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JEL Codes: O47, E61, E62.

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Do fiscal rules affect growth?*

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February 14, 2025

Abstract

Over the past three decades, many countries have adopted fiscal rules. This paper studies their impact on economic growth using an overlapping generations model with endogenous growth, where the government imposes both a debt rule and a budget balance rule. The model shows that fiscal rules are not neutral: their design and interaction, through an endogenously adjusting tax rate, directly shape savings, capital accumulation, and long-term growth. The model identifies conditions under which a balanced growth path exists and highlights the possibility of multiple steady states. When fiscal rules are too loose or initial debt is too high, the economy may converge to an unstable path. Tightening fiscal rules improves long-run welfare but can reduce current utility due to higher taxes. Empirically, we estimate a growth equation and address endogeneity using an instrumental variable strategy based on the geographical diffusion of fiscal rules. The results indicate that the adoption of fiscal rules boosts growth in developing and low-income countries. In Europe, only well-designed rules are associated with higher growth. Across specifications, debt rules consistently outperform budget balance rules, especially in less developed economies.

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1 Introduction

In the last few decades, economies all over the globe witnessed the emergence and persistence of large fiscal deficits and public debt. The sustainability of public finances has acquired new importance after the global financial crisis in the late 2000s. Since the early 90s, the adoption of fiscal rules has been increasing in the number of rules per country and in the number of countries that have adopted at least one rule.[1] Nowadays, there are over 90 countries with at least one fiscal rule in place.[2] According to Caselli et al. (2018), in the early 90s, the average number of rules per country was less than one. By 2015, this average increased to three rules per country, significantly accelerating after the 2008 financial crisis.[3]

Some criteria exist when choosing fiscal rules, such as sustainability, stabilization, simplicity, operational guidance, resilience, ease of monitoring, and enforcement (Kopits and Symansky, 1998). Nevertheless, a single fiscal rule might not achieve these criteria simultaneously. Some trade-offs are likely to emerge, such as stabilization versus simplicity: a more flexible rule to accommodate macroeconomic shocks is likely to have a more complex design. For example, rules that correct for the impact of business cycles by targeting cyclically adjusted balances. Well-designed fiscal structures are generally built over two pillars: (1) a fiscal anchor linked to the final objective of fiscal policy, and (2) one or more operational rules on fiscal aggregates (Andrle et al., 2015). The debt-to-GDP ratio acts as a fiscal anchor by setting a limit on how much public debt can grow and by shaping expectations about fiscal policy over the medium to long term. It can be designed to help ensure the long-term sustainability of public finances. However, it does not offer clear guidance for short-term fiscal decisions. In this regard, budget balance rules serve as more practical tools for day-to-day policy, as they are directly linked to debt developments and can be more easily managed by the government. Considering both types of rules together—one as a long-term anchor and the other as a short-term guide—can help strengthen fiscal discipline while supporting long-run economic growth.

This paper investigates whether the adoption and design of fiscal rules—specifically, debt and budget balance rules—have a measurable impact on long-run economic growth. We develop a theoretical framework based on an overlapping generations (OLG) model with endogenous growth through productive public spending, in which fiscal policy is constrained by both a debt rule and a budget balance rule. The debt rule sets an upper bound for the public debt. The deficit rule, which works as a budget balance rule, aims to keep budget deficits at a certain percentage of GDP. The tax rate is used to adjust the government budget constraint; thus, it is endogenously chosen. The model examines how these fiscal rules affect capital accumulation, interest rates, savings, and the long-run growth rate. It shows that fiscal rules significantly influence growth and welfare by shaping fiscal behavior and intertemporal resource allocation. A debt rule serves as a long-term anchor that curbs excessive borrowing and lowers future tax distortions, while a budget balance rule enforces short-term discipline but may restrict productive public spending.

[1] The International Monetary Fund (FMI) defines fiscal rules as longer-lasting constraints on fiscal policy through numerical limits on budgetary aggregates. The objective of such rules is to pursue debt sustainability and to ensure fiscal responsibility. They have been widely used to restrain fiscal policy discretion and bolster fiscal discipline.

[2] Eyraud et al. (2018).

[3] In the European Union, this average tripled from 2 to 6 fiscal rules between 2000 and 2015. Among countries outside the EU, the average number of fiscal rules went from zero during the same period.

Their interaction is key: a loose debt rule combined with a strict deficit rule can raise short-term welfare at the expense of long-run sustainability. In contrast, jointly enforced rules enhance both steady-state growth and intergenerational welfare, highlighting the importance of rule design in determining economic outcomes.

To assess the empirical relevance of the theoretical predictions, the second part of the study presents two empirical exercises using a cross-country panel. The first estimates a growth equation derived from the theoretical model, controlling for country and year fixed effects, to examine whether the observed correlations align with the model's predictions. This specification captures statistical associations without attempting to establish causality. In the second exercise, we address the potential endogeneity of fiscal rule adoption by employing an instrumental variables strategy that leverages the geographical diffusion of fiscal rules across countries, as proposed in Caselli and Reynaud (2019). This approach allows us to isolate the variation in fiscal rules that is plausibly exogenous to domestic growth conditions.

We implement this empirical strategy using a dataset covering 178 countries over the period 1985-2015, which enables us to examine heterogeneous effects across different income groups. The results show that the adoption of fiscal rules is associated with higher GDP per capita growth in developing and low-income countries. For European countries, fiscal rules that meet certain design criteria are also linked to higher growth. Across specifications, debt rules display a more consistent relationship with growth than budget balance rules. These findings suggest that the growth effects of fiscal rules depend not only on their existence but also on their structure and the broader institutional environment in which they operate.

This paper contributes to the literature on theoretical models that study fiscal sustainability under fiscal rules in overlapping generations (OLG) (Bräuninger, 2005; Yakita, 2008; Arai, 2011; Teles and Mussolini, 2014; Agénor and Yilmaz, 2017). These studies typically assume a constant deficit-to-GDP rule but omit debt constraints or the interaction between multiple fiscal rules, and none of them incorporate an empirical validation of their theoretical predictions. More recent research has clarified the channels through which fiscal rules affect growth, using OLG and AK-type models. For example, Carranza-Ugarte et al. (2023) develop an OLG model for Peru showing that well-calibrated, cyclically adjusted fiscal rules improve welfare and output. Our paper complements and extends this literature by introducing two distinct fiscal rules—a debt rule and a budget balance rule—into an endogenous growth OLG framework and analyzing their individual and joint effects on equilibrium dynamics, capital accumulation, and welfare. Unlike previous models that abstract from debt dynamics or treat investment as exogenous, our framework explicitly models the trade-offs involved and derives testable predictions that are then confronted with empirical evidence.

Several empirical studies have examined the impact of fiscal rules on fiscal outcomes (Debrun et al., 2008; Tapsoba, 2012; Bergman and Hutchison, 2015; Bergman et al., 2016; Caselli and Reynaud, 2019). However, there is less conclusive evidence regarding the effects of fiscal rules on economic growth (Castro, 2011; Afonso and Jalles, 2012; Gründler and Potrafke, 2020; Nabieu et al., 2021; Potrafke, 2025). While studies such as Gründler and Potrafke (2020) and Potrafke (2025) find a positive relationship between constitutional fiscal rules and long-run GDP growth, others—especially in developing country contexts—report more nuanced or negative outcomes. Nabieu et al. (2021), for instance, show that although fiscal rules improved fiscal balances in Sub-Saharan Africa, they also reduced growth by curbing critical public investment. Our empirical analysis adds to this debate by explicitly distinguishing between types of fiscal rules and their design quality, offering evidence that debt rules are more conducive to growth than budget balance rules, particularly in developing and low-income countries. Furthermore, our use of the geographical diffusion of fiscal rules as an instrumental variable is an important methodological innovation that improves causal inference in a context where endogeneity is a central concern.

Our paper also contributes to the literature that compares the effectiveness of debt rules and budget balance rules. Recent studies, including Nerlich and Reuter (2015) and Combes et al. (2017), find that debt rules alone are often too lax to discipline fiscal policy unless coupled with operational rules. Meanwhile, budget balance rules are shown to be more immediately binding, but potentially procyclical in downturns if not well designed. Our theoretical model confirms these trade-offs: it shows that relaxing the deficit rule may temporarily raise utility due to lower taxes, but at the cost of fiscal sustainability. Empirically, our IV analysis reveals that debt rules outperform budget rules in stimulating growth, particularly when designed flexibly and implemented in institutional environments that support credibility.

The quality and institutional anchoring of fiscal rules play a crucial role in their effectiveness, as emphasized in works by Eyraud et al. (2018), Bergman et al. (2016), and Manescu and Bova (2020), which highlight that well-designed rules—with legal backing, flexibility, and credible enforcement—are more likely to yield macroeconomic and growth benefits. Our paper builds on this insight by incorporating the IMF's rule strength index and demonstrating that rule quality significantly influences growth, particularly in advanced economies where the marginal effects of simply adopting rules have diminished. Unlike earlier empirical studies that often focused solely on the presence or absence of fiscal rules while ignoring heterogeneity in their type or enforcement, our approach adopts a more granular methodology using both binary and continuous measures of rule strength from the IMF Fiscal Rules Dataset. This allows us to capture the nuanced relationship between the stringency and credibility of rules and their impact on growth. While estimates based on dummy variables show limited significance, those using strength indices confirm our theoretical model's prediction that tighter debt rules are growth-enhancing. These results align with Vinturis (2023), who finds that countries with debt or balanced-budget rules allocate relatively more to public investment, a key growth driver. Furthermore, by isolating the impact of fiscal rules through external instruments based on neighboring countries' policies, and integrating fiscal dynamics into a general equilibrium structure, our paper provides both empirical evidence and theoretical grounding for the importance of rule design, credibility, and institutional support.

The rest of the paper is structured as follows: Section 2 presents the model; Section 3 analyzes equilibrium dynamics and welfare implications; Section 4 presents the first empirical exercise, where we estimate a growth equation derived from the model to analyze the direction of correlations; finally, Section 5 shows the second empirical exercise aiming to evaluate the effect of fiscal rule adoption and enforcement on GDP per capita growth rate and addressing the endogeneity problem. Finally, Section 6 concludes the paper.

2 Model

We consider an overlapping-generation model of endogenous growth, populated by two-period-lived generations. By assuming a Barro (1990)-type public flow expenditures, productive public spending can increase economic productivity, driving economic growth.

2.1 Individuals

Consider an OLG model of a one-sector economy populated by two-period-lived generations. Each generation consists of a continuum of identical individuals with unit mass. The discrete-time, infinite-horizon economy starts from period t=0. The utility function of an agent is:

$$U_t = \ln c_t^y + \beta \ln c_{t+1}^o \quad \text{with} \quad 0 < \beta < 1, \tag{1}$$

where c_t^y is the young-agent consumption of some generation and c_{t+1}^o is the consumption when old-aged.

Lifetime budget constraint of an agent:

$$\begin{aligned} c_t^y + s_t &\leqslant (1-\tau_t)w_t \\ c_{t+1}^o &\leqslant (1 + r_{t+1}(1-\tau_{t+1}))s_t \\ (c_t^y, c_{t+1}^o) &> 0 \end{aligned} \tag{2}$$

Moreover, young agents' savings are given by:

$$s_t = k_{t+1} + d_{t+1}, \tag{3}$$

where k_{t+1} is private capital and d_{t+1} are government bonds owned by private agents. Individuals take w_t (wage rate), r_{t+1} (interest rate) and τ (tax rate) as given.

