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Numerical simulations for the study of the methods for stellar kinematics and luminousity calibration

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Abstract. We created numerical simulations of a sample of K0 III stars with an ellipsoidal velocity distribution, an exponentially increasing density distribution and a prefixed solar motion. On the basis of the numerical samples we study the solar motion, using the known classical methods. We can assess in this way the external errors of the methods. We find that the dispersion of the results is rather large, except for large samples. Typical values of the uncertainty in the apex position for samples of one thousand stars are ± 0.7 km s^{-1} in V_0 and $\pm 2°$ in both l_0 and b_0.

We study the possible influence upon the solar motion of different factors which one encounters in practice like incomplete sky coverage, different ellipsoidal distributions and different values of the absolute magnitude dispersion. For the first factor, the uncertainty increases with decreasing sky coverage. For the two others we found that no important bias is produced, if these are not too far away from the normal values.

In the second part we use the artificial samples to compare the accuracy of different methods fo statistical parallax determinations. We conclude that the maximum likelihood method is definitely better than the classical methods, providing typically results with erros of ± 0.2, whereas in the classical methods the errors are of the order of ± 0.5.

Key words: Methods; numerical – stars; distances – stars kinematics – Galaxy: kinematics and dynamics

1. Introduction

In the classical methods for deriving the solar motion and the stellar luminosity calibration from statistical parallaxes there remains always a certain number of problems which are difficult to elucidate because no analytic solution is available.

A first example is given by the evaluation of the precision of the results of the solar motion. It is difficult to know what is the external error of the procedure used, which may be several times larger than the (internal) errors quoted.

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Another example is provided by the fact that often one has to work with samples of stars which are not distributed over the whole sky, but ony over part of it. It is not easy to assess if methods, which are valid for a whole sky coverage also apply to this case.

Another example is to see if the shape of the velocity ellipsoid interferes with the apex determination – in principle one would like to be sure that no mutual influence of this kind exists.

As remarked before, no analytic solution exists for these and other similar problems. We have thus constructed artificial samples, in which all parameters can be varied as one wishes, so that the influence of different factors can be analysed in detail. This is essentially what we have done in this paper.

We start constructing a realistic sample of stars, which imitates as closely as possible the real sample. We use then this sample to study the different problems one finds in practice.

2. The samples

We have constructed samples simulating K0 III stars, imposing the following conditions: Be x_i, y_i and z_i the space coordinates of star "i". We select random numbers x, y, z in such a way that the mathematical expectation fulfills the condition

$$\sum x_i = \sum y_i = \sum z_1 = 0 \, .$$

We request further, that the space density of the points be constant in the galactic plane ($z = 0$) and that outside there exists an exponentially decreasing density of $\exp(-|z|/\beta)$ with $\beta = 120$ pc.

We assume further that the sample has an average absolute magnitude $M_0 = 0.7$ and that the Gaussian dispersion is $\sigma = 0.5$. Individual M_i values are then assignated to each star through a random process.

Knowing M_i and x_i, y_i and z_i one may then calculate individual apparent magnitudes for each object.

We next assign X_i, Y_i and Z_i triples to each star, which represent its random space motion components. These values

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are selected from Gaussian functions for each X, Y and Z. We assume for the components that the mathematical components

$$\sum X_i = \sum Y_i = \sum Z_i = 0 \, .$$

For the dispersions we assume $\sigma(X) = 29$ $\sigma(X) = 18$ $\sigma(Z) = 16$ sec. These values were taken from Allen (1973).

Through a random process we assign then to each star its space motion components.

We add next to all stars a solar motion component

$$X_0 = 11.4 \quad Y_0 = 14.7 \quad Z_0 = 7.6 \text{ km s}^{-1}$$

equivalent to a solar motion with $V_0 = 20.1 \text{ km s}^{-1}$ toward $l_0 = 52.2$ and $b_0 = 22.2$. These values, except rounding errors, are very close to the result of the previous paper (Jaschek & Valbousquet 1993, Paper III).

We convert next the X_i, Y_i, Z_i with the solar motion included into proper motions and radial velocity. We simulate next the observational errors by adding a Gaussian function with zero average and a dispersion of $\pm 0''.007$ to each proper motion component and ± 3 km s^{-1} to the radial velocity. Through a random process we extract triples of observational errors and we add them to the calculated proper motions and radial velocity.

We have thus fabricated a sample of stars with realistic space density, velocity and luminosity distribution. We have not allowed for an error distribution in m_i, which is equivalent to assume that the apparent magnitudes are error free. This can be done since photoelectric measures of magnitudes areaccurate to 1 or 2% (at most). This is much better than the precision for the other parameters, like proper motions, radial velocities and absolute magnitudes so that the assumption of error-free magnitudes seems acceptable.

