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Is it Possible to Empirically Test a Metatheory?

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Abstract
In this paper, we examine the issue of the empirical or non-empirical status of philosophical metatheories. In particular, we ask whether a specific type of metatheoretical product, formal reconstructions of scientific theories, can be empirically tested. To answer this, we take Metatheoretical Structuralism as a metatheory and Classical Mechanics as our case studies. We show how classical mechanics can be reconstructed from structuralism. We then present a computer program, called Reconstructor, and show how it can be used to test the adequacy of the reconstruction. Finally, we discuss some philosophical points regarding these tests, namely, the issues of holism, circularity and metatheoretical predictions.

Keywords Empirical tests · Metatheoretical Structuralism · Classical Mechanics · Computer Program · Reconstructor

1 Introduction

The goal of this paper is to examine whether philosophical theories about science, and in particular, about the structure of scientific theories, are empirically testable. Additionally, if they are, to determine whether they are so in the same way that empirical theories are.

These goals are important for several reasons. Firstly, because metatheories purport to speak about certain parts of our (social/cultural) experience, i.e. scientific theories, which are culturally and historically situated entities. The fact that scientific metatheory is usually thought of as part of philosophy instead of empirical science is contingent, as disciplinary boundaries often are. Thus, metatheories should in principle be tested against what they intend to account for. Of course, philosophers of science do usually attempt some sort of empirical test of their metatheoretical products, in the sense that they do attempt to apply their frameworks to actual case studies, and in the process of doing so sometimes they adjust the former to make them better fit the latter. Yet, this is far from a systematic testing procedure, of the kind found in the empirical sciences. In this article, by empirical test we mean an actual procedure governed by explicit rules.

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A second reason our goal is important is that the question of empirical testability is a pressing issue for philosophers of science with an empiricist and/or a naturalistic stance. According to empiricists (of various tendencies) knowledge is either of the logical/mathematical/conceptual kind or of the empirical kind, there is no third category of philosophical knowledge. From a naturalistic perspective, science is part of the natural world and as such it may be (meta)scientifically studied.[1] Metatheoretical knowledge deals with natural phenomena and therefore should, in principle, be tested as any other knowledge with empirical scope.[2] For a naturalist, this does not exclude that some parts of philosophy of science be considered conceptual analysis/knowledge[3] (e.g. the explication of the notion of explanation, or of the notion of inductive/ampliative justification), but this is compatible with other parts being empirical research about cultural phenomena. In particular, we claim that this is so with metatheories about the structure of scientific theories (whether this implies a recursive self-application is a question we will not deal with here).

Our working hypotheses are, then, that metatheories are empirically testable, in a systematic way, and moreover, that these tests are relevantly similar to those of empirical scientific theories; therefore, metatheoretical studies, at least those that aim to reconstruct the structure of theories, can be thought of as part of an empirical science of science.

As a case study, we will take Sneedian or Metatheoretical Structuralism (MS) as our metatheory about the structure of scientific theories (Balzer et al., 1987; Díez & Lorenzano, 2002; Sneed, 1971). MS has been acknowledged as the most well-developed structural metatheory by philosophers as varied as Kuhn (2000), Cartwright (2008) or Frigg (2023). On the other side, it is by far the metatheory that has been applied to the reconstruction of more scientific theories in more varied fields, including physics and astronomy (Carman, 2010; Schmidt, 2014; Sneed, 1971), (bio)chemistry (Alleva et al., 2017; Caamaño, 2009; Donolo et al., 2007; Falguera & Donato-Rodríguez, 2016; C. Lorenzano, 2002; O'Lery, 2012), biology (Balzer & Lorenzano, 2000; Blanco, 2012; Díaz & Lorenzano, 2017; Díez & Lorenzano, 2013; Ginnobili, 2016; Méndez & Casanueva, 2006), sociology and linguistics (Abreu, 2012; Gonzalo & Balzer, 2012; Peris-Viñé, 2011), among

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[1] In this regard, our project can be considered part of this naturalistic or "(meta-)empirical" tradition in the philosophy of science (we thank an anonymous reviewer for this suggestion), from Whewell and Mach, through Carnap and Neurath, to Van Fraassen or Giere. This tradition, though, is complex and the different positions within it involve subtle differences in which we cannot enter here. What matters to our present concerns, is that it suggests that metascience (or at least some of its parts) could in principle be tested in an analogous manner to science itself. Our following testing proposal elaborates this suggestion in one of its possible directions, for there may be other "meta-testing" proposals that test other parts of meta-science (for instance, Estany, 1990, tests Kuhn's, Lakatos' and Laudan's different meta-scientific models of scientific change against different historical episodes).

[2] One might object that the project of empirically testing metatheories would make sense if the task of metatheories were descriptive, which is at least controversial; for instance, Moulines (1991) has defended that the task of metatheories is better characterized as interpretation than as description (we thank an anonymous reviewer for this comment). First, it is not clear that interpretations cannot be tested, they seem to be testable, to the extent that they generate (metatheoretical) predictions—see below. Second, and relatedly, even if metatheories are interpretations they may have a descriptive component (Moulines himself acknowledges this in a later work, see Díez and Moulines 1997). For example, all theories that are not non-directly self-confirmed (the usual case in science) are tested with data that are measured/gathered independently of the theoretical assumptions/laws used to make the relevant predictions, which is why those predictions can, and sometimes do, fail. Different metatheories may reconstruct this fact in partially different ways, and some ways may be more fruitful than others, but all metatheories (that have a minimal adequacy) should be suitable for our kind of meta-testing in this regard.

[3] As explicitly stated for instance by the philosophers of the so-called "Canberra plan" (see e.g. Jackson 1998, Chalmers 2012).

others (see Diederich et al., 1989, 1994 and Abreu et al., 2013, for a more complete reference of the theories reconstructed).

As a practical tool to perform the kind of tests we are interested in, we shall use Reconstructor (Roffé, 2019, available at https://arielroffe.quest/home#reconstructor), which is a computer program that allows the user to check the adequacy of various aspects of structuralist reconstructions (more on this in Sect. 3). Since any theory can only be empirically tested through its applications, and since an application of a metatheory is a reconstruction of a particular theory, being able to test particular reconstructions will serve as a test for the metatheory itself.

Put in different words, theories (including metatheories) make some general predictions, which can be empirically checked when one considers a particular application and fills all the relevant values/variables. For metatheories, it is not immediately obvious what these general predictions consist of, perhaps because metatheories are not usually presented very systematically, as other theories in science sometimes are. Instead of attempting such a systematic presentation from the outset, our dialectical proposal is to briefly present the metatheory in Sect. 2 (along with a case study) and to then show some clear ways of testing the adequacy of *particular* reconstructions in Sect. 3, with the aid of the computer program. We will only arrive at general metatheoretical predictions in Sect. 4. We believe this order of exposition will be clearer than if we parted from the general predictions. Section 4 also discusses further philosophical implications, related to holism and circularity.

2 Structuralist Rational Reconstruction

In order to make clear what exactly we propose to test, it is worth to start by identifying the different steps involved in a (structuralist) reconstruction of a scientific theory. The obvious departure point, previous to any reconstruction, is to be a competent user/knower of the theory in point, being able to recognize paradigmatic uses of it such as typical cases of successful applications, or typical failures or anomalies, etc. The properly reconstructive first step consists in informally identifying the exact boundaries of the object-theory. This is usually done by reference to standard textbooks and/or paradigmatic articles. This first task, though, is not always straightforward, since standard textbooks may sometimes include more than one theory or mix different theories. For instance, a well-used textbook in introductory physics courses (Halliday et al., 2014) mixes mechanics, optics, thermodynamics and others. This may in general be fixed by an exhaustive exploration of the literature and the sociology and teaching of the discipline and its theories. There is no general rule of application, the only methodology is a case by case (meta)study.

The next step is to choose, in case there exist different versions/formulations of the "same theory" (e.g. Newtonian and Lagrangian classical mechanics, or wave and matrix QM), the precise version one is willing to reconstruct. There is an issue regarding whether different versions are theoretically equivalent, or only empirically equivalent, in which we do not enter here. For the sake of this article, we treat different versions of theories, in the senses mentioned above, as different theories, so that in the reconstruction task one has to pick one. There is also the issue that theories change over time, and metatheories may provide the tools for a diachronic reconstruction (as e.g. MS has, cf. Balzer et al., 1987), but for the sake of simplicity we will focus here on synchronic aspects of the reconstruction only, so one must choose a specific stage of the theory to reconstruct.

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The third step consists in "informally reconstructing" the theory in point. This basically consists in identifying the conceptual apparatus (both primitive and defined) and the set of theory claims that utilize this apparatus. Among these, it is essential to distinguish those that describe the phenomena the theory "is about", or intends to "account for", identifying the concepts deployed in those statements, and the additional concepts (if any) and statements that the theory introduces in order to account for such phenomena. It is common to select among the latter some principles/laws that constitute the core or "primitive" content of the theory, such that the rest of the statements of the theory can be derived from them as theorems.[4]

This third step is methodologically instrumental to the fourth and final step, which consists in the formal reconstruction using a particular metatheoretical apparatus in a particular formal language (first order logic, set-theoretical predicates, category theory, etc.), Of course, depending on one's expertise one can skip the informal reconstruction, or just do it "in one's mind", and proceed directly to the formal reconstruction.

As just said, we take, for the reasons already given, as a metatheoretical case study a particular version of the model-theoretic and set-theoretic families, namely Sneedian structuralism. For a complete exposition of this metatheory see Balzer et al. (1987); for quite complete summaries cf. Diederich (1989), Moulines (1996, 2002), Frigg (2023). In what follows, we just recall the main elements of the structuralist apparatus, since this will be enough for our purposes.

As a member of the semantic or model-theoretic family, for MS, a reconstruction of a theory proceeds by defining its (different) classes of models. And as a member of the set-theoretic metalanguage family, such definitions are given by means of set-theoretic predicates.

