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The expansion rate of the intermediate universe in light of Planck

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ABSTRACT

We use cosmology-independent measurements of the expansion history in the redshift range $0.1 \lesssim z < 1.2$ and compare them with the Cosmic Microwave Background-derived expansion history predictions. The motivation is to investigate if the tension between the local (cosmology independent) Hubble constant H_0 value and the Planck-derived H_0 is also present at other redshifts. We conclude that there is *no tension* between Planck and cosmology independent-measurements of the Hubble parameter $H(z)$ at $0.1 \lesssim z < 1.2$ for the ΛCDM model (odds of tension are only 1:15, statistically not significant). Considering extensions of the ΛCDM model does not improve these odds (actually makes them worse), thus favouring the simpler model over its extensions. On the other hand the $H(z)$ data are also not in tension with the local H_0 measurements but the combination of all three data-sets shows a highly significant tension (odds ∼1:400). Thus the new data deepen the mystery of the mismatch between Planck and local H_0 measurements, and cannot univocally determine whether it is an effect localised at a particular redshift. Having said this, we find that assuming the NGC4258 maser distance as the correct anchor for H_0, brings the odds to comfortable values.

Further, using *only* the expansion history measurements we constrain, within the ΛCDM model, $H_0 = 68.5 \pm 3.5$ and $\Omega_m = 0.32 \pm 0.05$ (at 68% confidence) without relying on any CMB prior. We also address the question of how smooth the expansion history of the Universe is given the cosmology independent data and conclude that there is no evidence for deviations from smoothness on the expansion history, neither variations with time in the value of the equation of state of dark energy.

1. Introduction

The recent release of the determination of cosmological parameters from the Planck [1,2] Cosmic Microwave Background (CMB) observations, has shown that for a ΛCDM model, the extrapolated value at $z = 0$ of the Hubble parameter, H_0, is in tension with the one measured locally via astronomical observations of the local distance scale [3,4]. Such a difference could not be explained by cosmic variance in the standard ΛCDM model [5,6] and would require a highly inhomogeneous Universe. It could be due to unaccounted systematics in the data in either (or both) experiment, or to a failure of the adopted cosmological model to describe nature.

In a previous paper [3] we discussed the importance of local ($z = 0$) measurements in order to assess the consistency of the currently favoured cosmology model: ΛCDM. Because the CMB mostly probes the Universe at $z \approx 1100$, any cosmological parameter defined at any other redshift is necessarily model-dependent. Therefore, if cosmological parameters can be measured directly, precisely and robustly at other redshifts than $z \approx 1100$, the comparison with the same quantities derived from the CMB using the standard ΛCDM model can serve as a test of the model itself.

In Ref. [4] we concluded that local measurements of the expansion rate (H_0) and of the age of the Universe were in tension (odds 1:53) with the Planck-derived parameters from the CMB within the ΛCDM model. The tension was driven by H_0 and not by the age. With only the data-sets considered there, it was however not possible to determine whether this tension is a signature of systematics in either measurement or new physics: independent data are needed to make further progress.

One way to further investigate this is to "fill the gap" by focusing on $z > 0$ but still $z \ll 1100$. This can be done by using the recent measurements of $H(z)$ between redshift $0.1 \lesssim z < 1.2$ from the cosmic chronometers project [7–11].

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This method adds cosmology-independent measurements of the expansion rate back to when the Universe was only ∼1/3 of its current age (1/3 to the distance of last-scattering), thus significantly increasing the volume surveyed in the Universe to test the CMB-derived cosmology model. The cosmic chronometer method is the only method that provides cosmology-independent, direct measurements of the expansion history of the Universe. In fact, Supernovae data measure the luminosity distance–redshift relation, which is related to an integral of $H(z)$. However the necessary marginalisation over the (unknown) intrinsic magnitude of the standard candles is effectively equivalent to a marginalisation over an overall amplitude i.e., over H_0. Baryon acoustic oscillations must rely on the CMB measurement of a standard ruler (the sound horizon at radiation drag) to extract $H(z)$ information from radial clustering; angular clustering yields a combination of the angular diameter distance and the sound horizon at radiation drag and angle averaged clustering yields a combination of angular diameter distance, $H(z)$ and sound horizon. While the CMB determination of the sound horizon is robust e.g., Ref. [12] it is still somewhat model-dependent (e.g., [13–15]). Following Ref. [11] we concentrate on the redshift range $z < 1.2$.

In this paper we explore whether the expansion history at intermediate redshifts shows any signature of possible deviations from the CMB-inferred one. The paper is organised as follows: we first investigate (Section 3), in a mostly parameter-independent way via Gaussian Processes (GP), if the $H(z)$ data show any sign of the expansion not being smooth: we find none. We also find no evidence for variations of the dark energy equation of state parameter (Section 4). We then explore the constraints on the parameters of several cosmological models (the standard ΛCDM and its popular extensions) using *only* $H(z)$ data and no CMB prior (Section 5). Finally we compute the tension between the $H(z)$ data and the Planck-derived expansion history for different cosmological models (Section 6). We conclude in Section 7.

