Comment on "Association between quantum paradoxes based on weak values and a realistic interpretation of quantum measurements"

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In their paper [Phys. Rev. A **109**, 022238 (2024)], Aredes and Saldanha analyze several paradoxes related to weak values and present a "general argument" that aims to show that "realistic interpretations . . . of weak values lead to inconsistencies." Although we agree with the identified inconsistencies for the specific weak values analyzed there, in this Comment we demonstrate that the origin of these inconsistencies is not their general argument, which is formally incorrect. We use Bohmian mechanics as a counterexample to confirm that their conclusions are not valid for all weak values and quantum theories. In particular, we show that weak values postselected in position can in fact be interpreted within Bohmian mechanics as properties of quantum systems, detached from any measuring devices, in a consistent and meaningful way.

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I. INTRODUCTION

The paper in Ref. [1] initiates a discussion on whether weak values can be interpreted as referring to real properties within contextual quantum theories. The authors explore the meaning of weak values by analyzing several paradoxes. While we agree with Aredes and Saldanha that a realistic interpretation of weak values, as applied to the specific paradoxes they examine, leads to inconsistencies, we disagree with their explanation for the origin of these inconsistencies, as presented in the crucial section entitled "General Argument" of Ref. [1] (Sec. II therein).

The authors define a "realistic interpretation of a quantum measurement" (RIQM) as follows: "A measurement performed on a quantum system reveals the underlying ontological value of the measured quantity, that continues the same after the measurement." In addition, they define a "realistic interpretation of weak values" (RIWV) as follows: "The weak value $\langle O\rangle_w$ of an operator O reveals the objective reality of the physical quantity $\tilde{O}$ associated to this operator . . ." [2]. From these definitions, through the above-mentioned General Argument section, they conclude the following.

Conclusion 1. "The attribution of a physical reality to the weak value $\langle O\rangle_w$ is equivalent to adopting the cited realistic interpretation of quantum measurement." In other words, they consider that the RIQM and the RIWV are equivalent when dealing with weak values.

Aredes and Saldanha consider that the RIQM is "highly controversial" since it plainly contradicts the well-known quantum contextuality. Now, given this fact and the alleged equivalence between the RIQM and RIWV, the authors finally conclude the following.

Conclusion 2. "[R]ealistic interpretations . . . of weak values lead to inconsistencies . . . ."

In this Comment, we present three results to show that both Conclusions 1 and 2 are incorrect.

II. FALLACIOUS ARGUMENTATION

Let us carefully reconstruct Aredes and Saldanha's argument in Sec. II (entitled "General Argument") of Ref. [1] to show that it is fallacious. The authors invite us to consider a weak measurement procedure of an observable O, in which the quantum system is preselected in a state $|\psi_i\rangle$ and postselected in a state $|\psi_f\rangle$. The obtained weak value is therefore $\langle O\rangle_w = \langle\psi_f|O|\psi_i\rangle/\langle\psi_f|\psi_i\rangle$. Next the authors consider a situation in which the operator $\hat{O}$ can be written as the sum of an operator $\hat{P}$, from which the preselected state is an eigenvector with eigenvalue p, and an operator $\hat{Q}$, from which the postselected state is an eigenvector with eigenvalue q, that is, $\hat{O}$ satisfies $\hat{O} = \hat{P} + \hat{Q}$ with $\hat{P}|\psi_i\rangle = p|\psi_i\rangle$ and $\hat{Q}|\psi_f\rangle = q|\psi_f\rangle$ [see their Eq. (2) in Ref. [1]]. As a piece of terminology, the authors use the symbol $\tilde{O}$ to denote the "ontological value" corresponding to the property O associated with the operator $\hat{O}$. These ontological values are interpreted as objective properties of the considered system and are mentioned both in the RIQM and in the RIWV. Now we are prepared to present the crucial steps of Aredes and Saldanha's argument.

First, assuming the RIWV, they claim that, in the scenario above, the objective physical quantities associated with the operators $\hat{O}$, $\hat{P}$, and $\hat{Q}$ are $\tilde{O} = p + q$, $\tilde{P} = p$, and $\tilde{Q} = q$, respectively. Second, assuming the RIQM, and through slightly

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more elaborate reasoning, the authors claim to reach the same conclusion, namely, that $\tilde{O} = p + q$, $\tilde{P} = p$, and $\tilde{Q} = q$. So, up to this point, we can see that the authors' claim is that both the RIWV and RIQM entail the same consequence with respect to the ontological values $\tilde{O}$, $\tilde{P}$, and $\tilde{Q}$ in the considered scenario. Rather surprisingly, from this, they conclude the following.