This reconstructive task starts by identifying and categorizing the basic concepts/terms of a given theory T. T-terms/concepts are those used in the standard expositions of T. E.g., in Classical Mechanics (CM), the CM-concepts are "particle", "space", "time", "velocity", "acceleration", "mass", "force", "momentum", etc. We ignore those that can be defined in terms of others (velocity, acceleration, momentum, etc.) and concentrate on the primitive ones, "particle", "space" or "position", "time", "mass" and "force". The sets/relations/functions denoted by these primitive T-concepts are the constituents of the models of the theory. Hence, we take P, T, s, m and f as the corresponding denotations of "particle", "time", "space", "mass" and "force". For technical reasons that will become clearer later, we shall also add a set I that specifies the forces at stake in a given model. Therefore, CM theoretical models are of the form $\langle P, T, I, s, m, f \rangle$.

We call characterizations (or improper axioms) of the expositions of T, those formulae that determine the logical type of the components of the models, i.e. of the denotations of the T-concepts. E.g. (we proceed with some simplifications that will not affect the issue here): The set of particles P is a non-empty set; the time interval T is a subset of the continuum $\mathbb{R}$; The set of indexes I is a subset of $\mathbb{N}$; The position function s assigns a three-dimensional vector to every particle at a time; the mass function m assigns a positive real number to every particle; and the force functions, that for computational convenience we formalize

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[4] Note that this distinction between "primitive" and "derived" laws has nothing to do with the distinction in MS we introduce below between general guiding principles and their special laws, which, as we will clarify, are not "derived" from the guiding principles at all, but introduce new content by specifying some parameters left open in the guiding principles. In the sense of "primitive" used here, (some) specializations are primitive (and of course, some others are derived from the former).

as a single function f from $\mathbb{N} \times P \times T \times P \cup \{\varnothing\}$ into $\mathbb{R}^3$, where the first argument specifies the kind of force at stake (i.e. the numeric indices will represent gravitational, Hooke's, friction, electrostatic, etc. forces); $P \cup \{\varnothing\}$ serves to specify, when the force is an effect of another particle (e.g. gravitation, Coulomb's), the particle so responsible, and $\varnothing$ is used as a technical tool for cases of forces not caused by other particle (friction, Hooke, etc.). Then, for instance, $f(5, p_1, t, p_2)$ denotes the vector electrostatic force over (the charged particle) p_1 at t due to the (charged particle) p_2; and $f(3, p_1, t, \varnothing)$ the Hooke vector force over (the particle in a spring) at t. We call "potential models of T" the structures that satisfy these characterizations, and denote their set by $\mathrm{M_p}$(T), meaning that these are the structures with the appropriate logical type, such that it makes sense to ask whether they satisfy the laws of T. We can the define CM potential models as follows:

$x = \langle P, T, I, s, m, f\rangle \in \mathrm{M_p}(\mathrm{CM})$ iff:

1. $P \neq \emptyset$
2. $T \subseteq \mathbb{R}$
3. s is a function from $P \times T$ into $\mathbb{R}^3$ (twice differentiable)
4. m is a function from P into $\mathbb{R}+$
5. $I \subseteq \mathbb{N}$
6. f is a function from $I \times P \times T \times P \cup \{\emptyset\}$ into $\mathbb{R}^3$

We will also utilize some defined notions, for convenience's sake. For instance, we will write $a(p, t)$ for the acceleration of particle p at time t, with an understanding that the acceleration of the particle is just the second derivative of the position over time (i.e., $\mathrm{d}^2(s(p, t)) / \mathrm{d}(t)^2$).

We call proper axioms of the expositions of T, those formulas that impose additional, not merely typological constraints on the models, i.e. the "laws" of T; for instance in CM the Second law, the gravitational law, Hooke's law, etc. We call "(actual) models of T" those potential models that satisfy the proper axioms and denote their set by "M(T)". For instance, if for the sake of the example we consider CM constituted only by the following set of laws, M(CM) is defined as follows:

$x = \langle P, T, I, s, m, f\rangle \in \mathrm{M_p}(\mathrm{CM})$ iff $x \in \mathrm{M_p}(\mathrm{CM})$ and[5]:

1. $\forall p \in P, \forall t \in T: \sum_{i \in I} \sum_{p2 \in P \cup \{\varnothing\}} f(i, p, t, p_2) = m(p) \cdot a(p, t)$
2. $\forall p \in P, \forall t \in T: f(1, p, t, \varnothing) = m(p) \cdot g \ (g \in \mathbb{R}^3)$
3. $\forall p \in P, \forall t \in T: f(2, p, t, \varnothing) = -k\,(s(p, t) - s_{eq}(p)) \ (k \in \mathbb{R}+)$
4. $\forall p \in P, \forall t \in T: f(3, p, t, \varnothing) = -k_s \cdot m(p) \cdot g \cdot \cos(\Theta) \ (k_s \in \mathbb{R}+) \ (g \in \mathbb{R}+) \ (\Theta \in [0,360))$
5. $\forall p_1 \in P, \forall p_2 \in P, \forall t \in T: f(4, p_1, t, p_2) = k_e \cdot q(p_1) \cdot q(p_2) \,/\, |(s(p_1, t) - s(p_2, t))|^2 \cdot (s(p_1, t) - s(p_2, t)) \,/\, |(s(p_1, t) - s(p_2, t))| \ (k_e \in \mathbb{R}+)$

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[5] Since forces (symbolized by f) are 3-dimensional vectors, the right-hand side of the following equalities must also be vectors. In 3 this is already so, given the vectorial difference of positions. In 2 and 4, where the gravitational constant g intervenes, we take g to be a vector (of magnitude 9,81 m/s^2 in the MKS system, and a downward direction, perpendicular to Earth's surface—or towards the Earth's center from the particle's position, to be more precise). For the electrostatic case (axiom 5), we multiply by the unitary vector $(s(p_1, t) - s(p_2, t)) / |(s(p_1, t) - s(p_2, t))|$ in the direction from one particle to the other. For a technically more elegant formulation, cf, Balzer et al. ch 4.

Here, the first law formalizes Newton's Second Principle (SP), the second is the law of free-fall, the third is Hooke's law for springs, the fourth is the law of static friction for inclined planes and the last is Coulomb's law.[6]

Among the T-concepts, MS distinguishes between T-theoretical and T-non theoretical concepts. A T-concept is T-non theoretical iff it(s denotation) can be determined/measured without presupposing any law of T. E.g., space-position and time are MC-non theoretical concepts for, although sometimes they are determined or measured using dynamical laws, they can be determined with non-dynamical procedures (e.g., measuring distance by triangulations). A T-concept is T-theoretical iff it is not T-non theoretical, i.e., iff all of its determination methods presuppose the use of some T-law. E.g., mass and force are MC-theoretical. This distinction is relative to theories, a concept may be T-non theoretical for some T and T'-theoretical for some other T' (e.g. "pressure" is Thermodynamics non-theoretical but CM-theoretical). We call "partial (potential) models" the result of cutting out the T-theoretical components from T-potential models and denote their set by $M_{pp}(T)$. E.g., partial models of MC are of the form $\langle P, s, T\rangle$, i.e., they are purely cinematic structures ("the cinematic part" of mechanical models). $M_{pp}(T)$ aims to express the conceptual apparatus used by T but borrowed from previous theories, concepts used by T for describing the phenomena T wants to account for, whose individuation does not depend on the acceptance of T-laws. The idea is that T describes the phenomena it wants to explain/account for using T-non theoretical machinery, and then in order to account for such phenomena T introduces its own T-theoretical concepts relating them to the T-non theoretical ones through T-laws. Since phenomena in $M_{pp}(T)$ are determined without assuming any T-law, T's predictions/tests are fallible.[7] Some theories are purely phenomenological in that they do not introduce proper T-theoretical concepts ($M_{pp} = M_p$ in these cases), so their laws simply systematize phenomena, do not explain them (e.g., Galilean Kinematics or Keplerian astronomy).[8]

We call the "(mathematical/formal) core" of T the tuple $K = \langle M_p, M, M_{pp}\rangle$. K contains the formal part of T:[9] M_p is the model-theoretic version of the conceptual machinery, M of the laws, and M_{pp} of the conceptual machinery of the "empirical (testing) basis". Yet, the core K does not suffice for the individuation of T, for M contains "all" models, intended as well as non-intended (e.g., M(CM) contains "angelical models" (if any), or for sure "purely numerical" models (with P being a set of numbers)). For the individuation of T, the individuation of its "intended applications" (Kuhn's exemplars) is essential. We denote by "I(T)" this set of intended applications. Members of I(T) are singled out pragmatically, by means of intentional actions of scientists. Qua members of I(T), intended applications are described/identified only T-non theoretically, i.e. their set is a specific (pragmatically determined) subset of $M_{pp}(T)$. E.g., members of I(CM) are specific cinematic systems, such as

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6FL01 [6] Notice that there is an additional function here (the electrical charge function *q*), as well as several constants, that we did not introduce above as part of the basic language / potential models of the theory. This was for simplicity's sake. In the computer program below, we load everything at once.

7FL01 [7] Since a T-non-theoretical concept may be T*-theoretical relative to different theory T*, T-data may be "theory-laden", but laden by a theory that is not the one being tested against this data. This is why there are no (local) self-confirmations and T-tests are fallible. Important as these issues are, we are not going to deal with them here any further, since nothing in what follows, and in the examples below, depends on it.

8FL01 [8] See Díez (2013), Ginnobili and Carman (2016) and Roffé, Bernabé & Ginnobili (in press) for some discussions of this point.

9FL01 [9] MS also introduces two other sets representing two other kinds of theoretical restrictions on T-models: Constraints (C) and intertheoretical links (L), but since nothing in our case depends on these complexities we will skip them here.

