These are

- "[1] If in both intervals the Moon starts from the same speed (either both increasing or both decreasing), but does not return to that speed;
- [2] if in one interval it starts from its greatest speed and ends at its least speed, while in the other interval it starts from its least speed and ends at its greatest speed;
- [3] if the distance of [the position of] its speed at the beginning of one interval is the same distance from the [position of] greatest or least speed as [the position of] its speed at the end of the other interval, [but] on the other side."[29]

Jābir b. Aflaḥ applies the two additional conditions – equal initial and final speeds of a single interval and different initial speeds between intervals – he considers in order to avoid the previous situations.

The first situation is nullified by the condition that the initial speeds must be different, while the other two situations are nullified by the fact that the initial and final speeds of an interval must be the same. In fact, Jābir b. Aflaḥ's criticism is not of an error committed by Ptolemy, for the situations he states are derived from the conditions he originally suggests. Rather, the criticism is based on the fact that Ptolemy did not state the two

[29] Cf. PtA, p. 177.

additional conditions that were, as far as Jābir b. Aflaḥ was concerned, self evident.

There are also minor differences between the meanings expressed in the two versions. The Berlin version is more concise, while the Ms. Escorial 910 redaction is clearer.

4.7 On the positions of the Sun that provide equal increments in longitude from equal time intervals

Ptolemy states that increments of the solar anomaly during both intervals must be equal. Therefore, the Sun must traverse equal arcs in its excenter during equal time intervals.

Given that we are using lunar eclipses and that the increments in longitude and the intervals are equal, the arcs traversed by the Sun during equal time intervals must be the same since the Sun during the lunar eclipses is in opposition to the Moon. Therefore, if the Moon traverses equal increments in longitude during equal intervals, so must the Sun. We can see, then, that the four situations stated by Ptolemy are derived from the two conditions he suggested. As above, this is not a criticism of an error found in the *Almagest*, but rather of unnecessary redundancy.[30]

4.8 On the 'maximum difference in potency' between initial speeds

The mesogees are the initial points of both intervals that fulfil the solution described as 'maximum difference in potency' between initial speeds. Jābir b. Aflaḥ's criticism of this selection of eclipses is that it is difficult to obtain the anomaly from observed data when the Moon is close to a mesogee. The Ms. Escorial 910 version mentions that, even when the Moon is 3° or more from the mesogee, it is easy to consider it to be on the mesogee. This value of 3° is not given in the Berlin version. Jābir b. Aflaḥ's criticism is therefore based on the fact that it is difficult for an observer to determine whether the Moon is on the mesogee or close to it, and not on the incorrectness of the method *per se*.

Jābir b. Aflaḥ derives another criticism from this comment. He considers the somewhat implausible situation in which:

[30] Thābit ibn Qurra also points out this issue in the treaty mentioned above. Cf. Thābit ibn Qurra, *Oeuvres d'Astronomie. Text établi et traduit par Régis Morelon*, Paris, 1987, p. 87.

- the distance from one mesogee to the initial eclipse of the first interval is equal to the distance from the other mesogee to the final eclipse of the second interval
- the distance from one mesogee to the final eclipse of the first interval is equal to the distance from the other mesogee to the initial eclipse of the second interval

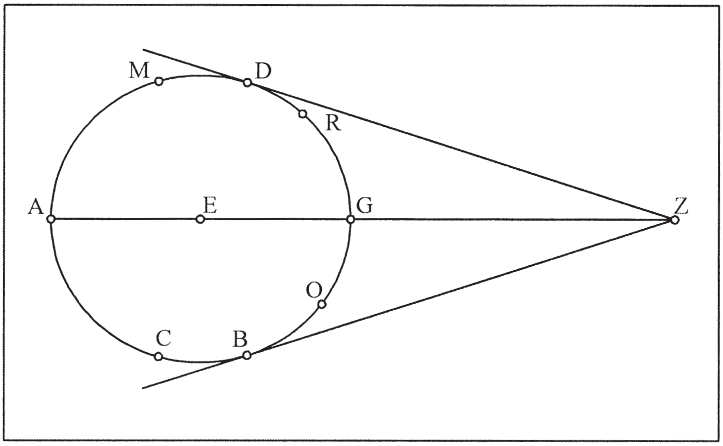

Figure 10: Ms. Berlin 39v

In order to clarify this, we will consider the description of this particular point as given in from the Berlin version. Let us consider Figure 10, in which points D and B are both mesogees of an epicycle. The Moon in E_1 is on point M close to D; in E2 on point R; in E3 on point C; and in E_4 on point O.

$\because MD = OB$

$\because DR = BC$

$\therefore MR = OC$

Therefore, as Jābir b. Aflaḥ states, this is the third situation that Ptolemy warned against and claimed should be avoided, due to the fact that the first interval is symmetrical to the second one, taking as a reference the apsidal line.

Later in the text, Jābir b. Aflaḥ gives another example of this situation, in which he considers the Moon to be in the first and third eclipses, each one in a different mesogee, and during the second and fourth eclipses at any of points R, C, O or M. The author concludes that "the Moon traverses in its epicycle in equal time intervals two equal arcs where its distances from the apogee and the perigee are the same". But this cannot be the case when the initial eclipses are each in a different mesogee. We can see, then, that a contradiction emerges.

The explanation of this problem in the Ms. Escorial 910 version is not as clear as in the Berlin version. In the Escorial 910 version, the author criticizes the fact that the Moon can in fact be 3° from a mesogee in both eclipses when we believe it to be exactly on the mesogee. Given that this can also occur during the final eclipses, the author concludes that this is similar to the situation highlighted by Ptolemy. In any case, this somewhat brief explanation would be difficult to understand without referring to the more detailed one given in the Berlin version. This is a possible case in which the Ms. Escorial 910 version is derived from the Berlin version.

4.9 Jābir b. Aflaḥ's conclusion on Ptolemy's description of the method

Jābir b. Aflaḥ is critical of the fact that Ptolemy had developed a new method, while he was using the values obtained by the four-eclipse method. First, the Berlin version quotes Ptolemy:

> As to what he says:
>
> > This is the method followed by those before us for obtaining such things. It is possible for you to know that this method is not easy to carry out, nor its procedure accessible, but requires a great deal of reflection and a deep insight on what I will show next.[31]
>
> What can be concluded is that these words in themselves do not require a deep insight. Ptolemy could make such a statement if he had provided another, easier method, if he did not need to apply the preventions (*taḥarruz*) required [for the method used by the

[31] Jābir b. Aflaḥ is making a reference to the following test from the *Almagest*: "That, then is the method which our predecessors used for the determination of such [periods]. It is not simple or easy to carry out, but demands a great deal of extraordinary care, as we can see of the following considerations". Cf. PtA, p. 176.

> ancient astronomers] and if he did not require the ancient method [for obtaining his own values]. But he could not fulfil any of these [requirements]. Instead he gave a correct method, but the enhancements introduced were lessened due to the observations the ancient [astronomers] used for determining the [lunar anomaly] period. He could not [provide a correct method] unless using the motion values the ancient astronomers obtained from this period. All that he provided relied on the [lunar anomaly] period the ancients provided by means of this method.[32]

Finally, Jābir b. Aflaḥ (at least in the Ms. Escorial 910 version) expresses his opinion on Ptolemy:

> What is truly deduced from such a man's issue is that he had not experience in the art of geometry, and for this reason he fell down in such things and in others we will point out in its proper place provided that God, glorified and exalted be, will.

In fact, Jābir b. Aflaḥ points out some details of the *Almagest* that are not errors in expression as such, but the result of a mathematically imperfect method. His criticism of the best method for selecting eclipses indicated by the 'maximum difference in potency' is valid from an astronomical point of view, but the later development in which it is compared with a lunar situation that Ptolemy warned against seems to be excessive. Why, therefore, is Jābir b. Aflaḥ so critical of Ptolemy?

5. Final conclusions

In this paper we have studied the first criticism of Ptolemy's *Almagest* which appears in Jābir b. Aflaḥ's *Iṣlāḥ al-Majisṭī*. This criticism focuses on the four-eclipse method used by ancient astronomers – such as Hipparchus – to find the lunar period in anomaly, as described by Ptolemy in *Almagest* IV.2.

[32] Jābir b. Aflaḥ is making a reference to the last part of *Almagest* IV.2 in which Ptolemy, after criticizing Hipparchus's method, bases his findings on Hipparchus's results: "But first, for convenience [of calculation] in what follows, we set out the individual mean motions [of the moon] in longitude, anomaly and latitude, in accordance with the above periods of their returns, and [also the mean motions] calculated on the basis of the corrections which we shall derive later". Cf. PtA, p. 179.

Jābir b. Aflaḥ points out some improvements on the four-eclipse method as described by Ptolemy, from a theoretical point of view. However, he does not change the value of the lunar period in anomaly obtained by Hipparchus and used later by Ptolemy in the *Almagest* while describing his lunar theory. The Andalusian mathematician and astronomer considers that the ancient astronomers developed a valid method for finding the lunar period in anomaly and that this method is the one he describes in the *Iṣlāḥ al-Majisṭī*. So he sadly concludes that Ptolemy's comments on the ancients' method show that the author of the *Almagest* did not understand it.

From a mathematical point of view, Jābir b. Aflaḥ's description of the four-eclipse method is far more elegant than the one in the *Almagest*. He also clarifies the obscure nuances in Ptolemy's description required for a full understanding of this method.

To do so, Jābir b. Aflaḥ divides the lunar epicycle in four sectors as defined by the lunar perigee and apogee and both mesogees. Jābir b. Aflaḥ used the sector theory extensively in the *Iṣlāḥ al-Majisṭī* since it also appears in the determination of Mercury's apogee[33] and in the determination of the eccentricity and direction of the apsidal line of a superior planet.[34]

In any case, the present criticism indicates Jābir b. Aflaḥ's thorough understanding of the *Almagest*. In fact he may well have been the first Western astronomer to understand it fully.

6. On the edition

The edition that follows the study is not a critical one but a working one that is based only on the Arabic manuscripts in Arabic script. Consequently, we have not used the Arabic manuscripts in Hebrew script, or the Hebrew or Latin manuscripts. Although the Latin edition of Apianus published in 1534 was consulted during the preparation of this study.[35] The three manuscripts used are:

[33] See J. Samsó [2000], p. 216 and ff.

[34] See N.M. Swerdlow [1987].

[35] Apianus, Petrus, *Instrumentum primi mobilis. Accedunt iis Gebri filii Affla Hispalensis Astronomi vetustissimi pariter et peritissimi, libri IX de astronomia, ante aliquot secula Arabice scripti, et per Giriardum Cremonensem latinitate donati, nunc vero omnium primum in lucem editi*, Nuremberg, 1534.

Escorial 910, abbreviated as Es[1]
Escorial 930, abbreviated as Es[2]
Berlin 5653, abbreviated as B

Whenever in an annotation appears a variant relating to a particular manuscript, it must be assumed that the published version considered to be correct is that of the manuscript (or manuscripts) that does not appear in the annotation. The extension of a variant has been indicated with keys. For instance, in

قال أبو محمّد جابر بن أفلح {الإشبيلي}[1]

the word al-Išbīlī has a variant in one or more manuscripts. We can also find nested keys with a parenthetical hierarchy. Lastly, whenever an annotation is found without keys in the Arabic text, the annotated variant is an addition and does not replace any word in the edition considered to be correct.

7. Translation

7.1 Ms. Escorial 910 version

[Es[1] f. 39v]

[1.] [On the anomalistic Moon's motion in longitude and latitude (ff. 39v-40r)]

When they found the Moon moving differently in longitude and latitude – [for instance,] in a given degree of the ecliptic the [lunar] motion (*ḥaraka*) is not [always] exactly the same, nor its latitude, but it can traverse any degree of the ecliptic with its mean motion, either the faster or the slower; and the same happens in latitude: for the Moon can be on its maximum latitude northward, southward or without latitude – they concluded that the return [period] in anomaly [Es[1] 40r] was different from the return [period] of the epicycle centre on the ecliptic and also that the [lunar] inclined orbit node was moving over the ecliptic.

It was also found that the time [the Moon] takes from its minimum motion to its mean motion is always greater than the time it takes from its mean motion to its maximum motion. This indicated the fact that its motion in its epicyclical apogee (*al-buʿd al-abʿad*) was towards the rear [of the order] of the signs. The ancient [astronomers] studied the way of determining [the lunar] return period along its epicycle and the return period of its epicycle centre along the ecliptic and concluded that [this research] must be based on lunar eclipses in order to avoid [the error] introduced by the lunar parallax, as we have said.

[2.] [Lunar epicycle division into four sectors depending on its true motion along them (fol. 40r)]

Since the Moon moves in an epicycle, its motion on the ecliptic faces four states (*ḥāl*, pl. *aḥwāl*). [Its motion in] the first state is increasingly fast and takes place when [the Moon] traverses from the mesogee (*al-majāz al-awsaṭ*) to the perigee (*al-buʿd al-aqrab*). The second is decreasingly fast and takes place when it traverses from the perigee to the second mesogee. The third is increasingly slow and takes place when it traverses from the mesogee to the apogee (*al-buʿd al-abʿad*). The fourth is decreasingly slow and takes place when it traverses from the apogee to the first mesogee. So we always know, depending on its motion state on the ecliptic, in which of the four sectors (*qiṭʿa*) of its epicycle [the Moon is located]; that is, the sectors limited by the apogee, the perigee and the two mesogees. And we will know, with good reason, in which sector the degree is located.

[3.] [The ancient astronomers' method for finding the lunar period in anomaly from two intervals defined by four eclipses]

[3.1.] [Brief description (fol. 40r)]

In order to find the [lunar] return period [in anomaly], the ancient [astronomers] looked for two lunar eclipses in which the Moon's speed (*sayr al-qamar*) was the same – that is, that the Moon was to be found in one of the four points with variable speed (*masīr*) mentioned – [and] therefore it was considered, with good reason, that [the Moon] had returned on its epicycle during the second eclipse to the [same] position it had [on its epicycle] during the first.