3. Magnitude limit of the sample

Many observational catalogues are limited in magnitude, either because of an (arbitrary) cut-off or because of observational constraints. A good example is the Brigth Star Catalogue, limited (arbitrarily) to apparent magnitude 6.5. A limitation in magnitude introduces a bias, the so-called Malmquist bias. In order to simulate a sample, we must limit our artificial sample in a similar way. Since we have taken one single spectral type (K0 III) for our sample, the difference between a sample limited in magnitude or in distance is due only to the dispersion in absolute magnitude and the errors in apparent magnitude (which we assumed to be zero). As a consequence of this dispersion there are in the sample stars of different brightness. For a given apparent magnitude limit the brighter stars can be seen up to greater distances than the faintest ones; as the bright stars are found in a greater volume of space they will be more easily found in the sample so biasing its mean absolute magnitude toward brighter values and producing the Malmquist bias (see Luri et al. 1993b).

To simulate this behavior in our artificial samples we have proceeded as follows: we know that stars further from the mean than three times the dispersion (σ) have a negligible probability of being in the sample, even if the effect of the Malmquist bias is taken into account for the brightest ones (at least for dispersions in M lesser than $1^{\text{m}}.5$). We will then generate artificial stars with a Gaussian distribution in the range $-0^{\text{m}}.8 < M < +2^{\text{m}}.2$. The distance limit ($R_{\text{lim}}$) for $M = -0.8$ given $m = 6.5$ can then be obtained so no stars will be found farther than R_{lim}. Our stars will then be generated in the range $0 < r < R_{\text{lim}}$. But these stars can still have apparent magnitudes greater than $m = 6.5$ so only the ones below this limit are retained. A magnitude limited sample is then obtained. We insist in describing this in detail because often one is tempted to use one sigma only, which is insufficient. With 3σ the number of stars to be rejected is rather large, but one is on the safe side.

We have used systematically two sample sizes, namely samples of 50 and 200 stars. The first number corresponds to the number of objects one has usually available for statistical parallax determinations, whereas the second corresponds to what is usually considered a "good sized" sample in current kinematic analysis. Each time we made N trials of either 50 or 200 stars, N being specified in each case. The results of the N trials are summarized by their average and dispersion. The latter value illustrates the variations around the average which are found in the N samples. Average and dispersion are defined as usually by the formulae

$$x = \frac{i}{n} \sum xi \qquad D = \sqrt{\sum (x - x_i)^2 / n - 1}$$

where x is a random variable.

4. Uses of the sample

We start using the sample for the determination of the solar motion (Sect. 4.1, 5, 6, 7, and 8) and we then apply it to the luminosity calibrations (Sect. 9).

4.1. Determination of the solar motion

We apply, as a first step, the classical Airy method to the radial velocities and the proper motions, to see if one recovers the solar velocity introduced. The resuts are summarized in Table 1.

As in all work with artificial samples, we have first checked that the dispersion of the results diminishes with increased sample size and that the number of trials does not change the dispersions. Both items are satisfied in our artificial samples.

The main result of Table 1 is that despite the use of samples of 200 stars – a sample size which is rather large and seldomly attained in practice – we do not recover the original values which we had introduced. From radial velocities we get from Table 1 for V_0 for the average from 50 samples the value of 20.3 as compared to the 20.1 km s^{-1} introduces. Futhermore L_0 and b_0 (50.3 and 21.2) fall respectivey 2° and 1° degrees from the true position. From space velocities the results are better – one gets 20.0–51.7 and 20.8 instead of 20.1–52.2 and 22.2.

When one compares these uncertainties (illustrated by the dispersions indicated in Table 1) with what is published in the

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Table 1. Solar motion from 50 trials

Method	N	V_0		l_0		b_0		$\sigma(V)$	
		Av.	Disp.	Av.	Disp.	Av.	Disp.	Av.	Disp.
RV	50	21.4	4.6	53.0	17.6	22.2	14.3	21.7	2.1
RV	200	20.3	3.1	50.3	7.4	21.2	8.1	22.0	1.4
PM	50			54.4	13.1	21.6	9.9		
PM	200			52.5	6.4	20.7	5.7		
SpV	50	20.4	3.3	54.0	9.8	22.2	8.6	38.6	2.8
SpV	200	20.0	1.6	51.7	5.2	20.8	4.1	39.2	1.2
I		20.1		52.2		22.2		37.7	