2. The $H(z)$ data for cosmic chronometers

The idea behind the cosmic chronometers approach is to exploit the fact that $H(z) = dt/dz(1+z)$ and that redshifts are easy to measure very accurately from spectroscopic data. The difficult quantity to measure is dt: the change in the age of the Universe as a function of redshift. This can be achieved by measuring (relative) ages of galaxies with respect to a fiducial model, thus circumventing the need to compute absolute ages, if one could find a galaxy population that is uniform enough (i.e. standard clocks, or cosmic chronometers). The cosmic chronometers approach relies on the fact that the most massive galaxies contain the oldest stellar populations: have less than 1% of their present stellar mass formed at $z < 1$, have formed their stellar population at high redshift, $z > 2$, and that since that time this population has been evolving passively. The differential ages of these galaxies can therefore be used to estimate the rate of change of the age of the Universe as a function of redshift and therefore $H(z)$ for $z < 2$. See Refs. [7–11] for more details. The $H(z)$ measurements so obtained are cosmology-independent, in the sense that they do not rely on assuming an underlying cosmological model, but rely on stellar population synthesis models to accurately model the effects of time on the integrated spectrum of the stellar population. Here and in what follows we use the expansion history measurements from Refs. [8,9,11] and in particular we start from the compilation reported in http://www.physics-astronomy.unibo.it/en/research/areas/astrophysics/cosmology-with-cosmic-chronometers of [16]. This compilation has data points that cover redshifts as high as $z = 1.8$. Ref. [11] showed that at $z > 1.2$ the dependence on the assumed stellar population model becomes important. Those authors considered two state-of the art models, "BC03" [17] and "MaStro" [18], and we use the "BC03" results for consistency with the other references where only results for this model are provided. A full treatment that marginalises over the model uncertainty will be presented elsewhere, here we only consider $z < 1.2$ (and therefore for the nature of the sample the data point at the highest redshift is at $z = 1.04$). Note that even in this sample there is one point that shows some dependence on the stellar population model, this is the highest redshift one at $z = 1.04$ from Ref. [11]. This dependence is mild: it is 1.6 times the purely statistical error but the final measurement errors include also sources of systematics (not including stellar model-dependence). We therefore increase slightly (20%) the (total) error bars of the point at $z = 1.04$ to encompass the stellar population model uncertainty. With this corrected error-bars the dependence on the stellar population model becomes insignificant and the results reported below do not depend qualitatively on this point.

3. Is the expansion history "smooth"?

We start by investigating if there are any signatures of deviation from smoothness using Gaussian Processes (GP) [19,20], which is a non-parametric Bayesian inference formalism. Authors of Ref. [21] have been the first ones to recently use GP on the same dataset to extrapolate the value of the Hubble parameter at $z = 0$ *assuming smoothness* of the expansion history. Here we use GP with a different purpose: to determine, in the least parametric way, if the data show any deviation from smoothness as a function of redshift.

We will not enter into a detailed description of GPs here but instead refer the interested reader to the literature (see e.g., [19,20] for standard references in the field). GP is a stochastic process that considers independent and identically distributed Gaussian measurement errors, the relations or correlations between measurements are defined via a covariance matrix. Using standard Bayesian methodology, the estimation of H at any value of z is determined by a probability distribution (posterior predictive). This results into a fully probabilistic estimation of the expansion history given the measurements. In our application the different $H(z)$ determinations are uncorrelated, therefore the off-diagonal terms of the covariance matrix determine the smoothness of the fitting function. In this approach, the only choice we have to make is the form of the off diagonal terms of the covariance matrix. We have experimented with various choices of the covariance matrix with similar results, but we report here the outcome of employing the most commonly used covariance matrix, the radial exponential:

$$K(z, z') = \sigma^2 \exp\left(-\frac{(z-z')^2}{2l^2}\right), \tag{1}$$

where z and z' are two different z values (i.e., $z \neq z'$) and l, s are the latent parameters or hyper-parameters[1] which are determined using $H(z)$ data themselves. Intuitively, l controls the correlation length and therefore the "smoothness" and σ controls the importance of the off-diagonal terms compared to the diagonal ones. The bigger the l is, the smoother the predictive function would look. Similarly, the higher the σ is the lower the signal-to-noise ratio is. The values of l and σ can be obtained using maximum likelihood estimation. Since we are dealing with hyper-parameters and not parameters, we maximise the marginal likelihood (over functions) and not the likelihood directly. Note that this approach is the same as the hierarchical Bayesian one [22]. To do so we calculate the partial derivative of the marginal likelihood with respect to the hyper-parameters and optimise the hyper-parameters using gradient based search [19].

[1] Hyperparameter is a parameter of a prior distribution; the term is used to distinguish them from parameters of the model for the underlying system under analysis.

