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Systematic review on the use of virtual and physical manipulatives by primary school teachers

Angela Ogbugwa Ochugboju, Javier Díez-Palomar *

Department of Linguistics, Science and Mathematics Education University of Barcelona, Campus Mundet, Edifici de Llevant, Planta 1, Despatx 165, 08035, Barcelona, Spain

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ABSTRACT

This study examines the use of virtual and physical manipulatives in mathematics instruction among primary school educators. The research adheres to a predetermined set of guidelines to systematically explore, select, evaluate, extract, and amalgamate information from 105 scientific sources. It employs a narrative methodology to elucidate and analyze the results. The findings revealed that educators utilise a variety of virtual and tangible manipulatives, including software, blocks, strips, tiles, and other resources, to instruct students in diverse mathematical concepts and skills. Educators employ these manipulatives for a range of educational purposes, including but not limited to introducing and reinforcing learning concepts, facilitating inquiry and discovery, and promoting problem-solving and reasoning abilities. Both virtual and physical manipulatives present advantages as well as challenges in the context of primary education. Implementing these strategies can enhance students' academic experiences, including their motivation, engagement, interest, understanding, fluency, communication, collaboration, and reflection. However, acquiring the necessary skills and resources, such as technical, pedagogical, or practical expertise, may pose challenges regarding availability and accessibility. Additionally, students may face cognitive, affective, or behavioural difficulties or potential hazards. The use of virtual and physical manipulatives is influenced by teacher education and professional development initiatives, which provide educators with knowledge, opportunities, and the empowerment to proficiently utilise both types of manipulatives within their teaching methodologies.

1. Introduction

Manipulatives are tangible items or visual representations that learners can physically manipulate to explore and understand topics and processes (Marley & Carbonneau, 2015). Educational physical objects include bricks, tiles, and coins, while virtual tools comprise computer-based simulations and animations. Manipulatives are widely used in primary education due to their perceived effectiveness in enhancing conceptual understanding, problem-solving skills, and communication abilities (Wang & Tseng, 2018). However, the effectiveness of manipulatives depends on their use and integration with other pedagogical methods (Crompton et al., 2021). Therefore, it is essential to examine how primary school teachers utilise manipulatives in their instructional settings and the various factors influencing their decision-making and implementation approaches (Lazonder & Ehrenhard, 2013).

This paper explores the use of physical and virtual manipulatives in teaching and learning mathematics in elementary classes. Consequently, the study aims to enhance students' conceptual understanding, problem-solving abilities, and communication skills. This research is significant as manipulatives influence students' mathematical learning and their potential as future mathematicians.

The use of manipulatives can be traced back to the counting boards of ancient civilisations and contemporary computer simulations. However, some teachers still fail to incorporate manipulatives into their classrooms, despite evidence highlighting their benefits (Crompton et al., 2021). Current literature indicates a consensus that observation and meaningful engagement in activities are essential for constructing mathematical knowledge. For instance, the National Council of Teachers of Mathematics (NCTM) has advocated for the use of manipulatives in mathematics instruction.

The purpose of this study is to examine the effectiveness, advantages, and challenges associated with manipulatives in elementary mathematics education. Instead of testing specific hypotheses, the research

* Corresponding author.
E-mail addresses: aochugoc92@alumnes.ub.edu (A.O. Ochugboju), jdiezpalomar@ub.edu (J. Díez-Palomar).

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adopts a qualitative perspective aimed at uncovering how manipulatives are perceived, applied, and experienced in classroom contexts. In particular, the study seeks to identify how manipulatives influence student engagement, conceptual development, and problem-solving strategies, while also considering practical barriers that may affect their consistent use. Overall, the findings are expected to provide educators and policymakers with evidence-based insights that can enhance teaching practices, promote active learning, and improve student outcomes in mathematics through the purposeful use of manipulatives.

2. Theoretical framework

Inclusion of a theoretical framework in this review provides a valuable basis for examining and interpreting manipulatives' role in mathematics instruction. Within systematic reviews, theoretical lenses not only direct evidence synthesis but may be used to draw boundaries around key concepts and relationships under investigation. By basing discussion within existing theories, this section identifies how learning with manipulatives is framed, how they act in anticipated ways upon students' mathematical development, and what aspects of influence might be considered meaningful. A theoretical framework operates as an analytical lens, confining the interpretation of findings and identifying patterns across the literature. For this review, a theoretical basis fixes discussion within cognitive and constructivist theories that remain highly relevant to how learners use physical and virtual tools. These theories highlight the importance of active engagement, dual-channel use, embodied experiences, and cognitive demand management as paramount within how manipulatives feature within mathematics pedagogy. By placing a study within these lenses, a theoretical basis ensures coherence between review aims and analysed evidence base while allowing a formal system to draw a basis for interpreting manipulatives ' practical consequences. Ultimately, this section underscores the importance of theory in linking empirical evidence to classroom practice and in advancing a deeper understanding of how manipulatives contribute to meaningful mathematics learning.

Having established the importance of a theoretical foundation for interpreting the role of manipulatives, we now outline the specific frameworks that have guided empirical research in this area.

Research on manipulatives in teaching has employed various theoretical frameworks to elucidate their impact on learning. Constructivism is a theoretical framework that significantly emphasizes learners' active engagement in developing their understanding through interactions with their environment (Otten et al., 2019). However, apart from constructivism, other theories contribute more to describing the construction of knowledge based on the child's interaction with the environment (Moyer-Packenham & Westenskow, 2013). Dual Coding Theory, introduced by Allan Paivio, posits that cognitive processing operates through two distinct but interconnected channels: verbal and non-verbal. This theory suggests that information is best learned when presented in two modes: words and images (Keldgord & Ching, 2022). According to this theory, manipulatives—tools that students can physically handle—facilitate learning by supporting both channels and allowing for deeper processing of information (Satsangi et al., 2018). Additionally, the Embodiment Theory further enriches this understanding by asserting that cognition is inherently linked with the body (Ladel & Kortenkamp, 2016, pp. 25–40). Manipulatives enhance this connection by enabling learners to touch or interact with materials, thus simplifying the comprehension and memorisation of concepts (Moyer-Packenham & Westenskow, 2013). Moreover, *Cognitive Load Theory* should be noted when explaining how manipulatives assist in understanding the materials. This is because manipulatives shift the cognitive demand to the characteristics of physical objects, allowing students to concentrate on the crucial factors that influence the learning process (Oymak & Ogan-Bekiroglu, 2021). The frameworks discussed, individually and in combination, collectively elucidate the nature of learning and the role of manipulatives in enhancing education. In this manner, educators can better understand and appreciate both virtual and physical manipulatives to cater to the diverse learning profiles of students.