Notes: RV = method based upon radial velocities. PM = method based upon proper motions. SpV = method based upon space velocities. I = introduced values. N = number of stars in the sample. $\sigma(V)$ = dispersion of the peculiar velocity of the stars (i.e. without solar motion). Av.= Average. Disp. = dispersion of the averages of fifty trials.

literature (apex longitude and latitude quoted to a tenth of a degree, space velocities to a thenth of km s^{-1}) one must conclude that this practice is unjustified, since the results have much large errors attached. It is easy to calculate from Table 1 that apex determinations based upon samples of even 1000 stars still carry uncertainties of ± 0.7 in V_0 and $\pm 2°$ in l_0 and b_0. Any definitive improvement passes through the use of large samples ($N > 1000$) and also a diminution of the observational errors.

5. Herschel's method

The Herschel method is based upon the fact that all proper motion vectors should converge toward one point on the sky, which is the solar apex. (For more details, see Paper I).

Since it is difficult to work with proper motions on the sphere, we have worked with the poles of these proper motion vectors. These poles should all lie on a great circle on the celestial sphere. We have calculated thus the inertial axis of the poles of the proper motions and the proper values (eigenvalues) associated to these axis. Theoretically the axis associated with the smallest eigenvalue should provide the direction of the apex.

The results show that we do not get close to the expected results. We have examined if this is due to the velocity ellipsoid used – perhaps if the axes of the velocity are large, the solar motion is "lost". We have redone the calculations with a sperical velocity distribution, but have found no improvement. We recovered the original values of the solar motion we had introduced only if we chose stars with no peculiar motion with respect ot the sun, the observed proper motions bing simple the reflect of the solar motion.

This confirms anew what we had pointed out already in Paper I, namely that the method does not work properly. Despite consierable numerical work we conclude that the method is unfitted for apex determinations and should thus be discarded.

6. Biased sky distribution of the sample stars

One is often confronted with the practical difficulty that the stars used for apex determinations are not distributed uniformly over the complete sky, but only over part of it. Does this influence the results?

We have selected several samples, corrsponding to varying limits in declinations from the equatorial pole southwards. The limits chosen are $-10°$, $0°$, $+30°$ and $60°$, which correspond respectively to 59, 50, 25 and 7% of the sky. Please observe that the limits of the zones are according to equatorial coordinates as it happens in practice because of observing constrants, whereas the distributions are symmetric in galactic coordinates. The results are given in Table 2 and can be compared with the results for the whole sky given in Table 1. A look at the Table 2 reveals various interesting facts. In the first place, the best method is the one of the space velocities, in the sense that it produces the least dispersed results, even in the case of a very small sky coverage. In the second place, the resulting solar motion is sensible to the extent of the sky coverage – the smaller the coverage is, the larger the dispersions are. The third, very welcome result is that a distribution over one hemisphere only does not produce a sensible bias, as compared with a distribution over the whole sky. Deviation from such a symmetric distribution tend to produce larger dispersions in the radial velocity method and the proper motion method.

7. Influence of the dispersion of the absolute magnitude

When one uses space motions for apex determinations, one has to know the distances fo the stars. If good trigonometric parallaxes are available, there exists no problem. However this is often not the case and one has to use distances derived from spectroscopical absolute magnitudes. The distances so derived are only good in the statistical sense because the average absolute magnitudes are affected by a (rather large) dispersion about the average. Most often the value of the dispersion is not well known, and a convenient value is simply assumed ($\sigma = \pm 0.5$). Could the adoption of an erroneous value of this dispersion affect the apex determination?

To check this, we have crated three more samples (besides the original one with $\sigma(M) = 0.5$), namely $\sigma(M) = 0.0$, $\sigma(M) = 0.2$ and $\sigma(M) = 1.0$. The results are given in Table 4. One can see that the $\sigma(M)$ has practically no influence on the results. This is what is expected for the methods using radial velocities and proper motions alone, since the distance enters only through the galactic rotation correction and the extinction correction. The result is not understandable for the method using space velocities, since there an increase in the dispersion of the absolute magnitudes creates distances which differ considerably from the true values, a fact which propagates directly into the space motions. According to the results of Table 3, it seems however that the influence of $\sigma(M)$ is very small as long as $\sigma(M)$ is inferior to $0\overset{m}{.}7$. Beyond this pont one finds an influence of the dipsersion. What happens is that one must take into account the Malmquist correction. Therefore the calculation of

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Table 2. Solar motion determination for biased distributions over the sky. 50 trials, sample size 200