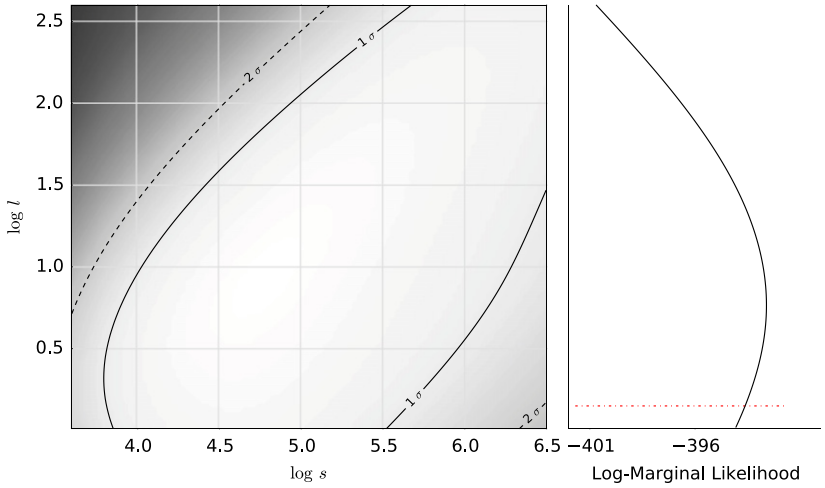

Fig. 1. Marginalised (over functions) likelihood given the data as a function of the two hyperparameters, l and σ. Right panel shows the marginalised (over σ) of the full likelihood shown on the left. The dotted red line indicates the 5% confidence of the l hyperparameter. This lower bound gives us the least smooth predictive posterior shown in Fig. 2.

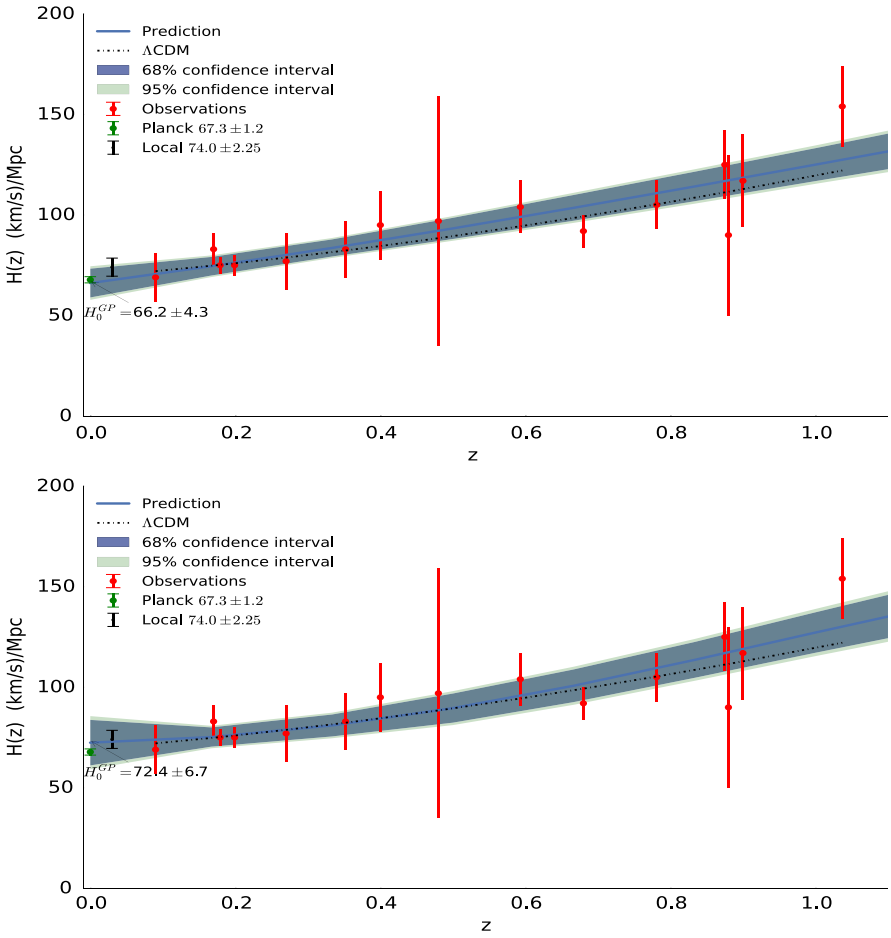

Fig. 2. Top panel, Gaussian Process prediction (solid regions) of the expansion history of the Universe from the cosmic chronometers data using the best estimate for the hyperparameters. Bottom panel: same prediction but using hyperparameters of low confidence (5%) that produce the least smooth features (this is achieved with low l values in the covariance matrix). Note the predicted value for H_0. There is no evidence for the expansion history not being smooth over the redshift range $0.1 \lesssim z < 1.2$ even at the extreme case of using very unlikely values of the hyperparameters. For reference the dash-dotted line is the (multi-dimensional) best fit from Planck data. The 68% confidence error around this line is ± 1 at $z = 1.1$.