2.1. Types and characteristics of manipulatives

Manipulatives can be classified into two main categories: virtual and physical. Virtual manipulatives refer to digital objects or representations that can be manipulated on various electronic platforms, such as computer screens or similar devices (Schnepel & Aunio, 2021). Physical manipulatives, on the other hand, refer to tangible objects or materials that can be manually handled. Both categories of manipulatives have distinct advantages and disadvantages depending on their specific qualities and benefits (Finti et al., 2016). Virtual manipulatives possess several characteristics that make them attractive for instructional purposes (Moyer-Packenham, 2016). These features include the ability to offer dynamic feedback, incorporate animation, provide scaffolding, present multiple representations, facilitate participation, ensure accessibility, enable customisation, and enhance motivation (Uribe-Flórez & Wilkins, 2017). For instance, virtual manipulatives can visually illustrate the immediate consequences of actions or modifications, such as adding or removing blocks, changing colours or shapes, and rotating or scaling objects (Fitzgerald, 2021). Furthermore, mathematical concepts or processes, such as fractions, decimals, or transformations, can be animated. Additionally, virtual manipulatives can offer learners assistance or tips to help them with problem-solving or reasoning tasks (Satsangi et al., 2018).

These tools can provide various representations of learning concepts or operations, including numerical, symbolic, graphical, and verbal formats (Pellas et al., 2018). Furthermore, virtual manipulatives enable learners to interact with them through different modalities, such as dragging, dropping, clicking, typing, or speaking (Kaur et al., 2022). Additionally, virtual manipulatives can be easily accessed from numerous locations and at any time, using various devices, including computers, tablets, and smartphones (Day & Hurrell, 2017). Moreover, these learning materials can be customised to meet the diverse needs or preferences of many learners, considering factors such as the level of complexity, linguistic requirements, or subject matter. Virtual manipulatives have the potential to enhance learners' motivation and engagement through the inclusion of visually appealing images, sound elements, and interactive games (Md Sullah et al., 2017).

Physical manipulatives offer numerous advantages in learning, especially in mathematical education (Delport, 2021). These educational tools can provide sensory-motor experiences, enhance spatial awareness, present concrete models, facilitate social engagement, and foster authenticity. For instance, using physical manipulatives can engage learners' sensory faculties and motor skills, allowing them to interact with mathematical objects or concepts through tactile experiences such as touching, feeling, or gripping. These tools can improve learners' spatial abilities, which include orientation, visualisation, and measurement (Shin et al., 2017). Physical manipulatives can represent mathematical concepts or scenarios tangibly and recognisably, as demonstrated by using coins to simulate monetary units or fractions. Moreover, the use of physical manipulatives has the potential to cultivate learners' social skills, including but not limited to communication, collaboration, and negotiation (Moyer-Packenham & Bolyard, 2016). Physical manipulatives are invaluable for establishing a connection between learning and real-world applications. For example, using clocks to measure time or maps to explore geography illustrates this connection. Throughout the learning process, physical manipulatives enable students to engage multiple senses, including tactile, visual, and auditory. Additionally, mathematical principles are represented through various means, comprising different shapes, colours, and sizes. This approach helps educators assist students in deepening their understanding of theoretical concepts and making connections to practical situations (Moyer-Packenham & Bolyard, 2016).

Nevertheless, it is essential to acknowledge that both virtual and physical manipulatives have certain limitations and present various challenges. Using virtual manipulatives may require technical expertise, specialised equipment, or resources that lack sufficient support for confident educators or learners (Verbruggen et al., 2021). Furthermore, these challenges may lead to cognitive or emotional impairments, including but not limited to distraction, confusion, frustration, or boredom (Bouck, Mathews, & Peltier, 2020). Physical manipulatives may also involve practical considerations, such as financial implications, storage needs, maintenance responsibilities, and distribution logistics. Additionally, pedagogical challenges may emerge, including oversimplification, misinterpretation, or misuse, which have been noted in previous research on mathematics education resources (Carbonneau et al., 2013; Moyer-Packenham & Westenskow, 2016).

2.2. Pedagogical approaches and strategies

Incorporating manipulatives into learning pedagogy is a complex decision that goes beyond merely selecting appropriate objects or representations in terms of kind or quantity. Additionally, it involves examining educational approaches and strategies that can optimise individuals' learning capabilities (Satsangi et al., 2018). Several crucial factors should be considered: congruence between the manipulatives, educational goals, and desired outcomes (Furner & Worrell, 2017). The selection and utilization of manipulatives should align with the stated mathematical objectives and competencies of the lesson or activity. The integration of manipulatives within instruction should be conducted in a way that ensures their connection and coordination with other instructional materials, including verbal explanations, textual symbols, diagrams, or questions (Pham, 2015). The manipulatives should be modified and tailored to align with the learners' existing knowledge and current requirements. In education, the instructor can provide instructional support and constructive feedback on the optimal utilization of manipulatives (Hott et al., 2020). Furthermore, the instructor may gradually decrease reliance on manipulatives as learners exhibit increased competence in the subject matter (Bouck et al., 2015). It is advisable for the instructor to actively monitor and assess the progress and achievements of the students by employing manipulatives. The teacher can utilise manipulatives to gather data on the learners' understanding or misconceptions (Anwar et al., 2019). Additionally, manipulatives can serve as a basis for providing feedback or implementing remedial measures (Pulos et al., 2023).