[3.2.] [Jābir b. Aflaḥ's conditions that the four eclipses must fulfil in order to find the period in anomaly (fol. 40r)]

Next, to verify that [the Moon] has returned [to the same epicyclical position] as they thought, they looked for two other eclipses such that

- the lunar variable speeds (*sayr al-qamar*) in both were the same, although
- [the variable speeds of the second pair of eclipses] must differ from the [lunar] variable speeds in the first two eclipses; and
- the time interval elapsed between the two [second eclipses] must be the same as the time interval elapsed between the first two; and
- the Moon must traverse two equal arcs [in longitude] on the ecliptic after completing [an integer number of] returns [in anomaly] in both time intervals.

When they found [the eclipses that fulfilled the conditions] according to this description, they knew therefore that the Moon had returned on its epicycle during the second eclipse to the [same] position it had [on its epicycle] during the first, and that it had returned during the fourth eclipse to the [same] position it had [on its epicycle] during the third. We will try to explain this with an example.

[3.3.] [Demonstration of the four-eclipse method for determining the period in anomaly (fol. 40r-41r)]

Let the circle ABG be the epicycle, its centre point E, the apogee point H, the perigee point L, the ecliptic centre point Z, and the line that passes through the apogee, the perigee and the ecliptic centre, line ZLEH [Es[1] 40v]. Let the mesogees be points T and K. Let the Moon be located during the first two eclipses in one of the arcs KH and HT and during the other two eclipses in one of the arcs KL and LT. And let all the conditions in all four eclipses be fulfilled as we have explained.

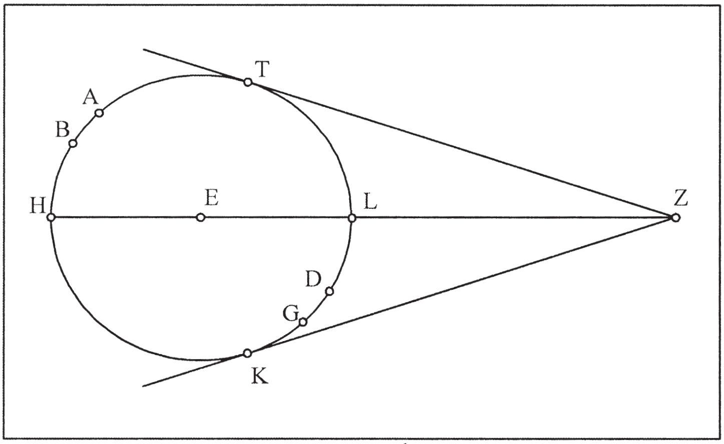

Figure 11. Ms. Es[1] fol. 40v.

I say: the Moon returns on its epicycle during the second eclipse to the [same] position it had [on its epicycle] during the first, and it returns during the fourth eclipse to the [same] position it had [on its epicycle] during the third.

Proof:

If this were not the case, then:

Let its position during the first [eclipse] be point A, during the second [eclipse] point B, during the third [eclipse] point G, and during the fourth [eclipse] point D. Since [we have considered as initial condition that] the two time intervals (*mudda*) are equal, the arc ALHB must be equal to the arc GHTD and the Moon must traverse equal arcs of the ecliptic in both equal time intervals with its mean motion after completing an integer number of returns [in anomaly].

As both equations (*zāwiyat al-ikhtilāf*) are subtended by arcs A[TLK]B and G[KHTL]D, one produces an increment in the [Moon's motion relative to its] mean motion and the second produces a decrement, so the true motion in longitude during both intervals differs. The arc [of the ecliptic] traversed [by the Moon] during the first time interval [exceeds] the mean motion by the equation (*zāwiyat al-ikhtilāf*) subtended by arc AB. The arc [of the ecliptic the Moon] traverses during the second time interval lessens the mean motion by the equation (*zāwiyat al-ikhtilāf*) subtended by arc GD. So the difference (*faḍl*) between the two depends on the sum (*majmūʿ*) of both equations. But we have considered as an initial condition that these were equal, so this conclusion is not possible. Consequently, the Moon does not return on its epicycle during the second eclipse to the [same] position it had [on its epicycle] during the first, and it does not return during the fourth eclipse to the [same] position it had [on its epicycle] during the third.

Likewise, if the Moon exceeds an [integer number of] returns in anomaly along arc AB during the first time interval and along arc GD during the second interval – i.e. if it is located at point B during the first eclipse, at point A during the second, at point D during the third, and at point G during the fourth – the arc of the ecliptic which exceeds an integer number of returns [in anomaly] during the first time interval lessens the mean motion by the equation (*zāwiyat al-ikhtilāf*) subtended by arc AB and the arc which exceeds [an integer number of returns in anomaly] during the second time interval exceeds the mean motion by the equation subtended by arc GD. The difference (*tafāḍul*) between the two arcs depends on the sum (*majmūʿ*) of both equations.

As before, [Es[1] 41r] the same consequence will be deduced if both time intervals take place in the same mid-epicycle: that is, in one of the two mid-epicycles HAL and HKL. This is what we intended to prove.

Similarly, as each of the two intervals contains two eclipses, each [interval] must enclose the same integer number of months, the same integer number of returns of the epicycle centre along the ecliptic, and equal arcs [of the ecliptic] which exceed the integer number of returns.

[3.4.] [Values obtained from the time intervals. (fol. 41r)]

When they found [the time intervals between eclipses], they divided the time of one of the two intervals by the number of the [lunar] returns in anomaly [during that time] and obtained the period of one [lunar] return [in anomaly]. When they divided the degrees of one circle – i.e. 360° – by the number of days [included in the lunar period in anomaly], they obtained the distance the Moon traverses in its epicycle during one day. Similarly, they divided the days in this time interval by the number of months and obtained the duration of the mean month. When this time was multiplied (*ḍūʿifa*) by the daily solar mean motion, the total value obtained was the distance the Sun traverses with its mean motion during one mean month until the Moon reaches [the Sun]. When the degrees of one circle – i.e. 360° – were added to [this last value], the total sum was the distance the Moon traverses in longitude during the mean month. Finally, when this value was divided by the number of days in the mean month, the result was the distance [the Moon] traverses in longitude during one day.

This is the procedure the ancient [astronomers] followed to obtain the [Moon's] period [in anomaly] and from which they obtained the [lunar] motions in longitude and anomaly.

[4.] [On the lunar eclipse positions which invalidate the four-eclipse method for finding the lunar period in anomaly]

[4.1.] [Ptolemy's description of the lunar eclipse positions that invalidate the four-eclipse method (fol. 41r)]

[Let us consider] Ptolemy's criticisms on the [ancients'] resolution of the period [in anomaly] and what he the need for exhaustive investigation and avoiding the lunar positions relative to the epicycle in which it can traverse equal arcs on the ecliptic in equal times without returning to the [same lunar] anomaly, which is possible [in the next occurrences, as Ptolemy states]:

- if the Moon in the first eclipse begins from the apogee and ends in the second eclipse in the perigee and if the third [eclipse] begins from the perigee and ends in the fourth [eclipse] in the apogee; or
- if it traverses an identical arc in its epicycle in both time intervals; or
- if it traverses two equal arcs in which its distances from the apogee and the perigee are the same – i.e. the distance of the [lunar] positions in the first and fourth eclipses is symmetrical to the epicycle apogee and perigee.

Therefore, in each of these three situations, it follows that the Moon traverses equal arcs on the ecliptic in equal time intervals, but [the Moon] does not complete a return in its epicycle.[36]

[36] Jābir b. Aflaḥ is making a reference to the following text: "Secondly, it is our opinion that we must pay no less attention to the moon's [varying] speed (*δρόμος*). For if this is not taken into account, it will be possible for the moon, in many situations, to cover equal arcs in longitude in equal times which do not at all represent a return in lunar anomaly as well. This will come to pass [1] if in both intervals the moon starts from the same speed (either both increasing or both decreasing), but does not return to that speed; or [2] if in one interval it starts from its greatest speed and ends at its least speed, while in the other interval it starts from its least speed and ends at its greatest speed; or [3] if the distance of [the position of] its speed at the beginning of one interval is the same distance from the [position of] greatest or least speed as [the position of] its speed at the

[4.2.] Jābir b. Aflaḥ's answer (fol. 41r-41v)]

[I say that] it is not necessary to avoid and investigate exhaustively what he mentioned for it is not possible for the Moon to be in one of these positions during the point at which the [ancients'] found the [lunar] period in anomaly. And this is due to the fact that what they considered first [Es[1] 41v] when they found the [Moon's] return period [in anomaly] is that the lunar variable speed (*sayr al-qamar*) during the second eclipse must be the same as its variable speed during the first for it to be considered that [the Moon] had completed a return in its epicycle, and likewise that during the fourth eclipse its variable speed must be the same as its variable speed during the third for it to be considered that [the Moon] had completed a return in its epicycle. But, [we ask ourselves:]

- how the [Moon's] variable speed can be the same at both ends of the same interval that the [ancient astronomers] established as condition when [the Moon] begins from the apogee in the first interval and ends in the perigee and begins from the perigee in the second interval and ends in the apogee [as the first occurrence that Ptolemy says must be avoided], since its variable speed at the beginning of the interval is therefore extremely different to its variable speed at the end of the interval and this [i.e. the first occurrence Ptolemy says must be avoided] is different from the condition established by [the ancient astronomers]; and
- how [the Moon] traverses an identical arc in its epicycle [in both time intervals] [as the second occurrence Ptolemy says it must be avoided], since its variable speed during the first eclipse is therefore exactly the same as during the third and its variable speed during the second eclipse is therefore exactly the same as during the fourth, but the [ancient astronomers] established a different condition, i.e. that its variable speed during the first and second eclipses must be different from its speed during the third and fourth [eclipses]; and

end of the interval, [but] on the other side. In each of these situations there will again be either no effect or the same effect [in both intervals] of the lunar anomaly, and hence equal increments in longitude will be produced [over both intervals], but there will be no return in anomaly at all. So the intervals adduced must avoid all the above situations if they are to provide us directly with a period of return in anomaly". Cf. PtA, pp. 177-8.

- how [the Moon] traverses two equal arcs in which the distances from the apogee and the perigee are the same [as the third occurrence that Ptolemy says must be avoided], since its variable speed during the first eclipse must therefore be the same as its variable speed during the fourth, and its variable speed during the second [eclipse] must therefore be the same as during the third.

Even if [the ancient astronomers] did not clearly state these conditions, from the practical procedure it can be inferred that they undoubtedly established them.

[5.] [On the Sun's positions for avoiding the solar anomaly]

[5.1.] [Ptolemy's description of the solar positions for avoiding the solar anomaly (fol. 41v)]

Similarly, [Ptolemy] also considered it essential for [the ancient astronomers] in their study of the solar positions in each of the desired eclipses that [the Sun] in each of [the eclipses] should be in one of the positions that must be avoided for the Moon, that is

- that [the Sun] begins from the eccentric apogee in the first eclipse and ends at its perigee in the second eclipse, and that it begins from the perigee in the third eclipse and ends at its apogee in the fourth; or
- that [in both intervals the Sun] traverses the exactly same arc of its eccentric; or
- that [the Sun] traverses two equal arcs provided its distances from the apogee and the perigee are the same; or
- that [the Sun] traverses an integer number of returns in its eccenter and also on the ecliptic in both intervals.[37]

[37] Jābir b. Aflaḥ points out the following test from the *Almagest*: "Therefore we define as the first necessary condition [for a return in lunar anomaly] that the intervals must exhibit one of the following characteristics with respect to the sun: [1] It must complete an integer number of revolutions [in both intervals]; or [2] traverse the semi-circle beginning at the apogee over one interval and the semi-circle beginning at the perigee over the other; or [3] begin from the same point [of the ecliptic] in each interval; or [4] be the same distance from the apogee (or perigee) at the first eclipse of one interval as it is at the second eclipse of the other interval, [but] on the other side". Cf. PtA, p. 177.

[5.2.] [Jābir b. Aflaḥ's answer (fol. 41v)]

[Against this, I say that] it is not necessary for them to investigate this for they only looked to verify

- two equal time intervals, both of which contain two eclipses; and
- that the Sun and the Moon traverse two equal arcs on the ecliptic in both intervals; and
- that the Moon in each of the eclipses [fulfils the conditions] we have described.

When [the ancient astronomers] found [two intervals that fulfilled the three previous conditions], the Sun must therefore have been in one of the four positions previously mentioned.

[6.] [On the best selection of eclipses for maximizing the difference in longitude of the two intervals when there is not a complete return in anomaly]

[6.1.] [Ptolemy's proposal (fol. 41v)]

[Ptolemy] also states, regarding the [eclipse] selection for determining these two intervals, that the Moon must begin the first and third eclipses at extremely different speeds (*sayrayn mukhtalifayn*) – i.e. that the two variable speeds were different according to [their] fastness (*surʿa*), slowness (*ibṭāʾ*) and [their] acceleration (*tazayyud*) or deceleration (*tanaqquṣ*).[38]

[6.2.] [Jābir b. Aflaḥ's proposal (fol. 41v-42r)]

[Against that, I say that] this is not essential and that [the ancient astronomers] have no need to apply this condition, for it makes it difficult

[38] Jābir b. Aflaḥ is making a reference to the following text: "On the contrary, we should select intervals [the end of which are situated] so as to best indicate [whether the interval is or is not a period of anomaly], by displaying the discrepancy [between two intervals] when they do not contain an integer number of returns in anomaly. Such is the case when the intervals begin from speeds which are not merely different, but greatly different either in size or in effect". PtA, p. 178.

to find eclipses that fulfil it. Instead, the fact that its speeds during the first [Es[1] 42r] and second eclipses were different to its speeds during the third and fourth [eclipses] in terms of the fastness (*surʿa*) and slowness (*ibṭāʾ*) constitutes for them [the previous condition]. As a result, one [lunar] speed would be greater than its mean speed and the second one would be smaller than its mean speed, to the point that if the Moon does not complete a return [in anomaly] in its epicycle, it causes a difference (*tafāḍul*) in the arcs of the ecliptic which implies an increment that depends on the sum (*majmūʿ*) of both anomaly angles. Therefore, the initial positions of the Moon must be very different so that the sum of its anomalies is clearly perceptible. This is only the case if the initial positions of the Moon are far from the mesogees, rather than what [Ptolemy] stated.