Method	V_0		l_0		b_0		$\sigma(V)$	
	Av.	Disp.	Av.	Disp.	Av.	Disp.	Av.	Disp.
(a) Over one hemisphere								
RV	20.3	2.4	52.2	7.9	21.7	7.2	22.2	1.2
PM			53.0	6.6	21.5	4.4		
SpV	20.2	1.6	52.7	5.2	21.7	3.0	39.3	1.3
(b) Over more than one hemisphere ($\delta > -10$)								
RV	20.1	2.3	55.1	9.1	22.0	9.3	22.3	1.1
PM			54.4	6.3	21.2	5.1		
SpV	19.9	1.5	54.7	5.5	21.6	4.3	39.2	1.4
(c) Over less than one hemisphere ($\delta > 30$)								
RV	20.2	2.8	53.5	8.2	22.7	9.5	21.8	1.1
PM			52.2	8.9	21.3	4.3		
SpV	20.3	1.4	52.7	6.3	21.7	3.8	39.2	1.5
(d) Over less than one hemisphere ($\delta > 60$)								
RV	22.3	5.0	51.3	10.9	22.8	16.0	21.2	1.2
PM			53.0	12.4	20.6	4.9		
SpV	20.4	1.7	51.9	5.4	21.0	2.8	39.3	1.3
I	20.1		52.2		22.2			

Notes: RV = method using radial velocities. PM = method using proper motions. SpV = method using space velocities. I = introduced values. $\sigma(V)$ = dispersion of the peculiar velocity of the stars (i.e. without solar motion). Av.= Average. Disp. = dispersion of the averages of 50 trials.

Table 3. Solar motion from samples with different $\sigma(M)$. 50 trials, 200 stars

Method	$\sigma(M)$	V_0		l_0		b_0		$\sigma(V_0)$	
		Av.	Disp.	Av.	Disp.	Av.	Disp.	Av.	Disp.
Rad. vel.	1.0	19.9	1.3	53.1	8.6	21.2	8.2	22.6	1.3
	0.5	20.3	3.1	50.3	7.4	21.2	8.1	22.0	1.4
	0.2	20.2	2.6	52.7	8.2	21.5	10.7	22.4	1.3
	0.0	21.0	2.4	52.7	8.9	22.0	7.0	22.0	1.1
Corr.	1.0	20.8	2.6	52.1	7.8	19.1	11.1	22.9	1.1
Prop. mot.	1.0			52.8	8.6	20.5	5.1		
	0.5			52.5	6.4	20.7	5.7		
	0.2			52.4	7.2	21.6	4.9		
	0.0			51.5	6.9	21.3	4.5		
Space vel.	1.0	15.6	1.3	52.9	6.3	19.4	3.9	32.4	1.7
	0.5	20.0	1.6	51.7	5.2	20.8	4.1		
	0.2	20.1	1.8	52.5	5.8	21.8	4.2	38.2	1.2
	0.0	20.2	1.4	51.9	5.7	21.5	3.6	37.1	1.3
Corr.	1.0	16.5	1.3	51.5	5.5	16.5	3.7	33.7	1.5

Notes: Rad. vel. = radial velocity. Prop. mot. = proper motion. Space vel. = space velocities. Av. = Average. Disp. = dispersion of the averages of 50 trials. "Corr." stands for the values of the first line, corrected for Malmquist bias.

the distances must not be based upon M, but upon the corrected M. If this is done, the influence of the dispersion disappears. The results of the calculations including the Malmquist correction are given in the last line for both the radial velocity and the space velocity method. We have given the results only for $\sigma = 1.0$; for smaller values of the differences are negligible.

The important point here is that one can apply the Malmquist correction only if one knows σ but this is just a poorly known quantity. What our numerical experiments show is that one needs not correct for the Malmquist bias as long as σ lies in the usually assumed range of 0.3–0.7.

8. Velocity ellipsoid

We considered next a possible influence of the velocity ellipsoid on the apex determinations. Is it possible that a spherical

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Table 4. Velocity ellipsoid and solar motion. 50 trials, sample size 200