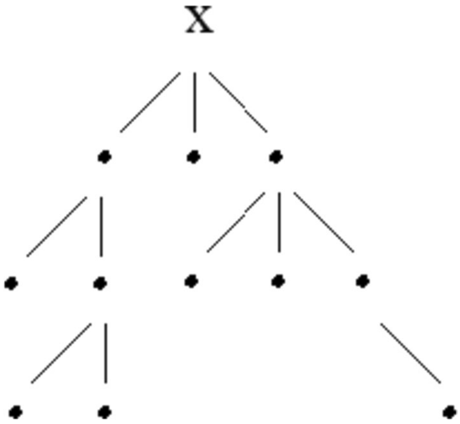

Fig. 1 Example shape of a theory net

space–time trajectories of planets around the sun, of the moon around the earth, of a pendulum, of a canyon-ball, of the body at the end of an elastic string, of a person skiing on a hill, etc. So, members of I(T) are the "T-facts" or "T-data" to be explained/predicted. Note that, as members of $M_{pp}(T)$, these intended applications can be identified and measured without presupposing the validity of T.

A Theory Element T is a pair $T = \langle K, I \rangle$. T is not a linguistic entity, but there is a linguistic entity associated with it. The "empirical claim" of T states that I is "embeddable" under K, that I "fits" with K: for every $x \in I$, there is an actual model $y \in M$ such that the non-theoretical concepts of y have the same extensions than those of x. That is, the "data" (i.e., the T-non theoretical values measured in the phenomenon) coincide with the empirical prediction of the theoretical model (i.e., the T-non theoretical values implied by the actual model)—which, remember, satisfies all the laws of the theory. E.g.: the empirical claim of CM states that the specific cinematic systems that belong to I(CM) do actually have the values they should have if there were masses and forces interacting with space and time in the specific ways dynamic laws specify. And this empirical claim is fallible, for remember that T- "data", i.e., the members of I, are identified/measured independently of the laws of T.[10]

At a given time a theory is identified with a (model-theoretic version of) "laws cum applications" entity. But the notion of Theory Element is too rigid to provide an adequate representation of that entity, for it does not pay attention to the fact (emphasized by Kuhn, and others) that "not all laws are at the same level". Some laws are more important or "central" than others. E.g., Newton's Second Law is very central and applies to all applications, the general law for elastic movements is less general, and the specific law for the simple pendulum is still less so. From the five laws presented above, only the first is fundamental, the rest are all special laws (which apply only to certain types of phenomena). At a given time, a theory has the structure of an inverted tree-like net, with its central parts/laws at the top, from where different specialization-branches open down making room for specific laws for specific applications (Fig. 1).

10FL01 [10] To avoid confusion later on, remember that this empirical claim describes how *the object theory* (in this
10FL02 case, CM) is to be tested, not the metatheory (structuralism).

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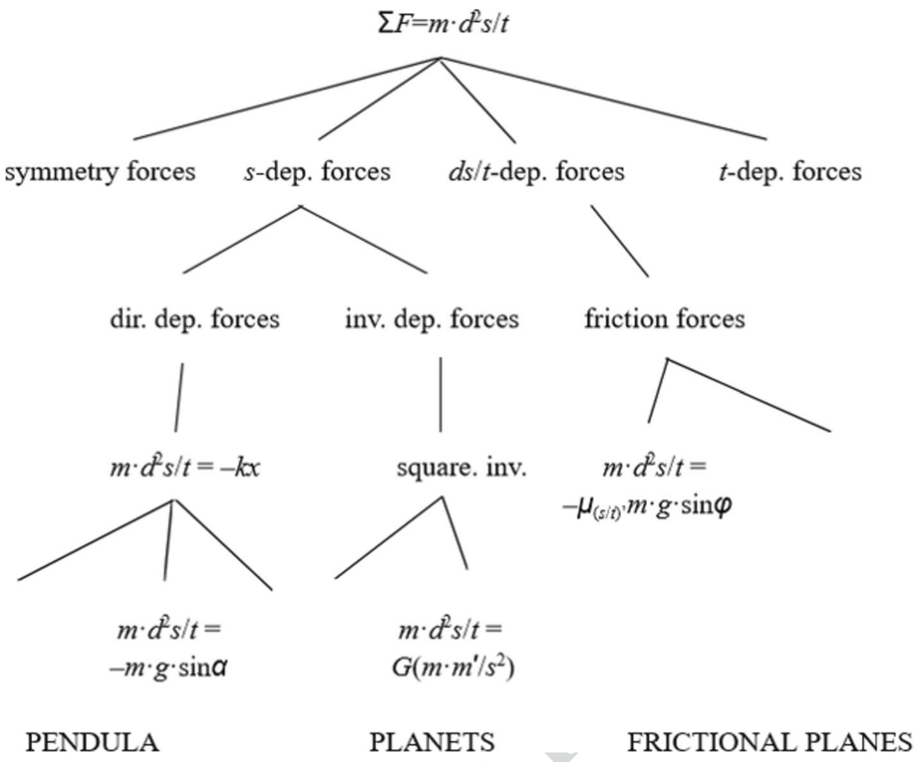


Fig. 2 Partial theory net of classical mechanics

The knots of this net are theory elements. A theory element that specializes another above it imposes additional empirical constraints (additional laws) on the actual models of the above theory element. So, the sets of actual models of lower elements of the net are subsets of the sets of actual models of upper elements in their branch. Every element of the net has its specific applications so that the sets of applications of upper elements include at least the applications of lower elements of the branch. We call Theory Net this tree-like structure made of theory elements connected by this "specialization" relation. E.g. the theory net of CM has a theory element whose actual models are constrained only by Newton's Second Law (and perhaps the Action-Reaction principle) on the top. At the second level, different theory elements impose additional constraints. One element imposes the constraint for all forces dependent on distance, i.e. it opens the "distance dependent forces" branch. Another imposes a different constraint for all forces dependent on velocity, opening the "velocity dependent" branch. And so on. At the third level, the first branch specializes into two additional sub-branches, one for forces that are directly-dependent on distance and another for forces inversely-dependent on distance. And so on. So, at the ends of these many sub-branches we find those theory elements with the more restricted sets of actual models, i.e. we find the most specific empirical constraints: the gravitation law, simple pendulum law, etc. We can show (part of) the structure of CM Theory net as follows in Fig. 2 (here 'd^2s/t' abbreviates 'd^2s/dt^2').

At a single moment, then, in the synchronic sense a theory can be identified with a Theory Net. But as we mentioned, theories, like (almost) everything else, change over time. During its history, in a diachronic sense, a theory can be identified with a sequence of theory nets so that posterior nets come from changes in anterior ones. We call such an entity a Theory Evolution. The changes are a consequence of falsified empirical claims (Kuhn's anomalies), and typically consist of (small) changes in old laws, finding out new specific laws, including new /excluding old applications, and so forth (Kuhn's normal science). The lower the position of a theory element in the net, the more likely it will be a target of the changes. The higher the position, the more general it is, the less empirical strength it has, the less likely it will be subject to change. The highest theory element is the least open to empirical refutation (e.g. Newton's Second Law is irrefutable taken in isolation). These top-net extremely general, quasi-vacuous "laws" typically behave as "guiding-principles" that have empirical import only when combined with other empirically-specific ones (cf. …). Abandoning a theory (i.e. a theory-evolution) consists in losing the previously held confidence in the possibility of solving those anomalies under the umbrella of its guiding-principle.

As we said, for our purposes, in this article, we need not enter in these diachronic complications here. We will also deal with a simplified version of CM theory net considering only the Second Law and four of its specializations (the ones formalized above).

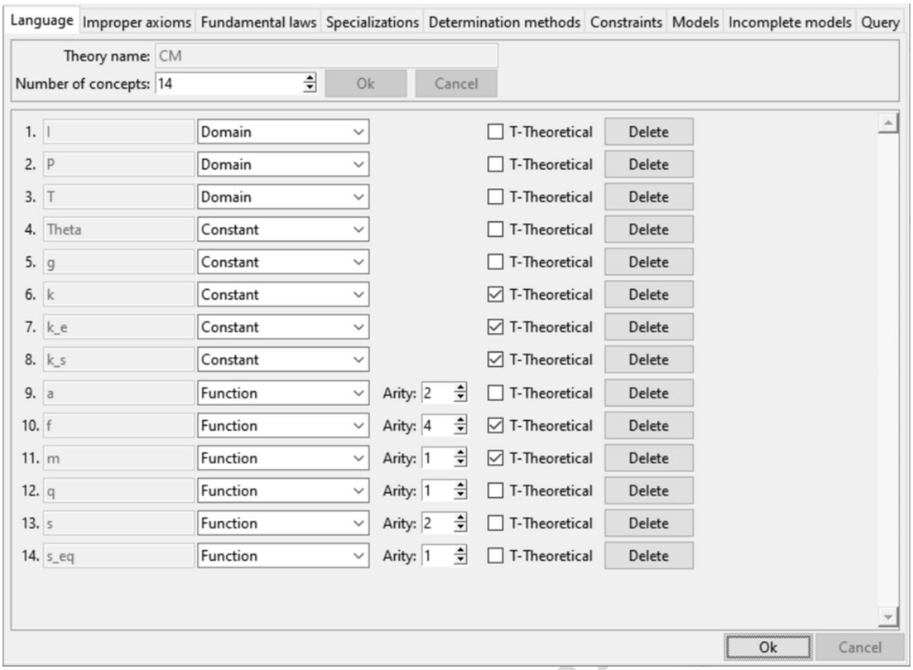
3 Testing a Reconstruction Using Reconstructor

In this section, we illustrate a systematic procedure by which one can test the adequacy of a structuralist reconstruction. Put briefly (since we will expand on this below), the procedure will consist of checking that the laws and applications of the theory "behave", according to theory's competent users, in the same way as their formalized/reconstructed counterparts. That is, that the codification of a theory's application satisfies the formalization of the laws if and only if the relevant scientific community considers the application to be a successful application of the theory. The test will be empirical in the sense that we will actually check, for various particular formalized applications, that the laws are dis/satisfied only when they are so considered by competent users (that is why we will take as examples cases in which the diagnosis by competent users is well known).

To carry out this procedure, we will utilize a computer program called Reconstructor to do the necessary calculations. As we shall see below, Reconstructor allows the user to load a structuralist formal reconstruction of a theory (by inputting a language, laws, models, etc.) and to then check, among other things, if a particular structure that interprets the language satisfies the laws (i.e. is an actual model). The full functionality of Reconstructor has been described in detail somewhere else (Roffé, 2019, 2020), so here we give a brief but sufficient exposition for our purposes. The file containing the loaded reconstruction will be made available as supplementary material (the attached CM.theory file).