There is an initial old generation endowed with k_0 units of capital. Generations are endowed with one unit of labor when young. This unit of labor is inelastically supplied in the first period. We assume that the labor supply is the same as the size of the younger generation. Agents consume a part of wage income when young and save the remainder for retirement. Older agents do not save.

2.2 Government

There are two components in government expenditure: z_t and g_t, that represents productive expenditures and consumption, respectively. Both z_t and g_t can be expressed as fraction of output:

$$z_t = \xi y_t \tag{4}$$

$$g_t = \theta y_t \tag{5}$$

We assume that g_t does not affect the utility of the private agent, θ is a fixed fraction and it is exogenously given. The fraction of production assigned for productive expenditure is ξ. The government also levies a tax rate (τ) on agents' income (wage and savings), and borrows from the private sector by issuing one-period risk-free domestic bonds (d_t) that pay interests $(1 + r_{t+1})$. Assume $d_0 = 0$ and $0 < \xi + \theta < 1$.

The inter-temporal government budget constraint is:

$$d_{t+1} - d_t = z_t + g_t - \tau_t(w_t + r_t s_{t-1}) + r_t d_t \tag{6}$$

Fiscal policy is subject to two numerical rules. The first is a debt rule:

$$d_t = \mu y_t \quad \text{with} \quad \mu \in (0, 1), \tag{7}$$

which imposes an upper bound on the stock of government debt relative to output. The second is a budget balance rule:

$$\frac{d_{t+1} - d_t}{y_t} = \gamma, \tag{8}$$

which constrains the size of the primary deficit (or surplus) as a share of output.

Given these two constraints, the government uses the tax rate τ as an adjustment instrument to satisfy both fiscal rules simultaneously. That is, for given values of ξ and θ, the required tax rate τ must adjust endogenously to ensure that both (7) and (8) hold in every period.

2.3 Firms

Consider a representative firm which maximizes profits in a perfect competitive market with production technology given by:

$$y_t = A\, k_t^{\alpha} (z_t\, l_t)^{1-\alpha} \quad \text{with} \quad 0 < \alpha < 1, \tag{9}$$

where y_t is the output, l_t and k_t are labor force and capital stock used in production, respectively, z_t are productive government expenditures. Capital law of motion is $k_{t+1} = (1 - \delta)k_t + i_t$. For simplicity, we assume $\delta = 1$.

Using equation (4) and since $l_t = 1$, we rewrite the production function:

$$y_t = A^{1/\alpha} \xi^{\frac{1-\alpha}{\alpha}} k_t = \tilde{A} k_t, \tag{10}$$

where $\tilde{A} := A^{1/\alpha} \xi^{\frac{1-\alpha}{\alpha}}$.

From equation (10), we get an AK-type technology. As a result, an increase in the rate of productive spending will increase productivity, which permanently raises the marginal product of capital and, consequently, its growth rate. Following Barro (1990), the productive public expenditure is a spending flow rather than public capital accumulation. Teles and Mussolini (2014) detail the implications of considering public expenditures as investments that increase the stock of public capital. There is a risk that a balanced growth path will not exist if we consider public spending as an investment that increases the growth rate of the public capital stock. The productivity growth rate could increase without limitation because of the nature of the AK model. As a result, investment in physical capital and economic growth takes an explosive path.

The AK production function displays, simultaneously, constant marginal returns for capital and decreasing returns for firms. This is a result of positive externalities of productive public spending. The firms do not realize that increasing their capital stock (which, in turn, increases output) will also increase productive public spending, leading to a rise in the marginal product of labor and capital. Thereby, the government's productive spending influences the level of aggregate productivity.

2.4 Competitive equilibrium

A competitive equilibrium for this economy consists of a sequence of allocations, $\{k_{t+1}, d_{t+1}, y_t, c_t^y, c_{t+1}^o, s_t^y\}$ and prices $\{w_t, r_{t+1}\}$, such that, given the initial conditions, k_0 and d_0, given fiscal variables, τ, θ, ξ, and fiscal rules, μ, γ, the following conditions hold:

1. Given prices $\{w_t, r_{t+1}\}$, the allocation $(c_t^y, c_{t+1}^o, s_t^y)$ solves the young agents maximization problem of each generation;
2. Given prices $\{w_t, r_t\}$, the allocation (y_t, k_t) solves the representative firm problem of profits maximization;
3. There is equilibrium in capital markets: $s_t = k_{t+1} + d_{t+1}$;
4. The resources restriction holds: $y_t = c_t + i_t + z_t + g_t$;
5. The government adopts the fiscal rules: $d_t = \mu y_t$ and $\frac{d_{t+1-d_t}}{y_t} = \gamma$

Individual agent solution consists of maximize equation (1) subject to equation (2), which yields:

$$s_t = \frac{\beta(1-\tau_t)w_t}{(1+\beta)} \tag{11}$$

From equation (11), it is clear that the tax rate τ has a negative effect on individual savings. Since τ adjusts endogenously to satisfy the government's intertemporal budget constraint under the debt rule (7) and the deficit rule (8), changes in fiscal rules directly influence savings behavior. In particular, a higher value of γ implies a more binding constraint on the deficit, requiring a higher τ to finance a fixed level of expenditure. This reduces s_t, and consequently lowers the accumulation of capital k_{t+1}, which in turn affects output and growth through the AK technology.

By contrast, a lower γ allows for more fiscal flexibility in the short run, potentially enabling the government to maintain a lower tax rate. This results in higher savings and capital accumulation, but may increase the risk of exceeding the debt limit defined by the debt rule (7). Thus, fiscal rules not only constrain government behavior but also shape household decisions and the trajectory of capital accumulation, which are central to the economy's long-run growth path.

Representative firm solution consists of profits maximization, resulting:

$$r_t = \alpha A^{1/\alpha}\xi^{\frac{1-\alpha}{\alpha}} = \alpha\tilde{A} \tag{12}$$

$$w_t = (1-\alpha)A^{1/\alpha}\xi^{\frac{1-\alpha}{\alpha}}k_t = (1-\alpha)\tilde{A}k_t \tag{13}$$

Since public productive expenditures affect the economy's productivity, they also affect interest rates and wages. The impact on interest rate means an increase in the marginal return of capital, which stimulate investment. From equation (11), we have that positive shocks on wages, due to increasing productive government expenditures, will increase savings. The AK model implies that savings affect growth in the long run. However, equation (11) also shows that the tax rate has a negative impact on savings. Thus, if the government needs to increase the tax rate to adjust its budget, the net gain in productive expenditures will decrease.

The interaction between the debt rule parameter μ and the deficit rule parameter γ has important implications for the equilibrium path of the economy, as the tax rate τ adjusts to ensure compliance with both rules, leading different combinations of μ and γ to generate distinct steady-state outcomes. A tighter debt rule (lower μ) limits the debt-to-output ratio, while a more stringent deficit rule (higher γ) demands faster fiscal adjustment via taxation. Since the model links capital accumulation to productivity growth through an AK-type production function, even small changes in fiscal policy parameters can yield persistent effects. Excessively sharp fiscal adjustments or highly restrictive borrowing constraints may push the economy toward a lower-growth steady state marked by reduced investment and capital, whereas more balanced rule combinations can sustain higher savings, investment, and stable growth. These dynamics point to the existence of multiple steady states and underscore the critical role of initial conditions and rule calibration in shaping long-run growth and welfare. Ultimately, the model demonstrates that fiscal rule design is not neutral, it actively shapes macroeconomic trajectories through its influence on intertemporal decisions and policy constraints.

3 Dynamics and long-term equilibrium

We now examine the case in which the government adopts both a debt rule, which limits the stock of public debt relative to output, and a deficit rule, which constrains the budget deficit as a share of output. To satisfy both rules, the government adjusts the tax rate endogenously each period. This implies that fiscal rules affect the intertemporal allocation of resources: changes in the tax rate influence private savings, which in turn affect capital accumulation and the long-run growth rate. To analyze these dynamics, we insert equations (3), (4), (5), (7) and (8) into the inter-temporal government budget constraint (6). Then, we rewrite the government budget constraint as:

$$\gamma y_t = \xi y_t + \theta y_t - \tau_t(w_t + r_t(k_t + \mu y_t)) + r_t d_t \tag{14}$$

By substituting equations (7), (10), (12), and (13) into equation (14), we obtain:

$$(1 - \tau_t) = \frac{1 + (\gamma - \xi - \theta)}{1 + \alpha\mu\tilde{A}} \tag{15}$$

Equation (15) demonstrates that a positive shock to both productive and unproductive expenditure must be accompanied by an increase in the tax rate (keeping fiscal rules unchanged). The deficit rule and the debt rule have opposite effects on the tax rate. An increase in the deficit limit (γ) reduces the government's need to finance its expenditure through taxation in the current period, thereby lowering the required tax rate. In contrast, a higher debt limit (μ) allows for greater borrowing, but over time it implies the need for higher revenue to service the accumulated debt, which puts upward pressure on the tax rate. As a result, the government has some degree of flexibility: by increasing both limits simultaneously, it can offset the opposing effects on the tax rate. However, we must also consider the effects of changing fiscal rules on the growth rate of public debt and private capital, as these parameters are strongly related to the existence of a balanced growth path. To see this, we derive one expression for the capital growth rate and one for the debt growth rate. Using equations (10), (13), and (15) in (11), the capital growth rate is written as:

$$\frac{k_{t+1} - k_t}{k_t} = \frac{\beta}{(1+\beta)} \left[\frac{1 + (\gamma - \xi - \theta)}{1 + \alpha\mu\tilde{A}}\right] (1-\alpha)\tilde{A} - \gamma\tilde{A} - \frac{d_t}{k_t} - 1 \tag{16}$$

Equation (16) shows that the capital growth rate depends on the fiscal rule parameters γ and μ. While a higher γ reduces the tax burden in the short term, it also increases the deficit-to-GDP ratio, which leads to faster debt accumulation. This, in turn, reduces the share of output available for private savings, thereby lowering capital accumulation. Since output grows proportionally with capital in the AK framework, the decline in the capital growth rate translates directly into slower output growth. Therefore, even if a higher γ appears beneficial by allowing for a lower tax rate, its overall effect on growth can be negative when the impact on savings and debt dynamics is fully taken into account.

Using the deficit rule and the production function, the debt growth rate is:

$$\frac{d_{t+1} - d_t}{d_t} = \gamma \tilde{A} \frac{k_t}{d_t} \tag{17}$$

The system dynamics can be represented by the capital growth rate (equation 16) and debt growth rate (equation 17), where d_{t+1} and k_{t+1} are state variables. A balanced growth path is defined as a trajectory in which the state variables grow at the same rate, i.e, for an equilibrium to exist, $\frac{d_t}{k_t}$ must be constant, such that $\frac{k_{t+1}}{k_t} = \frac{d_{t+1}}{d_t}$. Thus, we set (16) equal to (17) and obtain a quadratic equation with solution given by:

$$\frac{d}{k} = \frac{J \pm \sqrt{J^2 - 4\gamma\tilde{A}}}{2} \quad \text{where} \quad J \equiv \frac{\beta}{1+\beta}\left(\frac{1+(\gamma-\xi-\theta)}{1+\alpha\tilde{A}\mu}\right)(1-\alpha)\tilde{A} - \gamma\tilde{A} - 1$$

If there exists at least one positive root, there will exist one steady-state equilibrium.