		V_0		l_0		b_0		$\sigma(V)$	
		Av.	D.	Av.	D.	Av.	D.	Av.	D.
Sperical distr.									
29,29,29	VR	21.2	4.0	50.9	9.1	11.6	28.5	1.3	
	VS	20.9	2.0	51.2	5.6	21.8	6.2	52.1	1.5
	PM			51.7	3.1	20.9	6.9		
Plane distr.									
29,18,0	VR	20.4	2.7	52.3	9.0	21.1	7.7	20.4	1,2
	VS	20.3	1.4	53.2	5.2	21.7	1.5	35.4	1.5
	PM			53.6	6.2	21.7	3.2		
Ellipsoidal distr.									
29,18,16	VR	20.3	3.1	50.3	7.4	21.2	8.1	22.0	
	VS	20.2	1.6	52.7	5.2	21.7	3.0	39.3	1.3
	PM			53.0	6.6	21.5	4.4		
Sperical distr.									
58,58,58	VR	22.2	6.4	56.0	20.2	20.2	27.6	57.5	2.9
	VS	19.0	3.6	51.2	13.9	18.8	12.1	94.8	3.2
	PM			48.4	17.6	16.8	13.9		
Plane distr.									
58,36,0	VR	21.3	5.5	52.3	19.3	19.9	11.2	41.9	2.5
	VS	19.5	3.2	50.3	11.7	20.4	3.7	65.3	2.8
	PM			49.3	12.6	20.5	5.9		
Ellipsoidal distr.									
58,36,32	VR	21.8	4.4	50.5	17.7	23.8	15.7	44.4	2.6
	VS	19.7	2.5	54.3	10.9	22.0	7.4	71.5	2.8
	PM			53.6	12.0	21.5	9.4		

Notes: Av. = Average. D. = Dispersion of the averages from 50 trials. RV = radial velocities. SV = space velocities. PM = proper motions. $\sigma(V)$ = dispersion of the peculiar velocity of the stars (i.e without solar motion). In the second line of the first column are given the values of the axis of the velocity distribution in km s^{-1}.

distribution of the peculiar velocities influences the apex in a different way than a flat distribution? An influence of the velocity ellipsoid upon the apex seems contrary to theory, since one is determined from the quadratic terms whereas the other depends upon the lineal terms. However, if the quadratic terms are very large, an influence upon the solar motion, which is rather small, can not be excluded a priori.

We have made the calculation for samples composed by stars with two extremely different velocity distributions, namely

(1) a spherical velocity ellipsoid,
(2) a flat velocity ellipsoid.

We have also considered the size of the velocity ellipsoid (i.e. the numerical value of the axis), so as to have a complete coverage of the different possibilities. The results are given in Table 4. From an examination of the table one can draw two conclusions. First of all the shape and the size of the velocity ellipsoid do not influence the apex (V_0, l_0, b_0). Second, that the dispersions depend not so much upon the shape, but upon the size (i.e. the numerical value of the axes) of the velocity ellipsoid. Doubling the axes of the ellipsoid doubles almost the size of the dispersions of the average ellipsoid axes, as obtained from the same number N of numerical experiments. A larger velocity ellipsoid has thus larger sampling errors attached than a small one. In conclusion the apex valus are insensitive to (small) changes in the shape and the size of the velocity ellipsoid.

9. Luminosity calibration

For the velocity calibration with statistical parallaxes there exist in principle four methods – three classical ones (see Smart 1938) and a modern one.

The first three methods are called upsilon, tau and upsilon prime. In summary the methods suppose that the proper motion components of a star can be decomposed into one component directed toward the solar apex (upsilon) and one part perpendicular to the direction (tau). Whereas tau is free from the solar motion, the upsilon component can be decomposed into a part which is a reflex of the solar motion (υ_s) plus a peculiar component (υ_p) so that

$$\upsilon = \upsilon_s + \upsilon_p \ .$$

If the stars of a sample are well distributed over the whole sky, and the peculiar motion components are distributed randomly, one can assume that the sum

$$\sum \upsilon = \sum \upsilon_s \ ,$$

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since the sum of the peculiar components is zero. On the other hand the transversal motion T

$$T = 4.74\frac{\mu}{\pi} \ ,$$

where μ stands for proper motion and π for the parallax.

The sum $\sum v$ should thus be equal to the sum of the T, which can be ascertained from the radial velocities. After some transformations one ends up with

$$\overline{\pi}_v = \frac{4.74 \sum v \sin \lambda}{V_0 \sum \sin^2 \lambda} \tag{1}$$

where V_0 is the solar speed and λ the angular distance from the solar apex. $\overline{\pi}$ is the average parallax of the group from the upsilon method.

The *second method*, called the tau method, uses formula

$$\overline{\pi}_\tau = \frac{4.47\overline{|\tau|}}{\overline{|R|}} \tag{2}$$

where R is the peculiar radial velocity of the star after the solar component has been substracted from the observed radial velocity.

The *third* method, the so-called upsilon prime method, uses a similar formula, except that one uses not tau but $v' = v - v_s$. One has then

$$\overline{\pi}_{v'} = \frac{4.74\overline{|v'|}}{\overline{|R|}} \ . \tag{3}$$

In the τ and the v' methods, the average of the absolute value of the proper motion components has to be corrected for observational errors.