[6.3.] [Maximum difference in potency]

[6.3.1.] [Ptolemy's proposal (fol. 42r)]

[Ptolemy] states that the [motion of the] Moon in [the initial position of the] first interval [should] differ 'in potency' (*mukhālif fī 'l-quwwa*) from the initial position of the second [interval], i.e. that [the Moon] begins from one mesogee in the first interval and from the other mesogee in the second [interval].[39]

[6.3.2.] [Jābir b. Aflaḥ's criticism of Ptolemy's proposal (fol. 42r)]

[Against that I say that] this is extremely erroneous because it would not be possible to verify at any time the Moon's position in its epicycle. And were you able to verify it, it would not be useful for [obtaining] the [lunar anomaly] period. This is due to the fact that if we observe the Moon from its departure from a particular degree of its epicycle to its return to the same degree, we would not know its true position. [This is is case]

[39] Jābir b. Aflaḥ is making a reference to the following text: "By 'in effect' I mean when [the moon] starts from the mean speed in both positions, not, however, from the same mean speed, but from the mean speed during the period of increasing speed at one interval, and from that during the period of decreasing speed during at the other. Here too, if there is not a return in anomaly, there will be a great difference in the increment in longitude [over two intervals]; again, when the increment in anomaly is one or three quadrants of a revolution, the difference will again amount to twice the [maximum] equation of anomaly, and when the increment in anomaly is a semi-circle, the difference will be four times that amount". PtA, p. 178. Cf. HAMA Fig. 62, p. 1225.

particularly at both mesogees because of the difference (*tafāḍul*) between the anomaly angles in both. Its speed in terms of acceleration (*tazayyud*) or deceleration (*tanaqquṣ*) changes slowly [at such points] to the extent that it is possible for the Moon in each of the two eclipses to be 3° or more away from both mesogees to either side, though we continue to consider that it is located at the mesogee.

[6.3.3.] [Jābir b. Aflaḥ points out a contradiction in Ptolemy's discourse (fol. 42r)]

The same can be said of the other two eclipses. Therefore [the Moon] has traversed two equal arcs along its epicycle in both intervals and its distances from the apogee and the perigee are the same. But this is one of the three positions [Ptolemy] warned against, [for he says]: "Consequently, the Moon has traversed on the ecliptic in two equal intervals two equal arcs and it has not completed a return in anomaly".[40] [In short, Ptolemy] had prompted these positions without realizing that he had originally warned against them.

[7.] [Jābir b. Aflaḥ's opinion on Ptolemy's proficiency in geometry (fol. 42r)]

What is truly deduced from such a man's issue is that he had not experience in the art of geometry, and for this reason he fell down in such things and in others we will point out in its proper place provided that God, glorified and exalted be, so wills.

[8.] [On the anomaly period in latitude (fol. 42r)]

As for the Moon's motion in latitude, the ancient [astronomers] knew it by looking for two lunar eclipses with the same magnitude, exactly the same anomaly, the same occultation to both the north and south, and almost exactly the same node. By fulfilling these conditions, the Moon's nodal distance in the first of the two eclipses must necessarily be the same as its

[40] "In each of these situations there will again be either no effect or the same effect [in both intervals] of the lunar anomaly, and hence equal increments in longitude will be produced [over both intervals], but there will be no return in anomaly at all". Cf. PtA, p. 178.

distance in the other [eclipse] from the same node and the same side. Therefore, this interval contains [an integer number of] Moon returns in latitude and of the epicycle centre [returns related to the node]. When this interval was divided by the integer number of returns in latitude, the lunar mean motion in latitude was obtained. In this way, the ancient [astronomers] knew the lunar motions in longitude [Es[1] **42v**], anomaly and latitude.

[9.] [Daily motion in longitude, latitude and anomaly (fol. 42v)]

[The Moon's] daily motion in longitude is 13;10,34,58,33°; in anomaly, 13;3,53,56,29°; and in latitude, 13;13,45,39,48°.

7.2 Ms. Berlin 5653 version.

[B. f. 38v, Es[2] f. 43v]

[1.] [On the anomalous Moon's motion in longitude and latitude]

When they found the Moon moving differently in longitude and latitude – i.e. for any degree of the ecliptic the [Moon's] motion (*ḥaraka*) is not [always] exactly the same, nor is its latitude, but it is displaced in any degree of the ecliptic with its mean motion, either the faster or the slower; the same occurs in latitude since the Moon can be on its maximum latitude northward, southward or without latitude – they concluded that the return [period] in anomaly was different from the return [period] of the epicycle centre on the ecliptic and that the [lunar] inclined orbit node was moving over the degrees of the ecliptic. The ancients studied the way of determining [the lunar] return period in anomaly and the return period [of its epicycle centre] along the ecliptic and concluded that [this research] must be based on lunar eclipses in order to avoid [the error] introduced by the lunar parallax, as we have said.

[2.] [Lunar epicycle division into four sectors depending on its true motion along them]

As the Moon has different motions (*ḥarakāt*) – i.e. a fast motion, a slow motion and a mean motion – it must have four points (*nuqaṭ*) in its particular orbit (*al-falak al-khāṣṣ*). One is the point at which [the Moon's motion] is the fastest. The second is opposite the previous one and is the point at which [the Moon's motion] is the slowest. These two points are the apogee (*al-buʿd al-abʿad*) and the perigee (*al-buʿd al-aqrab*) of its particular orbit. The two points at which the Moon has a mean speed between the two previous ones are the mesogees (*al-majāz al-awsaṭ*) of its particular orbit. These four points divide this orbit [the epicycle] into four sectors (*qiṭʿa*). One is that in which [the Moon] changes from its fastest motion to its first mean motion: this motion is fast and decreasing (*sarīʿat mutanāqiṣa*). The second sector is that in which its motion is mean and decreasing. The third sector is that in which its motion is slow and increasing. Finally, the fourth sector is that in which its motion is mean and increasing.

[3.] [The ancient astronomers' method for obtaining the Moon's anomaly period]

[3.1.] [Brief description]

Therefore we know, with good reason, in which of the four sectors the Moon is located at any given time. [From this premise,] the ancient [astronomers] looked for two lunar eclipses, provided that the Moon's motion (*ḥaraka*) in both was one of these four types of motion [mentioned]. They considered then, with good reason, that [the Moon] during the second eclipse had [Es^2 f. 44r] returned to the same position in its particular orbit it had occupied during the first eclipse [B. f. 39r] and that the interval between the two eclipses contained an integer number of Moon's returns in its particular orbit.[41]

[3.2.] [Jābir b. Aflaḥ's conditions that the four eclipses should fulfil in order to obtain the lunar anomaly period.]

Given that they wanted to test and verify [whether these two eclipses were suitable], they looked for two other eclipses [that fulfilled the following

[41] "Hence the ancient astronomers, with good reason, tried to find some period in which the moon's motion in longitude would always be the same, on the grounds that only such a period could produce a return in anomaly. So they compared observations of lunar eclipses (for reasons mention above), and tried to see whether there was an interval, consisting of an integer number of months, such that, between whatever points one took that interval of [true synodic] months, the length in time was always the same, and so was the motion [of the moon] in longitude, [i.e.] either the same number of integer revolutions, or the same number of revolutions and the same arc". Cf. PtA, p. 175.

conditions]:

- that the lunar motion (*ḥaraka*) in these two other eclipses was the same
- that [the lunar motion in the second pair of eclipses] was different (*mukhālif*) from the motion (*ḥaraka*) of the first two eclipses
- that the two time intervals (*mudda*) between these four eclipses were the same
- that the Moon traverses two equal longitudes[42] along the ecliptic such only an integer number of cycles or an integer number of cycles plus additional equal arcs.

When they found [four eclipses] that fulfilled the conditions described, they knew that the Moon had returned to the same point of its particular orbit in the first pair of eclipses and that it had also returned to a second point [of its particular orbit] in the second pair of eclipses. Therefore, both time intervals contained an integer number of lunar returns in its particular orbit.

[3.3.] [On that Ptolemy did not clearly state this conditions (fol. 39r)]

Ptolemy mentioned this method from the ancient astronomers, but he did not clearly state the conditions on the lunar motions during the desired eclipses that we have mentioned here. However, even though he did not state them clearly, the meaning itself (*nafs al-maʿnà*) implies that these conditions are those required for the desired eclipses. Were it not as described, it would not be possible to complete an [integer number of] returns [in anomaly]. And as to [the following question:] from what is inferred that if the four eclipses fulfil these conditions, are both intervals that enclose an integer number of lunar returns in its particular orbit equal in number? This would be clear as I will explain [next] given God's power and help.

[3.4.] [Proof of these conditions]

Let circle ABGD be the Moon's epicycle and point E its centre. Let point Z be the ecliptic centre and line AEGZ the line which passes through the

[42] Literally: arcs.

apogee, the perigee and the ecliptic centre, where point A is the apogee and point G the perigee. From point Z we draw two tangents to circle AB on points B and D. We then obtain lines ZB and ZD. Therefore, points B and D are the mesogees. Let the Moon be at point H during the first eclipse and at point C during the third eclipse. Let the Moon's speed (*sayr al-qamar*) at both points [i.e. H and C] be as we have mentioned, i.e. [Es[2] f. 44v] that [its speed] should be different [at both points]. Let the Moon's speed (*sayr al-qamar*) at point H be equal to its speed during the second eclipse and its speed (*sayr*) during the third [eclipse] equal to its speed (*sayr*) during the fourth. Let both intervals be equal and the ecliptic sectors enclosed in both also be equal.

I say: the Moon returns exactly to point H during the second eclipse and exactly to point C during the fourth.

Proof:

If the Moon does not return to point H during the second [eclipse], let the Moon return to point T. And if it does not return to point C during the fourth [eclipse], let the Moon return to point O.

Given that both intervals are equal – i.e. the intervals between the first and second [eclipses] and between the third and fourth [eclipses] – arcs HT and CO are equal.

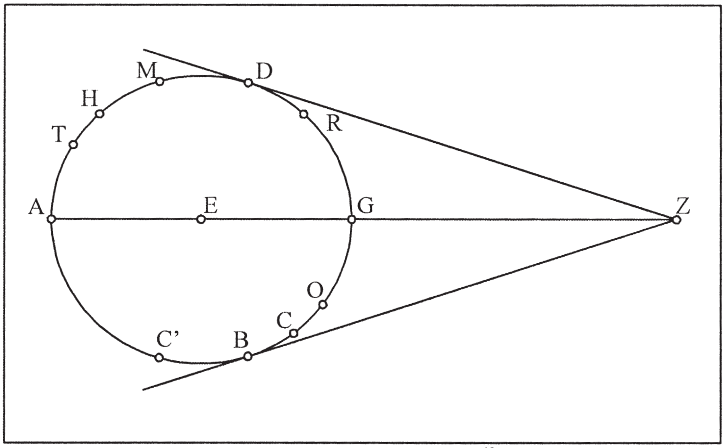

Figure 12: Fol. Berlin 39v[43]

[43] In the manuscript figure, point C appears where point C' is now located. However, this location is not coherent with the demonstration. Consequently, a new point C has been introduced for coherence with the demonstration appearing in the text.

Given that the Moon's speed (*masīr al-qamar*)[44] at points H and T is different (*mukhālif*) from its speed (*masīr*) at points C and O – i.e. its speed (*sayr*) is slow in one case and fast in the other, as stated as a condition – [B. f. 39v] there must necessarily be an increment in arc HT and a decrement in arc CO, both relative to the Moon's mean motion.

Given that both intervals are equal, the mean motion in both must be the same. Therefore, the true lunar motion during the first interval must be different from its [true] motion during the second [interval], depending on the sum (*majmūʿ*) of the two anomaly equations (*khilāf*) – i.e. the two angles with the ecliptic centre as vertex and subtended by arcs HT and CO. Thus, the Moon traverses two different arcs on the ecliptic in both equal intervals. The difference (*faḍl*) between the two is based on the sum of both equations (*ikhtilāf*) as subtended by arcs HT and CO. But we have established as a premise that the Moon traverses two equal arcs on the ecliptic in both equal intervals. Therefore, this conclusion makes it impossible for the Moon to be at any point other than H during the second eclipse and at any point other than C during the fourth eclipse. Consequently, during its second [eclipse] [the Moon] returns to its [epicycle] position during its first [eclipse], and during the fourth eclipse it returns to its position during the third. Both equal intervals contain an integer number of lunar returns in its epicycle, which is what we wanted to prove.

[4.] [On the best selection of eclipses for maximizing the difference between the Moon's speeds]

[4.1.] [Jābir b. Aflaḥ's proposal]

[Es^2 f. 45r] Given that the difference (*khilāf*) between the ecliptic arcs the Moon traverses in two equal intervals when the it does not return to its first position depends on the sum (*majmūʿ*) of the equations (*ikhtilāf*) produced by arcs HT and CO, the selected eclipses for obtaining the anomaly period must be those in which the lunar positions[45] produce a large difference between the mean and true motions. These positions are the apogee and perigee points and the [areas] next to them. Whenever the Moon's position during the eclipses withdraws from the apogee and

[44] The Ms. Es^2 uses *sayr* instead of *masīr* in this section.