Proposition 1. *There exists a critical value γ': if $\gamma < \gamma'$, then there are two steady states, x_1 and x_2. In these steady states, d_t and k_t grow at the same rate, thus d/k is constant. If $\gamma > \gamma'$ there is no steady state, and if $\gamma = \gamma'$ there is a unique steady state.*

Proof. See Appendix A.1. □

For a better understanding, we present the first steps of the proof of Proposition 1. First, we set equation (16) equal to (17):

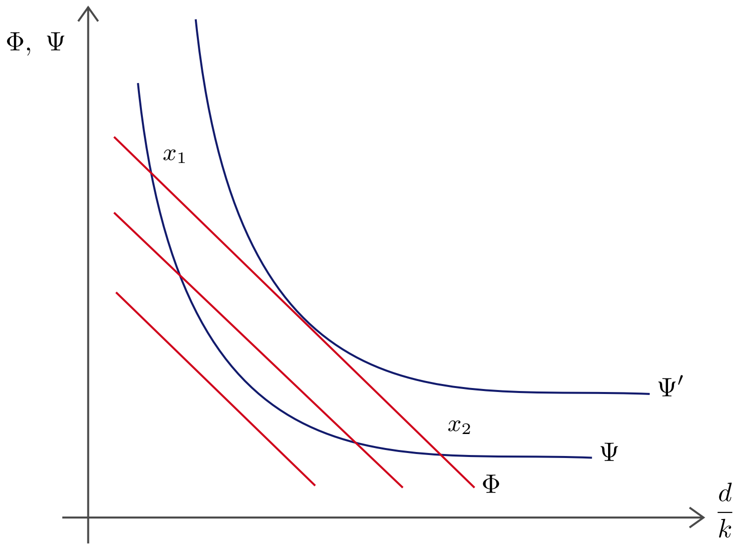
$$\frac{\gamma\tilde{A}}{d/k} = \frac{\beta}{(1+\beta)}\left[\frac{1+(\gamma-\xi-\theta)}{1+\alpha\mu\tilde{A}}\right](1-\alpha)\tilde{A} - \gamma\tilde{A} - \frac{d_t}{k_t} - 1$$

Then, we define x, Ψ and Φ.

$$\begin{aligned} x &\equiv \frac{d}{k} \\ \Psi(x,\gamma) &\equiv \frac{\gamma\tilde{A}}{x} + 1 \\ \Phi(x,\gamma) &\equiv \frac{\beta}{(1+\beta)}\left[\frac{1+(\gamma-\xi-\theta)}{1+\alpha\mu\tilde{A}}\right](1-\alpha)\tilde{A} - \gamma\tilde{A} - x \end{aligned}$$

Figure 1 depicts Proposition 1. Here, we illustrate the situations where there are two equilibrium, x_1 and x_2, ie., for values of γ and μ such that Ψ and Φ intercept. The point where Ψ and Φ' are tangents depicts the unique steady state case. Furthermore, an increase in γ shifts Ψ upwards and Φ downwards. A decrease in μ only shifts Φ upwards. Thereby, for a large γ and/or a sufficiently high μ, there won't exist a steady-state.

Figure 1 – Existence of BGPs



In this economy, an increase in μ represents a loosening of the debt rule, allowing for a higher public debt-to-GDP ratio. As shown in equation (16), this reduces the capital growth rate, since additional fiscal space can be used to expand both productive and unproductive spending, crowding out private savings. A similar mechanism applies to the deficit rule: a higher γ increases the allowable primary deficit, accelerating debt accumulation and reducing the share of output available for capital formation. These fiscal dynamics influence the trajectory of capital and debt, and are central to the existence of a balanced growth path.

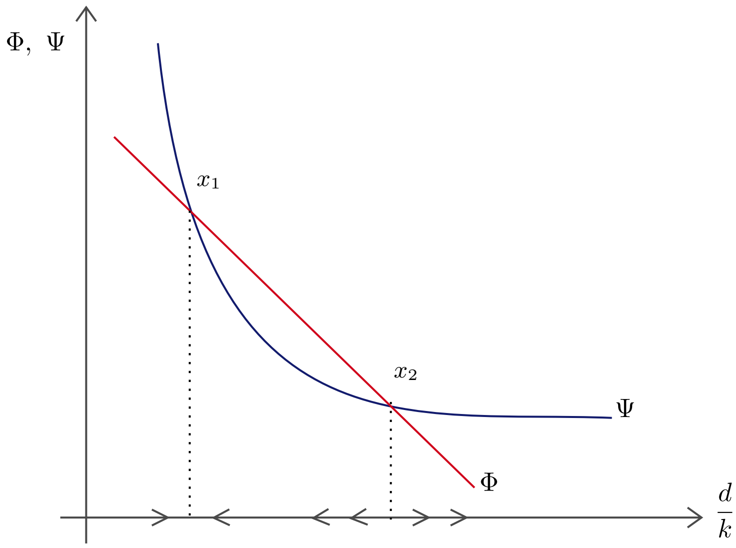
Proposition 2. *The steady state x_1, with low d/k, is locally stable. The steady state x_2, with high d/k, is unstable. Thus, fiscal rules γ and μ can be sustained around x_1, but for an initial d/k sufficiently large, it is not possible to comply with the rules.*

Proof. See Appendix A.2. □

Figure 2 depicts Proposition 2. As stated in the Proposition, x_1 is a stable equilibrium, so the fiscal framework is sustainable for plausible values of γ and μ and a sufficiently low initial d/k. In the unstable equilibrium in x_2 we have an upper bound for the initial d/k for which d/k converges to a finite level. That is, if the initial d/k is strictly greater than d/k of x_2, then the public-debt/capital ratio diverges. Thus, there is no equilibrium for a large initial public-debt/capital ratio, and the debt rule is pointless.

Taken together, Propositions 1 and 2 reveal that the long-run dynamics of the economy are highly sensitive to the initial debt-to-capital ratio and the design of the fiscal rules. The existence of multiple steady states implies that different fiscal trajectories are possible under the same structural parameters. The low-debt equilibrium (x_1) is locally stable and supports sustained capital accumulation and fiscal solvency, making it compatible with long-run growth. In contrast, the high-debt equilibrium (x_2) is unstable: when the initial public debt relative to capital exceeds a critical threshold, the economy diverges from the balanced path, and the fiscal framework becomes unsustainable. These results underscore the importance of both rule design and initial conditions in determining whether a fiscal regime supports growth and intertemporal stability.

Figure 2 – Stability of BGP



In Appendix A.3 and A.4, we analyze the cases with only one fiscal rule in place, revealing important limitations of rule design when applied in isolation. In the economy with a deficit rule only, the government has a mechanism to limit short-run fiscal imbalances, and—depending on parameters—multiple steady states may still emerge. However, without a constraint on the level of public debt, the long-run fiscal trajectory remains unconstrained. In particular, even when equilibrium exists, debt may grow persistently, leading to potential solvency risks. In contrast, under a debt rule alone, the stock of public debt is bounded relative to output, but the absence of a deficit constraint means that the government can run large deficits in the short run. The capital growth rate becomes a function of initial conditions, and the system lacks a unique, determinate balanced growth path unless specific paths for tax and expenditure are assumed. These findings reinforce the core insight from the combined-rules case: effective fiscal policy design requires complementary rules that simultaneously anchor long-term debt sustainability and guide short-term fiscal adjustments. Otherwise, reliance on a single constraint may produce indeterminate or unstable growth dynamics.

3.1 Welfare Analysis

In this section, we analyze the welfare implications of changes in fiscal rules. Specifically, we examine the effects of an unanticipated and permanent reduction in both the debt limit (μ) and the deficit limit (γ). This policy configuration reflects a tightening of fiscal discipline, as it imposes stricter constraints on public indebtedness and fiscal imbalances. Our objective is to quantify the impact of such a reform on the indirect utility of households.

To isolate the effects of the fiscal rules, we assume that the government keeps the share of productive expenditure (ξ) constant. As a result, changes in μ and γ do not affect the interest rate, but they do influence the tax rate and the debt-to-capital ratio. The indirect utility of an agent born in period t_1 is characterized by the following expression,[4] where k_{t-1} and d_{t-1} are predetermined and thus fixed in period t:

[4] See Appendix A.5 for derivation.

$$V_{t-1} = \beta \ln \beta + (1+\beta) \ln \left(\frac{1}{1+\beta} \right) + (1+\beta) \ln k_{t-1}$$
$$+ (1+\beta) \ln \left[\frac{1+\gamma-\xi-\theta}{1+\alpha\tilde{A}\mu} (1-\alpha)\tilde{A} \right] + \beta \ln \left[\frac{1+\gamma-\xi-\theta}{1+\alpha\tilde{A}\mu} \alpha\tilde{A} + 1 \right] \quad (18)$$

Proposition 3. *Assume that the economy has two equilibria of debt-capital ratio as in Propositions 1 and 2. An unanticipated, permanent decrease of μ, from period t, increases indirect utility of agents born in $t-1$. Yet, a permanent reduction of γ, decreases the indirect utility of agents born in $t-1$.*

Proof. Since $(1+\gamma-\xi-\theta) > 0$, the derivative of equation (18) with respect to μ is:

$$\frac{\partial V_{t-1}}{\partial \mu} = -\frac{\alpha\tilde{A}(1+\beta)}{1+\alpha\tilde{A}\mu} - \frac{\beta\alpha\tilde{A}^2(1+\gamma-\xi-\theta)(1-\alpha)}{1+\alpha\tilde{A}\mu+\alpha\tilde{A}(1+\gamma-\xi-\theta)} < 0$$

The derivative of equation (18) with respect to γ is:

$$\frac{\partial V_{t-1}}{\partial \gamma} = \frac{1+\beta}{1+\gamma-\xi-\theta} + \frac{\beta}{\mu+1+\gamma-\xi-\theta} + 1/\alpha\tilde{A} > 0$$

□

Put differently, a looser fiscal stance corresponds to an increase in either the debt limit (μ) or the deficit limit (γ). An increase in μ, the debt-to-GDP ceiling, reduces V_{t-1} due to a crowding-out effect: with γ held constant, the government must raise the tax rate to ensure long-term debt service, thereby reducing after-tax savings and lowering utility. In contrast, an increase in γ (with μ unchanged) raises V_{t-1}, as the government can run larger deficits in the short run, allowing for a lower tax burden and higher private savings. However, these gains are concentrated among the current generation. Over time, compliance with the debt rule may require fiscal adjustments—such as future tax increases—that shift the burden onto subsequent generations. Hence, the intertemporal distributional effects of loosening fiscal rules depend critically on whether both constraints are jointly enforced.

We conducted a similar welfare analysis for the case with only a deficit rule (see Appendix A.3). While both expressions for the derivative of welfare with respect to (γ) are strictly positive, indicating that a higher (γ) increases the utility of current agents, the mechanisms differ. With only the deficit rule, the welfare gain depends on the current debt-to-capital ratio d_{t-1}/k_{t-1}, making the benefit more sensitive to short-run fiscal conditions. In this case, the positive effect diminishes as (γ) and the debt burden increases.