2.3. Empirical evidence

The use of virtual and physical manipulatives is a common and effective method to enhance students' educational outcomes in mathematics. Manipulatives are tangible objects or visual representations that learners can physically engage with to explore and understand mathematical concepts and processes in a concrete and visual manner. Research has shown that manipulatives provide valuable support to students' learning in various ways, including offering scaffolding for their ideas, supplying physical tools to enhance memory, boosting motivation and engagement, and facilitating effective communication (Shin et al., 2021).

The existing body of studies on the impacts of diverse manipulatives on various dimensions of students' learning outcomes is extensive and varied. Numerous research studies have demonstrated that virtual and physical manipulatives can benefit students' academic performance, comprehension of concepts, proficiency in procedures, problem-solving capabilities, reasoning aptitude, and attitudes. For instance, Lafay et al. (2019) conducted a meta-analysis that revealed that utilising virtual manipulatives yielded a moderately favourable impact on students' academic performance and a substantial positive effect on their learning processes. Similarly, a recent investigation conducted by Lange (2021) found that the use of physical manipulatives had a modestly positive impact on students' academic achievement, as well as a somewhat beneficial influence on their disposition towards the learning process (Sarama & Clements, 2016).

2.4. Gap analysis

Despite considerable research on the role of manipulatives in mathematics education, gaps remain wide open, especially regarding virtual manipulatives. Most literature emphasizes their virtues, such as generating dynamic visualisations, scaffolding, and multiple representations (Pellas et al., 2018; Uribe-Flórez & Wilkins, 2017). Most studies, however, successfully demonstrate how these tools can facilitate learning, but not in a systematic exploration of under what circumstances and how they do this. For example, while constructivist, dual coding, and embodiment positions yield a strong theoretical argument in favour of virtual manipulatives (Satsangi et al., 2018; Ladel & Kortenkamp, 2016, pp. 25–40), empirical correspondences between these theories and targeted classroom practice remain patchy and inconsistent. Additionally, this problem's urgency transcends proof of effectiveness. Schools across the globe continue adopting digital technology at an exponential pace. Still, teachers often report a shortage of preparation, doubt about pedagogic strategies, or trouble finding a balance between the cognitive advantages virtual manipulatives confer and the hazard of distraction, oversimplification, or misuse (Bouck, Mathews, & Peltier, 2020; Verbruggen et al., 2021). Absent further exploration into these tensions, a risk is that virtual manipulatives will populate classrooms in a shallow manner that negates theoretical promise. Another gap is methodological breadth. Most studies use small-scale qualitative studies or single-classroom interventions, which are valuable. However, these are in pointing up immediate practice, but restricting generalisability and long-term effects upon conceptual development (Jimenez & Stanger, 2017). Large, longitudinal, mixed-method studies that evaluate immediate learning and retention, transfer, and equity effects across a range of student populations are a pressing research need. Lastly, teacher professional development research continues to be lacking. While evidence emphasizes teacher knowledge in using manipulatives effectively, only a few studies explore how training can be constructed to provide both technical and pedagogic competence. These gaps must be addressed if virtual manipulatives achieve their promise as mathematics education's transformative tools, rather than existing as unused digital supplements.

2.5. Research aim and questions

Nonetheless, the effects of employing various forms of manipulatives are not uniformly consistent across all research and contexts. Several studies have indicated negligible or negative impacts of using either virtual or physical manipulatives on specific aspects of students' educational achievements. An empirical study conducted by Gecu-Parmaksiz and Delialioglu (2019) found that utilising virtual manipulatives did not improve students' understanding of fractions compared to using physical manipulatives or no manipulatives at all. Similarly, research by Zacharia and Michael (2016) demonstrated that the use of physical manipulatives did not lead to a significant enhancement in students' spatial skills when compared to employing abstract symbols or no intervention, as noted by Satsangi et al. (2021).

Furthermore, numerous scholarly investigations have performed comparative analyses of the effects of virtual and physical manipulatives on various mathematical achievements, producing differing results based on factors such as the subject matter, educational level, and teaching approach. A study by Park et al. (2022) revealed that virtual manipulatives led to better outcomes than physical manipulatives when teaching algebraic concepts to primary school children (RodríguezRodríguez, Álvarez-Seoane, Arufe-Giráldez, Navarro-Patón, & Sanmiguel- Rodríguez, 2022). Conversely, a recent study by Johnson et al. (2021) showed that physical manipulatives outperformed virtual manipulatives in teaching geometric principles to preschool children.

Hence, one can contend that a conclusive and straightforward response to the inquiry regarding the superiority of virtual or physical manipulatives in students' learning results remains elusive. The results may exhibit variability contingent upon many elements requiring meticulous consideration and control in every study and circumstance. Furthermore, more comprehensive and methodical evaluations and meta-analyses are needed to consolidate the available data and ascertain the deficiencies and constraints within the present body of literature.

The research is guided by the following questions:

- How do primary school educators use virtual and physical manipulatives to enhance students' knowledge acquisition?
- What are the advantages and challenges of using virtual and physical manipulatives in primary lesson instruction?
- What are the possible consequences or effects of utilising virtual and physical manipulatives in mathematics instruction on teacher education and professional development, particularly in relation to teacher preparedness, instructional confidence, and the integration of innovative pedagogical strategies?

3. Methodology

3.1. Research design

This study employs a systematic review methodology to consolidate and critically evaluate existing literature on the use of virtual and physical manipulatives in primary school mathematics education. Unlike a conventional literature review, a systematic review follows a rigorous, transparent, and replicable protocol that specifies the inclusion criteria, search strategy, and appraisal methods used, thereby minimising bias and ensuring methodological rigor (Hwang & Riccomini, 2016).

The novelty of this review lies in its dual focus on both virtual and physical manipulatives within the same analytical framework, a perspective that is not commonly adopted in prior reviews, which often consider them separately or emphasise one kind over the other. By bringing these two types together, the study provides a more comprehensive comparative synthesis of their applications, benefits, and challenges. Furthermore, the methodology integrates multiple theoretical perspectives—including constructivism, dual coding, embodiment, and cognitive load theories—into the review protocol. This theoretical anchoring offers a deeper interpretive lens that extends beyond descriptive summarisation and strengthens the explanatory power of the findings.