In all three methods the link between the average parallax obtained and the absolute magnitude is made with the help of the formula

$$M = m + 5 - 5 \log \overline{\pi} + C_1 + C_2$$

where C_1 is a correction which depends upon the method used and C_2 a correction depending upon the velocity ellipsoid of the group. For more details the reader is referred to Smart (1938).

The underlying assumption of the methods is that the velocity distribution fo the stars is spherical, which is an oversimplification. For the case of an ellipsoidal distribution, one must add a correction which we have called C_2 (see Smart 1938). The preceding three methods habe been applied when radial velocities are not available for all stars of the sample and/or when nothing is known on the absolute magnitude. Such situation happens often (even today!), for instance when one is defining a new group of peculiar stars and one needs a first estimate of the average absolute magnitude.

The *fourth* method is based upon the maximum likelihood principle. Given a distribution function for the observable quantities, the best estimators of the parameters are taken as those which maximize the probability of the sample. This method was presented by Rigal (1958) and perfectioned by Jung (1970) and Heck (1978). A new approach is given by Luri (1933a; Luri et al. (1993b) and will be applied in what follows. Luri's method improves the other methods in several ways. It avoids some approximations that were introduced due to the limitations in computing power. The spatial distribution of stars – usually an exponential disk – is explicitly included, instead of the previously used homogeneous distribution. Futhermore, the very important effect of selection criteria on the sample is taken into account in a natural way. For instance, the effect of a limitation in apparent magnitude which leads to the Malmquist bias is eliminated directly, without the use of the approximated classical formula $M = -1.38\sigma^2$. Let us observe however that the ML methods can only be applied if for all stars one knows radial velocities and proper motions and apparent magnitudes. For a discussion of the limitations of the methods, see Jaschek (1988).

One difficult point with all methods is the evaluation of the errors of the absolute magnitude derived. The random sample we have created can be used to assess these in a completely independent way, since we can modify at will all intervening parameters.

9.1. Application to artificial samples

We have thus applied the formulae (1), (2) and (3) to derive the absolute magnitude for each sample. Table 5 provides the results.

In principle the results for the τ and the v' methods should still be corrected for the fact that the velocity distribution is an ellipsoid rather than a sphere. These corrections were tabulated by Smart (1938) in Tables 31 and 32 as a function of the axial ratio of the velocity ellipsoid, We can check the corrections by repeating our analysis on samples having different velocity ellipsoids. We maintain for all cases $M_0 = 0.7$ with a dispersion of ± 0.5. We have chosen large values for the axis of the velocity ellipsoid in order to vary them significantly. The results are given in Table 6. To our knowledge this is the first time that these corrections are checked.

It can be seen from an inspection of Table 6, columns 3, 4 and 5, that the results for different velocity ellipsoids do not behave in a systematic way. The dispersion of the averages (not given in the table) of the fifthy samples around the averages varies between ± 0.28 and ± 0.69. It is clear that Smart's corrections, listed in columns 6 and 7 of the table (usually ≤ 0.5, with one exception) are drowned in the uncertainty of the average absolute magnitude. We observe further that the average of the results for v, τ and v' (column 9) fall usually rather close to the true value (+0.7), irrespective of the velocity ellipsoid. The maximum likelihood method on the other hand (column 8) gives always the correct answer, also irrespective of the velosity ellipsoid.

We conclude thus that the corrections given by Smart (1938) do not improve the results and we have not applied them in this paper. A possible explanation of why they do not apply is that the underlying theory assumes that M has an unique value, whereas in reality (and in our artificial samples) it has a (Gaussian) distribution around its mean value. Probably this dispersion oblit-

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Table 5. Average absolute magnitude determined from the sample limited in magnitude. 50 trials, sample size 200

M	Upsilon	Ups. prime	Tau	Average	ML	Introd.
Average M	0.57	0.23	−0.05	0.25	0.72	0.70
Dispersion	0.24	0.29	0.22	0.44	0.15	

Note: "Dispersion" refers to the dispersion of the results of the 50 trials

Table 6. Average absolute magnitudes determined from samples of stars with different velocity ellipsoids limited in magnitude. 50 trials, sample size 200

		Calculated			Tabulated corrections			
	A.R.	υ	τ	υ'	τ	υ'	ML	Average
A	1.0	0.51	0.95	0.0	0.0	0.68	0.76	
B	1.45	0.71	1.04	1.04	0.25	0.14	0.70	0.93
C	1.93	0.66	0.64	0.76	0.46	0.25	0.71	0.69
D	2.9	0.78	0.63	0.64	0.65	0.50	0.70	0.68

Notes: Sample A: Axes 58–58–58. Sample B: Axes 58–40–40. Sample C: Axes 58–30–30. Sample D: Axes 58–20–20. A.R. = Axial ratio. ML = absolue magnitude from maximum likelihood method. Average = absolute magnitude from an average of the three statistical methods. The assumed absolute magnitude is +0.70.

erates completely the effect of the (small) corrections for the ellipsoid.