Under both rules, however, the derivative includes a term inversely proportional to (μ), reflecting the additional constraint imposed by the debt ceiling. This tempers the marginal gain from raising (γ), as future borrowing is restricted by (μ), reducing the scope for intertemporal smoothing. Notably, the presence of the term $1/(\alpha\tilde{A})$ in the combined-rule case makes the gain less dependent on the current debt level and more structurally anchored in model fundamentals. This comparison highlights that while loosening the deficit rule is always utility-enhancing in the short run, the magnitude and sustainability of the welfare gain differ depending on whether the fiscal framework includes a complementary debt constraint.

4 Empirical Analysis: Model Validation

This section presents an empirical exercise, in which we estimate a panel regression using cross-country data, following the specification derived from the theoretical model in Section 3. The aim is to examine whether the observed empirical patterns are consistent with the model's predictions. We do not seek to validate the model or to establish causal relationships. As noted by Teles and Mussolini (2014), endogeneity is a significant concern in this context. Accordingly, the analysis focuses on identifying the correlations implied by the theoretical framework.

Based on equation (16), we estimate a specification in which GDP growth is modeled as a function of the variables highlighted in that equation, along with lagged GDP, using a cross-country panel dataset. Since equation (16) does not represent an equilibrium condition, the results should not be interpreted as describing equilibrium relationships among the variables. Following Teles and Mussolini (2014), we adopt a non-linear empirical specification with interaction terms to better reflect the mechanisms identified in the theoretical model. A key distinguishing feature of our approach is the simultaneous consideration of two fiscal rules: a debt rule and a budget balance rule. Consequently, the empirical exercise focuses on examining the correlations between these fiscal rules and growth outcomes. We begin by estimating models where fiscal rules are represented by binary indicators, and then extend the analysis using a continuous index of fiscal rule strength.

The theoretical model suggests that productive public expenditures enhance productivity and stimulate economic growth. This effect arises because such expenditures increase the marginal return on private capital, thereby encouraging investment. Moreover, incorporating productive expenditures directly into the production function introduces a new channel through which public debt influences growth. The purpose of this empirical analysis is to assess the magnitude of these effects and their interaction with other fiscal variables. Additionally, we include measures of fiscal rules to test whether the empirical evidence aligns with the model's predictions.

4.1 Model and data

We estimate the following regression based on the theoretical model presented earlier:

$$
\begin{aligned}
gr_{it} = a_0 + \sum_{s=1}^{T} a_s gr_{it-s} + \beta_0 z_{it-1} + \beta_1 \frac{d_{it-1}}{y_{it-1}} + \beta_2 z_{it-1} * surplus_{it-1} \\
+ \beta_3 z_{it-1} * tax_{it-1} + \beta_4 z_{it-1} * \frac{d_{it-1}}{y_{it-1}} + \beta_5 \gamma_{it-1} + \beta_6 \mu_{it-1} + \alpha_i + \delta_t + \varepsilon_{it}, \qquad (19)
\end{aligned}
$$

where gr_{it} is the growth rate of GDP per capita of country i in year t and T is the number of lags of gr_{it} included in the dynamic panel. The variable z_{it} represents public productive expenditures. This variable is the sum of central government spending (as % of GDP) on education, health, transportation, communications, and energy. Public productive expenditures interacts with primary surplus ($surplus_{it}$), tax burden on income (tax_{it}) and debt-to-GDP ratio (d_{it}/y_{it}). These variables are all expressed as percentages of GDP. Country and year fixed effects are represented by α_i and δ_t.

The variables γ and μ capture the presence of budget balance (deficit) rules and debt rules, respectively. To assess their effect on GDP growth, we employ two alternative measures. First, we include binary indicators reflecting the existence of each type of fiscal rule. Second, we incorporate continuous indices measuring the strength of budget balance rules and debt rules. The data on fiscal rules are obtained from the *2017 Fiscal Rules Dataset* compiled by the International Monetary Fund. Further details on this dataset, along with the methodology used to construct the strength indices, are provided in the following subsection.

To estimate equation (19), we use an unbalanced panel dataset covering 97 countries over the period 1990–2015. Data on economic growth are sourced from the World Bank's World Development Indicators. Information on public debt and primary balances is obtained from the International Monetary Fund's Historical Public Debt Database, while data on productive expenditures and the income tax burden are drawn from the IMF's Government Financial Statistics.

4.1.1 Fiscal strength index

Data on fiscal rules are drawn from the 2017 Fiscal Rules Dataset, compiled by the Fiscal Affairs Department (FAD) of the International Monetary Fund. This dataset provides detailed information on national and supranational fiscal rules for IMF member countries. It covers 96 countries from 1985 to 2015 and includes data on 28 characteristics of national fiscal rules, organized into the following categories: (i) type of rule; (ii) year of implementation and major revisions; (iii) number of rules; (iv) legal basis; (v) government level covered; (vi) monitoring procedures; (vii) enforcement mechanisms; (viii) institutional supporting features (such as multi-year expenditure ceilings, fiscal responsibility laws, independent councils providing budgetary assumptions or monitoring implementation); and (ix) stabilization features (e.g., budget balance rules adjusted for the economic cycle or exclusion of public investment). Countries not included in the dataset are assumed to have had no fiscal rules in place during the period of analysis.

The Fiscal Rule Strength Index (FSI) is a composite measure that captures the design quality of four types of fiscal rules: budget balance rules, debt rules, expenditure rules, and revenue rules. For each rule type, a sub-index is constructed based on four key dimensions: legal basis, coverage, enforcement mechanisms, and supporting procedures and institutions.[5] To compute the overall index, the indicators associated with each dimension are summed to form a sub-index for each rule type. Most indicators are scaled between 0 and 1; where this was not originally the case, rescaling was applied. All sub-indices are then normalized to range from 0 to 1. The overall FSI, which ranges from 0 to 4, is obtained by summing the four sub-indices. A score of zero indicates the absence of any fiscal rules, while a score of four reflects a maximum-strength framework, characterized by well-designed rules across all four dimensions. Scores between 0 and 4 reflect varying degrees of rule quality, with any value above zero indicating the presence of at least one (possibly weakly designed) fiscal rule. It is important to note that the FSI captures the formal design strength of fiscal rules rather than their actual enforcement. As emphasized by Schaechter et al. (2012), higher index values indicate the presence

[5] The "supporting procedures and institutions" category includes multi-year expenditure ceilings, fiscal responsibility laws, independent bodies responsible for setting budgetary assumptions, and independent bodies monitoring budget implementation.

of institutional features that have been associated in the literature with greater rule effectiveness.[6]

4.2 Results

Table 1 presents the results of the baseline specification from equation (19), estimated using OLS. The analysis is based on a cross-country panel dataset with country and year fixed effects to account for unobserved heterogeneity across countries and over time. Comparing column (1) with the subsequent specifications reveals that including interaction terms significantly improves the explanatory power of the model. Columns (2) and (3) include dummy variables representing the presence of fiscal rules, while columns (4) and (5) use the fiscal strength index as an alternative measure of rule intensity.

Across all estimations, from columns (2) to (5), we observe consistent results regarding the direction and significance of fiscal variables and their interactions with GDP growth. A shock to productive public spending tends to have a positive effect, as it increases savings. However, this effect is partly offset by a rise in income taxation, which reduces the net return on savings. The sign of the interaction between productive spending and the tax burden aligns with the theoretical predictions of the model.

A similar rationale applies to the fiscal deficit. In the case of a primary surplus, the government does not need to absorb part of the increased savings to finance the deficit, which results in a positive effect on growth. Regarding public debt, an increase in debt—under certain fiscal conditions—can be associated with higher growth, as discussed in Teles and Mussolini (2014). However, the interaction between debt and public spending also matters. A positive shock to ξ (productive spending in the model) leads to higher interest rates, which increases the cost of servicing the debt. This, in turn, can crowd out private investment, dampening the positive impact of fiscal policy.

Turning to fiscal rules, the results are not statistically significant at the 5% level when using a dummy variable. However, when using the strength index for debt and deficit rules, the coefficients become statistically significant at the 5% level. The theoretical model predicts a positive sign for both rules, as stronger rules are associated with higher index values. For example, a stricter debt rule implies a lower value of μ, but a higher score in the index. The same logic applies to deficit rules.

Empirically, only the results for the debt rule are consistent with the model's predictions. In contrast, the deficit rule index may not fully capture the underlying concept it aims to represent. The index is based on the broader category of budget balance rules, which include diverse types such as structural balance rules, rules over the economic cycle, zero-balance rules, and primary balance targets. Furthermore, the index attempts to account for institutional features that enhance rule effectiveness, such as enforcement mechanisms and monitoring procedures. The original source, Schaechter et al. (2012), acknowledges this limitation, noting that a high score does not necessarily reflect effective implementation. As such, a negative coefficient on the deficit rule index might reflect cases where formally strong rules are poorly enforced, potentially hindering productive

[6] As Schaechter et al. (2012) do not provide full details on all the procedures involved in constructing the index, certain adaptations were required. For instance, the original methodology does not specify how to aggregate indicators from national and supranational fiscal rules. For a comprehensive overview of the IMF Fiscal Rules Index, see Schaechter et al. (2012).

growth. When considering its interaction with public debt, the results suggest that tighter deficit rules may support economic growth, depending on the level of public debt.

Table 1 – OLS estimation of the baseline specification

Variables	(1) gr_{it}	(2) gr_{it}	(3) gr_{it}	(4) gr_{it}	(5) gr_{it}
z_{it-1}	-0.0143	0.234*	0.257**	0.238*	0.324**
	(0.0182)	(0.133)	(0.126)	(0.134)	(0.125)
$Debt/GDP_{it-1}$	0.0135	0.0429**	0.0383***	0.0428**	0.0389***
	(0.00961)	(0.0168)	(0.0142)	(0.0173)	(0.0129)
$z_{it-1} * Debt/GDP_{it-1}$		-0.00211	-0.00279*	-0.00225	-0.00401**
		(0.00179)	(0.00160)	(0.00177)	(0.00162)
$z_{it-1} * TaxBurden_{it-1}$		-0.00593*	-0.00626**	-0.00601**	-0.00740**
		(0.00303)	(0.00296)	(0.00302)	(0.00291)
$z_{it-1} * PrimarySurplus_{it-1}$		0.00469**	0.00428**	0.00453**	0.00439**
		(0.00206)	(0.00193)	(0.00201)	(0.00188)
$D\mu_{it-1}$		0.738	0.811		
		(0.763)	(0.784)		
$D\gamma_{it-1}$		-0.726	-1.438*		
		(0.659)	(0.758)		
$D\gamma_{it-1} * Debt/GDP_{it-1}$			0.0202*		
			(0.0106)		
Index μ_{it-1}				0.0607**	0.0702***
				(0.0251)	(0.0265)
Index γ_{it-1}				-0.0466**	-0.0960***
				(0.0222)	(0.0306)
Index $\gamma_{it-1} * Debt/GDP_{it-1}$					0.000832***
					(0.000253)
Observations	1,292	1,121	1,121	1,121	1,121
Number of countries	91	84	84	84	84
Country FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES

Note: Robust standard errors, clustered at the country level, are reported in parentheses. Variable gr_{it} is the growth rate of GDP per capita of country i in year t; variable z_{it} represents public productive expenditures (sum of central government spending as % of GDP on education, health, transportation, communications, and energy). Public productive expenditures interacts with primary surplus ($surplus_{it}$), tax burden on income (tax_{it}) and debt-to-GDP ratio (d_{it}/y_{it}). The variables γ and μ represent the presence of budget balance (or deficit) rules and debt rules, respectively. Each of these variables is considered either as a binary indicator (D) or as a continuous measure (Index).