From a methodological standpoint, this study also introduces enhanced transparency and replicability. The use of explicit search statements (adapted for Scopus, Web of Science, and other databases), the adoption of PRISMA guidelines for study selection, and the application of the Mixed Methods Appraisal Tool (MMAT) for quality assessment, provide a level of procedural detail compared to previous reviews.

3.2. Scope

The scope of this review includes both empirical and theoretical works published between 2013 and 2023 that investigate the types, uses, impacts, and pedagogical consequences of manipulatives in primary school contexts (grades 1–7). Only peer-reviewed journal articles and conference proceedings published in English were included, ensuring both relevance and quality of evidence. By combining methodological rigor, theoretical integration, and comparative breadth, this review contributes a distinctive and novel perspective to the field of mathematics education research.

3.3. Search strategy

The search strategy for this review involved systematically identifying and obtaining relevant sources on the use of virtual and physical manipulatives by educators in primary schools. To achieve this, several academic databases were consulted, including Academic Search Complete, ERIC (Education Resources Information Center), Google Scholar, Scopus, and Web of Science. In addition, relevant literature was identified through manual searching of reference lists and the researchers' prior knowledge of the field, since relying exclusively on database keywords can risk overlooking valuable publications not readily indexed (Tucker, 2016).

The search terms were carefully selected to capture the core concepts of the study. Words such as *"manipulative," "object," "material,"* and *"tool"* were used to represent instructional resources, while *"virtual," "digital," "computer-based,"* and *"online"* reflected technology-mediated forms. The term *"physical"* denoted concrete, tangible resources. Boolean operators (AND, OR, NOT) and parentheses were used to combine these keywords into precise queries, with adaptations made to suit the syntax requirements of each database. Search filters were applied to restrict results by language (English), publication type (peer-reviewed journal articles), subject domain (education, mathematics), and date range (2013–2023).

To enhance transparency and replicability, examples of actual search statements used in different databases are presented in Table 1 below.

These queries illustrate how core terms were combined and adapted to capture literature on both virtual and physical manipulatives in the context of mathematics education for primary or elementary learners. This systematic approach ensured comprehensive coverage of relevant studies while maintaining a clear and replicable process.

3.4. Selection process

The selection method involves a meticulous assessment and subsequent selection of papers that meet the specified criteria for inclusion and exclusion in this study. The selection methodology employed here adheres to the guidelines outlined in PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) as shown in Fig. 1 below. However, it also incorporates the researchers' expertise in the subject area and familiarity with relevant literature (Pellas et al., 2018). This is necessary because automated searches using platforms like Google Scholar, Web of Science, Scopus, and others may overlook valuable sources that do not explicitly feature the keywords in their titles or abstracts yet remain pertinent to the topic under investigation. To address this limitation, the researchers also include alternative sources of information, such as:

- The reference lists of the papers in the review should be consulted, as they may contain citations to other relevant studies that need to be identified through the keyword-based search.
- Examining the citation network of the articles included in the review is crucial, as it has the potential to reveal additional papers that have either been cited or cited by them. Consequently, these papers may have a connection to the topic under investigation.

Table 1
Examples of database search statements.

Database	Example Search Query
Scopus	TITLE-ABS-KEY(("virtual manipulative*" OR "digital manipulative*" OR "computer-based tool*" OR "online manipulative*") AND ("mathematics" OR "math*") AND ("primary school*" OR "elementary school*"))
Web of Science	TS=(("physical manipulative*" OR "concrete object*" OR "mathematics tool*") AND ("teaching" OR "instruction") AND ("elementary education" OR "primary education"))

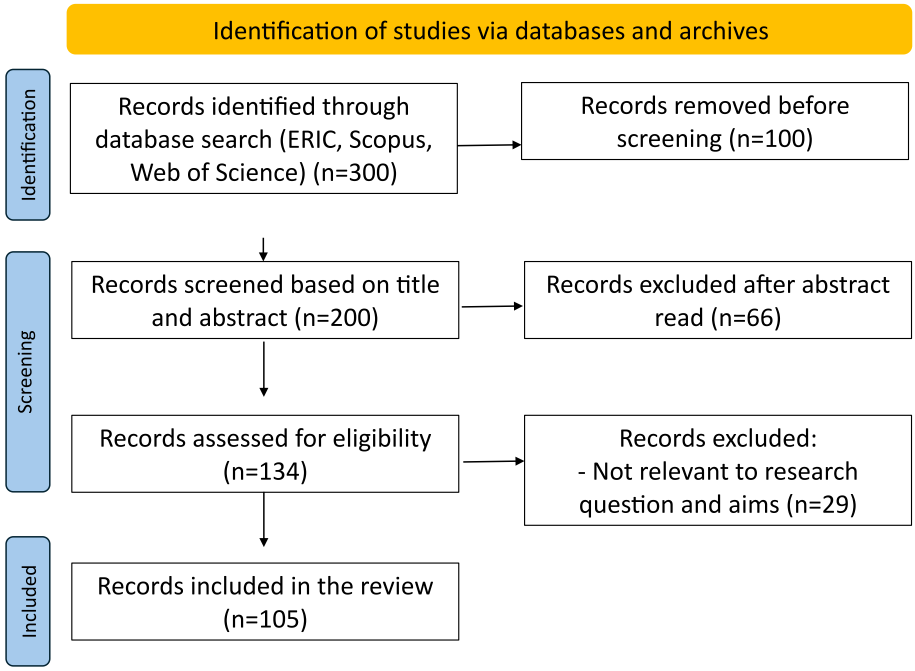

Fig. 1. Selection of studies for review.

- Incorporating recommendations or ideas from experts or peers in the relevant sector is beneficial, as they offer valuable insights and perspectives on selecting papers to be included in the review.