Before moving on to that, note that this is only one possible procedure for testing the adequacy of formal reconstructions. In other words, this only tests one (of several) aspects in which a reconstruction can be considered in/adequate. We discuss other possible criteria/procedures in Sect. 4 below. However, we believe a detailed presentation of the procedure at hand can serve as a blueprint for devising other, complementary, tests.

In our first illustration, Reconstructor checks a prediction that a mechanical model makes of the position in which a body hanging from a spring reaches equilibrium, that is,



Theory name: CM
Number of concepts: 14

1.	I	Domain		☐ T-Theoretical
2.	P	Domain		☐ T-Theoretical
3.	T	Domain		☐ T-Theoretical
4.	Theta	Constant		☐ T-Theoretical
5.	g	Constant		☐ T-Theoretical
6.	k	Constant		☑ T-Theoretical
7.	k_e	Constant		☑ T-Theoretical
8.	k_s	Constant		☑ T-Theoretical
9.	a	Function	Arity: 2	☐ T-Theoretical
10.	f	Function	Arity: 4	☑ T-Theoretical
11.	m	Function	Arity: 1	☑ T-Theoretical
12.	q	Function	Arity: 1	☐ T-Theoretical
13.	s	Function	Arity: 2	☐ T-Theoretical
14.	s_eq	Function	Arity: 1	☐ T-Theoretical

Fig. 3 Language of the theory entered into Reconstructor

stops after descending when gravitation and recuperation forces balance each other. This equilibrium model is then an application of two specialization branches, gravitation and Hooke's.

To make things simpler to follow, we will be inputting a simplified version of CM as an example. Firstly, the program allows us to define a language, as well as choose the lexical categories of the language items in question (Fig. 3).

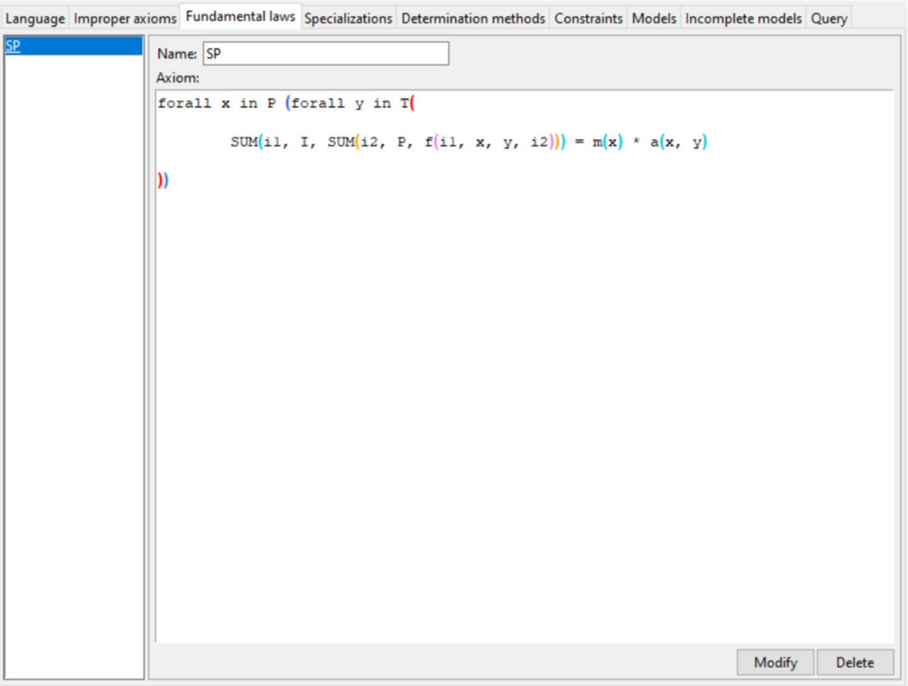
Note that we must enter the entire language of the theory at once (including all the constants used in all the special laws) for the program to be able to verify that the (fundamental and special) laws we enter next are well-formed formulae. Also note that we entered an acceleration function. As said above, this is not a primitive but a defined concept of the theory (the second derivative of space over time) and we could do without it, but it will be very convenient to have it at disposal of the program.

Once this is done, the user can load axioms (as either improper axioms or proper axioms— fundamental or special laws). In structuralist jargon, all the above would be referred to as defining the classes of potential and actual models and providing the theory-net. For example, the following Figs. 4 and 5 show how one can load the Second principle (SP) and Hooke's law in the program.

The syntax for the axioms is designed to closely resemble the one that structuralists use in practice but making it easy to input with a QWERTY keyboard (see Roffé, 2020, for a complete specification of the syntax).[11]

---

[11] Note the automatic coloring of parenthesis to make it easier to keep track of them, as well as the fact that the program ignores whitespaces.



Name: SP

Axiom:

```
forall x in P (forall y in T(

        SUM(i1, I, SUM(i2, P, f(i1, x, y, i2))) = m(x) * a(x, y)

))
```

Fig. 4 Newton's second principle entered into Reconstructor

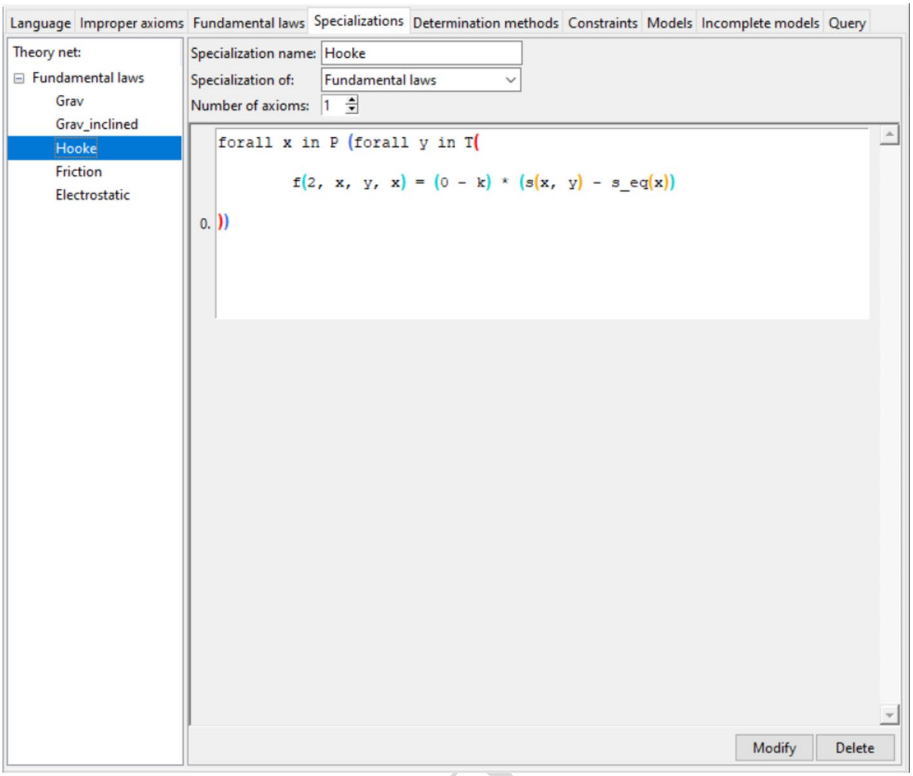
Once this is done, one may load particular models (in the sense of structures that interpret the language, which may or may not satisfy the laws). The interpretations we shall load codify different solutions for each of the following three (possible) applications of CM.[12]

In our first example, we attach a body of mass 1 kg to a massless spring, with a spring constant of 10N·m^{-1} (Fig. 6). The body's mass center is located at the spring's equilibrium point s_{eq}, which we denote as position 0 (in this and next, simplified examples, we take space to be single-dimensional; in this case positive numbers will indicate downward while negative numbers will indicate upward directions).

According to the law of free-fall, the gravitational force acting on the body will be 9,8N. When the body reaches equilibrium at time t_1, its acceleration will be 0, and according to the Second Law the sum of all acting forces must be 0 as well. Since the gravitational force of 9,8N is still present, the elastic force that counterbalances it must be $-$9,8N. Since, for this particle p, Hooke's law tells that it suffers a recuperation

---

[12] Note that, in the following examples, the intended applications are accounted for by combining theoretical laws that belong to different specialization branches of the theory-net (e.g., free fall and Hooke laws), which makes them technically "conjoin subspecializations". Whether these conjoin subspecializations may be represented in the theory-net as new "terminal" specializations, or it is better to consider their combination simply as a case of a practical joint application, is an interesting intra-structuralist issue we cannot enter here (just briefly: it is one thing to use different specializations to account for one intended application, it is another quite different thing to further specialize two different theory elements by conjoining their laws *and* adding some new ones, as it happens in a complete reconstruction of CM—as it is done e.g. in Balzer, Moulines & Sneed, 1987, ch. 4). Nothing, though, hinges on that for our present concerns.