> The attribution of a physical reality to the weak value value $\langle O\rangle_w$ [what we call the RIWV] is equivalent [3] to adopting the cited realistic interpretation of quantum measurements [what we refer to as the RIQM], since both assumptions lead to the same objective value for the physical quantity $\tilde{O} : \tilde{O} = p + q$ .... Since the cited realistic interpretation of quantum measurements is highly controversial, a reasonable way to avoid all the cited quantum paradoxes is to deny this realistic interpretation of quantum measurements, also denying the realistic view of the weak values.

Now it should be clear that this reasoning is fallacious. Two assumptions can share some consequences, yet this does not entail that they are logically equivalent. Given that Aredes and Saldanha reach Conclusion 1 in a fallacious manner, this conclusion is unfounded. In the next section we demonstrate that, moreover, it is false.

III. CONCLUSION 1 IS INCORRECT

Bohmian mechanics is a theory that accounts for all nonrelativistic quantum phenomena. Its ontology consists of particles with well-defined positions regardless of whether or not they are being observed [4–7]. The dynamics involves a (deterministic) equation of motion for the particles' positions (the so-called guiding equation [6]), which provides the velocity of each particle as a functional of the wave function. Additionally, the law of motion for the wave function itself is the (deterministic) Schrödinger equation.

A. Bohmian mechanics, a contextual theory

When Aredes and Saldanha mention that the RIQM is highly controversial, they explicitly refer to quantum contextuality and the Kochen-Specker theorem [8–13] to support this claim. One standard way to avoid the refutatory charge of this theorem is to accept that any hidden-variable theory aiming to be empirically adequate must be contextual. This amounts to denying that, if a quantum system possesses a property (the ontological value of an observable), it does so independently of any measurement context. It is worth noticing that Bohmian mechanics is contextual in this sense and that, as a consequence, is not refuted by the Kochen-Specker theorem and other similar results [12].

The RIQM affirms (or entails) what is known in the literature as the faithful measurement principle, which asserts that a measurement of an observable faithfully reveals the value which that observable had immediately prior to the measurement interaction [13]. Quantum mechanics violates both the faithful measurement principle and RIQM. In order to show this, consider a system in the state $\frac{1}{\sqrt{2}}(|\psi_a\rangle + |\psi_b\rangle)$, where $\hat{O}|\psi_a\rangle = a|\psi_a\rangle$ and $\hat{O}|\psi_b\rangle = b|\psi_b\rangle$. Now suppose one makes a (strong) measurement of observable O and obtains the eigenvalue a as the outcome. According to orthodox quantum mechanics and the eigenvalue-eigenstate link, this measurement does not faithfully reveal the preexisting value of O because, prior to the measurement, the ontological value of O was simply undefined. Therefore, in orthodox quantum mechanics, it is the very act of measuring O that "creates" a definite ontological value of the property O.

Bohmian mechanics also violates both the faithful measurement principle and RIQM. In this theory, the ontological value of a property after a measurement typically does not match its preexisting ontological value because the value of the property is modified during the interaction with the measuring apparatus. Notice that it is not the case that a definite property is created by the very act of measuring, as happens according to the orthodox quantum-mechanical approach. Simply, the (measuring) interaction between the two systems (the system itself and the measuring apparatus) may eventually lead to a change in some of their properties.

Overall, the RIQM is not only highly controversial but also incompatible with any quantum theory, as it contradicts the inherently perturbative nature of quantum measurements (and quantum interactions in general). This characteristic reveals a form of contextuality broader than that typically discussed within the framework of the Kochen-Specker theorem. Throughout the rest of our paper, we use the term "contextuality" in this broader sense.

B. Bohmian mechanics as a counterexample to Conclusion 1

We now consider an operator $\hat{O} = \hat{P} + \hat{Q}$ that satisfies the prerequisites of the general argument given in Ref. [1] and whose weak values are physically meaningful within the framework of Bohmian mechanics. As the operator $\hat{Q}$ we take the position operator $\hat{X}$ so that we consider for postselection the position eigenstate $|x_0\rangle$. We have that [14]

$$\langle X\rangle_w = \mathrm{Re}\frac{\langle x_0|\hat{X}|\psi_i(t)\rangle}{\langle x_0|\psi_i(t)\rangle} = x_0, \tag{1}$$

where $\hat{X}|x_0\rangle = x_0|x_0\rangle$. As operator $\hat{P}$ we take the velocity operator $\hat{V} = -i\frac{\hbar}{m}\frac{\partial}{\partial x}$, with m the mass of the particle. For the postselected position eigenstate $|\psi_f\rangle = |x_0\rangle$, we obtain the weak value $\langle V\rangle_w$ given by