The selection procedure comprises four distinct stages:

- **Identification**: Duplicate documents are removed from the files acquired through the search method and stored in a reference management programme such as Zotero.
- **Screening**: The inclusion and exclusion criteria are applied to assess the eligibility of article titles and abstracts. Papers that do not meet the requirements are excluded, and the reasons are documented.
- **Eligibility**: The eligibility of the complete texts of the remaining papers is determined using the inclusion and exclusion criteria. The documents that don't meet the requirements are excluded, and the reasons are noted.
- **Inclusion**: The papers meeting the requirements are added to the review, and the bibliographic data is collected.

The selection procedure was conducted by both authors, who operated independently. The first author created a preliminary list of references. Then, both authors analysed the set of studies collected, following the described selection procedure. They discussed the data collected until agreement on a final selection.

The search approach identified 105 papers that satisfied the predefined inclusion criteria for this investigation as shown in Fig. 1 below. The publications comprised 35 empirical research studies and 35 theoretical works (Moyer-Packenham, 2016). The empirical research encompassed 20 qualitative experiments, 10 quantitative studies, and five studies that employed mixed methodologies.

Prisma Flow Diagram.

3.5. *Quality assessment*

The quality evaluation process involves assessing and assigning a rating to the methodological quality and rigor of the papers included. The quality assessment employs the Mixed Methods Appraisal Tool (MMAT), a versatile instrument that evaluates various research methodologies, including qualitative, quantitative, and mixed methods approaches. It utilises a distinct set of criteria tailored to each study's specific design, ensuring the relevance and appropriateness of the appraisal process. The MMAT includes a pair of screening questions and a set of five distinct queries customised for each type of study. The collection of questions used for screening purposes encompasses:

- Do the research questions or objectives exhibit clarity?
- Does the study utilise an appropriate methodology to investigate and address its research questions or objectives?

The specific inquiries may vary according to the nature of the study. Nevertheless, they typically encompass aspects such as the collection and analysis of data, sample selection, assessment of validity and reliability, identification of bias, and consideration of ethical implications (Zacharia & de Jong, 2014). Each query is answered with an affirmative, negative, or indeterminate response (Otten et al., 2020). Calculating a quality score involves dividing the number of affirmative responses by the total number of relevant inquiries. The quality score is measured on a scale from 0 % to 100 %, with higher scores indicating better quality (Mumford & Birchwood, 2020).

3.6. *Data extraction and synthesis*

The data extraction and synthesis process involves extracting and summarising relevant information from the publications considered in the analysis (Moyer-Packenham & Bolyard, 2016). This synthesis process involves amalgamating and integrating the material acquired from various scholarly articles to address the research inquiries of this investigation. It employs a narrative methodology that utilises descriptive and thematic analysis to identify and present recurring patterns, themes, or trends found across different academic articles (Verbruggen et al., 2021). The use of a narrative technique in this study is deemed appropriate due to its capacity to enable a comprehensive and adaptable integration of diverse types of research.

4. Results

This section reports the findings of the systematic review, which are categorized based on the research inquiries of this investigation.

Primary school instructors utilise several virtual and physical manipulatives in their learning instruction.

4.1. Range of manipulatives in practice

Throughout the literature, both physical and virtual manipulatives emerged as key features within mathematics classrooms. Physical manipulatives such as blocks, counters, fraction strips, and geometry solids remain mainstays due to usability and congruence with long-held pedagogy involving hands-on learning. Virtual manipulatives have grown in popularity in recent decades. Examples include dynamic geometry programs such as GeoGebra and Geometer's Sketchpad, fraction tools available on the National Library of Virtual Manipulatives (NLVM) website, and integrated suites such as those available from the Math Learning Center (Osana & Duponsel, 2016). Whereas virtual manipulatives can be interactive and receive instant feedback, physical manipulatives can engage students tactically and sensori-motorally. Overall, both studies indicate a practice that seldom employs manipulatives in a standalone approach but rather is contingent upon instruction's design and achieves a balance somewhere between physical and virtual spaces (Karakırık, 2016).

4.2. The use of manipulatives

The primary topics identified in the publications indicate that educators utilise both virtual and tangible manipulatives as pedagogical tools to introduce or reinforce learning concepts or skills (Liggett, 2017). Teachers frequently employ dynamic geometry software to effectively demonstrate the characteristics of angles or polygons (Jimenez & Stanger, 2017). Similarly, fraction bars or strips are used to visually represent the equivalence or comparison of fractions. Additionally, base-ten blocks are commonly employed to clarify place value concepts or operations involving numbers. Educators utilise both virtual and tangible manipulatives as pedagogical tools to foster students' engagement in studying and comprehending mathematical concepts and their interconnections (Satsangi & Raines, 2023). Teachers make use of virtual manipulative environments to enable students to experiment with a variety of learning tools or representations. For instance, pattern blocks encourage students' abilities to generate or identify patterns, shapes, or symmetries. In contrast, algebra tiles facilitate students' manipulation or resolution of algebraic expressions or equations (Jimenez & Stanger, 2017). Educators utilised both virtual and tangible manipulatives as pedagogical tools to enhance students' problem-solving and reasoning skills (Satsangi & Miller, 2017). Teachers employed virtual and real manipulatives to improve students' problem-solving and reasoning abilities (Karakırık, 2016). Teachers utilised dynamic geometry software to present complex or open-ended problems that require students to provide justifications or explanations for their solutions. Additionally, fraction bars or strips were utilised to support students in developing strategies or representations for solving fraction problems (Đokić et al., 2022). Meanwhile, base-ten blocks were used to facilitate students' reasoning or computation when addressing number problems.