[45] There is a 'not' (*lā*) added at this point in the manuscript.

perigee points, it also withdraws from the [proper] selection [Ptolemy ought to choose]. So, rather than the positions mentioned by Ptolemy, it is convenient to avoid the situation in which the Moon's position during the first and third eclipses is on the mesogees or next to them.

[4.2.] [Jābir b. Aflaḥ's criticism of Ptolemy's proposal]

If the Moon were at point M during the first eclipse, next to point D (i.e. the mesogee), given that the Moon's motion when it is in an [area] next to point D is [more or less] the same, [the Moon] could be at point R during the second eclipse when we thought that [its position] during both eclipses was the same point. [Under these conditions,] it is also possible for [the Moon] to be at point C during the third eclipse – [the point] at which its distance from point B is the same as the distance from point R to point D – and at point O during the fourth [eclipse] – [the point] at which its distance from point B is the same as the distance between point M and point D – when we thought that [its position] during both eclipses was the [B. f. 40r] same point.

For this reason, the equation (*khilāf*) produced by arc RM must be equal to the equation (*khilāf*) produced by arc CO and both must be of the same kind, i.e. both [equations] must produce an increment or decrement in the true mean motion [relative to the mean motion].

Therefore, the Moon must traverse two equal arcs on the ecliptic during an integer number of cycles in two equal intervals and does not return [to its first point] in the epicycle. For this to be the case, the Moon's position during the first eclipse must be exactly the mesogee point, during the third [eclipse] the other mesogee point [Es^2 f. 45v], and during the second and fourth [eclipses] any of the points R, C, O or M. Therefore, the Moon traverses two equal arcs in its epicycle in equal intervals, in which its distances from the apogee and the perigee are the same. Yet this was one of the three situations Ptolemy had warned against. [Nevertheless,] we found him adopting this [forbidden] position as one of the selected positions for these observations.

[4.3.] [*Almagest*'s quote (fol. 40r-v) and Jābir b. Aflaḥ's criticism]

[Ptolemy] says in [*Almagest*] IV.2:

> So it is not convenient for the intervals to be used in these circumstances if we have established that they in fact produce a

period of return in anomaly. Instead, we should select a situation contrary to the [previously mentioned] situation, i.e. [those] intervals with the particularity of clearly being able to show the difference [between both intervals] when they do not enclose an integer number of returns in anomaly. So we will not confine to [the situation when] the intervals begin from speeds which are not merely different (*masīrāt mukhtalifa*), but greatly different either 'in size' (*fī 'l-miqdār*) or 'in potency' (*fī 'l-qūwa*).[46]

As for the [maximum difference] 'in size' – when the Moon begins in one of both intervals at its least speed (*sayr*) and does not end at the greatest speed (*sayr*) and in the other interval it begins at its greatest speed (*sayr*) and does not end at its least speed (*sayr*) – in such a case, the difference in the increment in longitude is extremely large when [the Moon] does not complete an integer number of cycles in anomaly. And, particularly when one difference [in anomaly] reaches a quadrant or three quadrants, the difference (*faḍl*) is twice the [maximum] equation of anomaly, which amounts to the difference of both intervals.[47]

As to [the maximum difference] 'in potency' – when [the Moon] begins at its mean speed (*al-masīr al-wasaṭ*) in both intervals, provided that [the Moon] does not begin from exactly the same [point with] mean [motion in both], but rather in one [interval] it begins from the increasing [mean] speed (*al-masīr bi-ḥaythu 'l-ziyāda*), while in the other it begins from the decreasing [mean] speed (*al-masīr bi-ḥaythu 'l-nuqṣān*) – in such a case, the differences in longitude [for each interval] are extremely great

[46] "So the intervals adduced must avoid all the above situations if they are to provide us directly with a period of return in anomaly. On the contrary, we should select intervals [the end of which are situated] so as to best indicate [whether the interval is or is not a period of anomaly], by displaying the discrepancy [between two intervals] when they do not contain an integer number of returns in anomaly. Such is the case when the intervals begin from speeds which are not merely different, but greatly different either in size or in effect". PtA, p. 177-8.

[47] "By 'in size' I mean when in one interval [the moon] starts from its least speed and does not end at the greatest speed, while in the other it starts from its greatest speed and does not end at its least speed. For in this case, unless the intervals contain an integer number of revolutions in anomaly, the difference in the increments in longitude over the two intervals will be very great; when the increment of anomaly is about one or three quadrants of a revolution, the intervals will differ by twice [maximum] equation of anomaly". PtA, p. 178. Cf. HAMA Fig. 62, p. 1225.

> when [the Moon] does not complete a return in anomaly. When one difference [in anomaly] also reaches a quadrant or three quadrants [as before], the difference (*faḍl*) is twice the [maximum] equation of anomaly, and when the [difference in anomaly] is a semi-circle, the difference will be four times that amount.[48]
>
> For this reason, we found that Hipparchus considered it necessary to be cautious as far as possible [Es^2 f. 46r] in the selection of the intervals to be used in such investigation (*faḥṣ*). Therefore, he used the [type of] difference (*faḍl*) [between both] lunar [intervals mentioned above] [B. f. 40v], so in one of the two intervals the Moon begins at its greatest speed (*akthar al-masīr*) and does not end at its least speed (*aqall al-masīr*), and in the other interval it begins at its least speed (*aqall al-masīr*) and does not end at the greatest speed (*akthar al-masīr*).[49]

This is the quotation in reference to Ptolemy's [*Almagest*].[50]

So Ptolemy, to obtain the [lunar anomaly] period, considered the selected eclipse for one interval to be in one of the two mesogees and for the other to be in the other mesogee. But we have explained previously that this is extremely far from the [fitting] selection, for this is one of the three situations [Ptolemy] had warned us against and had forbidden when

[48] Jābir b. Aflaḥ is making a reference to the following text from the *Almagest*: "By 'in effect' I mean when [the moon] starts from the mean speed in both positions, not, however, from the same mean speed, but from the mean speed during the period of increasing speed at one interval, and from that during the period of decreasing speed at the other. Here too, if there is not a return in anomaly, there will be a great difference in the increment in longitude [over two intervals]; again, when the increment in anomaly is one or three quadrants of a revolution, the difference will again amount to twice the [maximum] equation of anomaly, and when the increment in anomaly is a semi-circle, the difference will be four times that amount". PtA, p. 178. Cf. HAMA Fig. 62, p. 1225.

[49] Jābir b. Aflaḥ is making a reference to the following text from the *Almagest*: "That is why, as we can see, Hipparchus too used his customary extreme care in the selection of the intervals adduced for his investigations of this question: he used [two intervals], in one of which the moon started from its greatest speed and did not end at its least speed, and in the other of which it started from its least speed and did not end at its greatest speed". Cf. PtA, p. 178. Jābir b. Aflaḥ, in his refutation of Ptolemy, does not follow the order of the *Almagest*.

[50] Jābir b. Aflaḥ quotes Isḥāq b. Ḥunayn translation. Cf. Ms. Paris BN. Ar. 2482 f. 60r.

obtaining this interval. So he supported a selection that he didn't realize he had already warned against and ruled out.

[5.] [Jābir b. Aflaḥ's answer to Ptolemy's considerations on the difficulties of this method]

He says:

> This is the method followed by those before us for obtaining such things. It is possible for you to know that this method is not easy to carry out, nor its procedure accessible, but requires a great deal of reflection and a deep insight on what I will show next.[51]

What can be concluded is that these words in themselves do not rely on a deep insight. He could make such a statement if he had provided another, easier method, if he did not need to apply the preventions (*taḥarruz*) required [in the ancients' method] and if he did not require the ancients' method [to obtain his own values]. But he could not fulfil any of these [requirements]. Instead he provided a correct method, but the enhancements introduced were rendered less effective due to the observations the ancient [astronomers] used to determine the [lunar anomaly] period. He could not [provide a correct method] unless he used the motion values the ancients obtained from this period. The method he provided depended on the [lunar anomaly] period the ancients provided by means of this method.[52]

[51] Jābir b. Aflaḥ is making a reference to the following text from the *Almagest*: "That, then is the method which our predecessors used for the determination of such [periods]. It is not simple or easy to carry out, but demands a great deal of extraordinary care, as we can see of the following considerations". Cf. PtA, p. 176.

[52] Jābir b. Aflaḥ is making a reference to the last part of *Almagest* IV.2 in which Ptolemy, after criticizing Hipparchus's method, bases his findings in Hipparchus's results: "But first, for convenience [of calculation] in what follows, we set out the individual mean motions [of the moon] in longitude, anomaly and latitude, in accordance with the above periods of their returns, and [also the mean motions] calculated on the basis of the corrections which we shall derive later". Cf PtA, p. 179.

[6.] [On the solar positions for avoiding differences in the solar anomaly]

[6.1.] [Ptolemy's description of the solar positions for avoiding differences in the solar anomaly (fol. 40v)]

He says:

> Let us grant that both interval times are equal. For this reason, I also say, first of all, that there should not be a difference (*faḍl*) [between both intervals] due to the Sun's [equation of] anomaly. [Therefore, the Sun's equation of anomaly] in both intervals must be zero (*aṣl^an*) or must be exactly the same.[53]

[6.2.] [Jābir b. Aflaḥ's response (fol. 40v)]

The conclusion is different from what he states. For given what is established as a condition for the two intervals – i.e. [i.] that both were equal, and [ii.] that the Moon traverses equal arcs of the ecliptic in both [intervals] – and provided that the Sun in the mean time of each eclipse [Es^2 f. 46v] is in true opposition to the Moon, the Sun must only traverse two equal arcs of the ecliptic in both equal intervals. This is not so unless the difference (*faḍl*) due to the equation of anomaly [between both intervals] is zero, or unless it is exactly the same [in both intervals]. And this is not so unless [the Sun] is in on one of the four positions given by [Ptolemy]. Thus, [Ptolemy] suggested what is concluded from the premises. This is, then, a self-evident question.

[7.] [On the lunar positions that must be avoided]

[7.1.] [Ptolemy's description of the lunar positions that must be avoided (fol. 41r)]

Similarly, he mentions afterwards the necessity of avoiding certain lunar positions in its epicycle in the eclipses when finding these intervals [B. f. 41r], which are the positions in which [the Moon] can traverse equal arcs on the ecliptic during equal periods while [the Moon] does not complete a return in its anomaly, and this is the case when

[53] "Let us grant that [two] intervals [between pairs of eclipses] are found to be precisely equal in time". Cf. PtA, p. 176.

- the Moon begins from the apogee of its epicycle in the first eclipse and ends in the perigee in the second eclipse, and begins from the perigee in the third [eclipse] and ends in the apogee in the fourth [eclipse]; or
- it traverses an identical arc in its epicycle in both time intervals; or
- it traverses two equal arcs provided its distances from the apogee and the perigee are the same, i.e. the distance of the [lunar] positions in the first and fourth eclipses are symmetrical with respect to the apsidal line (*al-khaṭṭ al-mārr bi-l-buʿd al-abʿad wa 'l-aqrab*), as well as the [distance of the lunar] positions in the third and fourth [eclipses],

[Thus Ptolemy concludes that], in each of these three positions, the Moon must traverse two equal arcs on the ecliptic in two equal intervals, while not completing a return in its epicycle.[54]

[7.2.] [Jābir b. Aflaḥ's answer (fol. 41r)]

[Against that, I say that] it is not necessary to avoid [these positions] and to exhaustively investigate them because it is not possible for the Moon to be in one of these positions while they search for these intervals, for the first thing taken into consideration relative to the Moon is

[54] Jābir b. Aflaḥ is making a reference to the following text from the *Almagest*: "Secondly, it is our opinion that we must pay no less attention to the moon's [varying] speed (*δρόμος*). For if this is not taken into account, it will be possible for the moon, in many situations, to cover equal arcs in longitude in equal times which do not at all represent a return in lunar anomaly as well. This will come to pass [1] if in both intervals the moon starts from the same speed (either both increasing or both decreasing), but does not return to that speed; or [2] if in one interval it starts from its greatest speed and ends at its least speed, while in the other interval it starts from its least speed and ends at its greatest speed; or [3] if the distance of [the position of] its speed at the beginning of one interval is the same distance from the [position of] greatest or least speed as [the position of] its speed at the end of the interval, [but] on the other side. In each of these situations there will again be either no effect or the same effect [in both intervals] of the lunar anomaly, and hence equal increments in longitude will be produced [over both intervals], but there will be no return in anomaly at all. So the intervals adduced must avoid all the above situations if they are to provide us directly with a period of return in anomaly". Cf. PtA, pp. 177-8.

- that its speeds (*masīr*) during the first and second eclipses – i.e. those which contain a same interval – were the same (*masīr wāḥid*), thus they considered that [the Moon] had returned during the second eclipse to its position during the first, so that the interval contains an integer number of lunar returns in its epicycle;
- and [secondly] that its speeds (*masīr*) during the third and fourth eclipses were also exactly the same, thus they considered that [the Moon] had returned [to the same position] in its epicycle.

This condition makes it impossible (*baṭṭala*) for the Moon to be [Es[2] f. 47r] at the apogee during the first and fourth eclipses and at the perigree during the second and third [eclipses].

The two remaining positions – i.e. that the Moon traverses exactly the same arc in its epicycle in both intervals, and that it traverses two equal arcs [in its epicycle] provided that its distances from the apogee and the perigee were the same – are invalidated by the condition that the [Moon's] speed during the first two eclipses differs from that of the other two eclipses. In both positions that [Ptolemy warned against], the Moon's speed in the first two eclipses must be the same as that in the other two eclipses. However, this is in disagreement with the condition established. If these conditions on the Moon's speed are established for the suggested intervals, there is no basis for [Ptolemy's] claims about avoiding and exhaustively investigating [these positions] for the Moon, nor for the Sun. This is the method the ancient astronomers followed in order to find the [lunar anomaly] period.