$$\langle V\rangle_w = \mathrm{Re}\frac{\langle x_0|\hat{V}|\psi_i(t)\rangle}{\langle x_0|\psi_i(t)\rangle} = \frac{J(x_0,t)}{|\psi_i(x_0,t)|^2} = v^{\psi_i}(x_0,t), \tag{2}$$

where $J(x,t) = \frac{\hbar}{m}\mathrm{Im}\psi_i(x,t)\frac{\partial\psi_i^*(x,t)}{\partial x}$ is the quantum expression of the current density [15]. Within Bohmian mechanics, it is natural to interpret realistically x_0 in (1) and $v^{\psi_i}(x_0,t)$ in (2) as the position and the velocity, respectively, of a particle described by the wave function $\psi_i(x,t) = \langle x|\psi_i(t)\rangle$ and located at x_0.

In the case of a plane wave $\langle x|\psi_i\rangle \propto e^{ikx}$ (or an approximate plane wave), we get $v^{\psi_i}(x_0,t) = \hbar k/m$. Then we can construct the operator $\hat{O} = \hat{V} + \hat{X}$, which satisfies the requirements of Eqs. (2) and (3) in General Argument section of Ref. [1] for the given wave function. Within Bohmian mechanics, we obtain $\langle O\rangle_w = \hbar k/m + x_0$ and this weak value can be readily interpreted as the sum of the real velocity and the real position of the particle, satisfying the RIWV. Yet still, due to the contextual nature of the Bohmian theory, it does not satisfy Aredes and Saldanha's RIQM (i.e., the RIWV is possible without the RIQM), serving as a straightforward counterexample demonstrating that Conclusion 1 cannot be correct.

IV. CONCLUSION 2 IS INCORRECT

Orthodox and Bohmian quantum mechanics are empirically equivalent. Born's rule follows as a corollary from the equivariant motion of Bohmian trajectories and the quantum equilibrium hypothesis [16]. In particular, for a set of hypothetical trajectories $x^j(t)$, with $j \in \{1, \ldots, N\}$, each representing the trajectory of the system in a different experimental realization of the same single-particle wave function $\psi(x,t)$, one obtains that

$$|\psi(x,t)|^2 = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \delta(x - x^j(t)). \tag{3}$$

Apart from the position of a particle, it is natural to define additional properties subordinate to this position, as we have done for the velocity $v^\psi(x,t)|_{x=x^j(t)} = dx^j(t)/dt$ in Sec. III B. In general, for any function $S_B^\psi(x,t)$ of the wave function, one can define $S_B^\psi(x^j(t),t)$ as a Bohmian property subordinate to $x^j(t)$ (be it ontologically meaningful or merely informative). As happens to $x^j(t)$ and $v^\psi(x^j(t),t)$, this new Bohmian property refers to quantum systems that are not being measured. Yet still, within the Bohmian theory, using (3), one can compute the ensemble value of the property $S_B^\psi(x^j(t),t)$ over all $x^j(t)$ to be

$$\langle S_B\rangle = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} S_B^\psi(x^j(t),t) = \int dx\, S_B^\psi(x,t)|\psi(x,t)|^2. \tag{4}$$

Among the possible fields $S_B^\psi(x,t)$, there are some, the so-called local expectation values of Hermitian operators [17], which yield physically relevant properties for the trajectories because their ensemble average gives exactly the expectation associated with quantum measurements of the operator in question. Precisely, the local expectation value of the Hermitian operator $\hat{S}$ is defined to be

$$S_B^\psi(x,t) := \mathrm{Re}\frac{\langle x|\hat{S}|\psi(t)\rangle}{\langle x|\psi(t)\rangle}, \tag{5}$$

which is straightforwardly seen to satisfy $\langle S_B^\psi\rangle = \langle\psi|\hat{S}|\psi\rangle$. However, (5) is nothing but the position postselected weak value of $\hat{S}$ [14]. It turns out that not only for the position and velocity, but for many more observables, their position postselected weak values, i.e., local expectations (5), not only match the operator expectations, but happen to coincide with the Bohmian definitions of those properties, which are found independently of the discussion of a measurement of weak values (with no inconsistencies whatsoever). For instance, if one sets $\hat{S}$ to be the Hamiltonian operator H, the local expectation turns out to be exactly equal to the Bohmian energy (kinetic plus classical and quantum potentials [6]) of the particle. For the angular momentum operator, one gets the angular momentum of the Bohmian particle. And the list goes on [6,17,18].