5 Empirical Analysis: Addressing Endogeneity

In this section, we examine the impact of fiscal rule adoption and enforcement on GDP per capita growth, while addressing potential endogeneity concerns. To this end, we estimate a cross-country panel model with fixed effects, employing an instrumental variable approach. The instrument, recently introduced in the literature, exploits the geographic diffusion of fiscal rules across countries to capture exogenous variation in their adoption.

Most empirical studies employ GMM techniques within a cross-country panel data framework to estimate the effects of fiscal rules on various economic outcomes. The Arellano-Bond estimator and the system GMM estimator are commonly applied in panels where the dependent variable depends on its own past values (i.e., dynamic panels), and the explanatory variables are not strictly exogenous—that is, they may be correlated with past or current error terms. This is particularly relevant in the case of fiscal rules.

However, GMM estimation is not without limitations. Issues such as asymptotic bias due to the proliferation of instruments ("too many instruments") and weak performance when the autoregressive parameter is close to one can lead to invalid estimates. As highlighted in Acemoglu et al. (2019), the validity of the GMM results is based on effective treatment of endogeneity, which often arises from the omitted variable bias. In our context, this bias may occur if there are time-varying factors that simultaneously influence both the adoption of fiscal rules and GDP growth. While the inclusion of country fixed effects helps control for unobserved time-invariant heterogeneity, it does not eliminate biases arising from unobserved time-varying influences.

The instrumental variable (IV) approach is commonly used to address endogeneity concerns. The main challenge lies in identifying a valid instrument for the potentially endogenous variable. A valid instrument must satisfy two key conditions: relevance and exogeneity. Relevance requires that the instrument be strongly correlated with the explanatory variable of interest—in this case, fiscal rules. Exogeneity means that the instrument must provide variation in fiscal rule adoption that is unrelated to the outcome variable (GDP growth), i.e., it should not directly influence growth. However, the existing literature often relies on instruments that are either weak or questionable in terms of validity.[7]

To estimate the effect of fiscal rules on economic growth, we employ the instrument proposed by Caselli and Reynaud (2020), originally used to identify the causal impact of fiscal rules on fiscal balances. The instrument measures the prevalence of fiscal rules in neighboring countries. The underlying intuition is that the adoption and/or enforcement of fiscal rules in neighboring states can influence similar behavior in the domestic country.[8] This idea is rooted in the political science literature, which highlights various mechanisms through which policies diffuse across borders, including economic competition, policy learning, socialization, imitation, and coercion. In this context, the adoption of fiscal rules by neighboring countries may generate peer pressure or imitational effects, encouraging domestic adoption. A similar identification strategy is employed by Acemoglu et al. (2019), who use regional waves of democratization as an instrument to study the impact of democracy on economic growth, based on the observation that democratic transitions often spread regionally.

To capture the process of geographical diffusion, we follow Caselli and Reynaud (2020) and construct a contiguity-based instrument for fiscal rules in country i at time t, denoted by FR_{it}, as follows:

$$contiguity_{i,t} = \sum_{j \neq i}^{n-i} FR_{j,t} * X_{j,i,t}, \tag{20}$$

where j denotes a neighboring country of the domestic country i. The variable $FR_{j,t}$ is

[7] See Cherif et al. (2018) for a discussion on weak and "blunt"instruments.

[8] Caselli and Reynaud (2020) provide several anecdotal examples supporting the idea that countries often consider the experiences of their neighbors when introducing fiscal rules.

a binary indicator that equals 1 if country j has a fiscal rule in place at time t, and 0 otherwise. The term $X_{i,j,t}$ equals 0 when countries i and j do not share a border, and 1 otherwise. The denominator sums the number of countries that share a border with country i. As a result, the variable $contiguity_{i,t}$ reflects the proportion of neighboring countries that have adopted a fiscal rule at time t, thereby capturing the degree of geographical diffusion of fiscal rules around the domestic economy.

An alternative approach to instrumenting fiscal rules, proposed by Caselli and Reynaud (2020), considers the quality of fiscal rule design, as there is evidence that well-designed rules can significantly influence fiscal performance (Eyraud et al. (2018)). Rather than relying on a simple dummy variable indicating the presence of a fiscal rule, the contiguity instrument, $CFSI_{i,t}$, incorporates the Fiscal Strength Index (FSI) as follows:

$$CFSI_{i,t} = \frac{1}{\sum X_{j,i,t}} \sum_{j \neq i}^{n-i} FSI_{j,t} * X_{j,i,t}, \tag{21}$$

where $FSI_{j,t}$ denotes the Fiscal Strength Index of country j at time t (or 0 if no fiscal rule is in place), and $X_{j,i,t}$ equals 1 if countries i and j share a border, and 0 otherwise. The Fiscal Strength Index is a continuous measure that captures the overall strength of fiscal rules in a given country and year. Following the methodology outlined in Schaechter et al. (2012), we construct this index using data from the IMF Fiscal Rules Dataset. The index aggregates all fiscal rules in force within a country, accounting for their specific design features and institutional characteristics.

5.1 Model and data

We estimate a standard growth equation that can be derived from a Cobb-Douglas production function:

$$\Delta \text{ln}y_{it} = \beta_1 FR_{it} + \beta_2 \text{ln}y_{it-1} + \beta_3 \text{ln}k_{it} + \beta_4 \text{ln}g_{it} + \alpha_i + \delta_t \varepsilon_{it}, \tag{22}$$

where $\Delta \ln y_{it}$ represents the GDP growth of country i at time t, FR_{it} is the dummy variable indicating the presence of a fiscal rule in country i at time t, and y_{it-1} is the lagged real GDP per capita. The covariate g_{it} serves as a proxy for government size, while k_{it} denotes gross fixed capital formation. The term α_i captures country fixed effects, which account for the influence of any time-invariant characteristics specific to each country, and δ_t represents year fixed effects. The error term ε_{it} encapsulates all other unobservable, time-varying shocks to GDP growth. The instrument for FR_{it} is the variable $contiguity_{i,t}$.

We also estimate an alternative specification that incorporates the Fiscal Strength Index (FSI) as a measure of the strength of fiscal rules, as follows:

$$\Delta \text{ln}y_{it} = \beta_1 FSI_{it} + \beta_2 \text{ln}y_{it-1} + \beta_3 \text{ln}k_{it} + \beta_4 \text{ln}g_{it} + \alpha_i + \delta_t + \varepsilon_{it}, \tag{23}$$

where $FSI_{i,t}$ represents the overall strength of fiscal rules in country i at time t. This variable is instrumented using $CFSI_{i,t}$, the contiguity-based Fiscal Strength Index. In both specifications, the primary objective is to obtain consistent estimates of β_1.

In addition to fiscal rule data, presented in Section 4.1.1, we used the Country Borders dataset from *GeoDataSource.com* to build the two instruments mentioned above. This database provides a complete list of countries and associated land-border countries (the neighboring countries).

As our primary outcome variable, we use the GDP per capita growth rate, which we obtained from the World Bank Development Indicators. This measure is available for an unbalanced panel of 268 countries from 1960 to 2018. The proxy for government size is the government final consumption expenditure (as a percentage of GDP). That series and the gross capital formation (as a percentage of GDP) are also from the World Bank Development Indicators. As a result, we obtain a balanced panel of 178 countries covering the period from 1985 to 2015.

5.2 Estimation and results

We assume that neighboring countries' adoption of fiscal rules influences domestic economic growth solely through its effect on the domestic country's fiscal rule adoption. Formally, the exclusion restriction implies that fiscal rules and lagged GDP are orthogonal to contemporaneous and future GDP shocks, and that the error term ε_{it} is serially uncorrelated (Acemoglu et al., 2019). According to the authors, this assumption requires including a sufficient number of GDP lags in the estimation to eliminate residual serial correlation in ε_{it} and to control for any pre-existing GDP trends that may precede the adoption of fiscal rules. Therefore, by including lags of GDP per capita, we address two concerns: (i) the tendency for fiscal rules to be adopted in the aftermath of economic crises, and (ii) the potential influence of broader economic trends on both growth and fiscal performance.[9] We employ a fixed effects estimator to account for unobserved heterogeneity that may be correlated with both GDP growth and fiscal rule adoption. Year fixed effects capture common global shocks, while country fixed effects control for time-invariant characteristics such as geography.

Table 2 presents the first-stage results for the contiguity instrument. For all samples—except those restricted to Europe and advanced economies—the correlation between the instrument and the fiscal rule adoption dummy is statistically significant at the 1% level. This likely reflects the fact that Europe was among the early adopters of fiscal rules, reducing the variation captured by the instrument in that context.

While the F-statistic falls below the conventional threshold of 10 in some subsamples, we do not interpret this as evidence of a weak instrument. We conducted a range of diagnostic tests for weak and weakly-robust instruments, all of which support the strength of our instrument. In particular, we report the Sanderson and Windmeijer (2016) test, which is appropriate for models with a single endogenous regressor and a single instrument. Additionally, the Stock and Wright statistic is used to test the null hypothesis that the coefficients are jointly equal to zero, even in the presence of weak instruments.

Table 3 reports the second-stage results. Excluding the Europe and Advanced Economy subsamples, the coefficient on fiscal rules is positive and statistically significant at the 1% level, indicating a robust association between fiscal rule adoption and economic growth. The estimated coefficients on the convergence term (lagged GDP per capita) and capital accumulation are of the expected sign and statistically significant, consistent with standard growth theory.

[9] We estimated specifications including up to five lags of GDP per capita. However, only the first lag $(t-1)$ was statistically significant. Accordingly, we report results using only one lag.

The coefficient on government size—proxied by government final consumption expenditure as a percentage of GDP—is negative and statistically significant across all samples. Specifically, it is significant at the 1% level for the Europe and Advanced Economy subsamples, and at the 5% level for the remaining subsamples. This finding aligns with previous literature on the growth effects of government size (Fölster and Henrekson, 2001).

Regarding the impact of fiscal rules, the coefficients in Table 3 are interpreted in percentage points. For the full sample (column 1), the sample excluding European countries (column 2), and the sample excluding advanced economies (column 4), the results suggest that the adoption of fiscal rules is associated with an increase in the GDP growth rate of approximately 3 to 4 percentage points. Then, fiscal rules may enhance growth performance, particularly in low- and middle-income countries.

We also estimate the effects of debt rules and budget balance rules separately, as these are the most common types of rules in our sample and are the focus of our theoretical model. The corresponding results are reported in Tables B.1 to B.4 in Appendix B. In these estimations, debt rules show a slightly stronger positive effect on growth compared to budget balance rules across the three subsamples mentioned above.