[8.] [On the lunar periods obtained by Hipparchus]

Ptolemy mentioned that Hipparchus found this period to be 126,007 days plus one equinoctial hour. This period contained 4267 [lunar] months [B. f. 41v], 4573 complete [lunar] returns in its anomaly and 4612 [lunar] revolutions on the ecliptic less approximately 7 1/2° which are the degrees by which the Sun falls short of completing 345 revolutions, these revolutions being relative to the fixed stars. Hence when they divided the number of days found for this period by the number of months contained, the mean [synodic] month was obtained as approximately 29;31;50,8,9,20 days.[55]

When the number of days in a month is multiplied by the minutes the Sun traverses during one day with its mean motion – i.e. 0;59,8,17,13,12,31 – it gives the longitude the Sun traverses during a mean month. If the degrees of one cycle – i.e. 360° – are added to the previous longitude, [Es^2 f. 47v] this gives the longitude traversed by the Moon during a mean month with its mean motion. When this value is divided by the number of days in a month, the lunar mean motion in longitude during one day – i.e. approximately 13;10,34,58,30,33,30° – is obtained. When the solar mean motion during one day is subtracted from this, it gives the mean motion of the elongation between both during one day, i.e. 12;11,26,41,20,17,57°. When the number of complete revolutions in the anomaly contained in this period is multiplied by the degrees of one cycle and the result is divided by the number of days in this period, the [arc] of the epicycle the Moon traverses during one day – 13;3,53,56,29,38,30° approximately – is obtained.[56]

[9.] [On the lunar anomaly period in latitude (fol. 41v)]

As for the Moon's motion in latitude, the ancient [astronomers] determined it by looking for two lunar eclipses with the same magnitude, exactly the same anomaly, the same occultation both to the north and to the south and almost exactly the same node. By fulfilling these conditions, the Moon's nodal distance in the first of the two eclipses must necessarily be the same as its distance in the other [eclipse] from the same node and the same side. Therefore, this interval contains [an integer number of]

[55] "However, Hipparchus already proved, by calculations from observations made by the Chaldeans and in his time, that the above relationships were not accurate. For from observations he set out he shows that the smallest constant interval defining an ecliptic period in which the number of months and the amount of [lunar] motion is always the same, is 126007 days plus 1 equinoctial hour. In this interval he finds comprised 4267 months, 4573 complete returns in anomaly, and 4612 revolutions on the ecliptic less about 7° 1/2, which is the amount by which the sun's motion falls short of 345 revolutions (here too the revolution of sun and moon is taken with respect to the fixed stars). (Hence, dividing the above number of days by the 4267 months, he finds the mean length of the [synodic] month as approximately 29;31,50,8,20 days)". Cf. PtA, pp. 175-176 and *supra* n. 26.

[56] These values can be found in *Almagest* IV.3. Cf. PtA, p. 179.

Moon returns in latitude and of the epicycle centre in the inclined orbit. [Ptolemy] stated that Hipparchus found these [B. f. 42r] two eclipses by means of these conditions. He found that the period between the two [eclipses] contained 5458 months and 5923 returns in latitude.[57] When this interval is divided by the number of returns in latitude, the return period is obtained. When the number of degrees of a circumference [Es² f. 48r] – i.e. 360° – is divided by the previous value, the result is the distance the Moon traverses with its mean motion in latitude during one day – i.e. 13;13,45,39,40,17,19°. In this way, the ancient [astronomers] knew the lunar motions in longitude, anomaly and latitude.[58]

8. Edition

8.1 Ms. Escorial 910 39v - 42v

...[Es¹ f. 39v]

[1.] [في أنّ القمر يتحرّك على اختلاف في الطول وفي العرض]

ولمّا وجدوا القمر يتحرّك على اختلاف في الطول وفي العرض أعني أنّه ليس حركته في الجزء الواحد من فلك البروج حركة واحدة بعينها ولا عرضه فيه عرض واحد بعينه أبدا بل يتحرّك في الجزء الواحد أوسط حركاته وأعظمها وأصغرها وكذلك يكون عرضه أكثر ما يكون إلى الشمال وإلى الجنوب وقد يكون فيه لا عرض له استدلّ من ذلك على أن

[57] "Nevertheless, having already determined the period of return in anomaly, Hipparchus again adduces intervals containing [an integer number] of months which have at each end eclipses which were identical in every respect, both in size and in duration [of the various phases], and in which there was no difference due to the anomaly. Thus it is apparent that there is a return in latitude too. He shows that such a period is contained in 5458 months and 5923 returns in latitude". Cf. PtA, p. 176.

[58] Cf. *Almagest* IV.3. (PtA, p. 179).

عودته في فلك تدويره [Es[1] f. 40r] مخالفة لعودة مركز فلك التدوير في فلك البروج وأنّ العقدة من فلك المائل أيضا متنقلة على أجزاء فلك البروج وكذلك وجد الزمان الذي من حركته الصغرى إلى حركته الوسطى أعظم أبدا من الزمان الذي من حركته الوسطى إلى حركته العظمى فدلّهم ذلك على أنّ حركته في البعد الأبعد من فلك تدويره تكون إلى خلاف توالي البروج فنظر القدماء في جهة يصلون بها إلي معرفة زمان عودته في فلك تدويره وعودة مركز فلك تدويره في فلك البروج وتعمّدوا أن يكون ذلك بكسوفات القمر هربا ممّا يدخله اختلاف منظر القمر كما قلنا

[.2] [في أنّ لحركة القمر في الطول أربعة أحوال من أجل حركته في فلك تدويره]

ولما كان القمر تحرّك على فلك تدوير وجب أن تكون لحركته في فلك البروج أربعة أحوال فالحال الأوّل منها سرعة متزايدة وهي عند ذهابه من المجاز الأوسط إلى البعد الأقرب والثانية سرعة متناقصة وهي عند ذهابه من البعد الأقرب إلى البعد {المجاز}[59] الوسط الثاني والثالث بطوء متزايد وهي عند ذهابه من ذلك المجاز الوسط إلى البعد الأبعد والرابعة بطوء متناقص وهي عند ذهابه من البعد الأبعد إلى المجاز الوسط الأوّل فنعلم في كلّ وقت من الأوقات من قبل هذه الأحوال التي له في حركته في فلك البروج في أيّ قطعة هو من القطع الأربع من فلك تدويره أعني القطع التي يحدّها البعدان الأبعد والأقرب والمجازان الوسطان ونعلم على الجليل من

[59] In the margin in Ms. Es[1].

النظر في أيّ جزء هو من تلك القطعة

[.3] [في منهج القدماء للحصول على زمان القمر الدوريّ]

[.3.1] [وصف موجز]

فجعل القدماء عند طلبهم ذلك الزمان الدوريّ يطلبون كسوفين قمريين يكون سير القمر فيهما جميعا سيرا واحدا أعني من هذه المسيرات الأربعة المذكورة بحيث يظنّ به على الجليل من النظر أنّه قد عاد في فلك تدويره في الكسوف الثاني إلى موضعه منه في الأوّل

[.3.2] [شروط الكسوفات الأربع للحصول على زمان القمر الدوريّ]

ثمّ يختبر ذلك أعني هل عاد كما يظنّون

- بأن يطلبوا كسوفين أخرين سير القمر في كلّ واحد منهما سير واحد
- لاكنّه يخالف في السرعة أو الإبطاء لسيره في الكسوفين الأوّلين
- ويكون الزمان الذي بينهما مساويا للزمان الذي بين الأوّلين
- ويقطع القمر من فلك البروج في هذين الزمانين بَعد الأدوار التامّة قوسين متساويتين

فإذا وجدوا ذلك على هذه الصفة علموا بذلك أنّه قد عاد في فلك تدويره في الكسوف الثاني إلى موضعه منه في الأوّل وفي الكسوف الرابع إلى موضعه منه في الثالث ولنبيّن ذلك بمثال على هذه الصفة

[3.3.] [برهان هذه المنهج]

فليكن فلك التدوير دائرة ألف باء جيم ومركزها نقطة هاء ونقطة البعد الأبعد نقطة حاء والبعد الأقرب نقطة لام ومركز فلك البروج نقطة زاي والخطّ المارّ بالبعد الأبعد والأقرب ومركز فلك البروج خطّ زاي لام هاء حاء [Es[1] f. 40v] وليكن المجازان الأوسطان نقطتي طاء وكاف وليكن القمر في الكسوفين الأوّلين في إحدى قوسي كاف حاء وحاء طاء وفي الكسوفين الأخرين في إحدى قوسي كاف لام ولام طاء ولتكن جميع الشروط في الكسوفات الأربعة على ما ذكرنا

فأقول إنّ القمر قد عاد في فلك تدويره في الكسوف الثاني إلى موضعه منه في الأوّل وفي الرابع إلى موضعه منه في الثالث

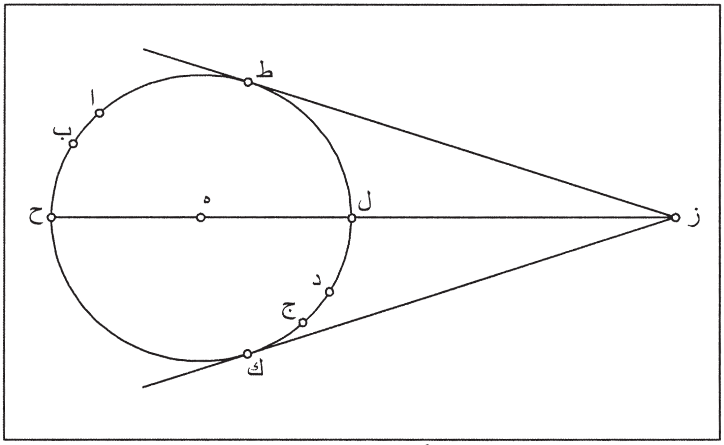

Figure 13: Fol. Es[1] 40r.

برهانه أنّه إن لم يكن ذلك كذلك فليكن موضعه في الأوّل نقطة ألف وفي الثاني نقطة باء وموضعه في الثالث نقطة جيم وفي الرابع نقطة دال فمن أجل أنّ المدّتين متساويتان يلزم أن تكون قوس ألف لام حاء باء مساوية لقوس جيم حاء طاء دال ويلزم أن يقطع القمر في هاتين المدّتين المتساويتين من فلك البروج بحركته الوسطى بَعد الأدوار التامّة قوسين متساويتين ومن أجل أن زاويتي الاختلاف التين توجبها قوسا ألف باء وجيم دال إحداهما توجب زيادة على الحركة المستوية والثانية توجب منها نقصانا تكون الحركة الحقيقية من فلك البروج في المدّتين مختلفتين فتكون القوس التي يقطعها في المدّة الأولى تزيد على الحركة الوسطى بزاوية الاختلاف التي توجبها قوس ألف باء والقوس التي يقطعها في المدّة الثانية تنقص عن الحركة الوسطى بزاوية الاختلاف التي توجبها قوس جيم دال فيكون الفضل بينهما بمجموع الزاويتين وقد فرضنا متساويتين فهذا خلف

لا يمكن فمن الخلف أن يكون القمر في فلك تدويره لم يعد في الكسوف الثاني إلى موضعه منه في الأوّل وفي الرابع إلى موضعه منه في الثالث

وكذلك يلزم من الخلف إن كان القمر قد زاد على العودة في فلك تدويره قوس ألف باء في المدّة الأولى وقوس جيم دال في المدّة الثانية أعني أن يكون في الكسوف الأوّل على نقطة باء وفي الثاني على نقطة ألف وفي الثالث على نقطة دال وفي الرابع على نقطة جيم فتكون لذلك القوس من فلك البروج التي تلحق زائدة على الأدوار التامّة في المدّة الأولى تنقص عن الحركة الوسطى بزاوية الاختلاف التي توجبها قوس ألف باء والقوس التي تلحق زائدة في المدّة الثانية تزيد على الحركة الوسطى بالزاوية التي توجبها قوس جيم دال فيكون التفاضل بين القوسين بمجموع الزاويتين

ومثل ذلك بعينه [Es[1] f. 41r] يلزم إن كان في المدّتين جميعا في نصف واحد أعني أحد نصفي حاء ألف لام وحاء كاف لام وذلك ما أردنا بيانه

وكذلك يلزم من أجل أن كلّ واحدة[60] من المدّتين يحيط بها كسوفان أن تكون كلّ واحدة منهما تحيط بشهور تامّة عدّتها واحدة وكذلك تحيط أيضا بعودات تامّة لمركز فلك التدوير في فلك البروج متساوية وزائد[ة] إلى ذلك القسي المتساوية التي تزيد على الأدوار التامّة

[3.4.] [مقادير أخرى تُخرَج من زمان القمر الدوري]

فلمّا وجدوا ذلك على هذه الصفة قسموا زمان إحدى المدّتين على عدد عودات الاختلاف خرج زمان العودة الواحدة فإذا قسم على عدد أيامها

[60] "وواحدة"

أجزاء دائرة واحدة وهي ثلاث مائة وستّون جزءًا خرج ما يقطعه القمر من فلك تدويره في اليوم الواحد وكذلك قسموا أيام تلك المدّة على عدد ما فيها من الشهور خرج زمان الشهر الوسط فإذا ضوعف ذلك الزمان لحركة الشمس الوسطى في اليوم الواحد كان المجتمع من ذلك ما تقطعه الشمس بحركتها الوسطى في زمان الشهر الوسط حتى يلحقها القمر فإذا أضيف إلى ذلك أجزاء دائرة واحدة وهي ثلاث مائة وستّون جزءًا كان المجتمع ما يقطعه القمر في الطول في زمان الشهر الوسط فإذا قسم ذلك على عدد أيام الشهر الوسط خرج ما يقطعه في الطول في اليوم الواحد

فهذا هو الطريق الذي سلكه القدماء في استخراجهم الزمان الدوري الذي يصلون منه إلى مقادير حركاته في الطول والاختلاف

[.4] [في الأوضاع القمر التي يجب التجنّب عنها]

[.4.1] [وصف بطلميوس في هذه الأوضاع]