Fig. 1 shows the marginalised likelihood given the data as a function of the two hyperparameters, l and σ. The right panel shows the marginalised full likelihood (left panel) over σ. The dotted red line indicates the 5% confidence of the l hyperparameter. This lower bound gives us the least smooth predictive posterior shown in Fig. 2.

The top panel of Fig. 2 shows the 68% and 95% confidence bound of the predicted posterior using the GP with the radial exponential

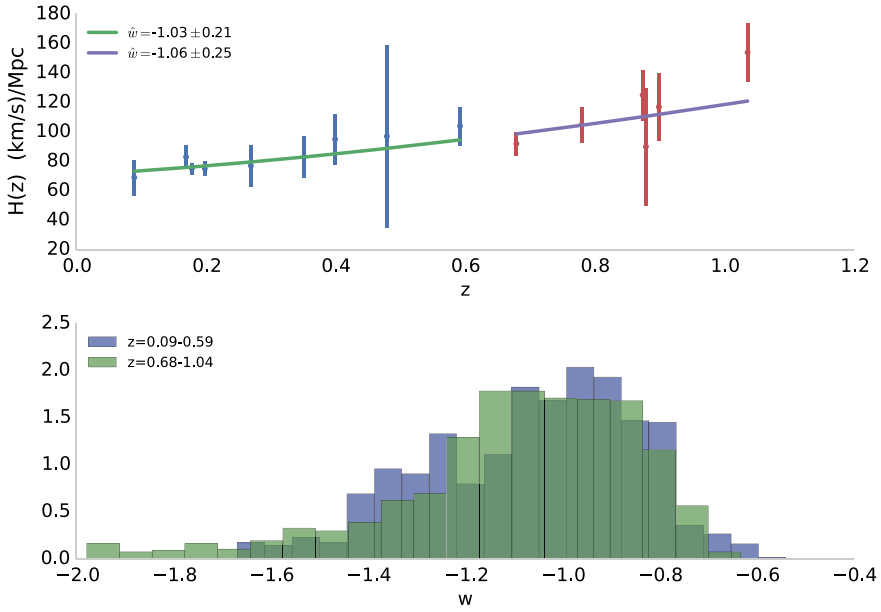

Fig. 3. Top panel, mean value of the posterior distribution of H as a function of z. In this test, data were divided into two different subsets: $0.1 \leq z < 0.6$ and $0.6 < z \leq 1.04$. Both tests were treated independently but we used the same priors. Bottom panel shows the posterior for the w for the two data sets. Clearly the two estimates are consistent.

covariance and the best estimate for the hyper-parameters, as described above. The bottom panel shows the same prediction but using hyper-parameters that produce the 5% least smooth features.

These regions are fully compatible with the Planck CMB derived expansion rate (dot-dashed line). But more interestingly is the fact that there is no signature for deviations from smoothness (over the redshift range explored) even when we considered the least likely value of l that creates the least smooth curve. Any sharp or asymmetric variation of the equation of state parameter, w, that results in a non-smooth expansion history, does not seem to be present at the accuracy of the data.

To be more quantitative there is no evidence for small values of l, large values of l – which couple different redshift bins all across the redshift range considered and therefore give smooth curves – are not only allowed but mildly preferred.

Another interesting feature is the value of the extrapolation at $z = 0$ (as already pointed out and studied in Ref. [21]). While the error at $z = 0$ is larger than the one from Planck,[2] it is fully compatible with it. It however points to the central value of the locally determined H_0 being slightly high. Here we use Ref. [3] "world average" value for H_0 obtained from the determinations of [23,24].

4. Does w vary with time?

In this section, we examine possible variations of the dark energy equation of state parameter w as a function of z. We use the ΛCDM model as the true model and we simply ask if for different domains of z the best estimates of w are statistically different or similar.[3] We first estimate all three parameters for the dataset and then we divide the data into subsets and compare the posteriors of w looking for significant statistical differences. For easy reference, in the Appendix we report useful formulae for the expression of $H(z)$ as a function of cosmological parameters.

We perform multiple tests by dividing the data into different subsets and repeat the experiment many times. None of these tests produce any significant evidence that w varies above the noise of the measurement given the ΛCDM model. In order to enhance the power of our test, we fixed the values of H_0 and Ω_m to the values estimated from the whole data set and repeat the test looking for statistically different values of w, an indication of a varying w. This is shown in Fig. 3. In this particular test the data were divided into two sub-sets and we find the marginalised posteriors for w for both sub-sets. In the top panel we show the results of the fit using Bayesian linear regression, whereas in the bottom panel we compared the marginalised posteriors. All tests we performed show similar results. We conclude that given the measurement error w is invariant in time for the redshift range considered $z \leq 1.04$.