In summary, interpreting weak values postselected in position as values of properties of Bohmian particles offers a consistent and paradox-free framework for understanding experiments on the weak values defined as $S_B^\psi(x,t) = \mathrm{Re}\langle x|\hat{S}|\psi(t)\rangle/\langle x|\psi(t)\rangle$. This consistency is directly inherited from the consistency of the Bohmian theory in accounting for all nonrelativistic quantum phenomena [6,7,19]. Therefore, we have shown that Conclusion 2 is also incorrect as a general claim. At least a realistic interpretation of weak values postselected in position within Bohmian mechanics does not lead to any contradiction [20].

V. NONCONTEXTUAL WEAK VALUES FROM CONTEXTUAL BOHMIAN MECHANICS

We have claimed that weak values (postselected in position), as formal quantities (independently of experimental protocols estimating them), can be consistently interpreted within Bohmian mechanics as properties determinately possessed by particles, such as position and velocity. Now, as explained later, there exist experimental protocols to estimate some of these weak values for microscopic systems. So some of their Bohmian properties, which are *a priori* independent of any context in the laboratory, can be determined empirically. If we call this a measurement, it will seem there is a contradiction with the fact that Bohmian mechanics is a contextual quantum theory; however, there is no contradiction when we clarify that the word "measurement" can have slightly different meanings. In quantum mechanics, it usually means an experiment conducted on a single copy of the system. However, a protocol to determine a weak value uses an ensemble of copies of the system, with a contextual intervention on each of them. Such intervention will alter the subsequent dynamics of each copy for the inherent backaction of quantum interactions, but within Bohmian mechanics, the obtained number (the weak value) will indeed coincide with each copy's property at the time right before our intervention started. The weak value is "measured" in this statistical sense.

To be specific, the empirical protocol to measure a weak value involves an ensemble of N identically prepared microscopic systems, all described by the same initial wave function ψ. On each individual microscopic system, a weak measurement [21] of $\hat{O}$, giving o_a, is performed, followed by a projective measurement of an $\hat{F}$ giving f_a (with eigenvector $|f_a\rangle$). This yields a large set of pairs $\{o_a, f_a\}_{a=1}^N$. The average of the o_a's conditioned on a fixed $f_a = f$ then estimates $\mathrm{Re}\langle f|\hat{O}\psi\rangle/\langle f|\psi\rangle$ (to leading order in both the device-subsystem coupling and the time between each o_a and f_a measurement) [6,7,14].

In particular, the number estimated by considering $\hat{O} = \hat{v}$ and $\hat{f} = \hat{X}$ is $v^\psi(x,t)$ [as in Eq. (2)], i.e., the Bohmian velocity that a microscopic system with wave function $\psi(x,t)$ (free from the perturbation of a measurement backaction) would have. Note that even if this will approximate the actual Bohmian velocity of the individual microscopic systems used to compute $v^\psi(x,t)$ right before the measurements started in the laboratory, typically, it does not allow the determination of their Bohmian velocity between the two measurements or at later times. A quantum measurement (weak or not) of an ensemble of identically prepared microscopic systems implies an entanglement between each system and measurement apparatus. As soon as this interaction begins, the Bohmian velocity of each microscopic system needs to be considered in each joint system-measurement apparatus configuration space [namely, each system's velocity will no longer be only a function of x, as in $v^{\psi}(x,t)$, but will also depend on external variables in the laboratory]. Note that none of these should be surprising, since the experimental protocol yielding the theoretical expectation value $\langle\psi|O|\psi\rangle$ also satisfies that (i) the outcome is obtained by averaging, (ii) it characterizes how the measured copies of the microscopic system were right before they were intervened, but (iii) not (usually) how they were after that.

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[1] A. M. Aredes and P. L. Saldanha, Association between quantum paradoxes based on weak values and a realistic interpretation of quantum measurements, Phys. Rev. A **109**, 022238 (2024).

[2] We must clarify an ambiguity in Aredes and Saldanha's definitions. An experimental weak value involves three times: the preselection time t_1, when the initial state of the quantum system is prepared; the time t_2, when the weak measurement is conducted; and the postselection time t_3, when the projective measurement is done. However, the definition of the weak value $\langle O\rangle_w$ in Eq. (1) of Ref. [1] reflects the assumption $t_3 = t_2 = t_1$, as the absence of time evolution operators between the preparation and the weak measurement, or between the weak measurement and the postselection, testifies. This makes Aredes and Saldanha's clarification of the RIWV stating that the weak value "$\langle O\rangle_w$ ... reveals the objective reality ... at a time between the pre- and postselections" rather vague and inconsistent with their own definition of $\langle O\rangle_w$ because such times are not present there. Hence, we also consider $t_3 = t_2 = t_1$ in the definition of the RIWV in the interest of consistency between the RIWV and the formula of a weak value. Another reading of their clarification may suggest that the "time between the pre- and postselections" refers to an instant between t_1 and t_2, when the quantum system is already preselected yet still disentangled from the measuring apparatus. In the latter case, their statement aligns with a weak-value formula defining a property of unperturbed subsystems, confirming our decision to omit it from the RIWV.