Table 2 – First stage contiguity instrument results

Variables	(1) FR_{it}	(2) FR_{it}	(3) FR_{it}	(4) FR_{it}	(5) FR_{it}
Contiguity_{it-1}	0.136***	0.153***	0.0619*	0.148***	0.0958*
	(0.0202)	(0.0217)	(0.0322)	(0.0193)	(0.0481)
Observations	4,555	3,509	1,046	3,489	1,066
R-squared	0.305	0.293	0.444	0.325	0.353
No. countries	177	137	40	140	37
Sample	Full	No europe	Europe	No Advanced	Advanced
F-stat 1st Stage	8.387	5.792	14.73	7.001	12.52
Stock-Wright p-value	0.0001	0.0001	0.0001	0.0000	0.3468
SW F-stat	45.37	50.27	3.69	58.66	3.97
SW p-val	0.0000	0.0000	0.0621	0.0000	0.054

Note: This Table presents the first-stage results for the contiguity instrument. The variable $contiguity_{i,t}$ reflects the proportion of neighboring countries that have adopted a fiscal rule at time t (see equation 20). Robust standard errors, clustered at the country level, are reported in parentheses. Other covariates not reported.

Equation (23) presents an alternative specification that employs a continuous measure of fiscal rule strength—the Fiscal Strength Index (FSI)—instead of a binary indicator for fiscal rule adoption. The rationale behind this approach is that merely enacting a fiscal rule may not have a meaningful impact on economic growth if the rule is weakly designed and fails to influence fiscal behavior. To address this, we incorporate a measure that captures the design features of fiscal rules, under the assumption that well-designed rules are more likely to be effective. The contiguity-based Fiscal Strength Index (CFSI) is used as an instrument for the FSI.[10]

[10] One limitation of the FSI is that it reflects only the design characteristics of fiscal rules, without accounting for their implementation or compliance. As such, it does not capture the political will or public support that may be essential for the rules' effectiveness.

Table 3 – Second stage contiguity instrument results

Variables	(1) $\Delta\ln Y$	(2) $\Delta\ln Y$	(3) $\Delta\ln Y$	(4) $\Delta\ln Y$	(5) $\Delta\ln Y$
FR_{it}	3.340***	3.078***	9.040*	4.046***	-0.668
	(1.027)	(1.038)	(5.127)	(1.227)	(1.105)
$\ln Y_{it-1}$	-5.538***	-4.777***	-15.22**	-5.189***	-3.857***
	(0.859)	(0.822)	(5.949)	(0.937)	(0.984)
$\ln K_{it}$	3.607***	3.277***	5.639***	3.350***	2.126
	(0.660)	(0.735)	(1.835)	(0.713)	(1.788)
$\ln G_{it}$	-2.212**	-1.824**	-11.58***	-1.860**	-9.786***
	(0.909)	(0.897)	(3.530)	(0.886)	(2.636)
Observations	4,555	3,509	1,046	3,489	1,066
No. countries	177	137	40	140	37
Country FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES
Sample	Full	No europe	Europe	No Advanced	Advanced

Note: This Table is based on equation 5.1. The variable Y_{it} is the GDP per capita; FR_{it} is a dummy for fiscal rule presence; G_{it} proxies government size; K_{it} denotes gross fixed capital formation. Robust standard errors, clustered at the country level, are reported in parentheses. Constant term and year fixed effects not reported. Coefficients are p.p. since ΔlnY is multiplied by 100.

Table 4 presents the first-stage results. Although the F-statistic is below the conventional threshold of 10, additional diagnostic tests confirm that the instrument is not weak. In contrast to the binary case, the CFSI instrument remains relevant for the Europe and Advanced Economy subsamples, as the index captures changes in the strength of existing rules—not merely the adoption of new ones. For example, reforms to the Stability and Growth Pact in 2005 and 2011, which enhanced its design, are reflected in the index and may influence neighboring countries' fiscal frameworks through diffusion effects.

Table 5 reports the second-stage results. The effect of stronger fiscal rule design on economic growth is positive and statistically significant for all samples, except the Advanced Economy subsample. In particular, for the *No Europe* and *No Advanced* samples—which include developing and low-income countries—the estimated coefficients are larger in magnitude and more statistically significant. This suggests that these countries may benefit more from improvements in fiscal rule design.

Tables B.5 to B.8 present the results for the debt rule subindex and the budget balance rule subindex. When comparing these two, debt rules appear to have a stronger association with economic growth, both in terms of statistical significance and coefficient magnitude. This finding suggests that debt rules may be a more effective policy tool for fostering growth in developing and low-income countries.

Overall, our findings indicate that the adoption of fiscal rules is positively associated with economic growth in developing and low-income countries. In these subsamples, the estimated coefficients for fiscal rule adoption are positive and statistically significant, in contrast to the results for Europe and advanced economies. One possible explanation is that developing and low-income countries still have significant room to benefit from the introduction of fiscal rules—even when such rules are weakly designed or not fully enforced—as their mere adoption may signal a commitment to fiscal discipline. In the case of European countries, instruments based on the Fiscal Strength Index (FSI) appear to be relevant, and the estimated effect on growth is also positive. Given that Europe

Table 4 – First stage for the contiguity fiscal strength index (FSI)

Variables	(1) FSI_{it}	(2) FSI_{it}	(3) FSI_{it}	(4) FSI_{it}	(5) FSI_{it}
$CFSI_{it-1}$	0.705***	0.407***	0.597***	0.566***	0.695***
	(0.0883)	(0.105)	(0.115)	(0.117)	(0.134)
Observations	4,555	3,509	1,046	3,489	1,066
R-squared	0.361	0.173	0.540	0.251	0.508
No. countries	177	137	40	140	37
Sample	Full	No europe	Europe	No Advanced	Advanced
F-stat 1st Stage	7.244	4.066	8.085	4.503	5.253
Stock-Wright p-value	0.002	0.0098	0.0005	0.0007	0.0264
SW F-stat	63.86	15.09	27.08	23.39	27.07
SW p-val	0.000	0.0002	0.000	0.000	0.000

Note: This Table presents the first-stage results for the CFSI instrument. The variable $CFSI_{it-1}$ is an index that aggregates all fiscal rules in force within a country, accounting for their specific design features and institutional characteristics (see equation 21). Robust standard errors, clustered at the country level, are reported in parentheses. Other covariates not reported.

Table 5 – Second stage for the contiguity fiscal strength index (FSI)

Variables	(1) $\Delta \ln Y$	(2) $\Delta \ln Y$	(3) $\Delta \ln Y$	(4) $\Delta \ln Y$	(5) $\Delta \ln Y$
FSI_{it}	1.424**	3.880**	2.367*	3.340***	0.719
	(0.570)	(1.714)	(1.217)	(1.231)	(0.576)
$\ln Y_{it-1}$	-4.752***	-4.584***	-8.817***	-4.757***	-5.273***
	(0.759)	(0.820)	(2.522)	(0.892)	(0.972)
$\ln K_{it}$	3.672***	3.371***	4.539***	3.450***	2.548
	(0.655)	(0.726)	(1.337)	(0.701)	(1.906)
$\ln G_{it}$	-2.088**	-1.835**	-10.03***	-1.808**	-9.954***
	(0.942)	(0.905)	(2.454)	(0.910)	(2.878)
Observations	4,555	3,509	1,046	3,489	1,066
No. countries	177	137	40	140	37
Country FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES
Sample	Full	No europe	Europe	No Advanced	Advanced

Note: This Table is based on equation 23. The variable $FSI_{i,t}$ represents the overall strength of fiscal rules in country i at time t; Y_{it} is the GDP per capita; G_{it} proxies government size; K_{it} denotes gross fixed capital formation. Robust standard errors, clustered at the country level, are reported in parentheses. Constant term and year fixed effects not reported. Coefficients are p.p. since ΔlnY is multiplied by 100.

was among the first regions to adopt fiscal rules in the early 1990s, current efforts are focused more on reforming and adapting existing rules to contemporary challenges, such as elevated public debt and slower economic growth. When disaggregating by rule type, our results show that debt rules have a stronger positive association with economic growth than budget balance rules, particularly in developing and low-income countries. This suggests that debt rules may be more effective in promoting macroeconomic stability and growth in these contexts.

6 Conclusion

This paper develops a novel overlapping generations model with endogenous growth, incorporating two fiscal rules—a debt rule and a budget balance rule—to examine their individual and joint effects on capital accumulation, macroeconomic dynamics, and welfare. The theoretical model shows that these rules are not neutral: they influence intertemporal decisions through their effect on tax rates, savings, and productive public spending. Our analytical results reveal conditions under which a balanced growth path exists and demonstrate that the sustainability and welfare implications of fiscal frameworks depend critically on the calibration of fiscal rules and initial debt levels.

The empirical contribution of the paper is twofold. First, we estimate a growth equation derived from the theoretical model using a cross-country panel. Fiscal rules are measured both as binary indicators and through a continuous Fiscal Strength Index (FSI). While simple rule dummies yield inconclusive results, the use of the FSI confirms that stronger fiscal rules—especially debt rules—are associated with higher economic growth, in line with the model's predictions. In contrast, budget balance rules show weaker and less consistent effects.

Second, we address endogeneity concerns using an innovative instrumental variable approach based on the geographical diffusion of fiscal rules. This method, inspired by Caselli and Reynaud (2020), exploits the idea that fiscal rule adoption or reform in neighboring countries can influence similar developments domestically. This approach enables us to identify the causal effect of fiscal rules on growth across heterogeneous country groups.

Our results show that fiscal rule adoption significantly enhances growth in developing and low-income countries. These countries appear to benefit even from the signaling effect of rule adoption, regardless of strict enforcement or design. In contrast, for advanced economies and European countries—where fiscal rules are more widespread and longstanding—the growth effects are only evident when rule quality is explicitly considered. Notably, the FSI-based instrument yields significant and positive effects even for European countries, suggesting that reforms to strengthen existing rules can still enhance growth.

Finally, we show that debt rules consistently outperform budget balance rules in promoting growth—both theoretically and empirically. This suggests that, particularly for developing and low-income economies, well-calibrated debt rules offer a more robust fiscal policy tool for fostering long-term growth.

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A Appendix: Proofs and derivations

A.1 Proof of Proposition 1

Set equation (16) equal to (17):

$$\frac{\gamma\tilde{A}}{d/k} = \frac{\beta}{(1+\beta)}\left[\frac{1+(\gamma-\xi-\theta)}{1+\alpha\mu\tilde{A}}\right](1-\alpha)\tilde{A} - \gamma\tilde{A} - \frac{d_t}{k_t} - 1$$

Define:

$$x \equiv \frac{d}{k}$$

$$\Psi(x,\gamma) = \frac{\gamma\tilde{A}}{x} + 1$$

$$\Phi(x,\gamma) = \frac{\beta}{(1+\beta)}\left[\frac{1+(\gamma-\xi-\theta)}{1+\alpha\mu\tilde{A}}\right](1-\alpha)\tilde{A} - \gamma\tilde{A} - x$$

Differentiating with respect to x:

$$\frac{\partial\Psi}{\partial x} = -\frac{\gamma\tilde{A}}{x^2} \;<\; 0$$

$$\frac{\partial\Phi}{\partial x} = -1 \;<\; 0$$

Functions Ψ and Φ are decreasing and convex in x. If $x \to 0$, $\Psi(x) \to \infty$ and if $x \to \infty$, $\Psi(x) \to 1$. If $x \to \infty$, $\Phi(x) < 0$ and, if the condition $\frac{\beta(1-\alpha)}{1+\beta}\left(\frac{1+\gamma-\xi-\theta}{1+\alpha\tilde{A}\mu}\right) > \gamma$ is satisfied, $\Phi(0) \in (0,\infty)$.