5. What does the expansion history tell us about the cosmological model?

It is interesting to investigate what constraints on cosmology (cosmological models and parameters) one can impose by using *only* the cosmic chronometers data. While these data have been used in cosmological analyses by several authors, when scanning such a large parameter space they were almost always used in combination with other data sets, especially CMB and/or supernovae/BAO, e.g., [25,16]. This is shown in Table 1 and Fig. 4. The models we consider are: the standard ΛCDM, $w\Lambda$CDM where the (constant) parameter describing the equation of state of dark energy can differ from -1, $o\Lambda$CDM where we allow the curvature to be non-zero and finally w_0w_aCDM, where the Universe is spatially flat but the equation of state of dark energy can change as a function of time as $w(a) = w + w_a(1 + a)$ where a is the scale factor and w, w_a parameters of the model [26,27]. The constraints are qualitatively similar (and in agreement with) to the ones obtained from Supernovae type 1 A data, perhaps not surprisingly given that cosmic chronometers and supernovae are probes of the expansion history. As illustrated in Fig. 4 the constraints and degeneracy direction in parameter space, for models that are generalisations of the ΛCDM are complementary to the ones obtained from the CMB.

When considering marginalised constraints on single parameters, Table 1 shows that the ΛCDM model is fairly well constrained

[2] The Planck determination reported here assumes a flat ΛCDM model.

[3] We use weakly informative priors: uniform prior on Ω_m between 0 and 1, Gaussian prior on H_0 centred around 68 km s^{-1}Mpc^{-1} with a width of 10 km s^{-1}Mpc^{-1} and uniform prior on w between -2 and 0.

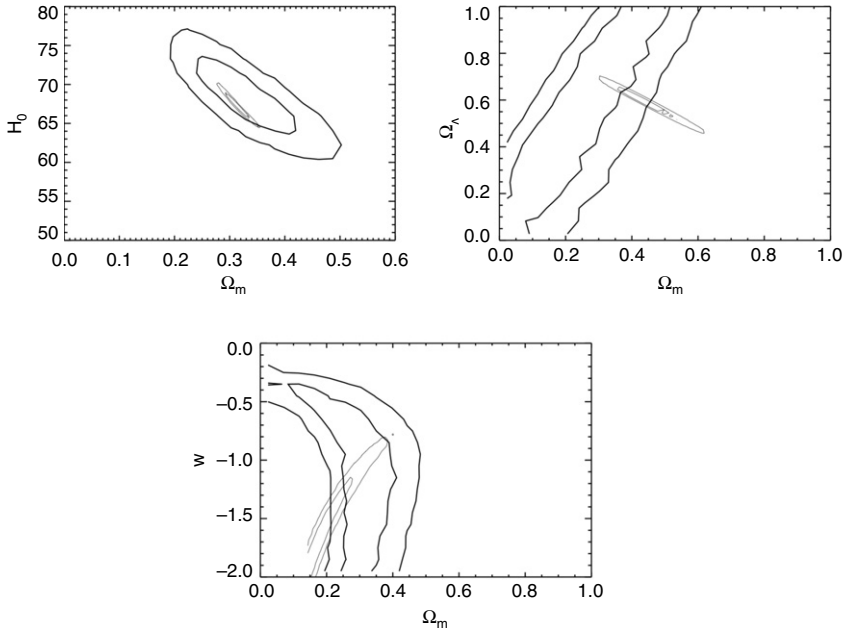

Fig. 4. 68% and 95% joint confidence contours for different cosmological models. Top left panel: ΛCDM, top right panel: $o\Lambda$CDM and bottom panel: $w\Lambda$CDM. The thick lines correspond to constraints obtained with expansion history data *only*, while the thin contours are using Planck temperature data *only*. Note the nice orthogonality and complementarity of both datasets.

Table 1
Parameter constraints on different cosmology models using the cosmology-independent expansion history determined by the cosmic chronometers compilation of Ref. [11]. Unless otherwise indicated the error-bars reported correspond to 68% confidence. When no error-bars are reported the parameter is kept fixed at the reported value. The "–" symbol denotes no meaningful constraint.

Model	H_0	Ω_m	Ω_Λ	Ω_k	w_0	w_a
ΛCDM	68.5 ± 3.5	0.32 ± 0.05	$1-\Omega_m$	0	-1	0
$w\Lambda$CDM	70 ± 7	<0.4 (95%)	$1-\Omega_m$	0	<-0.3 (95%)	0
oCDM	>50 (95%)	–	–	–	-1	0
$w_0 w_a$CDM	>45 (95%)	–	–	–	–	–

for the two parameters that affect the expansion history (H_0 and the density parameter Ω_m). Once the ΛCDM is extended, the cosmic chronometers data by themselves are not very constraining, being able to provide only upper(lower) limits on the parameters of the cosmology model. It is worth noting that for the most general extension of the ΛCDM model ($w_0 w_a$CDM) H_0 has to be larger than 45 km s^{-1} Mpc^{-1}.

6. Comparing with the Universe at $z = 1100$

In Ref. [4] we introduced a new statistical tool to determine the odds of two distributions being in tension and thus decide if one could combine the two distributions to further restrict the parameter space of the model. We briefly review here our main argument for completeness and to clarify notation. We refer the reader to Ref. [4] for more detailed explanations.