Specialization name: Hooke

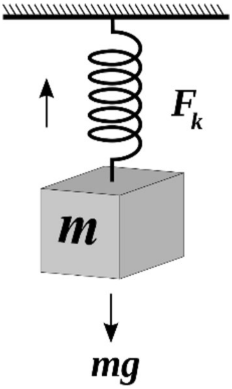
Specialization of: Fundamental laws

Number of axioms: 1

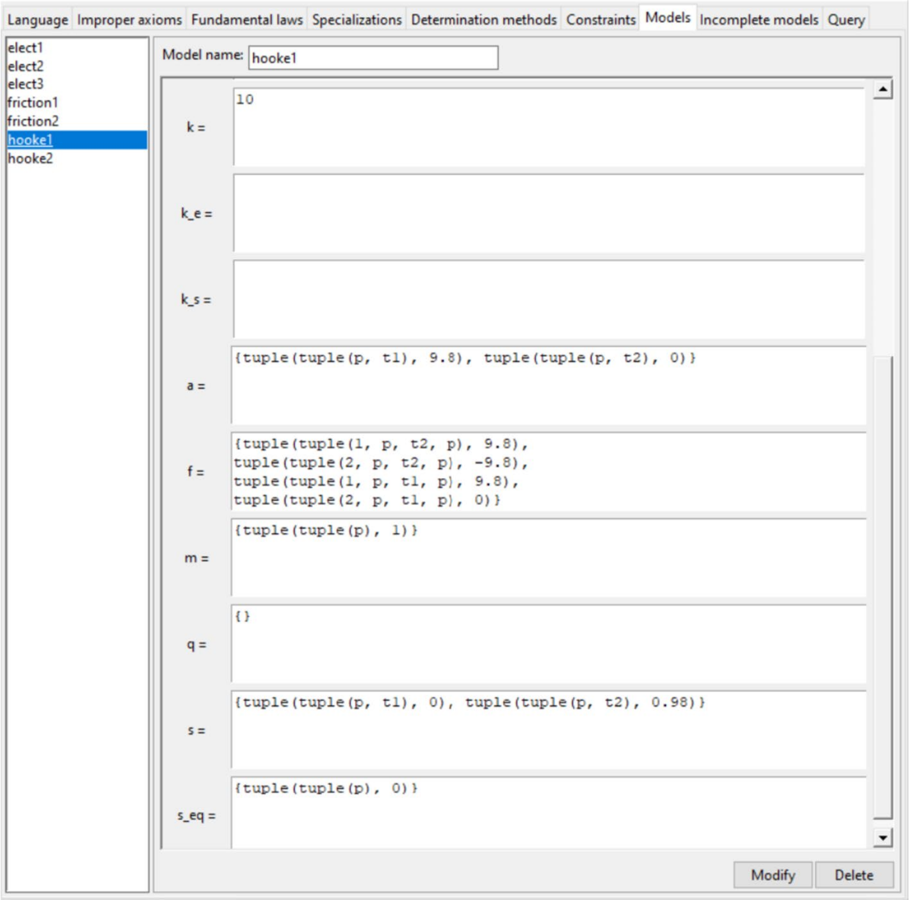
```
forall x in P (forall y in T(

        f(2, x, y, x) = (0 - k) * (s(x, y) - s_eq(x))

0. ))
```

Fig. 5 Hooke's law entered into Reconstructor

Fig. 6 Graphical representation of our first example



force $- k \cdot (s(p, t_2) - s_{eq})$, we obtain $9.8\text{N} + (\text{-}10\text{N}\cdot\text{m}^{-1}\ (s(p, t_2) - 0) = 0$, and we can infer that the correct solution for the equilibrium point should be $s(p, t_2) = 0{,}98$ m. Now suppose that we have two proposed solutions for the equilibrium, one 0,98 m and the other 1 m. Figure 7 shows the first of these two (potential) models loaded into Reconstructor:



Model name: hooke1

```
k = 10
k_e =
k_s =
a = {tuple(tuple(p, t1), 9.8), tuple(tuple(p, t2), 0)}
f = {tuple(tuple(1, p, t2, p), 9.8),
tuple(tuple(2, p, t2, p), -9.8),
tuple(tuple(1, p, t1, p), 9.8),
tuple(tuple(2, p, t1, p), 0)}
m = {tuple(tuple(p), 1)}
q = {}
s = {tuple(tuple(p, t1), 0), tuple(tuple(p, t2), 0.98)}
s_eq = {tuple(tuple(p), 0)}
```

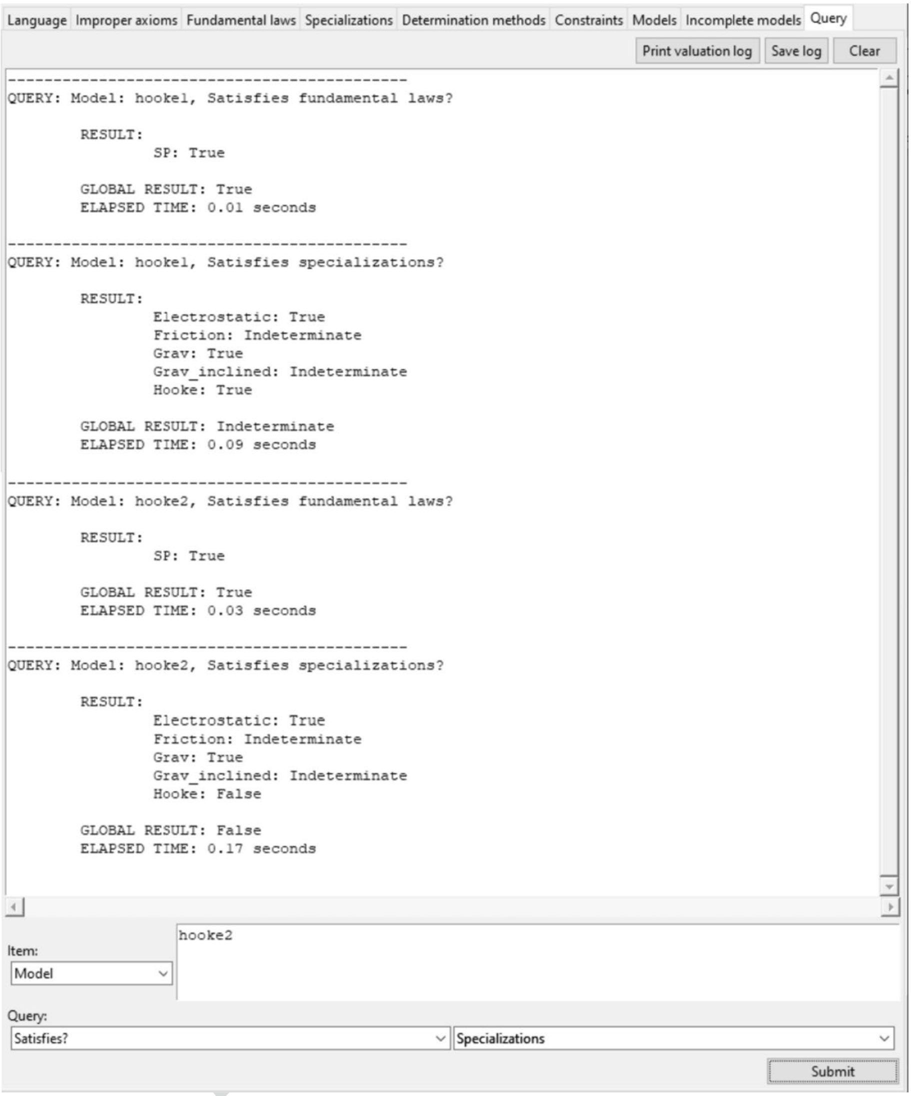
Fig. 7 A particular interpretation entered into Reconstructor. Notice that in the “Models” tab of the program one loads interpretations for the language which may or may not satisfy the laws (i.e. they may not be actual models of the theory)

Note that the constants pertaining to frictional or electrostatic models were left with an empty interpretation.[13]

All this is useful because one may then query the program for whether the given interpretations satisfy the laws (i.e. if the interpretation is a potential and/or an actual model of the theory). For instance, Reconstructor determines that the first interpretation above (the one that has a correct value for the final position, named “hooke1” in the system) satisfies both the second principle and Hooke’s law, while the second (the one that

---

[13] If, later on, we ask Reconstructor to evaluate sentences in this model that make use of those constants, they will likely get the indeterminate value as an output. In fact, Reconstructor uses a 3-valued paracomplete logic to evaluate sentences in models. The semantics are such that for classical inputs, the program always returns classical values (true and false), but we might get the indeterminate value if some denotation is empty or incomplete. See Roffé (2019, 2020) for a complete specification of the semantics that the program uses.



```
--------------------------------------------
QUERY: Model: hooke1, Satisfies fundamental laws?

        RESULT:
                SP: True

        GLOBAL RESULT: True
        ELAPSED TIME: 0.01 seconds

--------------------------------------------
QUERY: Model: hooke1, Satisfies specializations?

        RESULT:
                Electrostatic: True
                Friction: Indeterminate
                Grav: True
                Grav_inclined: Indeterminate
                Hooke: True

        GLOBAL RESULT: Indeterminate
        ELAPSED TIME: 0.09 seconds

--------------------------------------------
QUERY: Model: hooke2, Satisfies fundamental laws?

        RESULT:
                SP: True

        GLOBAL RESULT: True
        ELAPSED TIME: 0.03 seconds

--------------------------------------------
QUERY: Model: hooke2, Satisfies specializations?

        RESULT:
                Electrostatic: True
                Friction: Indeterminate
                Grav: True
                Grav_inclined: Indeterminate
                Hooke: False

        GLOBAL RESULT: False
        ELAPSED TIME: 0.17 seconds
```

Item: Model
hooke2

Query: Satisfies? Specializations

Fig. 8 Responses to queries of whether the two interpretations, hooke1 and hooke2, satisfy the fundamental and special laws. The output of the program appears in the big white box at the top. (The reason why the law of electrostatics comes out true is simply that (as we shall see below) it applies to at least two different particles, i.e. the law begins by stating "for all p_1 in P, for all p_2 in P (if $p_1 \neq p_2$ then …)". Since these interpretations contain only one particle, the antecedent of the conditional is always false, and the law comes out as (vacuously) true. Nothing substantive for our case hinges on this, though.)

has an incorrect value for the final position, named "hooke2") does not satisfy Hooke's law (see Fig. 8). We could also load a third interpretation that satisfies Hooke's law but does not satisfy the second principle, with the same incorrect distance (by changing the denotation of the force function, we leave this as an exercise to the reader).

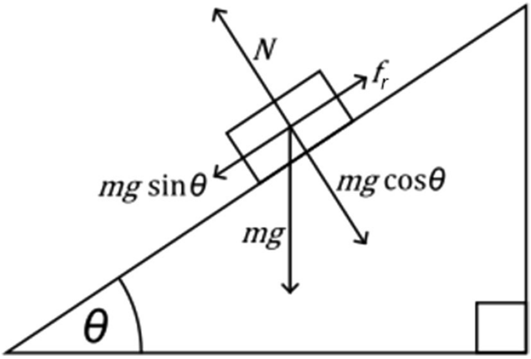


Fig. 9 Graphical representation of our second example

This is exactly what we would expect. The first interpretation depicts/codifies a situation where the laws of the theory hold, while the second depicts/codifies a situation where they do not hold. In fact, this is one of the procedures that can be used to test the adequacy of a formal reconstruction. Interpretations that represent paradigmatically successful applications of the theory should satisfy the formalization of the laws; and vice-versa, interpretations that codify non-successful applications should not satisfy all the laws.