[3] Throughout this Comment, we assume that when Aredes and Saldanha use the word "equivalent" they are attempting to show that the RIQM and RIWV are "logically equivalent" (Conclusion 1). In Sec. II we demonstrate that the authors establish this logical equivalence in a fallacious manner. Aredes and Saldanha might eventually respond to this analysis by claiming that they are not committed to showing that the RIQM and RIWV are logically equivalent, but merely that they are equivalent in a (much weaker) sense: that both lead "to the same objective value for the physical quantity $\tilde{O}$: $\tilde{O} = p + q$." However, then nothing controversial about the RIQM beyond this specific consequence can serve as a reason to reject the RIWV. Moreover, as mentioned earlier, the authors acknowledge that the negation of the RIWV has already been presented as a resolution of the cited paradoxes (which is not a very illuminating thesis since it does not help explain why the RIWV leads to problematic results in some cases but not in others as shown in Sec. III B). In summary, either the word equivalent means logically equivalent, in which case Aredes and Saldanha have a potentially interesting (though, as we show, false) claim or they fail to contribute anything novel to the physics of weak values in their paper.

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[14] The expression $\langle x|\hat{S}|\psi(t)\rangle/\langle x|\psi(t)\rangle$ in Eq. (5) is a complex number and it can be written as $\mathrm{Re}\langle x|\hat{S}|\psi(t)\rangle/\langle x|\psi(t)\rangle + i\,\mathrm{Im}\langle x|\hat{S}|\psi(t)\rangle/\langle x|\psi(t)\rangle$. Thus, one can define one weak value for the real part of (5) and another weak value for the imaginary one. The experimental determination of these two weak values involves two different protocols in the laboratory. In this Comment, we only deal with the weak value associated with the real part of (5), but a similar discussion, if needed, can be done for the imaginary part [7]. Notice, in addition, that similar conclusions can be obtained for weak values preselected (instead of postselected) in position defined as $\mathrm{Re}\langle\psi(t)|\hat{S}|x\rangle/\langle\psi(t)|x\rangle$.

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[19] Since $S_B^{\psi}(x^j(t), t)$ represents the value of a property of a quantum system detached from any measuring apparatus, it is, by construction, a context-free value, as its definition does not depend on any measurement context. The property $S_B^{\psi}(x^j(t), t)$ (and all the weak values in general) is not required to satisfy the Bell-Kochen-Specker theorem because the hypotheses of such a theorem apply only to measured values of an individual quantum system. In contrast, an experimental weak value always involves several quantum measurements, as described in Sec. V.

[20] For example, the three-box paradox simply disappears when the arbitrary postselected state $|\psi_f\rangle = (|A\rangle + |B\rangle - |C\rangle)/\sqrt{3}$ mentioned in Ref. [1] is replaced by the position postselected state $|\psi_f\rangle = |x\rangle$. In this case, the weak value postselected in position, $J_B^{\psi_i}(x) := \mathrm{Re}\langle x|j\rangle\langle j|\psi_i\rangle/\langle x|\psi_i\rangle$ with $j = \{A, B, C\}$, can indeed be interpreted as "being in box j" without any inconsistency. For a Bohmian system defined by the initial state $|\psi_i\rangle = (|A\rangle + |B\rangle + |C\rangle)/\sqrt{3}$, plus a (primitive) property x_A inside box A, the subordinate property of being in box j defined by weak values postselected in position $J_B^{\psi_i}(x_A)$ follows trivially as $A_B^{\psi_i}(x_A) = 1$, $B_B^{\psi_i}(x_A) = 0$, and $C_B^{\psi_i}(x_A) = 0$. Corresponding results apply when the particle position is x_B inside box B and x_C inside box C. The particle will never be present in two boxes simultaneously.

[21] The weak measurement of $\hat{O}$ is described by a positive-operator-valued measure given by a projective measurement on an ancillary system weakly coupled to the object quantum system. The measurement collapses the ancilla into an eigenstate of the observable coupled to the operator $\hat{O}$ of the subsystem, yielding the eigenvalue o_a of the former as the outcome of the experiment.