وأمّا ما شنعه عليهم بطلميوس في استخراجهم هذا الزمان الدوري وما ذكر أنّهم يحتاجون فيه من الاستقصاء والتحرّز من أن يكون القمر من فلك تدويره في أوضاع يمكن أن يقطع فيها من فلك البروج في الأزمنة المتساوية قسيا متساوية ولا يعود في اختلافه وذلك يتهيأ

- بأن يكون القمر في الكسوف الأوّل يبتدئ من البعد الأبعد وينتهي في الكسوف الثاني إلى البعد الأقرب ويكون في

الثالث يبتدئ من البعد اللأقرب وينتهي في الرابع إلى البعد الأبعد أو

- أن يقطع في كلّ واحدة من المدّتين من فلك التدوير قوسا واحدة بعينها أو
- أن يقطع قوسين متساويتين ومتساويتي البعد من البعد الأبعد أو الأقرب أعني أن يكون موضعاه في الكسوف الأوّل والرابع بُعدهما عن جنبي بعد واحد أعني الأبعد أو الأقرب بعدا سواء

فيلزم في كلّ واحد من هذه الأوضاع الثلاث أن يكون القمر يقطع من فلك البروج في المدّتين المتساويتين قوسين متساويتين ولا يعود في فلك تدويره

[4.2.] [ردّ جابر بن أفلح]

فليس يحتاجون إلى التحرّز والاستقصاء الذي ذكره لأنّه ليس يمكن أن يكون القمر في طلبهم للزمان الدوري على واحد من هذه الأوضاع لأنّ أوّل ما ينظرون [Es[1] f. 41v] إليه في طلب هذا الزمان الدوري أن يكون سير القمر في الكسوف الثاني مثل سيره في الأوّل على الجليل من النظر حتّى يظنّ أنّه قد عاد في فلك تدويره وكذلك في الكسوف الرابع أن يكون سيره فيه مثل سيره في الثالث حتّى يظنّ به أيضا أنّه قد عاد في فلك تدويره

- فكيف يكون سيره في طرفي المدّة الواحدة سيرا واحدا على

ما شرطوه ويكون يبتدئ في المدّة الأولى من البعد الأبعد وينتهي في أخرها إلى البعد الأقرب ويبتدئ في الثانية من البعد الأقرب وينتهي في أخرها إلى البعد الأبعد فيكون إذاً سيره في أوّل المدّة في غاية الخلاف لسيره في أخرها وذلك خلاف ما شرطوه فيهما

- وكيف يكون يقطع من فلك التدوير قوسا واحدة بعينها فيلزم لذلك أن يكون سيره في الكسوف الأوّل هو بعينه في الثالث وسيره في الثاني هو بعينه في الرابع وقد شرطوا خلاف ذلك أعني سيره في الكسوف الأوّل والثاني مخالف لسيره في الثالث والرابع
- وكيف يقطع قوسين متساويتين ومتساويتي البعد عن البعد الأبعد أو الأقرب ويلزم عن هذا أن يكون سيره في الكسوف الأوّل مثل سيره في الرابع وسيره في الثاني مثل سيره في الثالث

وإن كانوا لم يصرحوا هذه الشروط فإنّه يخرج من عملهم هذا إنّهم شرطوها ضرورة

[.5] [في الأوضاع الشمس التي يجب أن تكون فيها]

[.5.1] [وصف بطلميوس في هذه الأوضاع]

وكذلك ما ألزمهم أيضا من نظرهم إلى مواضع الشمس في الكسوفات

المطلوبة أن تكون[61] في كلّ واحد منها في الأوضاع التي تجتنب للقمر أعني

- أن تكون في الكسوف الأوّل قد ابتدأت من البعد الأبعد من الخارج المركز وانتهت في الكسوف الثاني إلى البعد الأقرب منه وتكون في الكسوف الثالث قد ابتدأت من البعد الأقرب وانتهت في الرابع إلى البعد الأبعد منه أو
- تكون تقطع قوسا واحدة بعينها من فلكها الخارج المركز أو
- تقطع منه قوسين متساويتين ومتساويتي البعد عن البعد الأبعد أو الأقرب أو
- تكون تقطع في كلّ واحدة من المدّتين دورات تامّة من فلكها الخارج المركز

فتكون[62] أيضا في فلك البروج كذلك

[5.2.] [ردّ جابر بن أفلح]

فليس يحتاجون إلى البحث عن شيء من ذلك لأنّهم

- إنّما يطلبون مدّتين متساويتين يحيط بكلّ واحدة منهما كسوفان

[61] يكون

[62] يكون

- ويكون الشمس والقمر قد قطعا في كلّ واحدة من المدّتين من فلك البروج قوسين متساويتين
- ويكون القمر في الكسوفات على ما وصفنا

فإذا وجدوا ذلك لزم عنه ضرورة أن تكون الشمس على أحد تلك الأوضاع الأربعة المذكورة آنفا

[.6] [في اختيار الكسوفات الأفضل لكي يكون لسير القمر فيها غاية الاختلاف]

[.6.1] [اقتراح بطلميوس]

وما ذكروه أيضا من الاختيار في طلب تينك المدّتين من أن يكون ابتداء القمر في الكسوف الأوّل وفي الكسوف الثالث من سيرين مختلفين غاية الاختلاف أعني سيرين مختلفين في السرعة والإبطاء وفي التزيّد والتنقّص

[.6.2] [اقتراح جابر بن أفلح]

ليس ذلك بضروري ولا بهم حاجة إلى هذا الاشتراط لأنّه يعسر وجود كسوفات تكون هذه حالها بل يجزيهم في ذلك أن يكون سيره في الكسوف الأوّل [Es[1] f. 42r] والثاني مخالفا في السرعة والإبطاء لسيره في الثالث والرابع بحيث يكون أحد سيره أعظم من سيره الوسط والثاني أصغر من سيره الوسط حتّى أنّه إذا لم يعد القمر في فلك تدويره كان التفاضل في القسي من فلك البروج التي يلحق زائدة بمجموع زاويتي الاختلاف فيجب

لذلك أن تكون مبادئ القمر من مواضع كثيرة الاختلاف ليكون مجموع الاختلافين له قد ظاهر محسوس وهذا يكون بأن تكون مبادئ القمر بعيدة من المجازين الأوسطين على خلاف ما ذكر هو

[6.3.] [في أن يكون مبدأ القمر في المدّة الأولى مخالفا في القوّة لمبدئه في المدّة الثانية]

[6.3.1.] [اقتراح بطلميوس]

وأمّا قوله أن يكون هذا القمر في المدّة الأولى مخالفا في القوة لمبدئه في الثانية أعني أن يبتدئ في الأوّل من المجاز الأوسط ويبتدئ في الثانية من المجاز الأوسط الأخر

[6.3.2.] [نقد جابر بن أفلح على اقتراح بطلميوس]

فذلك غاية الخطاء لأنّه ليس يتحقّق في وقت من الأوقات موضع القمر من فلك تدويره ولو تحقّق ذلك لم يحتج إلى الزمان الدوري لأنّا كنّا نرصد القمر من مفارقته لجزء ما من فلك تدويره حتّى يعود إليه لاكنّا لا نعلم حقيقة موضعه لا سيّما في المجازين الأوسطين لعلّة تفاضل زوايا الاختلاف فيهما فيكون سيره بطيء الاختلاف في التزيّد والتنقّص حتّى أنّه يمكن أن يكون بعد القمر عن جنبي المجاز الأوسط بثلاثة أجزاء وأكثر في كلّ واحد من الكسوفين ونحن نظنّ به أنّه في المجاز اللأوسط

[6.3.3.] [في أن يتناقض بطلميوس على رأي جابر بن أفلح]

وكذلك في كلّ واحد من الكسوفين الأخرين فيكون قد قطع في كلّ واحدة

من المدّتين من فلك تدويره قوسين متساويتين ومتساويتي البعد عن البعد الأبعد والأقرب وهو أحد الأوضاع الثلاثة التي حذّر عنها فيلزم من ذلك أن يكون القمر قد قطع في المدّتين المتساويتين من فلك البروج قوسين متساويتين ولم يعد في اختلافه فقد حضّهم وهو لا يشعر على ما نهاهم عنه وحذّرهم من أن يقعوا فيه

[.7] [في رأي جابر بن أفلح على تمرين بطلميوس في صناعة الهندسة]

والذي أتحقّق من أمر هذا الرجل أنّه لم يكن له تمرين في صناعة الهندسة ولذلك سقط في مثل هذا وفي غيره ممّا سننبّه عليه في موضعه إن شاء الله عزّ وجلّ

[.8] [في زمان القمر الدوري في العرض]

وأمّا حركته في العرض فإنّ القدماء أدركوها بأن طلبوا مدّة بين كسوفين قمرين يكون مقدار المنكسف من قطر القمر فيهما واحدا ويكون القمر فيهما في نقطة واحدة بعينها من فلك التدوير ويكون المنكسف في جهة واحدة من الشمال أو الجنوب وعند عقدة واحدة بعينها فإنّ باجتماع هذه الشروط يلزم ضرورة أن يكون بعد القمر في أوّل كسوفيه من العقدة مساويا لبعده في أخرهما من تلك العقدة بعينها في تلك الجهة نفسها فتكون تلك المدّة محيطة بعودات القمر في العرض وبمركز فلك تدويره فإذا قسمت تلك المدّة على عدد عودات لعرض خرجت حركة القمر الوسطى في العرض فبهذا

الطريق أدرك القدماء حركات القمر في الطول [Es[1] f. 42v] والاختلاف والعرض

[.9] [مقادير حركة القمر في الطول والاختلاف والعرض في اليوم الواحد]

فأمّا حركته في الطول في اليوم الواحد فثلاثة عشر جزءًا وعشر دقائق وأربع وثلاثون ثانية وثمان وخمسون ثالثة وثلاث وثلاثون رابعة {وثلاثون خامسة وثلاثون سادسة}[63] وفي الاختلاف ثلاثة عشر جزءًا وثلاث دقائق وثلاث وخمسون ثانية وستّة و خمسون ثالثة وتسع وعشرون رابعة وفي العرض ثلاثة عشر جزءًا وثلاث عشرة دقيقة وخمس وأربعون ثانية وتسع وثلاثون ثالثة وثماني وأربعون رابعة

8.2 Berlin version

... [Es[2] f. 43v, B. f. 38v]

[.1] [في أنّ القمر يتحرّك على اختلاف في الطول وفي العرض]

ولمّا وجد القمر يتحرّك على اختلاف في الطول والعرض أعني أنّه ليس حركته في الجزء الواحد من فلك البروج حركة واحدة بعينها ولا عرضه فيه عرض واحد بعينه أبدا بل يتحرّك في الجزء الواحد أوسط حركاته وأعظمها وأصغرها وكذلك يكون عرضه عنه أكثر ما يكون إلى الشمال وإلى الجنوب وقد يكون فيه لا عرض له استدلّ من ذلك على أن عودته

[63] In the margin in Ms. Es[2].

في {اختلافه}[64] مخالفة لعودته[65] في فلك البروج وأنّ {العقدة}[66] من فلكه المائل أيضا منتقلة على أجزاء فلك البروج فنظر القدماء في جهة يصلون بها إلى معرفة زمان عودته في اختلافه وعودته في فلك البروج وتعمّدوا أن يكون ذلك {بالكسوفات القمرية}[67] هربا ممّا يدخله اختلاف منظر القمر كما قلنا

[2.] [في أن يجب التقسيم فلك تدوير بأربع قطع من أجل حركته في الطول]

ولما كان للقمر حركات مختلفة أعني حركة سريعة وحركة بطيئة وحركة متوسطة {يجب}[68] أن يكون له في فلكه الخاصّ به أربع نقط إحداهنّ يكون فيها أسرع ما يكون والثانية مقاطرة لها يكون فيها أبطأ ما يكون وتكون هاتان النقطتان هما البعد الأبعد والبعد الأقرب من فلكه الخاصّ و{نقطتان}[69] تكون حركته فيهما متوسطة بين {هاتين}[70] الحركتين وهما المجازان الأوسطان من هذا الفلك الخاصّ فتكون هذه النقط الأربعة تقسم هذا الفلك بأربع قطع إحداهنّ هي التي تكون حركته فيها من أسرع حركته إلى حركته المتوسّطة الأولى وهي حركة سريعة متناقصة والقطعة الثانية

[64] Interlineal correction of "فلك التدوير" by "اختلافه" in Ms. Es^2.

[65] Ms. Es^2 "مركز فلك التدوير".

[66] Ms. B. "العودة".

[67] Ms. Es^2 "بكسوفات القمر".

[68] Ms. Es^2 "وجب".

[69] In the margin in Ms. Es^2; Ms. B. "نقطتين".

[70] Ms. B. "هداتين".

حركته فيها حركة وسطى متناقصة أيضا والقطعة الثالثة حركته فيها حركة بطيئة متزيّدة والقطعة الرابعة حركته فيها حركة متوسّطة متزيّدة أيضا

[3.] [في منهج القدماء للحصول على زمان القمر الدوريّ]

[3.1.] [وصف موجز]

فيجب لذلك أن نعلم على الجليل من النظر في كلّ وقت محدود أيّ قطعة هي من هذه القطع الأربع فجعل القدماء يطلبون كسوفين قمرين تكون حركة القمر في كلّ واحد {منهما}[71] حركة واحدة من هذه {الحركات}[72] الأربع فيعطي بذلك على الجليل من النظر أنّه قد [Es² f. 44r] عاد في الكسوف الثاني من فلكه الخاصّ إلى موضعه منه في الكسوف الاوّل [B. f. 39r] وأنّ تلك المدّة التي بين الكسوفين محيطة بعودات تامّة للقمر في فلكه الخاصّ به

[3.2.] [شروط الكسوفات الأربع للحصول على زمان القمر الدوريّ على رأي جابر بن أفلح]

ولمّا أرادوا امتحان ذلك وتحقّقه طلبوا كسوفين أخرين

- تكون حركة القمر في كل واحد {منهما}[73] حركة واحدة

[71] Ms. B. "منها".