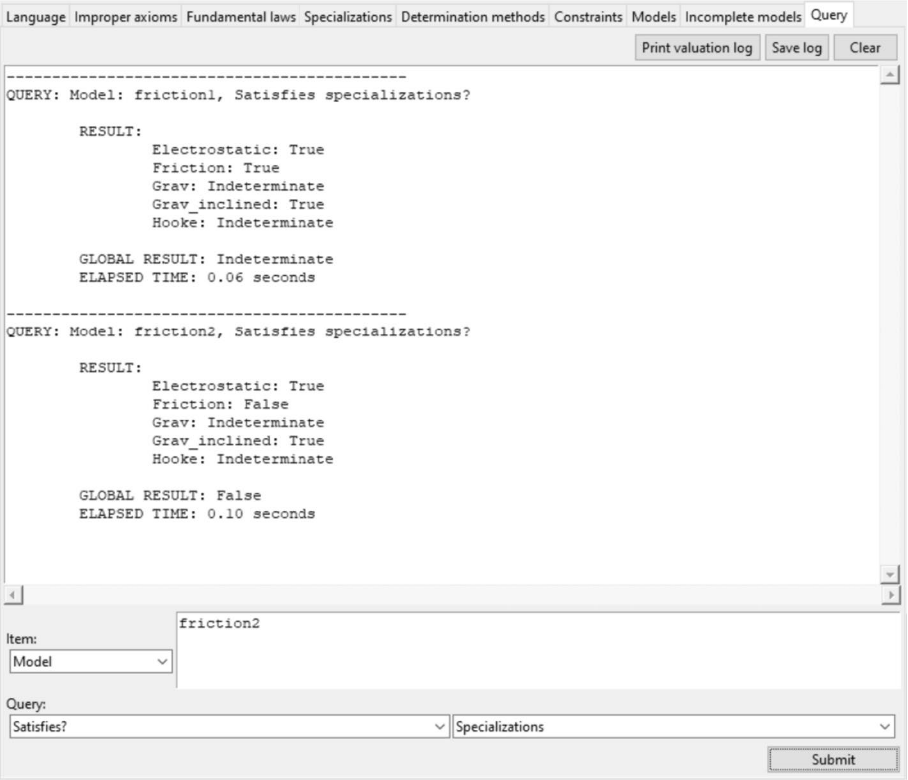
Our second illustration involves an application of dynamical equilibrium to frictional planes. To simplify the exposition, we do not take a case of kinetic friction that may depend on velocity but a case of static friction that depends only on the surface. In this case actual models are the potential models that satisfy the Second Law, the action-reaction principle, the free fall gravitation law and the static friction law. These actual models are again applications of different specializations within CM's theory net under the Second Law as the common general principle.

Now what Reconstructor checks is the prediction that a mechanical model makes of the friction coefficient of an inclined plane, such that any lower value will cause a body to descend. The system involves three forces, the gravitational weight that decomposes into one component parallel to the plane and other component perpendicular towards the plane; the normal reaction force N opposite to the latter gravitational component, and the frictional force $f_r = k_s \cdot N$ that depends on the static coefficient of friction k_s for the given surface (Fig. 9).

The limit point before the particle starts moving is the equilibrium point in which, given the Second Law, all forces add up to 0. Since the normal force N is equal to $-m(p) \cdot g \cdot \cos(\Theta)$ (i.e. the reaction of the component of the gravitational force orthogonal to the plane), the equilibrium reduces to the case in which f_r equals the component of the gravitational force parallel to the plane: $m(p) \cdot g \cdot \sin(\Theta) = -k_s \cdot N$ (here we take the center of mass in equilibrium as the 0 spatial point and downward parallel to the plane as positive and upward as negative displacements).

To load all this into Reconstructor we must be careful, since computers operate with floating point arithmetic, which may introduce slightly incorrect results for the usual trigonometric operations. Thus, the law is loaded as $|f(4, p, t, \emptyset) - k_s \cdot m(p) \cdot g \cdot \cos(\Theta)| < 0.00000000001$.

To take some concrete values (i.e. to add some particular interpretations), let the body have a mass of 1 kg and the angle Θ be 30°. Since $\sin(30^\circ) = ½$, the gravitational force acting on the body will be 4.9N. Again, since it is in equilibrium, its acceleration will be 0, and according to the Second Law the sum of all acting forces must be 0 as well. Hence, friction must be acting with a force of -4.9N. Thus, we get that $f_r = k_s \cdot N = -4.9$N.



```
--------------------------------------------
QUERY: Model: friction1, Satisfies specializations?

        RESULT:
                Electrostatic: True
                Friction: True
                Grav: Indeterminate
                Grav_inclined: True
                Hooke: Indeterminate

        GLOBAL RESULT: Indeterminate
        ELAPSED TIME: 0.06 seconds

--------------------------------------------
QUERY: Model: friction2, Satisfies specializations?

        RESULT:
                Electrostatic: True
                Friction: False
                Grav: Indeterminate
                Grav_inclined: True
                Hooke: Indeterminate

        GLOBAL RESULT: False
        ELAPSED TIME: 0.10 seconds
```

```
friction2
```

Query: Satisfies? | Specializations

Fig. 10 Responses to queries of whether the two models, friction1 and friction2, satisfy the special laws

Therefore, $4.9\text{N} = k_s - m(p) \cdot g \cdot \cos(30°)$. The solution for k_s will be $1/(2 \cdot \cos(30°))$, which is approximately 0.5773502691896257. This (potential) model is loaded with the name "friction1" into Reconstructor. A second (potential) model, called "friction2" is identical but has an incorrect value of 0.5 for k_s.

If, once again, we ask Reconstructor whether these two interpretations satisfy the special laws, we get the expected output (Fig. 10).

The fact that Reconstructor outputs the values we expect it to, is, once again, evidence that the reconstruction is adequate (more on this in the next section).

According to MS, both cases are different applications of several specializations of the Second Law. This means that, in both cases, a failing prediction would not be fixable by changing the Second Law, since this would affect models in other specialized branches that also presuppose it. It would be possible, however, to change some other more specific laws while preserving the Second Law. One could say that the same applies in these two cases with respect to the gravitational free-fall law, since we would not be permitted to change it in one case but not in the other. This is so only by the nature of the examples chosen, since both make essential use of the free fall force/weight too. However, this is not the case in general, as the following example, in which the free fall law plays no role, shows. In this case, we have two springs with two particles with electrical charges of different sign, horizontally exerting attractive electrical force pulling each other according to Coulomb's law;

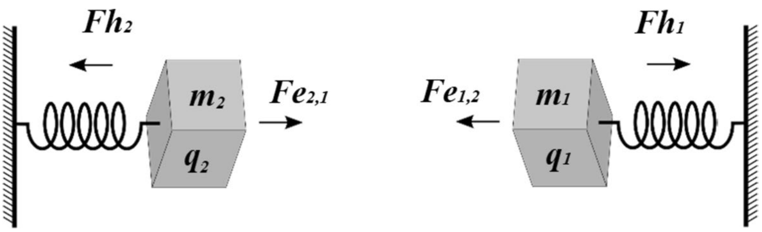


Fig. 11 Graphical representation of our third example

here weight and friction are negligible, and the only other force intervening is the recuperation elastic force of the springs. The free fall law then plays no role but the Second Law still does (Fig. 11).

We have, on a frictionless surface, a particle p_1 of mass m_1 and positive charge q_1 at the extreme of the right spring with elasticity constant k_1, and a particle p_2 of mass m_2 and negative charge q_2 at the extreme of the left spring with elasticity constant k_2. With given values for charges and elastic constants, and taking the 0 point as the initial position of p_2, and 102 m as the initial position of p_1, at initial time t_1; the problem to solve is to find out the positions in which each charge reaches equilibrium at final time t_2, that is the displacement x (final minus initial positions, that is $s(p, t_2) - s(p, t_1)$) of each particle for reaching equilibrium. Since there is no friction, weight has no dynamical effect and as we said the only relevant forces are the electrostatic and the elastic forces acting on each particle. Thus, here actual models are potential models satisfying Coulomb's and Hooke's laws and the Second Law.

In this situation, and given the Second Law, each particle i reaches equilibrium when the electric force $f_e = 9 \cdot 10^9$ (N·m^2·C^{-2}) $q_i \cdot q_j$ / $dist_{ij}^2$ it suffers and the elastic recuperation force $-k_i \cdot x$ of its spring add up to 0.[14] For the sake of simplicity, we take both masses to equal 1 kg, the charges to be of opposite sign and equal module, say 0,01 C for p_1 and -0.01 C for p_2, and the springs to have equal elastic constants, say $k_1 = k_2 = 90$ N.m^{-1}. In such conditions, the displacements of x_1 and x_2 before equilibrium coincide. With these specific values, we obtain $x_1 = x_2 = 1$ m; that is, the final equilibrium positions are $s(p_1, t_2) = 101$ and $s(p_2, t_2) = 1$.

Once loaded into Reconstructor (with the name "elect1"), the program outputs that this interpretation satisfies both the second principle, Coulomb's law and Hooke's law. If, however, one modifies some values (for instance, the final positions or the charges of the particles) then some laws come out false in the program. This, again, is exactly what we would expect.

An example of what we just mentioned can be found in the complementary CM.theory file. There, we also load an interpretation, named "elect2", that modifies the charges to 0.001C and − 0.001C, which does not satisfy Coulomb's law (because the corresponding electrostatic forces would need to be 0.9N and − 0.9N instead of the loaded 90N and − 90N).