Moreover,

$$\frac{\partial\Psi}{\partial\gamma} = \frac{\tilde{A}}{x} \;>\; 0$$

$$\frac{\partial\Phi}{\partial\gamma} = -\tilde{A} + \frac{\beta(1-\alpha)\tilde{A}}{(1+\beta)(1+\alpha\mu\tilde{A})} \;<\; 0$$

since $\frac{\beta(1-\alpha)\tilde{A}}{(1+\beta)(1+\alpha\mu\tilde{A})} \in (0,1)$.

Thus, an increase in γ shifts Ψ upwards and Φ downwards. Thereby, for low values of γ, there exists two steady-states, and for a large γ, there is no steady-state. Therefore, $\exists\, \gamma'$ such that if $\gamma = \gamma'$, there will be one steady-state where Ψ and Φ tangent.

Besides,

$$\frac{\partial\Psi}{\partial\mu} = \;0$$

$$\frac{\partial\Phi}{\partial\mu} = -\frac{\beta}{1+\beta}\frac{(1-\alpha)\tilde{A}^2\alpha}{(1+\alpha\tilde{A}\mu)^2}[1+\gamma-\xi-\theta] \;<\; 0$$

So an increase in μ has no effect over Ψ but shifts Φ downwards. A decrease in μ shifts Φ up. Thus, for a sufficiently high μ, there won't exist a steady-state.

A.2 Proof of Proposition 2

The definition of $x = d/k$ implies that

$$\frac{x_{t+1}}{x_t} = \frac{d_{t+1}/k_{t+1}}{d_t/k_t} = \frac{d_{t+1}/d_t}{k_{t+1}/k_t}$$

Define $\hat{D} := \dfrac{d_{t+1}}{d_t}$ and $\hat{K} := \dfrac{k_{t+1}}{k_t}$. Thus, $\dfrac{x_{t+1}}{x_t} = \dfrac{\hat{D}}{\hat{K}} \implies x_{t+1} = \dfrac{\hat{D}}{\hat{K}} x_t$.
The differential is

$$\frac{\partial x_{t+1}}{\partial x_t} = \frac{\partial \hat{D}}{\partial x_t}\frac{x_t}{\hat{K}} + \frac{\hat{D}}{\hat{K}} - \frac{\partial \hat{K}}{\partial x_t}\frac{\hat{D}}{\hat{K}^2}x_t$$

Around the steady-state, we have $\hat{D} = \hat{K}$ and $x_{t+1} = x$, then

$$\frac{\partial x_{t+1}}{\partial x_t} = 1 + \frac{1}{\hat{K}}\left(\frac{\partial \hat{D}}{\partial x}x_t - \frac{\partial \hat{K}}{\partial x}x_t\right)$$

Using equations (16) and (17), we have the following differentials:

$$\frac{\partial \hat{K}}{\partial x} = -1 \quad \text{and} \quad \frac{\partial \hat{D}}{\partial x} = -\frac{\gamma \tilde{A}}{x^2}$$

Thus, we have that

$$\frac{\partial x_{t+1}}{\partial x_t} = 1 + \frac{1}{\hat{K}}\left(x_t - \frac{\gamma \tilde{A}}{x_t}\right)$$

Taking into account the term in parentheses in the above equation, if $x \to 0$, this term goes to $-\infty$. If $x \to \infty$, the term goes to ∞. Then, $\frac{\partial x_{t+1}}{\partial x_t} < 0$ for low levels of x, and $\frac{\partial x_{t+1}}{\partial x_t} > 1$ for higher levels of x.

Therefore,

$$\frac{\partial x_{t+1}}{\partial x_t} < 1 \quad \text{in the equilibrium with low } x$$
$$\frac{\partial x_{t+1}}{\partial x_t} > 1 \quad \text{in the equilibrium with high } x$$

Hence, x_1 is a stable equilibrium and x_2 an unstable equilibrium.

A.3 Economy with a deficit rule

Let us now consider the case in which the government adopts only one fiscal rule: a budget balance rule that limits the size of the deficit. As in previous sections, the tax rate is the policy instrument used to satisfy the government's intertemporal budget constraint. Substituting equation (8) into (6), the government budget constraint becomes:

$$\gamma y_t = \xi y_t + \theta y_t - \tau_t(w_t + r_t(k_t + \mu y_t)) + r_t d_t$$

Using equations (12), (13) and (10), we can express the tax rate as a function of model parameters and state variables:

$$(1-\tau_t) = \frac{1+(\gamma-\xi-\theta)}{1+\alpha d_t/k_t} \tag{24}$$

The public debt growth rate remains as in the previous section and is given by equation (17). Substituting (13), (10) and (24) into the savings equation (11), we obtain the capital growth rate:

$$\frac{k_{t+1}-k_t}{k_t} = \frac{\beta}{(1+\beta)}\left[\frac{1+(\gamma-\xi-\theta)}{1+\alpha d_t/k_t}\right](1-\alpha)\tilde{A} - \gamma\tilde{A} - \frac{d_t}{k_t} - 1 \tag{25}$$

In a balanced growth path, the debt-to-capital ratio remains constant over time. Setting equations (17) and (25) equal to each other yields the following cubic equation in the ratio $x = d_t/k_t$:

$$\alpha x^3 + x^2(\alpha\gamma\tilde{A}+\alpha+1) - x\left[\frac{\beta}{1+\beta}(1+\gamma-\xi-\theta)(1-\alpha)\tilde{A} - \gamma\tilde{A}(1+\alpha) - 1\right] + \gamma\tilde{A} = 0 \tag{26}$$

Proposition 4. *Since equation (26) is cubic, it has at least one real root. Given that condition* $\frac{\beta}{1+\beta}(1-\alpha)(1+\gamma-\xi-\theta) > \frac{1}{\tilde{A}} + \gamma(1+\alpha)$ *is satisfied, then it has two (or none) positive real roots.*

Proof. Let equation (26) be written as a function: $m(x) = x^3 + \Gamma_1 x^2 - \Gamma_2 x + \Gamma_3$. It follows that:

$$\lim_{x\to-\infty} m(x) = -\infty \quad \text{and} \quad \lim_{x\to\infty} m(x) = \infty$$

Since $m(x)$ is cubic, this implies that it is continuous. Thus, by the intermediate value theorem, $\exists\, x'$ such that $m(x') = 0$. Hence, at least one real root exists. Given the model parameters, coefficients Γ_1 and Γ_3 are both greater than zero. Thus, if the following condition is satisfied, Γ_2 is also positive

$$\frac{\beta}{1+\beta}(1-\alpha)(1+\gamma-\xi-\theta) > \frac{1}{\tilde{A}} + \gamma(1+\alpha)$$

Thus, the signals of the coefficients of $m(x)$ are: $+\ +\ -\ +$, which implies that two sign changes exist, so there are either two or no positive roots. For negative roots, we look at $m(-x)$ signs: $-\ +\ +\ +$, which implies only one sign change, therefore, there exists one negative root. □

If equation (26) has two positive real roots, the behavior of the system mirrors that in Proposition 1. However, unlike the case with combined fiscal rules, the model now lacks a debt rule to anchor the level of public debt. As a result, while multiple steady states may exist, the government has no mechanism to control the growth trajectory of public debt over time. This fact highlights a limitation of relying solely on a deficit rule: it constrains the flow of fiscal imbalance but does not ensure long-run sustainability of debt dynamics.

Welfare implications under a deficit rule. Now we evaluate the welfare effects of the deficit rule. Substituting the optimal savings decision from equation (11) into the utility function, we derive the following expression for the agent's indirect utility at time t:

$$V_t = \ln\left[(1-\tau)w_t - \frac{\beta}{1+\beta}(1-\tau)w_t\right] + \beta\ln\left[\frac{\beta}{1+\beta}(1-\tau)w_t(1+r_{t+1}(1-\tau))\right] \tag{27}$$

By substituting the equilibrium tax rate (equation 24) into the utility function (27), we have:

$$V_t = \beta \ln \beta + (1+\beta)\ln\left(\frac{1}{1+\beta}\right) + (1+\beta)\ln\left[\frac{(1+\gamma-\xi-\theta)}{1+\alpha d_t/k_t}(1-\alpha)\tilde{A}k_t\right]$$
$$+\beta \ln\left[1+\alpha\frac{d_t}{k_t}+\alpha\tilde{A}(1+\gamma-\xi-\theta)\right] - \beta\ln\left(1+\alpha\frac{d_t}{k_t}\right)$$

Evaluating this expression for agents born in $t-1$, where k_{t-1} and d_{t-1} are predetermined, we get:

$$V_{t-1} = \beta \ln \beta + (1+\beta)\ln\left(\frac{1}{1+\beta}\right) + 2(1+\beta)\ln k_{t-1} - (1+2\beta)\ln\left(k_{t-1}+\alpha d_{t-1}\right)$$
$$+(1+\beta)\ln\left[(1-\alpha)\tilde{A}(1+\gamma-\xi-\theta)\right] + \beta\ln\left[(k_{t-1}+\alpha d_{t-1}+\alpha\tilde{A}k_{t-1}(1+\gamma-\xi-\theta)\right]$$

Taking the derivative of V_{t-1} with respect to γ, we obtain:

$$\frac{\partial V_{t-1}}{\partial \gamma} = \frac{(1+\beta)}{1+\gamma-\xi-\theta} + \frac{\beta\tilde{A}}{1/\alpha + d_{t-1}/k_{t-1} + \tilde{A}(1+\gamma-\xi-\theta)} > 0$$

This derivative confirms that increasing the deficit limit (γ) raises the utility of agents born in (t-1), since it allows the government to reduce the contemporaneous tax rate by financing a larger share of its spending through borrowing. However, the magnitude of this welfare gain is diminishing in both (γ) and the debt-to-capital ratio d_{t-1}/k_{t-1}: as either increases, the denominator of each term in the expression rises, reducing the marginal utility benefit of further relaxing the deficit rule.

In contrast, when both fiscal rules are in place, the welfare effects operate differently. A reduction in the debt ceiling μ increases $V_{t-1}$by limiting future tax distortions and promoting capital accumulation. However, tightening the deficit limit (γ) reduces V_{t-1}because it requires higher taxation in the short run without altering the long-run debt path (already constrained by μ). Therefore, while a looser deficit rule improves short-term welfare when no debt rule is present, its effectiveness diminishes with fiscal slack and rising debt levels, highlighting the trade-off between immediate relief and long-term fiscal sustainability.