[72] Ms. Es² "الحركت".

[73] Ms. B. "منها".

- مخالفة لحركته في الكسوفين الأوّلين
- وتكون المدّتان اللتان بين هذه الكسوفات الأربعة متساويتين
- ويقطع القمر في كلّ واحدة من هاتين المدّتين من فلك البروج قسيا متساوية إمّا دورات تامّة فقط وإمّا دورات تامّة وقسي زائدة على الدورات متساوية

وإذا وجد ذلك على هذه الشروط الموصوفة علموا أنّ القمر قد عاد في الكسوفين الأوّلين إلى نقطة واحدة من فلكه الخاصّ به وأنّه قد عاد أيضا في الكسوفين الأخرين إلى نقطة ثانية منه أيضا فيكون كلّ واحدة من المدّتين المتساويتين تحيط بعودات تامّة للقمر في فلكه الخاصّ به

[3.3.] [في أنّ بطلميوس لم يفصح بهذه الشروط]

فذكر بطلميوس عن القدماء هذا الطريق ولم يفصح بهذه الشروط التي ذكرناها في هذه الحركات للقمر في الكسوفات المطلوبة وإن كان لم يفصح بذلك فنفس[74] المعنى يعطي أنّ هذه الشروط مطلوبة في هذه الكسوفات وإن لم يكن على هذه الصفة لم يبنى عن تمام العودات وأمّا من أين يتبيّن أنّه إذا كانت هذه الكسوفات الأربعة على هذه الشروط {بأن}[75] كلّ واحدة من

[74] Ms. Es^2 "هذا".

[75] Ms. Es^2 "فإن".

المدّتين التين بينهما تحيط بعودات تامّة للقمر في فلكه الخاصّ به {بأنّه}[76] متساوية العدّة فذلك يتبيّن على ما أصف {بحول الله وقوّته}[77]

[.3.4] [برهان هذه الشروط]

ليكن القمر يتحرّك على فلك {تدوير}[78] وهو دائرة أ ب ج د حول مركز ه ومركز فلك البروج نقطة ز والخطّ المارّ بالبعد الأبعد والأقرب وبمركز {فلك}[79] البروج خطّ أ ه ج ز والبعد الأبعد نقطة أ والأقرب نقطة ج ولنخرج من نقطة ز خطّين يماسّان دائرة أ ب على نقطتي ب د وهما خطّا زب و زد فتكون نقطتا ب د المجازين الأوسطتين وليكن القمر في الكسوف الأوّل على نقطة ح وفي الكسوف الثالث على نقطة ص {وسيره}[80] في هاتين النقطتين على ما ذكرنا أعني [Es^2 f. 44v] أنّهما مختلفان وسيره في نقطة ح هو سيره على الجليل من النظر في الكسوف الثاني وسيره في الثالث هو سيره {أيضا}[81] في الرابع والمدّتان متساويتان والقطع فيهما من فلك البروج {متساوية}[82]

[76] Ms. Es^2 "تامّة".

[77] Not in Ms. Es^2.

[78] Ms. Es^2 "تدويره".

[79] In the margin in Ms. B.

[80] Ms. Es^2 "وسيراه".

[81] Not in Ms. B.

[82] Ms. Es^2 "مستاو".

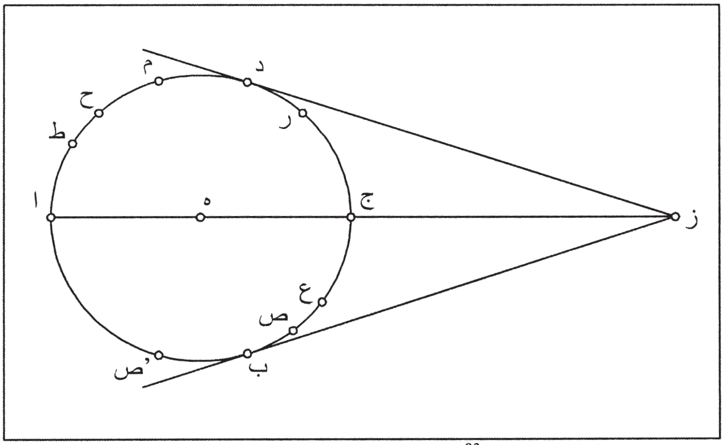

Figure 14: Fol. B. 39v.[83]

فأقول إنّه قد عاد في الكسوف الثاني إلى نقطة ح بعينها وفي الرابع إلى نقطة ص بعينها

برهانه

أنّه إن لم يعد في الثاني إلى نقطة ح فليكن فيه على نقطة ط وإن لم يعد في الرابع إلى نقطة ص فليكن فيه على نقطة ع فمن أجل أنّ المدّتين متساويتين أعني التي بين الأوّل والثاني وبين الثالث والرابع {فيجب}[84] لذلك أن {تكون}[85] قوسا ح ط وص ع متساويتين ومن أجل أنّ {مسير}[86] القمر في نقطتي ح وط مخالف {لمسيره}[87] في نقطتي ص ع أعني أنّ أحدهما سير بطيء والأخر سريع على ما اشترط تكون إحدى قوسي ح ط

[83] Cf. *supra* n. 20.

[84] Ms. Es^2 "يجب".

[85] Ms. Es^2 "تكونا".

[86] Ms. Es^2 "سير".

[87] Ms. Es^2 "لسيره".

وص ع [B. f. 39v] توجب في حركة القمر الوسطى زيادة والثانية توجب فيها نقصانا ولما كانت المدّتان متساويتين وجب أن تكون الحركة الوسطى {فيهما}[88] {متساوية}[89] فيجب لذلك أن تكون حركة القمر الحقيقية في المدّة الأولى مخالفة لحركته في الثانية {لمجموع}[90] الخلافين معًا أعني أنّ الزاويتين اللتين عند مركز فلك البروج التين {توتّرانهما}[91] قوسا ح ط وص ع فيكون القمر قد قطع في تينك المدّتين المتساويتين من فلك البروج قوسين مختلفين والفضل بينهما بمجموع الاختلافين اللذين توجبانهما قوسا ح ط ص ع وقد فرضنا أنّ القمر قد قطع من فلك البروج في تينك المدّتين المتساويتين قوسين متساويتين {فهذا}[92] خلف لا يمكن[93] أن يكون القمر في الكسوف الثاني على غير نقطة ح وكذلك الكسوف الرابع على غير نقطة ص فهو {إذًا}[94] قد عاد في الكسوف الثاني إلى موضعه في الأوّل وفي الرابع إلى موضعه في الثالث وكلّ واحدة من المدّتين المتساويتين محيطة بعودات تامّة للقمر في فلك تدويره وذلك ما أردنا {أن نبيّن}[95]

[88] Ms. Es^2 "فيها".

[89] Corrected. Ms. B., Es^2 "متساويتين".

[90] Ms. Es^2 "بمجموع".

[91] Ms. Es^2 "توترهما".

[92] Ms. B. "هذا".

[93] Ms. Es^2 "فمن الخلف". Crossed out in Ms. B.

[94] Ms. Es^2 "إذن".

[95] Not in Ms. Es^2.

[.4] [في اختيار الكسوفات الأفضل لكي يكون لسير القمر فيها غاية الاختلاف]

[.4.1] [اقتراح جابر بن أفلح]

[Es² f. 45r] ولما كان {الخلاف}[96] بين القوسين التين قطعهما القمر من فلك البروج في المدّتين المتساويتين إن لم يعد القمر إلى موضعه الأوّل {فهو}[97] مجموع الاختلافين الذين توجبانهما قوسا ح ط وص ع وجب أن تكون الكسوفات المختارة في طلب هذا الزمان الدوري هي الكسوفات التي مواضع القمر[98] توجب اختلافا كثيرا بين الحركة الوسطى والحقيقية فهذه المواضع هي نقطتا البعد الأبعد والأقرب وما قرب منهما وكلّما بعدت مواضع القمر في الكسوفات من نقطتي البعد الأبعد والأقرب كانت أبعد من الاختيار فينبغي أن يتجنّب {ضرورة}[99] أن {يكون}[100] موضعا القمر في الكسوف الأوّل وفي الثالث في المجازين الأوسطين أو {في}[101] قريب منهما على خلاف ما ذكر بطلميوس

[.4.2] [نقد جابر بن أفلح على اقتراح بطلميوس]

فإنّه إن كان القمر في الكسوف الأوّل على نقطة م وهي قريبة من نقطة د

[96] Ms. Es² "اختلاف".

[97] Ms. Es² "هو".

[98] Interlineal addition of "لا" in Ms. B.

[99] Ms. B. "ضرة".

[100] In the margin in Ms. Es².

[101] Not in Ms. B.

التي هي المجاز الأوسط فمن أجل أنّ حركة القمر فيما يقرب من نقطة د[102] حركة واحدة لا تتغيّر كثير {[الــ]ــتغيّر يمكن}[103] أن يكون في الكسوف {الثاني على نقطة ر ونحن نظنّ أنّه في الكسوفين على نقطة واحدة ويمكن أيضا أن يكون في الكسوف}[104] الثالث على نقطة ص التي هي بعدها من نقطة ب كبعد نقطة ر من نقطة د {ويكون}[105] في الكسوف الرابع على نقطة ع التي {هي}[106] بعدها من نقطة ب كبعد نقطة م من نقطة د ونحن نظنّ أنّه في الكسوفين في [B. f. 40r] نقطة واحدة

فيجب لذلك أن يكون الخلاف الذي توجبه قوس ر م متساويا للخلاف الذي توجبه قوس ص ع ويكونا من جنس واحد أعني أنّهما توجبان معا في الحركة الحقيقية زيادة أو نقصانا

فيلزم عن ذلك أن يكون القمر قد قطع في فلك البروج بَعد الأدوار التامّة في المدّتين المتساويتين قوسين متساويتين ولم يعد في فلك تدويره وذلك بعينه يلزم أن يكون موضع القمر في الكسوف الأوّل نقطة المجاز الوسط بعينها وفي الثالث نقطة المجاز الوسط [Es^2 f. 45v] الأخر وفي كلّ {واحد}[107] من الثاني والرابع إحدى نقط ر ص ع م بحيث يقطع في فلك تدويره في المدّتين المتساويتين قوسين متساويتين متساويتي البعد من البعد

[102] Ms. B “إلى”.

[103] Ms. Es^2 “تغيّر ممكن”.

[104] In the margin in Ms. B.

[105] In the margin in Ms. B.

[106] Not in Ms. B.

[107] Ms. Es^2 “واحدة”.

الأبعد أو الأقرب وهذا هو أحد الأوضاع {الثلاث}[108] التي حذّر منها بطلميوس ونهى عنها ونجده قد حوّل هذا الموضع من المواضع المختارة للقمر في هذه الأرصاد

[4.3.] [اقتراح بطلميوس في مسيرات عظيمة الاختلاف إمّا في المقدار وإمّا في القوّة في نصّ المجسطي ونقد جابر بن أفلح]

وذلك أنّه يقول في النوع الثاني من المقالة الرابعة ما هذا نصّه

فليس ينبغي {إذاً}[109] أن يكون في المدد التي تستعمل شيء من هذه الأعراض إن كنّا نقرّر فيها أنّها تكون في الحقيقة مشتملة على زمان عودة الاختلاف بل إنّما ينبغي أن نتخيّر منها ما كانت حاله ضدّ هذه الحال[110] إذا أعني المدد التي بها خاصّة يمكن أن تظهر الاختلاف إذا لم تحط بعودات تامّة من عودات الاختلاف أعني ألاّ نقتصر على أن تكون مبادئها من مسيرات مختلفة فقط بل من مسيرات عظيمة الاختلاف إمّا في المقدار وإمّا في القوّة

أمّا في المقدار فمثل أن يبتدئ في إحدى المدّتين من أقلّ السير ولا ينتهي إلى أعظم السير ويبتدئ في المدّة الأخرى من أعظم السير ولا ينتهي إلى أقلّ السير فإنّ {هذا الوجه}[111] يكون فضل الزيادة في الطول غاية الفضل

108 Ms. Es^2 "الثلاثة".

109 Ms. Es^2 "إذن".

110 Annotation in Ms. B.: "أعني أن يكون أحدهما بطيئا والأخر سريعا".

111 Ms. Es^2 "هذه الوجوه".

وذلك أنّه إن لم يكمل الاختلاف أدوار تامّة وخاصّة متى كان يلحق في اختلاف واحد ربع واحد أو ثلاثة أرباع كان الفضل الذي من قبل الاختلاف {حينئذ}[112] فضلين[113] بهما كانت المدّتان غير متساويتين

وأمّا في القوّة فمثل أن يبتدئ[114] في كلّ واحدة من المدّتين من المسير الوسط إلاّ أن الابتداء لا يكون من وسط بعينه بل يكون في إحداهما في المسير بحيث الزيادة ويكون في الأخر من المسير بحيث النقصان فإنّ على هذا الوجه أيضا خاصّة تكون فضلات الطول تخالف بعضها بعضا غاية الخلاف من غير أن يكون الخلاف قد عاد إذ كان متى لحق في اختلاف واحد ربع أيضا أو ثلاثة أرباع كان الفضل الذي من قبل الاختلاف فضلين ومتى كان الذي يلحقه نصف دائرة كان الفضل أربعة ومن أجل ذلك وجدنا إبرخس أيضا قد يظنّ أنّه قد احتاط {بغاية}[115] ما تمكن من [Es^2 f. 46r] الاحتياط في اختيار المدد المستعملة في هذا الفحص فاستعمل[116] الفضل في القمر [B. f. 40v] على أن مبدأ إحدى المدّتين من أكثر المسير ولا ينتهي إلى أقلّ المسير ومبدأ المدّة الأخرى من أقلّ المسير وانتهاؤها ليس عند أكثر المسير

فهذا هو نصّ قول بطلميوس

112 Interlineal addition in Ms. B.

113 Annotation in Ms. B.: "أعني ضعف الاختلاف".

114 Annotation in Ms. B.: "أعني القمر".

115 Ms. Es^2 "غاية".

116 Annotation in Ms. B.: "أعني أن لا يستعمل نصف دائرة بل إمّا أقلّ أو أكثر كما قال ربع أو ثلاثة أرباع".

فقد جعل من الكسوفات المختارة في طلب هذا الزمان الدوري في الكسوفات التي يكون القمر {فيها}[117] في {إحدى}[118] المدّتين في أحد المجازين الأوسطين ويكون في المدّة الثانية في المجاز الأوسط الأخر وقد بينّا آنفا أنّ ذلك بعيد جدّا في الاختيار وإنّه أحد الأوضاع الثلاثة التي حذّر منها في طلب {هذه}[119] المدّة ونهى عنها فقد اختار وهو لا يشعر ما حذّر منه ونهى عنه

[.5] [في ردّ جابر بن أفلح على صعوبات هذا المنهج]

وأمّا قوله فهذا هو الطريق الذي {سلكه}[120] من كان قبلنا في استخراج هذه الأشياء وقد يمكنك أن تعلم أنّ هذا الطريق ليس سهل المرام ولا قريب المأخذ بل يحتاج فيه إلى تأمّل شديد وتحصيل مستقصى ممّا أنا واصفه
وما يتّصل بهذا فكلام غير محصل وذلك أنّه إنّما كان ينبغي أن يقول مثل هذا القول لو كان هو قد أتى بطريق أخر أسهل {من هذا}[121] وليس يحتاج فيه إلى ما يحتاج في هذا الطريق من التحرّز ويكون مع هذا غير مفتقر إلى الطريق التي أتى بها القدماء ولم يمكنه شيء من ذلك بل إنّما أتى بطريق صحيح به وقلّل الفضل الداخل من قبل الأرصاد التي استعملها

[117] Not in Ms. B.