To illustrate how Reconstructor represents the centrality of the Second Law, as the structuralist metatheory claims, consider the following scenario. Imagine a scientist that

---

[14] In Reconstructor, we multiply the previous version of Coulomb's law by a factor of $(s(p_1, t) - s(p_2, t)) \,/\, |s(p_1, t) - s(p_2, t)|$. This is just to give the corresponding force a direction.

looks at an empirical setup similar to the above, with the two bodies at equilibrium 100 m apart, and she measures (independently of CM) that the electrical charges of the bodies are 0.001C and − 0.001C. She then has at least three options to fix this situation: (i) consider her data to be incorrect (perhaps she measured the charges or the distance incorrectly), (ii) consider either Coulomb's or Hooke's law to be incorrect / in need of modification and (iii) consider the Second Law to be incorrect / in need of modification. Of course, these options are non-exclusive, but for simplicity's sake let us consider that our scientist wants to change as little as possible in the theory, so she will only be willing to take one of those routes.

Assume she is convinced that here data are correct. Option (ii) could consist of, for example, changing Coulomb's law to the following:

$$5b.\ \forall p_1 \in P, \forall p_2 \in P, \forall t \in T : f\left(4, p_1, t, p_2\right) = k_e \cdot \mathbf{100} \cdot q\left(p_1\right) \cdot q\left(p_2\right) / (s(p_1, t) - s\left(p_2, t\right))^2$$

Thus, the electrostatic forces will be 90N and − 90N and all will be well (the bodies are at equilibrium, with the springs counteracting them).

Option (iii) would entail leaving Coulomb's law as is and changing the Second Law. Since Coulomb's law is unmodified, the electrostatic forces will actually be 0.9N and − 0.9N. Thus, for the bodies to be at rest (to have zero acceleration), the Second Law will need to be changed to something like:

$$1b.\ \forall p \in P, \forall t \in T : \left|\Sigma_{i\in\mathbb{N}}\Sigma_{p2\in P\cup\{\emptyset\}} f\left(i, p, t, p_2\right)\right| - \mathbf{89.1N} = |m(p) \cdot a(p, t)|$$

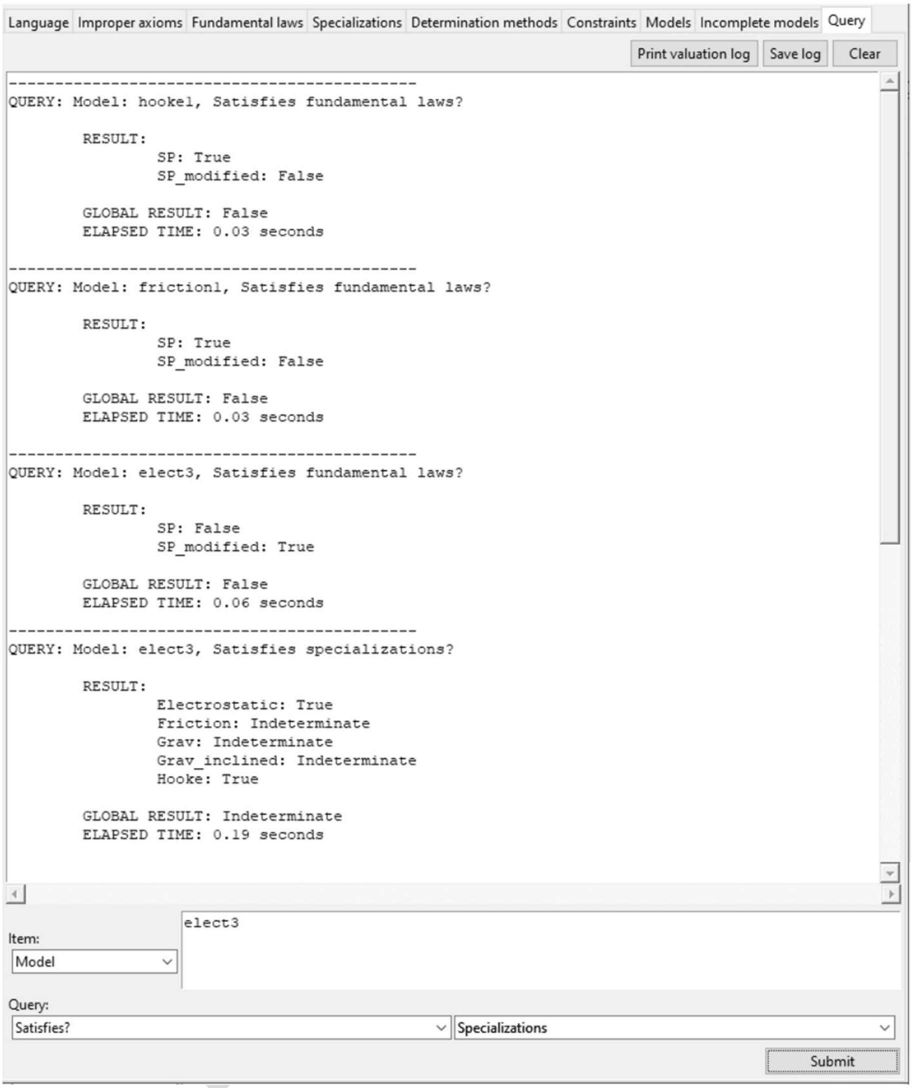
What will obviously happen if one goes down this last path is that many more applications of the theory will be affected, including those that have nothing to do with electrostatic forces (such as the two previous applications above).

All this is illustrated by Reconstructor. One can actually load these two modified laws and test what happens with the various models one has. For instance, if we modify Coulomb's law as above, then the models "hooke1" (the model from the first situation, with all values correct), "friction1" (the model from the second situation, with all values correct) and "elect2" (the interpretation with 0.001C and − 0.001C charges, and 90N and − 90N forces) all come out as satisfying the specializations and fundamental laws.

If, however, one changes the Second Law instead of Coulomb's law (we now test a model "elect3" with 0.001C and − 0.001C charges, and 0.9N and − 0.9N forces), the following happens (Fig. 12):

Notice that the interpretation "elect3" satisfies both the modified version of the Second Principle ("SP_modified") as well as Coulomb's and Hooke's law, and all is well *for this particular model*. However, the interpretations "hooke1" and "friction1" now do not satisfy the modified version of the Second Law.

Thus, MS predicts, and the MS program in Reconstructor implements, that in case of anomalous, failing predictions, CM scientists will first try to fix them by changing special laws instead of the Second Law. This (meta) prediction might fail, though, if scientists started by trying more radical solutions. For instance, in the case of the anomaly of Mercury, CM scientists could have tried to fix it by changing the Second Law accordingly *only for the case of Mercury*. But this never happened, and instead they tried several alternatives (the postulation of Vulcan, of celestial dust claws, even the modification of the gravitational law—see Giné, 2008), but within CM they never tried to change the Second Law (actually, following Kuhn, one can say that working within CM consists, among other things, in being committed to fix anomalies without changing the Second Law).



```
--------------------------------------------
QUERY: Model: hooke1, Satisfies fundamental laws?

        RESULT:
                SP: True
                SP_modified: False

        GLOBAL RESULT: False
        ELAPSED TIME: 0.03 seconds

--------------------------------------------
QUERY: Model: friction1, Satisfies fundamental laws?

        RESULT:
                SP: True
                SP_modified: False

        GLOBAL RESULT: False
        ELAPSED TIME: 0.03 seconds

--------------------------------------------
QUERY: Model: elect3, Satisfies fundamental laws?

        RESULT:
                SP: False
                SP_modified: True

        GLOBAL RESULT: False
        ELAPSED TIME: 0.06 seconds

--------------------------------------------
QUERY: Model: elect3, Satisfies specializations?

        RESULT:
                Electrostatic: True
                Friction: Indeterminate
                Grav: Indeterminate
                Grav_inclined: Indeterminate
                Hooke: True

        GLOBAL RESULT: Indeterminate
        ELAPSED TIME: 0.19 seconds
```

Item: Model

elect3

Query: Satisfies? Specializations

Fig. 12 Responses to various queries including the modified version of the second principle (SP_modified)

As said above, one can postulate different procedures to check the adequacy of other aspects of formal reconstructions. The procedure just outlined looks mainly (though not only, as we will discuss in the next section) at the formalization of the laws and the construction of the (potential) models. Other procedures could look at the determination methods, intertheoretical links, constraints, etc. Reconstructor could also facilitate the execution of some of those other possible tests, but we shall not go into these complications here (for that, see Roffé, 2020). In the next section, we discuss some philosophical issues related to these reconstruction tests.

4 Discussion

4.1 Holism

The first point to note is that the above empirical test of a formal reconstruction (and indirectly, of the metatheory with which this reconstruction has been built) is as subject to the phenomenon of holism as any test of any empirical theory.

To see this, suppose that the procedure delineated in the previous section fails. That is, that Reconstructor determines that at least one of the laws of the theory is false for an interpretation (i.e. a potential model) we informally believe should satisfy them. For instance, suppose that in Fig. 8, Reconstructor outputs that the interpretation $hooke_2$ does satisfy Hooke's law (i.e. it outputs "true" instead of "false"). What must we conclude out of this fact?

A first answer would be that our reconstruction is inadequate. But, and here the phenomenon of holism makes a first appearance, it could be inadequate in different ways. Firstly, it could be the case that we informally thought about the law correctly but formalized it incorrectly. For instance, perhaps we informally thought "the sum of forces for a given particle at a given time equals its mass times its acceleration at that time" but then used a product instead of a sum. This would be an error in the move from an "informal reconstruction" to a formal one, a formalization mistake.

It could also be the case that the informal reconstruction was wrong in the first place, and that the formal law was a correct formalization of this inadequate initial understanding. Or it could be that *both* our informal understanding and its corresponding formalization are inadequate. The point is that a failure in a test like that of Sect. 3 tells us that there is something wrong somewhere, but it doesn't tell us precisely where. We are *at the same time* testing our informal understanding of the theory and its formal representation.