Coming back to Table 5, one finds the discomforting results that even after 50 trials of a large sample (200 stars) one gets absolute magnitudes which differ by several tenth from the correct value. The υ method is the one which approximates most closely the correct result and this is in line with what other astronomers found also. Table 5 provides also the result from the maximum likelihood (ML) method, which comes out much closer to the correct value than the other three method, for the same sample size. This can also be seen very clearly in the dispersions. Whereas for each of the three methods the average dispersion is ±0.25, for the ML method the dispersion is only ±0.15. This confirms the superiority of the ML method.

9.2. *Application to real samples*

We have next examined if our conclusion about the (large) uncertainty of the statistical parallax methods is confirmed by an application to real samples. We have thus calculated the average absolute magnitude for all groups of the Bright star catalogue, Fourth edition (BS4), which we have used in the first three papers of this series. For comparison we have used the Schmidt-Kaler (1982) compilation of absolute magnitudes of stars because it has been used very often in the literature. We have retained all groups, for which Schmidt-Kaler provides an average absolute magnitude (or for which such a value can be constructed), despite the fact that many of our groups are rather small. From these list of groups we have retained only those groups for which at least two of the three methods give a result (i.e the statistical parallax gave a non-negative value). As

Table 7. Average absolute magnitude from samples of real stars from BS4

Group	N	M	M_{corr}	M(SK)	Difference
Giants (III)					
B 5–7	54	−1.69	−1.34	−1.60	+0.26
B 8	64	−2.00	−1.65	−1.90	+0.25
B 9	63	−1.50	−1.15	−0.60	−0.55
G 8	112	+0.70	+1.05	+0.80	+0.25
K 0	148	+0.98	+1.25	+0.70	+0.55
K 1	85	−0.01	+0.34	+0.60	−0.26
K 2	93	+0.35	+0.70	+0.50	+0.20
K 3	84	−0.34	+0.03	−0.30	+0.33
K 5–7	50	−0.98	−0.65	−0.25	−0.40
				+0.07 ± 0.37	
Dwarfs					
B 3	60	−1.69[a]	−1.34	−1.6	+0.26
B 6–7	58	−0.91	−0.56	−0.9	+0.34
B 8	93	−0.75[a]	−0.40	−0.25	−0.15
b 9	135	−0.18[a]	+0.17	+0.2	−0.03
A 0	132	+0.55[a]	+0.90	+0.65	+0.35
A 1	127	+0.89	+1.24	+1.0	+0.24
A 2	103	+0.78	+1.13	+1.3	−0.17
A 3	88	+1.27	+1.62	+1.5	+0.12
A 4–5	60	+0.41	+0.76	+1.85	−1.09
A 6–9	58	+1.49	+1.84	+2.3	+0.46
F 2–3	51	+2.26	+2.61	+3.6	−0.99
F 4–5	62	+2.74	+3.09	+3.5	−0.41
F 8–9	60	+3.57	+3.92	+4.1	−0.18
G 0–1	50	+4.15	+4.50	+4.5	−0.00
				−0.16 ± 0.45	

Notes: The last line of each part of the table provides the average difference and its dispersion.
[a] stands when only two of the three methods gave a positive parallax.

mentioned above we have not applied the corrections for the velocity ellipsoid, We have given in Table 7 the average results from the three methods, next their magnitude corrected by the Malmquist correction ($\Delta M = 1.38\sigma^2$, with sigma = 0.5) and the average magnitude taken from Schmidt-Kaler's compilation. The last column provides the difference between both. As can be seen, the overall comparison shows that both calibrations agree, despite some large discrepancies. This can be seen by the average differences (0.07 and −0.16 and its dispersion (±0.37 for the giants and 0.45 for the dwarfs). In fact the agreement is better than what can be expected, since both procedures have large uncertainties attached. We have seen that statistical parallaxes have uncertainties of the order of ±0.44 for samples of 200 stars (and thus about ±0.6 for samples of 100 stars), and Schmidt-Kaler estimates the uncertainty of his results of the order of ±0.3 for the main sequence and ±0.7 outside it.