[118] Ms. B. "أحد".

[119] Interlineal addition in Ms. B.

[120] Ms. B. "سلك".

[121] Ms. Es^2 "منها".

القدماء في استخراج هذا الزمان الدوري ولم يمكنه ذلك إلاّ بأن استعمل فيه مقادير {الحركات}[122] التي استخرجها القدماء بهذا الزمان الدوري فكلّ ما أتى به إنّما هو مبني على هذا الزمان الدوري الذي استخرجه القدماء بهذا الطريق

[6.] [في الأوضاع الشمس التي يجب أن تكون فيها]

[6.1.] [وصف بطلميوس في هذه الأوضاع]

وأمّا قوله
فلنترك أوّلا أنّ أزمان المدد توجد مساوية على الصحّة [و]أقول أيضا أوّلا إنّه ليس ينبغي بذلك ما لم يكن الفضل الذي من قبل اختلاف الشمس أيضا إمّا ألاّ يكون أصلا في كلّ واحدة من المدّتين وإمّا أن يكون واحدا بعينه

[6.2.] [ردّ جابر بن أفلح]

وما يتّصل به فكلام خلف وذلك أنّ الذي يشترط في المدد المطلوبة

- وهو أن تكون متساوية
- ويكون القمر يقطع فيها من فلك البروج قسيا متساوية

122 Ms. Es^2 "الحركت".

وإذا كان ذلك {كذلك}[123] وكانت الشمس في وسط زمان كلّ كسوف [Es² f. 46v] مقاطرة على الحقيقة للقمر يجب أن تكون الشمس {إنّما}[124] قد قطعت في تلك المدّتين المتساويتين من فلك البروج قوسين متساويتين وذلك لا يكون إلاّ بأن لا يكون فضل من {قبل}[125] الاختلاف أصلا وإمّا بأن يكون الفضل واحد بعينه وذلك أيضا لا يكون إلاّ بأن يكون على أحد {الأربع}[126] الأوضاع التي ذكر فقد جعل ما يلزم عن المفروض مطلوبا وهو أمر بيّن بنفسه

[7.] [في أوضاع القمر التي يجب التجنّب عنها]

[7.1.] [وصف بطلميوس في هذه الأوضاع]

{وكذلك}[127] ما ذكر بعد هذا من أنّه يلزم أن يتحرّز من مواضع القمر في فلك تدويره في الكسوفات المستعملة في طلب هذه المدد [B. f. 41r] وهي المواضع التي يمكن فيها أن يقطع من فلك البروج في الأزمنة المتساوية قسيا متساوية ولا يعود في اختلافه وذلك

[123] Ms. B. "لذلك".

[124] Not in Ms. B.

[125] In the margin in Ms. B.

[126] Ms. Es² "الأربعة".

[127] Ms. B. "لذلك".

- بأن يكون القمر في الكسوف الأوّل يبتدئ من البعد الأبعد من فلك تدويره وينتهي في الكسوف الثاني إلى البعد الأقرب ويكون في الثالث يبتدئ من البعد الأقرب و ينتهي في الرابع إلى البعد الأبعد أو
- أن يقطع في كلّ واحدة من المدّتين من فلك تدويره قوسا واحدة بعينها أو
- أن يكون يقطع منه قوسين متساويتي البعد من البعد الأبعد أو الأقرب أعني أن يكون موضعاه في الكسوف الأوّل والرابع بعدهما عن جنبي الخطّ المارّ بالبعد الأبعد والأقرب بعدا متساويا وكذلك أيضا موضعاه في الكسوف الثاني والثالث

فيلزم في كلّ واحد من هذه الأوضاع الثلاثة أن يكون القمر يقطع من فلك البروج {في}[128] المدّتين المتساويتين قوسين متساويتين ولا يعود في فلك تدويره

[.7.2] [ردّ جابر بن أفلح]

فليس يحتاج إلى التحرّز والاستقصاء لأنّه ليس يمكن أن يكون القمر في {طلبهم}[129] هذه {المدّة}[130] على واحد من هذه الأوضاع لأنّ أوّل ما ينظر

[128] In the margin in Ms. Es[2].

[129] Ms. B. "طهم".

إليه من أمر القمر أن يكون {مسيره}[131] في الكسوف الأوّل والثاني أعني {اللذان}[132] يحيطان بمدّة واحدة {مسيرا}[133] واحدا على الجليل من النظر حتّى يظنّ به أنّه قد عاد في فلك تدويره في الكسوف الثاني إلى موضعه منه في الأوّل كي تكون المدّة محيطة بعودات تامّة للقمر في فلك تدويره وكذلك أيضا يكون {مسيره}[134] في الكسوف الثالث والرابع {مسيرا}[135] واحدا أيضا بعينه على الجليل من النظر حتّى يظنّ به أيضا أنّه قد عاد في فلك تدويره فهذا الشرط يبطّل أن يكون القمر [Es[2] f. 47r] في الكسوف الأوّل والرابع في البعد الأبعد ويكون في الثاني والثالث في القرب الأقرب وأمّا الموضعان الباقيان أعني الذي يقطع القمر في أحدهما من فلك تدويره في المدّتين قوسا واحدة بعينها والموضع الذي يقطع منه في المدّتين قوسين متساويتي البعد عن البعد الأبعد والأقرب فإنّه يبطّلهما ما اشترط أيضا وهو أن يكون مسير القمر في الكسوفين الأوّلين مخالفا لمسيره في الكسوفين الأخرين لأنّ في كلّ واحد من هذين الموضعين يلزم أن يكون سير القمر في الكسوفين الأوّلين هو سيره في الكسوفين الأخرين وهذا خلاف ما شرط فإذا اشترطت في المدد المطلوبة هذه الشروط في مسير القمر لم يحتج[136]

[130] Ms. Es[2] "المدد".

[131] Ms. Es[2] "سيره".

[132] Ms. Es[2] "اللذين".

[133] Ms. Es[2] "سيرا".

[134] Ms. Es[2] "سيره".

[135] Ms. Es[2] "سيرا".

[136] Crossed out in Ms. B.

إلى شيء ممّا ذكر من التحرّز والاستقصاء لا في القمر ولا في الشمس فهذه هي الطريق التي سلكها القدماء في استخراج هذا الزمان الدوري

[.8] [مقادير أخرى تُخرَج من زمان القمر الدوري]

وذكر بطلميوس عن إبرخس أنّه وجد مقدار هذا الزمان مائة ألف يوم وستّة وعشرون ألف يوم وسبعة أيام وساعة واحدة من ساعات الاستواء ويستكمل فيه من الشهور أربعة آلاف ومائتان وسبعة وستّون شهرا [B. f. 41v] ومن عودات الاختلاف التامّة أربعة {آلاف}[137] وخمس مائة و ثلاثة وسبعون عودة ومن أدوار فلك البروج أربعة آلاف وستّمائة واثنتى عشرة عودة إلاّ سبعة أجزاء ونصف جزء بالتقريب وهي الأجزاء التي ينقصها {الشمس}[138] في الثلاثة مائة والخمسة والأربعين الدورة وهذا على أن عودات هذه الأشياء إنّما تعمل فيها على القياس إلى الكواكب الثابتة فلمّا قسموا هذه الأيام وجدت لهذا الزمان الدوري على عدّة الشهور فيه خرج زمان {الشهر الوسطي}[139] تسعة وعشرون يوما وإحدى وثلاثون[140] دقيقة وخمسون ثانية وثماني ثوالث وتسع روابع وعشرون خامسة بالتقريب وإذا ضوعفت أيام الشهر بالدقائق التي تقطعها الشمس بحركتها الوسطى في اليوم الواحد وهي تسع وخمسون دقيقة وثماني ثوان وسبع عشرة ثالثة

[137] In the margin in Ms. B.

[138] Ms. B. "الشهر".

[139] Ms. Es^2 "الشهور الوسطى".

[140] In Ms. B., addition in the margin crossed out: "وثمان".

وثلاث عشرة رابعة واثنتى عشرة خامسة وإحدى وثلاثون سادسة كان من ذلك ما تقطعه الشمس في زمان الشهر الوسطي وإذا أضيف إلى ذلك أجزاء دورة واحدة وهي ثلاث مائة وستّون جزءًا [Es^2 f. 47v] كان ذلك ما يتحرّك القمر في الطول بالوسط في زمان الشهر الوسطي وإذا قسم ذلك على عدد {أيام الشهر}[141] خرجت حركة القمر الوسطى في الطول في اليوم الواحد وذلك ثلاث عشر جزءًا وعشر دقائق وأربع وثلاثون ثانية وثمان وخمسون ثالثة {وثلاثون رابعة}[142] {وثلاث}[143] وثلاثون خامسة وثلاثون سادسة بالتقريب فإذا نقص من ذلك حركة الشمس الوسطى في اليوم الواحد بقيت حركة البعد بينهما بالوسط في اليوم وذلك اثنا عشر جزءًا وإحدى عشرة دقيقة وستّ وعشرون ثانية وإحدى وأربعون ثالثة وعشرون رابعة وسبع عشرة خامسة {وسبع وخمسون سادس}[144] و{إنّما}[145] إذا ضوعف عودات الاختلاف التي تحتوي عليها ذلك الزمان الدوري بأجزاء دائرة واحدة وقسم المجتمع على عدد أيام ذلك الزمان الدوري خرج ما يقطع القمر في اليوم الواحد من فلك تدويره وذلك ثلاثة عشر جزءًا وثلاث دقائق {وثلاث}[146] وخمسون ثانية وست وخمسون ثالثة وتسع وعشرون رابعة وثمان وثلاثون خامسة وثلاثون سادسة بالتقريب

[141] Ms. Es^2 “الأيام للشهر”. Explanation in the margin “بيان: أيام الشهر”.

[142] Not in Ms. B.

[143] Not in Ms. Es^2. Illegible correction in Ms. B.

[144] Not in Ms. B.

[145] In the margin in Ms. B.

[146] Not in Ms. Es^2.

[.9] [في زمان القمر الدوري في العرض]

وأمّا حركة القمر في العرض فإنّ القدماء أدركوها بأن [يــ]ــطلبوا مدّة بين كسوفين قمريين يكون مقدار المنكسف من قطر القمر فيهما واحدا ويكون القمر فيهما في نقطة واحدة بعينها من فلك تدويره ويكون المنكسف من صفحة القمر في جهة واحدة من الشمال أو الجنوب وعند عقدة واحدة بعينها فإنّ باجتماع هذه الشروط يلزم ضرورة أن يكون بعد القمر في أوّل كسوفيه من العقدة متساويا لبعده في الثاني من تلك العقدة بعينها في تلك الجهة بعينها فتكون تلك المدّة محيطة بعودات تامّة للقمر في العرض ولمركز فلك تدويره في الفلك المائل فذكر أنّ إبرخس وجد هذين [B. f. 42r] الكسوفين على هذه الشروط ووجد الزمان الذي بينهما يحيط بخمسة آلاف شهر وأربعمائة وثمانية وخمسون شهرا ومن أدوار العرض خمسة آلاف دورة وتسعمائة دورة وثلاث وعشرون دورة فإذا قسمت تلك المدّة على عدد {عودات}[147] العرض خرج زمان {العودة}[148] الواحدة فإذا قسم على ذلك العدد عدد أجزاء دائرة واحدة [Es² f. 48r] وهو ثلاث مائة وستّون جزءًا خرج ما يقطعه القمر لحركته الوسطى في العرض في اليوم الواحد وذلك ثلاثة عشر جزءًا وثلاث عشرة دقيقة وخمس وأربعون ثانية وتسع وثلاثون ثالثة وأربعون رابعة وسبع عشرة خامسة وتسع عشرة سادسة فبهذا الطريق أدرك القدماء حركات القمر في الطول والاختلاف والعرض

[147] Ms. B. "عوات".

[148] Ms. B. "عدة".