4.3. Advantages or challenges in using virtual and physical manipulatives in primary lesson instruction

Utilising both virtual and physical manipulatives has significantly enhanced students' motivation, engagement, and interest in learning (Bouck et al., 2019). The use of virtual and physical manipulatives was well received by students due to their engaging, interactive, and visually appealing nature (Karakuş & Peker, 2015). The students also displayed favourable attitudes and emotions towards learning, including self-assurance, curiosity, and contentment. Employing virtual and physical manipulatives has been found to improve students' understanding of learning concepts, proficiency in executing procedures, and problem-solving abilities (Poongodi & Periasamy, 2020). Teachers utilised virtual and physical manipulatives to boost performance and achievement among students in tests or tasks (O'Meara et al., 2020). In their learning, students exhibited enhanced understanding and increased cognitive adaptability while engaging with various concepts and procedures (Hwang et al., 2018). Utilising virtual and physical manipulatives facilitated effective communication, collaboration, and reflection among students in the learning context (Crompton et al., 2021). Virtual and physical manipulatives have been observed to enable more substantial dialogues and explanations among students, both with their peers and professors (Pires et al., 2019). By employing virtual and physical manipulatives, students engaged in collaborative efforts and actively exchanged their thoughts and strategies with their peers (Arufe-Giráldez et al., 2023). Students were involved in critical self-reflection and evaluation, wherein they analysed and assessed their work or the work and feedback of their peers while utilising both virtual and physical manipulatives (Bouck, Mathews, & Peltier, 2020).

However, the use of virtual and physical manipulatives presents also several challenges. These challenges include the requirement for technological, pedagogical, or practical skills and resources, which may not always be readily accessible or available to teachers and students (López-Gámez et al., 2020). Educators have faced challenges when using virtual and physical manipulatives, primarily due to inadequate training, support, and confidence levels (Kabel et al., 2021). They have also struggled to incorporate virtual and physical manipulatives into other instructional components, such as curriculum, assessment, and classroom management (Simon, 2022). Teachers require sufficient equipment, resources, and time to effectively utilise virtual and physical manipulatives (Arufe-Giráldez et al., 2022). The use of virtual and physical manipulatives may lead to cognitive, emotional, or behavioural challenges or potential hazards for students (Moyer-Packenham & Westenskow, 2016). Students have experienced a range of negative emotions, including confusion, frustration, and boredom, when employing virtual and physical manipulatives that exceeded their level of complexity, difficulty, or relevance to their particular educational needs (Vágová, 2021). They may develop misconceptions, inaccuracies, or dependencies when using virtual and physical manipulatives that lack alignment or connection with mathematical symbols or representations. Additionally, students exhibited various forms of distraction, cheating, and off-task behaviour when utilising virtual and physical manipulatives without adequate teacher supervision or regulation (Peltier et al., 2019).

4.4. Consequences or effects of utilising virtual and physical manipulatives in mathematics instruction on teacher education and professional development

A common thread running across the evidence base is that manipulatives–physical or virtual–help students achieve conceptual understanding, visualize abstract concepts, and become active participants in the learning process. Most studies showed gains in problem-solving, fraction knowledge, and geometric reasoning. Virtual manipulatives, especially, were lauded for their ability to create dynamic visualizations and instant feedback that maintain student engagement and facilitate exploration (Azmidar et al., 2021). Physical manipulatives, on the other hand, were appreciated for facilitating collaborative learning and promoting discussions between peers during group work. Their use was also seen to be especially valuable among younger students and special education students, whose tactile interaction helped reinforce knowledge retention (Ladel & Kortenkamp, 2016, pp. 25–40). Through this manner, both virtual and physical manipulatives help provide multi-sensory learning experiences that venture beyond procedural calculation to further conceptual development. However, levels of impact ranged dramatically between contexts. Some studies reported few or even zero gains. Thus, Gecu-Parmaksiz and Delialioglu (2019) reported that virtual manipulatives failed to improve fraction learning noticeably compared to physical or control manipulatives. Zacharia and Michael (2016) also reported no noticeable impact of physical manipulatives on spatial reasoning abilities, raising concerns parallel to those reported by Satsangi et al. (2021). Such mixed evidence implies that manipulatives need not be successful across all contexts but rather become successful depending upon how teachers use manipulatives in contexts that work easily with teacher instruction and help achieve learning targets.

Teacher education and professional development must offer instructors the opportunity to acquire knowledge regarding the theoretical foundations and empirical evidence supporting the use of virtual and physical manipulatives in lesson delivery (Reiten, 2018). Educators should understand the advantages and challenges associated with employing virtual and physical manipulatives across various mathematical domains and proficiencies. They must have a comprehensive understanding of evidence-based concepts and strategies for proficiently using these manipulatives. Teacher education and professional development should enable instructors to engage in and assess the application of virtual and physical manipulatives in learning instruction (Ladel & Kortenkamp, 2016, pp. 25–40). Educators ought to be provided with a variety of virtual and physical manipulatives tailored to their instructional context and learning objectives. They must be able to evaluate the impact of these manipulatives on their own or their colleagues' teaching practices. Teacher education and professional development should offer teachers the opportunity to conceive and implement the use of virtual and physical manipulatives in educational instruction (Lehnert et al., 2022). Educators need to be capable of carefully selecting and employing appropriate virtual and physical manipulatives that align with their educational aspirations and desired outcomes (Lee & Chen, 2016). Beyond their core responsibilities, educators should effectively incorporate and integrate virtual and physical manipulatives with other instructional components, such as verbal explanations, written symbols, diagrams, or questions (Azmidar et al., 2021). To adequately support and assess the use of virtual and physical manipulatives across varying levels of understanding, educators must be able to scaffold and evaluate these instructional tools.

4.5. Implications for teachers and pedagogy

A critical theme that emerges from this review is that manipulatives' efficacy rests greatly on how teachers incorporate them into practice within the class. Research universally stressed that manipulatives should not be employed in isolated ways but within a sequenced plan of instruction, scaffolding, and discussion guides. Teachers' pedagogical choices, such as how to shift from concretes to abstractions, were critical in determining student results (Azmidar et al., 2021). Direct applications follow concerning teacher education and professional development. A number of studies pointed out those numerous teachers, while appreciative of manipulatives' prospect, did not receive adequate preparation or felt a lack of confidence in employing them efficiently (Ladel & Kortenkamp, 2016, pp. 25–40). Particularly, virtual manipulatives were not exploited adequately due to technology obstacles or low familiarity. Consequently, professional development programs that tackle both pedagogy and technology-based aspects come into immediate relevance to help equip teachers to utilise manipulatives to manipulatives' full capacity (Peters et al., 2020). Concurrently, the research warned against misuse, such as oversimplification or manipulative misinterpretation, due to poor guidance. Teachers consequently require not only technical competence but a firm footing in mathematics education theory to circumvent pitfalls and optimise benefits (Carbonneau, Marley, & Selig, 2013; Moyer-Packenham & Westenskow, 2016).