The largest differences occur for the A 4–5 V and the F2–3 V groups. We observe that in general four groups later than A 3 and earlier than F 5 our absolute magnitudes are brighter than Schmidt-Kaler's. Since we hope to publish a detailed discussion of the absolute magnitudes based upon statistical parallaxes using the ML method, we shall not pursue the discussion.

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Let us add simply that we have spent a good deal of computing time in examining various possibilities for improving the results and/or explaining the large incertitude of the results. Let us mention briefly the factors examined and our conclusions.

(a) Formulas. Since in formulas (1), (2) and (3) one uses the solar motion derived from the (small) sample it could be that by using the "grand average" value of the slar motion (derived in Paper III) one could decrease significantly the scatter. No improvement was observed.

(b) Since in formulas (2) and (3) one uses the average of the absolute values of tau and upsilon prime, we have substituted this by the square root of the dispersion. This was done because the latter is less affected by errors. The results turn out to be quite insensitive to such a change.

(c) Data. Since the BS4 proper motions are in the SAO system, it is tempting to think that the large scatter is a consequence of the somewhat low precision of the data. We have therefore (as in Paper II) set up a sample, within the BS4, of all stars which have proper motions in the PPM+FK5 catalogues. We have recalculated the statistical parallaxes for all groups, using the (improved) proper motions. A detailed comparison shows that the scatter between the results of the three methods subsists, which leads to conclude that the fault lies not with the proper motion system, but probably with the method itself.

(d) Sample size. The most obvious factor is the sample size. If we consider the six most populous samples of Table 7, with $N > 100$, the average difference "average – Schmidt-Kaler" is 0.20 ± 0.26 from 5 groups. For the smaller groups ($N < 100$), the average difference is -0.11 ± 0.46 from 14 groups. The dispersion is thus related to the sample size, as one could expect. Using the latter value for an average group size of 70, one could expect for $N = 200$ a dispersion of about ± 0.3 to ± 0.4.

Nevertheless, in practice one is selcom in presence of very large samples with, say, $N = 200$. One has to use the methods on small samples and in this case the uncertainty of the result is large. The conclusion drawn from the artificial sample, namely that for the same sample size, the three statistical methods are inferior in precision to the ML method is thus entirely applicable.

As a last point we should notice that the maximum likelihood method provides the solar motion as part of the results. Since the ML method produces much better absolute magnitudes than the traditional methods, one could ask if the solar motion it produces is also much better than the results from the traditional methods. We provide in Table 8 the results of the classical methods and of the ML method. The conclusion is that the errors are quite similar in both methods.

Table 8. Comparison between solar motion determinations from space velocities. 50 trials, 200 stars

Apex	V_0		l_0		b_0		$\sigma(V)$	
	Av.	D.	Av.	D.	Av.	D.	Av.	D.
Classical	19.8	1.4	52.3	6.4	21.7	3.2	38.0	1.2
ML	19.6	1.7	52.6	6.3	21.6	4.5	36.1	1.3

Av. = average. D. = dispersion of the averages of 50 trials. ML = maximum likelihood method.

10. Conclusions

We have set up a procedure to fabricate artificial samples of K0III stars which simulates as well as possible the real sample. We are thus able to produce an arbitrary number of star samples, limited either in volume or in magnitude.

In the first part we apply the methods used in Papers I, II and III of this series to determine the solar motion fromour artificial samples.

The most important results abtained are:

(a) One does not recover the input values except if very large samples are used, with $n \gg 200$ stars. Since most solar motions derived so far are based on small samples, the precisions usually quoted by authors overestimate the attainable precision. Typical values for the precision of the apex determination based upon a sample of one thousand stars are ± 0.7 km s^{-1} for V_0 and $\pm 2°$ for both l_0 and b_0.

(b) Herschel's method for determining the solar motion does not give consistent results and should be abandoned.

(c) When the solar motion is derived from incomplete sky coverage, it is best to use one hemisphere only and the space velocity method. Other methods are subject to increased dispersion when the sky coverage decreases.

(d) The values of the solar motion are relatively insensitive to the exact shape of the ellipsoidal distribution function of the residual velocities of the stars.

(e) Small changes in the assumed dispersion of the absolute magnitudes used for the determination of stellar distances do not influence the solar motion parameters, determined from the space motions.

(f) The ML methods for absolute magnitudes do not produce improved values of the solar motion.

In the second part we have used the samples to check the accuracy of the different methods used to derive the absolute magnitude, namely the tau, the upsilon, the upsilon prime and the maximum likelihood method.

(g) We found the important result that the ML method is definitely superior to the other methods. The other methods should only be used if the ML method is not applicable because of the lack of observational data.

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