A.4 Economy with a debt rule

Let us now consider the case in which the government adopts only a debt rule, that is, a constraint on the stock of public debt relative to output. As in previous scenarios, the tax rate is the policy instrument used to ensure that the intertemporal budget constraint is satisfied. Substituting the debt rule (7) into the government budget constraint (6), we obtain:

$$\mu y_{t+1} - \mu y_t = \xi y_t + \theta y_t - \tau_t(w_t + r_t(k_t + \mu y_t)) + r_t d_t$$

Using equations (12), (13) and (10), and defining the capital growth rate as $\pi \equiv k_{t+1}/k_t$, the tax rate can be expressed as:

$$(1-\tau_t) = \frac{1+\mu\pi-(\mu\xi+\theta)}{1+\mu\alpha\tilde{A}} \quad (28)$$

The capital growth rate is then obtained by substituting (13), (10), and (28) into the savings equation (11):

$$\frac{k_{t+1}}{k_t} = \frac{\beta(1-\alpha)\tilde{A}[1-(\mu+\xi+\theta)]}{(1+\mu\tilde{A})(1+\beta)(1+\alpha\tilde{A}\mu)-\beta(1-\alpha\tilde{A})\mu} \tag{29}$$

To compute the debt growth rate, we return to the government budget constraint and divide equation (6) by d_t again using (13), (10) and (28). This yields:

$$\frac{d_{t+1}}{d_t} = 1+\alpha\tilde{A} \tag{30}$$

It is worth noting that, in this setup, the equilibrium path is not uniquely determined. Since the tax rate in equation (28) depends on the capital growth rate π, the existence of a balanced growth path (BGP) is conditional on initial conditions. If the economy converges to the BGP from the first period, it depends on the initial values k_0 and d_0. More generally, convergence in some later period s would imply that the long-run path depends on the levels k_{t-s} and d_{t-s} . The absence of a deficit rule means that there is no mechanism directly limiting the fiscal deficit, which can generate indeterminacy in the short-run dynamics and ambiguity in the transition to the steady state.

A.5 Derivation of equation 18

Inserting equation (11) in the utility function yields the following indirect utility function:

$$V_t = \ln\left[(1-\tau)w_t - \frac{\beta}{1+\beta}(1-\tau)w_t\right] + \beta\ln\left[\frac{\beta}{1+\beta}(1-\tau)w_t(1+r_{t+1}(1-\tau))\right] \tag{31}$$

Inserting equations (13) and (15) above yields:

$$\begin{aligned} V_t = {} & \beta\ln\beta + (1+\beta)\ln\left(\frac{1}{1+\beta}\right) + (1+\beta)\ln k_t \\ & + (1+\beta)\ln\left[\frac{1+\gamma-\xi-\theta}{1+\alpha\tilde{A}\mu}(1-\alpha)\tilde{A}\right] + \beta\ln\left[\frac{1+\gamma-\xi-\theta}{1+\alpha\tilde{A}\mu}\alpha\tilde{A}+1\right] \end{aligned}$$

B Tables

Table B.1 – Debt rule - first stage contiguity instrument results

Variables	(1) DR_{it}	(2) DR_{it}	(3) DR_{it}	(4) DR_{it}	(5) DR_{it}
ContiguityDR_{it-1}	0.146***	0.148***	0.0908***	0.139***	0.143***
	(0.0181)	(0.0258)	(0.0216)	(0.0232)	(0.0285)
Observations	4,555	3,509	1,046	3,489	1,066
R-squared	0.309	0.257	0.490	0.273	0.452
No. countries	177	137	40	140	37
Sample	Full	No europe	Europe	No Advanced	Advanced
F-stat 1st Stage	7.898	4.492	8.841	4.523	10.56
Stock-Wright p-value	0.0002	0.0012	0.000	0.0002	0.9342
SW F-stat	64.95	32.92	17.73	35.91	25.24
SW p-val	0.0000	0.0000	0.0001	0.0000	0.0000

Note: Robust standard errors, clustered at the country level, are reported in parentheses. Other covariates not reported.

Table B.2 – Debt Rule - second stage contiguity instrument results

Variables	(1) $\Delta \ln Y$	(2) $\Delta \ln Y$	(3) $\Delta \ln Y$	(4) $\Delta \ln Y$	(5) $\Delta \ln Y$
DR_{it}	2.980***	2.927**	6.880*	4.219**	0.0400
	(1.146)	(1.418)	(3.881)	(1.708)	(0.782)
$\ln Y_{it-1}$	-5.206***	-4.510***	-12.82***	-4.925***	-4.472***
	(0.833)	(0.805)	(4.485)	(0.929)	(0.795)
$\ln K_{it}$	3.583***	3.246***	5.235***	3.294***	2.188
	(0.669)	(0.754)	(1.521)	(0.736)	(1.825)
$\ln G_{it}$	-1.890*	-1.525	-10.09***	-1.471	-9.940***
	(0.995)	(0.990)	(2.839)	(0.990)	(2.771)
Observations	4,555	3,509	1,046	3,489	1,066
No. countries	177	137	40	140	37
Country FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES
Sample	Full	No europe	Europe	No Advanced	Advanced

Note: Robust standard errors, clustered at the country level, are reported in parentheses. Constant term and year fixed effects not reported. Coefficients are p.p. since ΔlnY is multiplied by 100.

Table B.3 – Budget balance rule - first stage contiguity instrument results

Variables	(1) BBR_{it}	(2) BBR_{it}	(3) BBR_{it}	(4) BBR_{it}	(5) BBR_{it}
$ContiguityBBR_{it-1}$	0.143***	0.160***	0.0638*	0.153***	0.102*
	(0.0219)	(0.0210)	(0.0343)	(0.0200)	(0.0510)
Observations	4,555	3,509	1,046	3,489	1,066
R-squared	0.307	0.283	0.475	0.318	0.390
No. countries	177	137	40	140	37
Sample	Full	No europe	Europe	No Advanced	Advanced
F-stat 1st Stage	7.514	5.400	16.10	6.769	5.847
Stock-Wright p-value	0.0009	0.0014	0.0005	0.0002	0.233
SW F-stat	42.67	58.24	3.46	58.18	3.97
SW p-val	0.0000	0.0000	0.0704	0.0000	0.0539

Note: Robust standard errors, clustered at the country level, are reported in parentheses. Other covariates not reported.

Table B.4 – Budget Balance Rules - second stage contiguity instrument results

Variables	(1) $\Delta \ln Y$	(2) $\Delta \ln Y$	(3) $\Delta \ln Y$	(4) $\Delta \ln Y$	(5) $\Delta \ln Y$
BBR_{it}	2.391***	2.501**	6.393	3.190***	-0.786
	(0.917)	(1.094)	(4.226)	(1.166)	(1.025)
$\ln Y_{it-1}$	-5.042***	-4.478***	-12.41**	-4.736***	-3.740***
	(0.760)	(0.761)	(5.073)	(0.835)	(0.944)
$\ln K_{it}$	3.646***	3.326***	5.493***	3.395***	2.045
	(0.650)	(0.729)	(1.647)	(0.703)	(1.814)
$\ln G_{it}$	-2.080**	-1.720*	-10.08***	-1.721*	-9.878***
	(0.920)	(0.909)	(2.637)	(0.895)	(2.641)
Observations	4,555	3,509	1,046	3,489	1,066
No. countries	177	137	40	140	37
Country FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES
Sample	Full	No europe	Europe	No Advanced	Advanced

Note: Robust standard errors, clustered at the country level, are reported in parentheses. Constant term and year fixed effects not reported. Coefficients are p.p. since ΔlnY is multiplied by 100.

Table B.5 – Debt rule - first stage for the contiguity fiscal strength sub-index

Variables	(1) DR $index_{it}$	(2) DR $index_{it}$	(3) DR $index_{it}$	(4) DR $index_{it}$	(5) DR $index_{it}$
CDR $index_{it-1}$	0.685***	0.462***	0.490***	0.568***	0.682***
	(0.0843)	(0.113)	(0.110)	(0.121)	(0.127)
Observations	4,555	3,509	1,046	3,489	1,066
R-squared	0.351	0.189	0.532	0.245	0.522
No. countries	177	137	40	140	37
Sample	Full	No europe	Europe	No Advanced	Advanced
F-stat 1st Stage	6.705	3.973	7.965	3.898	6.451
Stock-Wright p-value	0.0006	0.0118	0.000	0.0004	0.0155
SW F-stat	66.2	16.68	19.76	22.13	28.73
SW p-val	0.0000	0.0001	0.0001	0.0000	0.0000

Note: Robust standard errors, clustered at the country level, are reported in parentheses. Other covariates
not reported.

Table B.6 – Debt rule - second stage for the contiguity fiscal strength sub-index

Variables	(1) Δln$Y$	(2) Δln$Y$	(3) Δln$Y$	(4) Δln$Y$	(5) Δln$Y$
DR index$_{it}$	5.969***	11.67**	13.52**	12.03***	2.899
	(2.212)	(5.942)	(5.786)	(4.620)	(2.176)
lnY_{it-1}	-4.944***	-4.567***	-11.78***	-4.925***	-5.426***
	(0.800)	(0.832)	(3.397)	(0.941)	(1.040)
lnK_{it}	3.674***	3.320***	5.314***	3.411***	2.613
	(0.660)	(0.737)	(1.700)	(0.713)	(1.950)
lnG_{it}	-2.055**	-1.738*	-10.43***	-1.736*	-9.843***
	(0.952)	(0.925)	(2.590)	(0.930)	(2.907)
Observations	4,555	3,509	1,046	3,489	1,066
No. countries	177	137	40	140	37
Country FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES
Sample	Full	No europe	Europe	No Advanced	Advanced

Note: Robust standard errors, clustered at the country level, are reported in parentheses. Constant term and year fixed effects not reported. Coefficients are p.p. since ΔlnY is multiplied by 100.

Table B.7 – Budget Balance rule - first stage for the contiguity fiscal strength sub-index

Variables	(1) BBR index$_{it}$	(2) BBR index$_{it}$	(3) BBR index$_{it}$	(4) BBR index$_{it}$	(5) BBR index$_{it}$
CBBR index$_{it-1}$	0.685***	0.462***	0.490***	0.568***	0.682***
	(0.0843)	(0.113)	(0.110)	(0.121)	(0.127)
Observations	4,555	3,509	1,046	3,489	1,066
R-squared	0.369	0.180	0.538	0.248	0.513
No. countries	177	137	40	140	37
Sample	Full	No europe	Europe	No Advanced	Advanced
F-stat 1st Stage	8.650	4.536	8.391	4.799	6.065
Stock-Wright p-value	80.12	18.09	24	29.76	30.28
SW F-stat	0.0000	0.0000	0.0000	0.0000	0.0000
SW p-val	0.0325	0.0263	0.0035	0.0017	0.459

Note: Robust standard errors, clustered at the country level, are reported in parentheses. Other covariates not reported.

Table B.8 – Budget Balance rule - second stage for the contiguity fiscal strength sub-index

Variables	(1) Δln$Y$	(2) Δln$Y$	(3) Δln$Y$	(4) Δln$Y$	(5) Δln$Y$
BBR index$_{it}$	2.788*	9.560*	5.366	9.739**	0.645
	(1.617)	(5.300)	(3.269)	(3.904)	(1.492)
lnY_{it-1}	-4.548***	-4.431***	-8.013***	-4.676***	-4.708***
	(0.731)	(0.780)	(2.418)	(0.857)	(0.875)
lnK_{it}	3.617***	3.320***	4.229***	3.396***	2.268
	(0.649)	(0.731)	(1.220)	(0.702)	(1.830)
lnG_{it}	-2.076**	-1.821**	-9.740***	-1.797**	-9.983***
	(0.938)	(0.898)	(2.413)	(0.901)	(2.789)
Observations	4,555	3,509	1,046	3,489	1,066
No. countries	177	137	40	140	37
Country FE	YES	YES	YES	YES	YES
Year FE	YES	YES	YES	YES	YES
Sample	Full	No europe	Europe	No Advanced	Advanced

Note: Robust standard errors, clustered at the country level, are reported in parentheses. Constant term and year fixed effects not reported. Coefficients are p.p. since ΔlnY is multiplied by 100.