Imagine that we have performed two experiments: A, B and for each experiment we produce a posterior $P_{A,B}(\theta|D_{A,B})$ where θ represents the parameters of the model and $D_{A,B}$ represents the data from experiments A, B respectively. Let us also assume that for producing both posteriors we have used the same, uniform priors over the same "support", x, i.e., $\pi_A = \pi_B = \pi, \pi = 1$ or 0 and therefore $\pi_A \pi_B = \pi$. Let H_1 be the (null) hypothesis that both experiments measure the same quantity, the models are correct and there are no unaccountable errors. In this case, the two experiments will produce two posteriors, which, although can have different (co)variances, and different distributions, have means that are in agreement. The alternative hypothesis, $H_{\neg 1}$ is when the two experiments, for some unknown reason, do not agree, either because of systematic errors or because they are effectively measuring different things or the model (parameterisation) is incorrect. In this case, the two experiments will produce two posteriors with two different means and different variances. To distinguish the two hypotheses we use the Bayes factor, imagine that we can perform a *translation* (shift) of (one or both of) the distributions in x and let us define $\bar{P}_A$ the shifted distribution. This translation changes the location of the maximum but does not change the shape or the width of the distribution. If the maxima of the two distributions coincide then

$$\int \bar{P}_A \bar{P}_B dx = \bar{\mathcal{E}}|_{\max A=\max B}. \tag{2}$$

This can be considered as our "straw man" null hypothesis (H_1). As the distance between the maxima increases (but the shape of the distributions remains the same),

$$\int \bar{P}_A \bar{P}_B dx = e < \bar{\mathcal{E}}, \tag{3}$$

and eventually $e \longrightarrow 0$ as the two distributions diverge. Clearly the Evidence ratio for the (null) hypothesis E_1 is $\mathcal{E}/\bar{\mathcal{E}}|_{\max A=\max B}$, as the normalisation factors λ cancel out, and the Evidence ratio for the alternative $H_{\neg 1}$ is its reciprocal. We therefore introduce:

$$\mathcal{T} = \frac{\bar{\mathcal{E}}|_{\max A=\max B}}{\mathcal{E}}, \tag{4}$$

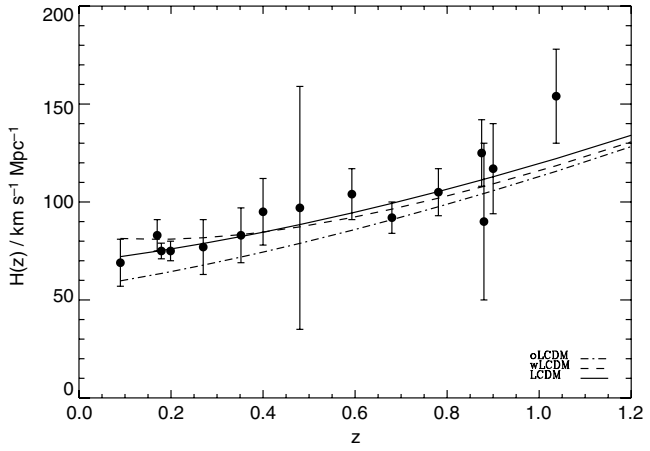

Fig. 5. Expansion history from the cosmology-independent cosmic chronometers project compared to different best-fit Planck cosmology models in the redshift range $0.1 \lesssim z < 1.2$. Error-bars on the data points correspond to 68% confidence.

which denotes the degree of tension that can be interpreted in the widely used (slightly modified, [28]) Jeffrey's [29] scale. $\mathcal{T}$ indicates the odds: 1:$\mathcal{T}$ are the chances for the null hypothesis. In other words, a large tension means that the null hypothesis ($\max A = \max B$) is unlikely.

Continuing on with the logic outlined above, let us consider that experiment A gives us the prior and experiment B gives us the data. $\mathcal{T}$ is the evidence ratio between the integrated *posterior* in two cases. In the numerator: a "just so" case where the prior from experiment A has a maximum that coincides with the maximum likelihood from experiment B; in the denominator: the prior from experiment A has a maximum that happened to be where it is.[4] Note that since the evidence is translation-invariant it does not matter which data set is considered as prior and which one as "data": the result is symmetric.

This argument can be extended to as many measurements as we want: the relevant integral just becomes higher-dimensional. For example let us consider the 15 $H(z)$ measurements in light of a Planck CMB prior for $H(z)$.

Table 2 shows the tension and odds for three different cosmology models while Fig. 5 shows the corresponding expansion history.

The first result to note is that the odds of the expansion history data and the Planck CMB ΛCDM model derived parameters being in tension are very low: just 1:15, which is above 1 σ but below 2 σ; we conclude that for the redshift range $1.04 \geq z \gtrsim 0.1$ there is no evidence for tension between "local" data and CMB derived expansion parameters. We also note that extensions to the ΛCDM model are in tension with the "local" data, thus disfavouring extensions to the minimal ΛCDM model. These conclusions do not depend on the value chosen for the data point at $z = 1.04$ ("BC03" or "MaStro") or even the inclusion of this point. In particular the very low odds for the non-flat LCDM model are driven by the data at $z \leq 0.6$.