4.6. Methodological considerations and bias

In interpreting these results, methodological quality across reviewed studies demands sensitive consideration. Whereas some studies were sound, with large samples and strong designs, others were constrained by small numbers of participants, the absence of a control condition, or inconsistent achievement measures (Jimenez & Stanger, 2017). Such shortcomings make it possible that effects might be overstated or underrated in favour of manipulatives. Further, publication bias was implicated, whereby studies reflecting positive findings were widely disseminated compared to null or negative findings that attracted minimal attention (Azmidar et al., 2021). To compensate for this, both published and unpublished work was considered here, including grey literature, in concordance with PRISMA recommendations. Nonetheless, variability across methodological quality and reporting practice makes it difficult to draw firm inferences about overall effect magnitude. Such variability makes it imperative to conduct further large-scale studies that will be stringent, such as randomised controlled trials (RCTs), to identify situations in which manipulatives will be highly effective. Absent such evidence, pronouncements about universal manipulatives' benefits remain guarded and sensitive to context (Jimenez & Stanger, 2017).

4.7. Synthesis of insights

Taken in aggregate, evidence concludes that manipulatives—the physical and computer-based varieties—are productive instruments of primary mathematics teaching, provided that they are incorporated thoughtfully into instruction. They increase levels of engagement, facilitate conceptual understanding, and can be a positive force on a variety of abilities in pupils. However, effectiveness is anything but a given. Outcomes range widely across studies to attribute such influences as instruction design, teacher knowledge, the abilities of students, and subject matter. Manipulatives should then be considered not a silver bullet but rather teaching materials whose effectiveness is a function of context. Success in making use of them rests upon well-prepared teachers, positive classrooms, and careful alignment between instruction and learning objectives. Future research will need to go beyond lone studies to contribute stronger, large-scale evidence to support effectiveness and be informed by a more consistent base of methodology.

5. Discussion

5.1. Addressing RQ1: educators' use of virtual and physical manipulatives

Primary school educators use a variety of virtual and physical manipulatives in their teaching practices, including dynamic geometry software, fraction bars or strips, virtual manipulative environments, base-ten blocks, pattern blocks, and algebra tiles (Crompton et al., 2018). They employ these manipulatives to introduce or reinforce learning concepts and skills, support students' exploration of learning ideas and relationships, and enhance their problem-solving and reasoning abilities. Both virtual and physical manipulatives offer benefits and pose challenges in primary mathematical education (Yousef, 2021). The use of both types has significantly improved various aspects of students' academic performance (Debodinance et al., 2017).

Following our systematic review of the literature, one conclusion from the consulted studies is that primary school teachers (RQ1) employ both virtual and tangible manipulatives as pedagogical tools to motivate and engage pupils in learning mathematics. There is a consensus that the use of manipulatives facilitates the comprehension of mathematical concepts and their interrelationships. These manipulatives have positively influenced students' motivation, engagement, interest, conceptual understanding, procedural fluency, problem-solving skills, communication abilities, collaboration, and reflective thinking (Paola & Ordiales, 2023). Moreover, primary educators utilise both types of manipulative resources to create learning environments that enable pupils to experiment and discover connections among different mathematical objects. This, in turn, supports the development of pupils' problem-solving and reasoning abilities. Therefore, the response to RQ1 would be affirmative, asserting that primary school teachers use both virtual and physical tools to facilitate effective, enhanced exploratory and collaborative learning among students.

5.2. Addressing RQ2: advantages and challenges of manipulative use

The employment of both virtual and physical manipulatives has been found to have a positive impact on pupils' motivation (RQ2)—a finding consistently reported across all analysed studies. Virtual and physical manipulatives can present cognitive, emotional, or behavioural challenges and potential drawbacks for students (Pellas et al., 2018). These resources encourage greater interaction with mathematics, increase pupils' involvement, and enhance their interest in the subject. The reviewed studies further indicate that the use of manipulatives contributes to a deeper understanding of mathematical concepts and to the development of key competences such as problem-solving. Additionally, manipulatives promote critical thinking, self-reflection, and evaluative skills, fostering reflective and critical dialogue among learners.

However, these benefits come with certain challenges (RQ2). Using both virtual and physical manipulatives requires technical, pedagogical, or practical skills and resources, which may not always be readily accessible or achievable for educators or learners (Linder & Simpson, 2017). Integrating physical and virtual manipulatives into classroom practice also requires training in designing didactic tasks. Teachers and students may face issues such as lack of resources, inadequate training, or poor integration of technologies (López-Gámez et al., 2020). Overload, misunderstandings, and distractions can result from a misalignment between activities and goals, as well as insufficient support from the teacher's intervention. Therefore, it is vital for primary teachers to develop professional competencies that enable them to analyze and design mathematics teaching activities that effectively utilise manipulatives. Without such preparation, manipulatives might become distractions or even tools for superficial engagement, undermining their potential to foster meaningful mathematical understanding. This underscores the importance of providing adequate teacher training to effectively use technology in mathematics instruction. Therefore, the response to RQ2 emphasizes that the effective use of both virtual and physical manipulation tools depends on the teacher's competence.