And this is not all that we are testing. Note that we also translated the informal description "we attach a body of mass 1 kg to a massless spring, with a spring constant of $10\text{N}\cdot\text{m}^{-1}$…" into a formal model (a set-theoretic structure with domains, relations, and so on). It could also be the case that both the informal and formal understanding of the laws were correct, but that our error lied in the construction of the model. Or, again, there could be mistakes in any combination of these steps. And, again, the procedure outlined above would not tell us where the error lies.

Or, still, the error could lie in Reconstructor itself, so that the informal understanding, the formalization of the laws and the construction of the formal model are all ok, but the program contains a bug in its code. Likewise, the code of the program itself could be ok, but the error could lie in the programming language it was written in, or even in the hardware of the system it is running on.[15] In the same way, if the calculations were done by hand instead of by a computer, correct execution of the calculations is what would be presupposed.

Finally, as Quine (1951) and others have noted, mathematics and/or logic are also at stake in these tests, although modifying them would come at a great cost, so we tend not to

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[15] As a side note, the second point is worth mentioning since the use of computer programs in science is ubiquitous, and computational tools are not the kind of thing that is usually thought of as being a part of what is being tested in discussions of holism. Also, to reassure the reader, both Reconstructor and the programming language it is written in (Python) have been extensively tested (which is not to say that some corner cases may originate bugs).

take this route. All the above shows that when testing a reconstruction (and a metatheory) the phenomenon of holism is as present as in any other test of any empirical theory.

Also note that if Reconstructor does behave as we expect (i.e. for a particular reconstruction, it outputs "true"/ "false" when we expect it to) we cannot definitely conclude that the reconstruction is adequate, for it is possible that there exist two errors that counterbalance each other. Disappointing as this underdetermination might seem, it has nothing specifically to do with our case, it is just a feature as characteristic of metatheories as it is of empirical theories.

4.2 Circularity

A second interesting point is the following. Putting aside the issue of holism, let us assume for a moment that all the background knowledge is fixed (the code works as expected, the translation of the empirical situation into a model is adequate, etc.) and that all we are testing is the adequacy of the laws. One could argue that, even then, the test is circular, because Reconstructor is built using the structuralist metatheory. So, it seems that we are testing Structuralism with a program that assumes it.

Our first straightforward answer would be that the computational design of Reconstructor does not actually assume Structuralism. All it does is some basic logic stuff. As we saw in Sect. 3, all it does (or at least all we used) is to allow one to define a language, axioms and interpretations for that language, and check if the models satisfy the axioms. Structuralism is much more than this.

The objector could insist that there are more assumptions baked into Reconstructor. For instance, the fact that all (fundamental and special) laws can be evaluated in all models. One cannot (as was exemplified above with the case of Mercury) load a modified version of the second law and tell the program that it should only be evaluated in one model. In other words, one cannot change the laws from one model to another. To this we would reply that, although it is true that this is more than a purely logical assumption, it still does not presuppose a specifically structuralist thesis, but something much weaker shared by many metatheoretical frameworks (we could call it trans-modelic nomological coherence). We then affirm that the program itself does not assume specific structuralist (meta) principles, although one of course can use it in a way that mirrors structuralist ideas. For instance, concepts that are used only in some special laws were left with an empty interpretation in models that represent cases in which they do not apply. In contrast, the concepts that pertain to the fundamental law are always assigned a denotation, because we want the fundamental law to always come out true. Notice that the above-mentioned fact that changing the Second Law will affect all models (as a consequence of the way we load them), but changing a special law will not, is not a feature of the program, but of the way we use it, following the structure of CM itself as MS understands it.

This issue is related to another relevant point. If all Reconstructor does is some basic logic stuff, are we only testing the move from an informal understanding to a formal reconstruction? Let us clarify this. A usual *modus operandi* of structuralists when making a reconstruction is to first think informally about the concepts, laws, etc. of the theory, and only then codify them into formal axioms and theory structure. This last part, one may object, seems like the only thing we did in Sect. 3. Although according to the above description of what the program does it could seem that it merely checks the final part of this process, this is not so. As noted in the previous subsection, the result that Reconstructor outputs is a function not only of the adequacy of the formalization per se, but of all the steps involved before it (including the correct informal understanding of the theory and its informal reconstruction). So even if the program itself only does some basic logic calculations, it is testing the output of a more complex series of steps.

4.3 Predictions

So far, we have shown that metatheories (Structuralism in particular) are empirically testable. We also have argued, in Sect. 4a, that they resemble tests of scientific theories in being subject to holism. One additional point in which metatheories could be similar to empirical theories is related to the predictions both make. In a typical test of an empirical theory, one makes a prediction and then "observes" (in some sense) if it holds or not. The additional point worth considering is this: What is the prediction at stake in a (meta-)test like this (if any)? The question can be restated in approximately this way: if one were to consider Structuralism as an empirical framework, what would it tell us we should find about which parts of our experience?

This is a difficult question, and one for which we do not yet have a complete answer. But part of the answer is clear. For instance, it is clear that the domain of application of Structuralism (what it speaks about) are standard expositions of empirical theories, as they can be found in textbooks and specialized papers. That is what the metatheoretician takes as their "observed" input when they face the task of building a reconstruction. What structuralism tells us is that, given a standard exposition, we should find, among other things, the following:

1. Potential and actual models (this is like saying a language and some laws).
2. An application of the theory, represented as a potential model, should satisfy the fundamental and special laws if and only if the application is informally considered to be successful.
3. If the theory is nontrivial, then the actual models should be a proper subset of the potential models (at least in the terminal specializations, see below). That is, non-trivial theories should put some restriction on what can happen.
4. If the theory unifies many different types of phenomena, then it should include some special laws, that restrict the actual models differently for each branch under a common theory element. Moreover, this process is recursive, as special laws can be added to restrict already specialized theory elements, forming a tree-like structure (in structuralist terms, a theory-net).
5. If the theory is explanatory and not purely phenomenological, then the potential models should be ampliative over the partial models.[16]
6. The fundamental law is rarely put into question by the users working within the theory. If an application apparently falsifies the theory, scientists will attempt to restore empirical adequacy by modifying specialized elements first.[17]

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[16] This is debatable and assumes Díez's account of explanation (see Díez, 2013). There is also a debate as to whether all or some of the new concepts in the *explanans* have to be T-theoretical, see the papers mentioned in footnote 8.

[17] This is not to say that fundamental laws are never revised, but only that when they are, then, in Kuhnian terms, a scientific revolution is taking place (i.e. the researchers are not working with the same theory anymore).

If something like the above is what structuralism is about, then the prediction(s) being made are clearer (given a standard exposition, they consist in at least those six statements). Note that 2 and 6 look a lot like what we postulated as the testing procedure of Structuralism in Sect. 3. As was emphasized at various points in the paper, one could perhaps devise other testing procedures for structuralism using the other predictions.

Also note that these predictions are expressed in the specific formulation they assume in the structuralist metatheory. One could perhaps find equivalents to some of them in other metatheories (e.g. prediction 6 in Kuhnian or Lakatosian frameworks; prediction 2 in any metatheory that attempts to use formal tools to reconstruct empirical theories)[18]—which is unsurprising, since MS takes inspiration from some of these other accounts. Debatably, the tests we introduced in this paper can be seen as confirming not just MS but also other metatheoretic proposals as well (again, one would have to see if the specific formulations of the relevant predictions in them allows this conclusion). In contrast, other predictions (e.g. numbers 4 and 5, especially the discussion mentioned in footnote 17) are unique to MS. If one's goal was to make a crucial test between different metatheories (which is not our goal here) then these kinds of unique predictions would take a special importance.

5 Conclusions

Philosophical metatheories like Sneedian Structuralism intend to speak about parts of our socio-cultural reality (empirical theories). As such, they should be subject to rigorous empirical testing, in the same way that any other systematic body of empirical knowledge is. Additionally, if one takes either an empiricist or a naturalistic stance, the question of empirical testability of metatheoretical products becomes more pressing. In this paper, we have attempted to illustrate how a systematic testing procedure for metatheories might look like. As a case study, we focused on Structuralism as our metatheory, but the procedure can be easily adapted to other metatheories. We began by showing how an empirical theory, CM, can be formally reconstructed within MS, and used the example to briefly introduce some central tenets of structuralism itself.

We presented the testing procedure, together with a computer program (Reconstructor) that can aid in performing the necessary calculations. The general idea for the procedure is to check whether the formal translation of our informal understanding of the theory behaves as we expect. In more specific structuralist terms, to see if the translation of an informally successful application into a model satisfies the formalization of the laws (and vice-versa for unsuccessful ones). The way in which Reconstructor helps with this process is by allowing one to load a formal language, laws and models, and also to query for whether the models satisfy the laws, all with a syntax close to the one used in practice by structuralists and with a friendly graphical interface (that does not assume the user knows how to code).

Note that the issue of the empirical testablity of metatheories and the specific testing procedures we have proposed are in principle independent of the computer program we presented to implement/aid those tests. All the necessary calculations could be made by hand, although that would be both more prone to error and incredibly tedious, since it would involve recursively calculating thousands of valuations (the reader can click the

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[18] We thank an anonymous reviewer for bringing this to our attention.

"Print valuation log" in the Query module to see just how many valuations are being calculated by the program). We hope that the practical usefulness of the program has been thoroughly illustrated in the examples above.

We have shown that the proposed empirical tests for metatheories suffer from the same holistic underdetermination we find in empirical tests of standard (i.e., non-meta-) empirical theories: a failure in the procedure does not locate where error lies (it could be in the formalization of the laws, in the representation of the application as a model, in our informal understanding of the theory, in the computer code, etc.). We have also argued that, though holistic, the test is not circular: the Reconstructor program does not presuppose specific structuralist principles; even if it does presuppose some metatheoretical theses (e.g. what we called transmodelic nomological coherence), these are a lot weaker than specific structuralist tenets.

Finally, with regard (meta)predictions, that is the aimed agreement between what the metatheory predicts and what we "observe" in scientific practice, we identified six central structuralist theses from which (meta)theoretical predictions can be derived, and pointed to two among them on which our testing procedure focused. We conclude that, other differences notwithstanding, our case study offers several interesting respects in which testing meta-empirical theories is relevantly similar to testing standard empirical theories.

6 Ethical Statement

All authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

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