Both the Planck CMB results and the local H_0 ones have been recently re-examined [30,31].

Ref. [31] excludes the 217 GHz channel from the analysis with the motivation that it has been shown to fail a number of null tests and therefore could be affected by some systematics. They also independently re-do the cleaning and analysis. As a result some

[4] Note that here one could loosely exchange "likelihood" with "posterior" for experiment B since we assume uniform priors for experiments A and B that are either 1 or 0 over the same "support" and therefore one can absorb the prior for experiment B into the A prior.

Table 2
Tension (in the Jeffrey's scale) and odds that the expansion history data from the cosmic chronometers ($1.2 < z < 0$) and the Planck CMB-derived expansion in the selected cosmological model are in tension. Note that there is no significant tension with the Planck ΛCDM model. Extensions to this model increase the tension – dramatically in the $o\Lambda$CDM case – with the model-independent expansion history $H(z)$ data; thus no extension of the model is justified.

Model	$\ln \mathcal{T}$	Odds
ΛCDM	2.73	1:15
$w\Lambda$CDM	3.27	1:26
$o\Lambda$CDM	4.93	1:138

Table 3
Tension and odds within the ΛCDM model, for different combinations of the datasets considered: Planck orig. refers to the Planck team's analysis, Planck revisited refers to Ref. [31], $H_0^{\rm WA}$ refers to the "word average" of Refs. [23,24] following [3]. $H_0^{\rm Efs,all}$ and $H_0^{\rm Efs,NGC4258}$ are from Ref. [30]. Clearly both re-analysis lower the tension. Note that the numbers for the combination Planck + $H_0^{\rm WA}$ are different from Ref. [4] because here we do not include the measurement of the age of the Universe.

Data combination	Planck orig.		Planck revisited	
	$\ln \mathcal{T}$	Odds	$\ln \mathcal{T}$	Odds
Planck + $H(z)$	2.73	1:15	2.71	1:14
Planck + $H_0^{\rm WA}$	3.55	1:35	2.85	1:17
Planck + $H_0^{\rm Efs,NGC4258}$	0.44	1:1.6	0.27	1:1.3
Planck + $H_0^{\rm Efs,all}$	1.77	1:6	1.33	1:3.7
Planck + $H_0^{\rm WA}$ + $H(z)$	6.17	1:473	5.5	1:245
Planck + $H_0^{\rm Efs,NGC4258}$ + $H(z)$	3.1	1:22	3.0	1:20
Planck + $H_0^{\rm Efs,all}$ + $H(z)$	4.4	1:81	4.0	1:55

of the parameters show some shifts, most notably Ω_m (down by 1σ) and H_0 (up by 0.6σ): $\Omega_m = 0.302 \pm 0.015$; $H_0 = 68.0 \pm 1$ do not have access to their full posterior therefore we shift the Planck posteriors to be centred to these values and we refer to this as Planck "revisited". This underestimates the final error-bars by some 20%–30% but it is a conservative estimate of the agreement of the new analysis with lower redshift measurements. Ref. [30] studies the dependence of the H_0 measurement on the choice of distance scale anchor and on the procedure for outliers rejection. LMC and MW cepheids distance anchor yield values of H_0 in agreement with each other but NGC 4258 maser distance gives lower values.

Using all distance anchors he obtains $H_0^{\rm Efs,all} = 72.5 \pm 2.5$ km s^{-1}Mpc^{-1} while using the maser distance yields $H_0^{\rm Efs,NGC4258} = 70.6 \pm 3.3$ km s^{-1}Mpc^{-1}. The combination LMC + MW yields a value very close to the "world average" ($H_0^{\rm WA}$ in this table) used above (we do not consider this combination below as results are virtually unchanged). In Table 3 we report the results of the various combinations; the tension between CMB and cosmic chronometers $H(z)$ remains virtually unchanged between the original Planck and the revisited Planck constraints.

Clearly both re-analyses lower the tension. If only Planck and the local H_0 measurements are considered either reanalysis brings the tension to comfortable values, but if all measurements are considered (i.e. we consider a 16 dimensional tension) both re-analyses are needed and only the maser distance brings the odds to comfortable values. Note that $H(z)$ + $H_0^{\rm WA}$ are consistent with each other (odds 1:2.4, $\ln \mathcal{T} = 0.9$). What we see here is the power of combining many different determinations and therefore evaluating the tension in high-dimensions. When degeneracies are present (as for example in the case of the Planck extrapolated values of $H(z)$, H_0) single determinations might be in agreement but the multi-dimensional distribution might be less so; this is illustrated in Fig. 6.