5.3. Addressing RQ3: implications for teacher education and professional development

Regarding RQ3, the literature review indicates that teacher training is a vital factor influencing the effective integration of manipulatives in mathematics instruction. Educators should be well-trained and knowledgeable about the theories behind the practical use of manipulative tools for effective knowledge acquisition (Ladel & Kortenkamp, 2016, pp. 25–40). Training programs should expose instructors to both types of tools to enhance their curriculum development for teaching the subject (Lehnert et al., 2022). Therefore, it is essential that teacher education programs incorporate the pedagogical use of manipulatives along with reflective practice on their implementation as a core part of the skills and knowledge required for future mathematics teachers. The scope of teacher education and professional development significantly impacts how manipulatives are used in instructional settings (Pavlou et al., 2021). These programs must equip educators with insights into both the theoretical foundations and empirical evidence supporting the use of virtual and physical manipulatives in lessons (Bosse, 2016). Additionally, professional development programs should create opportunities for teachers to engage with and critically assess the use of virtual and physical manipulatives in instruction (Lehnert et al., 2022). They also help teachers in designing and implementing these resources within their teaching practices (Shurr et al., 2021).

6. Limitations

This study is constrained by the scope and criteria of the systematic review, potentially resulting in the exclusion of pertinent or significant studies that do not meet the predetermined inclusion and exclusion criteria (Kozan et al., 2023). This analysis excluded papers not authored in the English language or not published in peer-reviewed journals or conference proceedings. Furthermore, it is essential to note that this analysis only encompasses publications released between 2013 and 2023. The study is limited by the quality and thoroughness of the included literature, which may impact the validity and reliability of the study's findings and conclusions (Desoete et al., 2016). For example, certain publications included in the study exhibited subpar quality scores or ratings, indicating potential methodological deficiencies or limitations (Williams, 2020). Several publications in the analysis presented inconsistent or contradictory findings, suggesting the existence of heterogeneity or bias among the sources (Bouck et al., 2019). The study is further constrained by the synthesis and analysis of the extracted data, which could introduce subjectivity or errors in interpreting and integrating the findings and conclusions (Otten et al., 2019). For instance, certain aspects of the collected data were incomplete and less precise, necessitating the use of assumptions or subjective judgments. Some components of the synthesised data were derived through descriptive or thematic analysis, possibly omitting specific nuances or features.

7. Future directions

To enhance the rigor and comprehensiveness of the synthesis and evaluation of the impact of virtual and physical manipulatives on students' mathematical performance, it is imperative to conduct a meta-analysis or meta-synthesis of the quantitative or qualitative data extracted from relevant scholarly articles (Lee & Chen, 2016). This involves employing statistical or thematic approaches to amalgamate and juxtapose the findings of numerous studies that have assessed or elucidated the effects of virtual and physical manipulatives on students' learning or performance. This will facilitate a more thorough and reliable evaluation of the extent, orientation, coherence, and variation of the impacts of virtual and physical manipulatives across various contexts, populations, experimental designs, and assessment tools (Paliwal, 2022).

To enhance the comprehensiveness and inclusivity of a systematic review, it is necessary to include a broader range of scholarly articles on the use of virtual and physical manipulatives in education (Gunasegar et al., 2021). For example, considering papers written in languages other than English or published in alternative sources, such as books, reports, or dissertations, can be beneficial. Additionally, it is vital to include scholarly articles published before 2013 or after 2023. This requires expanding the search method and refining the selection criteria to include a larger number of scholarly articles that investigate the use of virtual and physical manipulatives in the context of mathematical education. Incorporating a more comprehensive range of sources and perspectives, including additional evidence, will enhance the overall comprehensiveness and inclusivity of the existing body of knowledge, thus reducing the risk of inherent biases or limitations in the literature review (Macrides et al., 2022). Furthermore, this encourages the inclusion of scholarly articles that provide diverse viewpoints or novel insights into the application of virtual and physical manipulatives within mathematical education.

To conduct a more focused and specialised systematic review, it is essential to investigate a specific feature or topic regarding the use of virtual and physical manipulatives in educational settings (Reneau, n.d.). For instance, the investigation will focus on using virtual and physical manipulatives across various mathematical domains, such as fractions, geometry, or algebra (Cao & Hsu, 2022). Additionally, it is crucial to explore the application of virtual and physical manipulatives across diverse groups or situations, including children with special needs, students with lower academic performance, and schools in rural areas (Marley & Carbonneau, 2014). This involves refining the study's inquiries and goals to focus on a particular subject or issue related to the use of virtual and physical manipulatives in educational contexts. This approach would yield a comprehensive and thorough examination and integration of the scholarly works regarding the use of virtual and physical manipulatives concerning the specified subject matter (Bouck & Park, 2018). Furthermore, this would facilitate the exploration of using virtual and physical manipulatives for various subgroups or circumstances that may present distinct requirements or challenges in mathematical education (Silva et al., 2021).

8. Contributions

This research provides a thorough analysis of both virtual and physical manipulative use in primary school instruction, with a clear focus on pedagogical application and educational effectiveness. Everything in this project was completed collaboratively by the research team. The study's conception and design were developed collectively to ensure both practical use of manipulatives in schools and a solid understanding of the theoretical background from existing literature. Data collection involved rigorous, systematic searching, screening, and selecting relevant studies, which were then shared and carefully evaluated by all authors. Both authors participated in analyzing and interpreting the results to maintain consistency and ensure accurate identification of themes, patterns, and gaps in knowledge. Draft preparation was a team effort. The first author drafted the initial version of the paper, while the second author critically reviewed it to enhance its intellectual content, sharpen arguments, and clarify ambiguities. Both authors contributed to defining the findings and recommendations, especially regarding pedagogical implications and future research. They also gave final approval for submission and agreed to take responsibility for the integrity of the work. Both authors meet authorship criteria, having participated in conception, preparation, revisions, and the study's overall responsibility. This collaborative effort ensures that the paper not only builds on existing knowledge but also contributes meaningfully to advancing evidence-based practices in primary education.

CRediT authorship contribution statement

Angela Ogbugwa Ochugboju: Writing – original draft, Formal analysis. **Javier Díez-Palomar:** Writing – review & editing, Supervision, Methodology, Investigation, Conceptualization.

Data availability statement

The data that support the findings of this study are available on request from the corresponding author.

Ethics approval and consent to participate

Ethical approval is not applicable to this manuscript.

Declaration of generative AI and AI-assisted technologies in the Writing process

During the preparation of this work the author(s) used Grammarly in order to edit the language. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

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