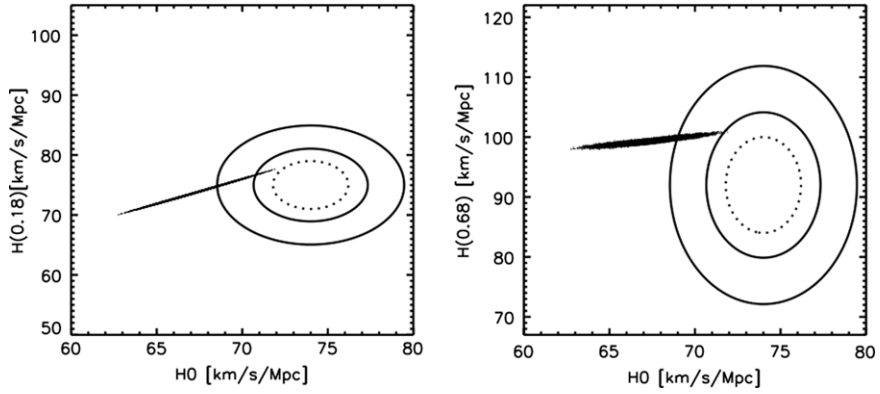

Fig. 6. Two-dimensional projection of the posteriors considered. In the left panel H_0 vs $H(z = 0.17)$ and in the right panel H_0 vs $H(z = 0.68)$. The points represent the Planck extrapolated values within the ΛCDM model sampled via Markov Chain Monte Carlo, the ellipses are the direct determinations.

7. Discussion and conclusions

Using the cosmology independent expansion history data ($H(z)$) from the compilation of Ref. [11] we have explored if the intermediate redshift universe is in tension with the Planck ΛCDM derived one. To do so we have generalised the two-dimensional tension introduced in Ref. [4] to an arbitrary number of dimensions. The inclusion of new data adds cosmology-independent measurements of the expansion rate back to when the Universe was only $\sim$1/3 of its current age (1/3 to the distance of last-scattering), thus significantly increasing the volume surveyed to test the CMB-derived cosmology model. We find no tension (odds only 1:15) between Planck and $H(z)$. This is in contrast with the local universe ($z = 0$) which is in significant tension [4] (1:57) with Planck. However $H(z)$ and H_0 are not in tension within a ΛCDM model: the central value for the Hubble constant obtained from $H(z)$ happens to be very near the Planck-CMB inferred one, but the error-bars are larger. When considering the three datasets – Planck, $H(z)$ and H_0 – the tension becomes highly significant (odds become uncomfortably low $\sim$1:400). Thus the addition of $H(z)$ deepens the mystery of the mis-match between Planck and local H_0 measurements, and cannot univocally determine whether it is an effect localised at $z \sim 0$ or $z \sim 1100$ or if it shows as a smooth change of the measured value of H_0 as a function of the redshift of the measurement.

Both the Planck and the local H_0 measurements have been independently re-visited and re-analysed. Either re-analysis brings the tension between Planck and local H_0 measurements to a comfortable level. On the other hand, when the three datasets are considered the Planck re-analysis has very little effect, and only assuming the NGC4258 maser distance as the correct anchor for H_0 brings the odds to comfortable values. However at present there are no compelling reasons to discard the other anchors [23,30].

The highly significant tension for the combined data set within the ΛCDM model however warrants further investigation: if we were to interpret the number in terms of Gaussian standard deviations it would correspond to 4.9 σ. Of course at this level it is very important to know extremely well the tails of the distributions, here we have assumed that all errors in measurements of H_0 and $H(z)$ are Gaussian and this might not be exact. The intermediate redshift measurements from the cosmic chronometers project, seems not to be explicitly in tension either with the $z = 0$ nor with the CMB. Elsewhere [32] we will consider other probes of the expansion history that cover a similar redshift range: baryon acoustic oscillations and supernovae. Moreover significant improvements in the $H(z)$ measurements is forthcoming [33], the significantly smaller error-bars might be able to localise or to point to a possible reason for the tension.

We have also explored how smooth the expansion history data are. This is motivated by the fact that we wanted to investigate if with cosmology independent expansion history data we could see departures from a cosmological constant prediction. Using Gaussian processes we concluded that the data are smooth with no sign of sharp deviations from a constant equation of state of dark energy over the redshift range $0.1 \lesssim z \leq 1.04$.

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Appendix. Useful formulas

For completeness we report here the expressions for $H(z)$ for a homogeneous and isotropic FRW Universe.

For a flat Universe ($\Omega_\Lambda = 1 - \Omega_m$, and curvature parameter $\Omega_k = 0$), generic equation of state parameter for dark energy

$$H(z) = H_0(1+z)^{3/2}\sqrt{\Omega_m + \Omega_\Lambda \exp\left[3\int_0^z \frac{w(z')}{(1+z')}dz'\right]}; \quad (5)$$

for a non-flat Universe ($\Omega_k = 1 - \Omega_m - \Omega_\Lambda$), equation of state parameter for dark energy given by the w_0, w_a parameterisation:

$$H(z) = H_0\left\{\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda(1+z)^{3(1+w_0+w_a)}\exp[-3w_a z/(1+z)]\right\}^{1/2}; \quad (6)$$

of course for a flat ΛCDM model we have:

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}. \quad (7)$$

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